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Synchronization risk and delayed arbitrage*

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Abstract

We argue that arbitrage is limited if rational traders face uncertainty about when their peers will exploit a common arbitrage opportunity. This synchronization risk - which is distinct from noise trader risk and fundamental risk - arises in our model because arbitrageurs become sequentially aware of mispricing and they incur holding costs. We show that rational arbitrageurs "time the market" rather than correct mispricing right away. This leads to delayed arbitrage. The analysis suggests that behavioral influences on prices are resistant to arbitrage in the short and intermediate run.

JEL classification: G14, D82

Keywords: Limits to arbitrage; Synchronization risk; Market timing; Efficient market hypothesis; Behavioral finance.

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1 Introduction

It is puzzling that professional traders often mutually agree that certain assets are overvalued or undervalued and yet do not trade accordingly. An often-cited example of an overpriced stock is Priceline.com, whose market capitalization reached $30 billion in April 1999, surpassing the combined market capitalization of all major U.S. airlines.[1] Stocks can also be underpriced. Brennan (1990) refers to "latent assets" and considers large takeover bid premia as evidence of underpricing. Mispricing may arise in other asset markets, too. Dammon, Dunn, and Spatt (1993) report large and persistent mispricing in the bond market based on a comparison of three high-yield bonds of RJR Nabisco that differ only in the form in which interest is paid. The cash-paying bond traded at a huge premium compared to identical pay-in-kind bonds and deferred-coupon bonds over a two-year period. Standard market imperfections cannot explain such substantial mispricing. Similarly, certain currencies in the foreign exchange market are often known to be under/overvalued over many years, but arbitrageurs are afraid to trade on this mispricing too early. The objective of this paper is to provide a rationale for why this type of mispricing can persist even when professional arbitrageurs are present in the market.

One possible explanation is that not all market participants are fully rational. For example, behavioral traders can trade based on investor sentiment and ignore relevant information. Even though there is little controversy in the literature about the presence of behavioral traders, there is disagreement about whether these boundedly rational traders actually affect prices. Proponents of the *efficient markets hypothesis*, like Fama (1965) and Ross (2001), maintain that rational arbitrageurs will undo any mispricing caused by behavioral traders. Hence, "the price is right." Our paper disputes this claim, offering a new reason - "synchronization risk" - for why mispricing can persist despite the presence of rational arbitrageurs.

The efficient markets hypothesis is self-evident when arbitrage strategies are riskless and professional traders are willing to take unbounded positions. In reality, though, any arbitrage involves some risk since markets are not complete. Whenever a mispriced asset is not redundant, an arbitrage strategy is risky even if rational traders care only about the final payoff of the arbitrage strategy. In other words, an arbitrage trade is riskless only if a perfect substitute for the mispriced asset exists. Therefore, arbitrageurs can rarely fully hedge their arbitrage strategies. The recent literature on the limits to arbitrage has identified two broad categories of risk: fundamental risk and noise trader risk. An arbitrage strategy can be risky because the fundamental value of a partially hedged portfolio might change over time. In addition, arbitrageurs understand that their model might not coincide with the true data-generating process.

[1] Likewise, the market capitalization of eBay exceeded that of RJR Nabisco, Yahoo that of Boeing, and Amazon.com that of Borders, Barnes & Noble, Kmart and JC Penney combined. Ofek and Richardson (2001) provide a detailed account of overpricing of Internet stocks during the 1990s.

Thus, arbitrageurs have to bear a *fundamental risk* even if they can sustain the arbitrage strategy until the final payoff is realized. While these fundamental shocks are permanent, the activity of behavioral noise traders might lead to temporary price movements. These price changes temporarily reduce the value of the arbitrage portfolio if the price moves even further away from the fundamental value. If arbitrageurs are compelled to liquidate their positions in the intermediate term, then they are forced to take losses exactly when the arbitrage opportunity is greatest. DeLong, Shleifer, Summers, and Waldmann (1990a) call this *noise trader risk*. There are many reasons why arbitrageurs have to liquidate their position before the arbitrage finally pays off. Clearly, if arbitrageurs are short-lived, as in models with overlapping generations, they only care about the price at which they will sell the asset in their final period of consumption. In practice, professional fund managers have (endogenous) short horizons because their clients evaluate them on short-term performance and relatively poor performance leads to an outflow of funds (Shleifer and Vishny, 1997). Finally, arbitrageurs might find it optimal to (partially) liquidate their position early in an incomplete market setting when the investment opportunity set changes.

An important additional risk has been ignored in the literature. It derives from arbitrageur's uncertainty about *when* other arbitrageurs will start exploiting a common arbitrage opportunity. We term this risk *synchronization risk*. Note that in contrast to noise trader risk, synchronization risk does not primarily stem from the activity of other noise traders, but from uncertainty about the market timing decisions of other rational arbitrageurs. In other words, while noise trader risk reflects the risk that the price might move even further away from the fundamental value, synchronization risk pertains to uncertainty regarding the *timing* of the price correction.

Our model builds on the framework developed in Abreu and Brunnermeier (2001). It has three key ingredients. First, we assume that a single arbitrageur alone cannot correct the mispricing. In our model, the mispricing only disappears if a *critical proportion* of $\kappa < 1$ arbitrageurs have traded based on their information.[2] Any trade imbalance of rational arbitrageurs will be absorbed by behavioral traders who interpret it as a random fluctuation in order flow. The price correction only occurs when the aggregate order imbalance of arbitrageurs exceeds the behavioral traders' absorption capacity. The critical mass requirement introduces an element of *coordination* among the arbitrageurs in our model. In addition, rational arbitrageurs are also *competitive* since $\kappa < 1$. An arbitrageur who waits too long misses the profit opportunity if the price correction occurs in interim.

A second element of our model is that arbitrageurs' opinions about the timing of the price correction are dispersed. In particular, we assume that arbitrageurs become *sequentially aware* of the mispricing. Some early-informed arbitrageurs immediately learn of the mispricing when the price departs from the fundamental value. Others

[2]This critical mass assumption is similar to that used in the speculative currency attack literature (Obstfeld, 1996; Morris and Shin, 1998).

receive this information later. After some time all arbitrageurs know that the price does not reflect the fundamental value. Note that in our setting, as is arguably the case in reality, arbitrageurs do not know how early they receive this information relative to other traders. This sequential awareness element, together with the critical mass requirement, introduces a temporal dimension to the coordination problem. A synchronization problem arises because no individual arbitrageur knows when other traders will trade based on their information and hence they do not know when the price correction will occur.

Finally, we argue that arbitrageurs incur explicit and implicit *holding costs* in order to exploit an arbitrage opportunity. For example, several explicit costs arise when an overpriced asset is sold short. Short-sellers must hold the short-sale proceeds in a margin account that pays minimal or no interest. Moreover, if the stock is "on special," that is, if it is difficult to locate shareholders who are willing to lend the share, short-sellers will receive a negative interest rate on their short-sale proceeds. In other words, the short-seller is indirectly paying a lending fee. Furthermore, margin requirements force short-sellers to put additional money into low interest bearing margin accounts. This can bind a large amount of an arbitrageur's capital, which could be used for alternative investment opportunities. Note that short-sale costs also arise if an asset is undervalued. In this case, the arbitrageur goes long on the underpriced asset, but shorts assets with correlated payoffs in order to hedge the position. In addition, arbitrageurs also face implicit holding costs. For example, they cannot fully hedge their arbitrage strategy in a world where a perfect substitute for the mispriced asset does not exist. Other examples include the relative performance evaluation of fund managers and the risk that the lender of a security might recall the asset.[3]

The fundamental result of our analysis is that the combination of synchronization risk with holding costs typically causes arbitrageurs to delay acting on their information. However, they eventually do trade and while arbitrage does occur, it can be significantly delayed. This result holds despite our assumption that the price correction occurs with certainty within a finite horizon. Thus, the classical backward induction argument does not apply. The reason is that at no point in time is the mispricing common knowledge among the arbitrageurs. It might be the case that all arbitrageurs know of the mispricing, and all arbitrageurs know that all know that the price is too high or too low, but it is never the case that all arbitrageurs know that everybody knows that everybody knows and so on ad infinitum.

We also obtain a variety of comparative statics results. As one would expect, we find that the duration of the mispricing increases with the size of the holding costs, the dispersion of opinion among arbitrageurs, and the absorption capacity of the behavioral traders. The larger the mispricing, the larger is the incentive of each individual

[3] We initially only stressed the former interpretation in the form of a reputational penalty. We are grateful to Ming Huang and Harrison Hong for emphasizing the broader holding cost perspective adopted here.

arbitrageur to trade based on that information, and the sooner the price will revert towards the fundamental value. When the mispricing exceeds bounds, determined by dispersion of opinion among arbitrageurs and other model parameters, arbitrageurs will act on their information without delay. The fundamentals serve as an anchor around which the price can fluctuate. Thus, Black's (1986) (half-serious?) suggestion that the fundamental value is between half and twice the current value is a description that is consistent with our model.

The model can be extended to allow for asymmetric holding costs which, as has been widely noted, are higher for a short position than for a long position. Since arbitrageurs who receive bad news have to sell an asset short in order to exploit its overpricing and conversely for good news, our model provides a simple explanation of why "bad news travels slowly."

Our model also explains why fads and fashions in the acquisition and use of information are compatible with rational optimizing behavior. The coordination aspect of our model creates a strong incentive for individual arbitrageurs to act only on information that other arbitrageurs are acting upon, and to ignore information that is being generally ignored, even when such information is more important in terms of fundamentals. It does not pay to research or use "sleepy news" even when such news is known to be important.

Our results are also robust in a generalized model in which partial price adjustments occur whenever the trading pressure exceeds a smaller threshold. Arbitrageurs update their information based on price movements. However, this inference is imperfect since no arbitrageur knows whether the price movement was due to trading pressure from rational arbitrageurs or a shift in investor sentiment. Therefore, opinions remain dispersed. Note that trading immediately after a partial price adjustment can be costly since the price might reverse if it was only due to a temporary shift in investor sentiment. In this setting, arbitrageurs might trade against each other. Some arbitrageurs bet on a price reversal and buy (sell) the asset even though they know that the asset is overpriced (undervalued). Other arbitrageurs, who became aware of the mispricing earlier, think that it is more likely that the price move will trigger a full price correction, and trade in the direction of the fundamentals.

In summary, our analysis shows that arbitrage ultimately works, but that it occurs with a delay. While markets are not fully informationally efficient, the departure from the fundamental value is constrained. Prices are governed by the fundamental value in the long term, but behavioral biases might affect prices in the intermediate and short term.

2 Related literature

The existing literature on limited arbitrage, as summarized in Barberis and Thaler (2001) and Shleifer (2000), offers alternative rationales for the persistence of mispricing. DeLong, Shleifer, Summers, and Waldmann (1990a) introduce the concept of noise trader risk. In their model, mispricing persists because risk-averse, short-lived arbitrageurs only worry about next period's price - which is affected by noise trader demand - instead of the riskless long-run fundamental value. In Shleifer and Vishny (1997), fund managers limit their arbitrage out of fear of a drawdown. Fund managers are afraid that their investors will withdraw their money if they suffer intermediate short-term losses even though the arbitrage provides a riskless positive payoff in the long-run. These papers build on the insight that distorted prices might become even more distorted in the short run before eventually returning to their normal long run values. The logic of our model is different. In our framework, arbitrage is limited because of problems of synchronization and an individual incentive to time the market. This synchronization risk not only provides an alternative explanation for persistent mispricing, it also amplifies the effects described in the earlier literature on limited arbitrage. Furthermore, these limits to arbitrage are derived even though we assume that aggregate resources of all arbitrageurs suffice to correct a given mispricing.

Bubbles are a special form of mispricing. Before a bubble bursts, the stock price grows at a high rate even though it is mutually known that the assets are overpriced. In Allen and Gorton (1993), professional fund managers "churn" bubbles at the expense of individual investors. Abreu and Brunnermeier (2001) show that arbitrageurs prefer to ride a growing bubble until its size reaches a critical level. One common element of these papers and the current one is that traders do not know how early they sell out relative to other professional traders. In DeLong, Shleifer, Summers, and Waldmann (1990b), rational arbitrageurs push up the price to induce behavioral feedback traders to aggressively buy stocks in the next period. This delayed reaction by feedback traders allows the arbitrageurs to unload their position at a profit. Unlike the latter papers, the mispricing in this paper persists even though behavioral traders do not increase it over time. There are numerous other papers on bubbles; some of the important contributions are summarized in Brunnermeier (2001).

Morris (1995) develops a dynamic coordination game based on the idea of "asynchronous clocks" of Halpern and Moses (1990), which has some similarities to the model we develop. The papers by Morris and Shin (1998) on currency attacks and Morris (1995) exploit the global games approach of Carlsson and van Damme (1993). Our model does not belong to the class of games to which the results of Carlsson and van Damme apply. These issues are discussed further in Abreu and Brunnermeier (2001).

Tuckman and Vila (1992) highlight the central role of explicit holding costs. They solve an arbitrageur's dynamic portfolio choice problem wherein risk aversion prevents fully exploiting the mispricing. They argue that holding costs are more substantial than transactions costs, which is supported by their empirical study of the U.S. Treasury bond market. Arbitrageurs' trading activity has no impact on the mispricing in their paper since it is fully exogenously specified, in contrast to our analysis.

3 The model

We begin with an overview of our model; a detailed discussion of individual modeling assumptions appears later. There is a single risky asset whose price is denoted by p_t. Time $t \in [0, \infty)$ is continuous. The fundamental value of the risky asset is denoted v_t. Prior to the arrival of a shock at a random time $t_0 \in [0, \infty)$, the fundamental value of the asset is $v_t = e^{rt}$ and from t_0 onwards $v_t = \left(1 + \tilde{\beta}\right) e^{rt}$.[4] The distribution of t_0 is exponential: $F(t_0) = 1 - e^{-\lambda t_0}$ and

$$\tilde{\beta} = \begin{cases} -\beta & \text{with prob. } \frac{1}{2} \\ +\beta & \text{with prob. } \frac{1}{2} \end{cases}, \text{ with } \beta > 0.$$

There are two types of agents: rational arbitrageurs and behavioral traders. The focus of the analysis is on the decisions of the former. Prior to the shock at t_0, $p_t = v_t$. Thereafter, the price can deviate from fundamentals. Between t_0 and $t_0 + \eta$, rational arbitrageurs become *sequentially* aware of the new fundamental value at a uniform rate. The mass of arbitrageurs is normalized to one. Ex ante, any arbitrageur is equally likely to become aware at any $t \in [t_0, t_0 + \eta]$. An individual arbitrageur who becomes aware of the change in fundamentals at t_i thinks that t_0 is distributed between $t_i - \eta$ and t_i. We denote this type of arbitrageur by $\hat{t}_i$. If $t_0 = t_i$, arbitrageur $\hat{t}_i$ is in fact the first to realize that the fundamental value has departed from the price. At the other extreme, if $t_0 = t_i - \eta$, she is the last to become aware of the mispricing. Since she does not know t_0, she does not know how early she receives the information relative to other arbitrageurs. Note that while informed arbitrageurs disagree about t_0, they have the same estimate of the fundamental value and, hence, of the size of the mispricing. The exponential prior distribution of t_0, combined with the uniform arrival of this information to arbitrageurs, guarantees that each arbitrageur's problem is symmetric relative to the time t_i the arbitrageur becomes aware that the price does not coincide with the fundamental value of the asset.

Behavioral traders persist in thinking that there has been no shock and support a price of $p_t = e^{rt}$ even after t_0. Suppose that there has been a negative shock, so that $\tilde{\beta} = -\beta$. Behavioral traders can support the mispricing as long as the aggregate selling pressure by rational arbitrageurs lies below a threshold κ. When this threshold

[4] As will be apparent soon, this analysis can be easily generalized to a setting where $v_t = w_t \left[1 + E_t\left(\tilde{\beta}\right)\right]$, with $w_t = \left[\int_0^t r w_s ds + M_t\right]$, $w_0 = 1$, and M_t following any martingale process. A particular example is a generalized geometric Brownian motion, where $w_t = \left[\int_0^t r w_s ds + \int_0^t \sigma w_s dB_s\right]$.

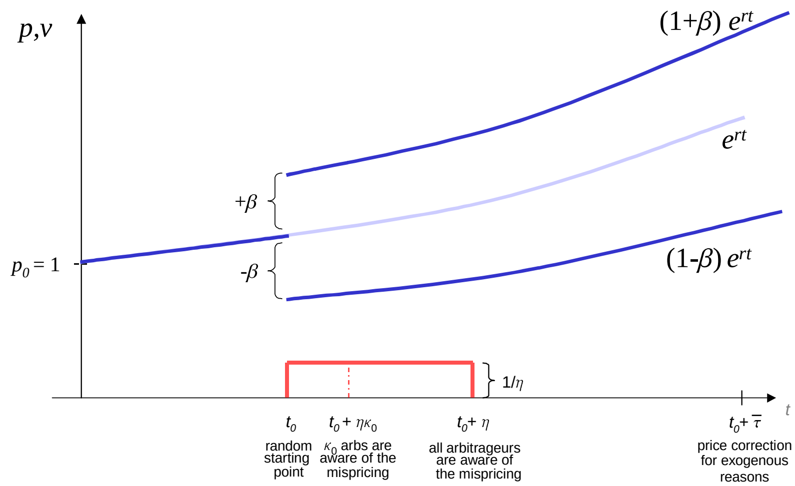

Figure 1: Price and fundamental value

is crossed, the price adjusts and coincides with the fundamental value of the asset. A parallel statement applies to the case of a positive shock $\tilde{\beta} = +\beta$. This price process is illustrated in Fig. 1.

For simplicity, we assume that all arbitrageurs are risk neutral. The drawback of this assumption is that arbitrageurs may find it optimal to submit orders of infinite size. To preclude this possibility, we also assume that each arbitrageur can only trade within a maximum long position and a maximum short position in the number of shares. Without loss of generality, we normalize the maximum number of shares an arbitrageur can hold (short or long) to $x_i \in [-1, 1]$, where $x_i = -1$ reflects arbitrageur $\hat{t}_i$'s maximum short position and $x_i = 1$ the maximum long position. Such limits to the positions arbitrageurs can take have been rationalized in the literature in a variety of ways, including appeals to risk aversion, wealth constraints, and asymmetric information. Note that trading activity is risky in our framework and that there are no risk-free arbitrage opportunities. For each period that an arbitrageur departs from the diversified (neutral) portfolio (normalized to $x_i = 0$), holding costs are $cp_t\,|x_i|$. We restrict our analysis to parameter values where $c > \frac{\lambda}{1-e^{-\lambda\eta\kappa_0}}\beta$. Absent this restriction, arbitrageurs always trade right after they become aware of the mispricing.

The price correction occurs as soon as the aggregate order imbalance of all arbitrageurs exceeds $\kappa(\cdot)$, where the latter is a function of $(t - t_0)$. To obtain closed-form solutions, we specify $\kappa(t - t_0) = \kappa_0 \left[1 - \frac{1}{\bar{\tau}}(t - t_0)\right]$ to be linearly decreasing, with $\bar{\tau} > \max\{\eta, \frac{1}{\lambda}[\frac{1}{\ln\frac{c}{c-\lambda\beta}} - \frac{1}{\lambda\eta\kappa_0}]^{-1}\}$. The inequality guarantees that κ does not decline "too fast." The main results carry over to any decreasing κ-function as long as $-\frac{\partial\kappa}{\partial(t-t_0)} < \frac{1}{\eta}$

for all $(t - t_0)$.[5] The linear specification applies to t such that $t - t_0 \leq \bar{\tau}$. At $t_0 + \bar{\tau}$, $\kappa(\bar{\tau}) = 0$ and the mispricing is corrected even in the absence of trading pressure from the rational arbitrageurs. When a whole mass of arbitrageurs submit their orders exactly in the same trading round and the accumulated order flow strictly exceeds κ, we assume that each order is only executed with a probability ψ at the pre-correction price. With probability $(1 - \psi)$, the order will be executed at the new corrected price v_t.[6]

There are many rationales for our modeling choices. For example, imagine that behavioral traders are confident that they become aware of (the) mispricing as early as the first rational arbitrageurs. They view any aggregate trading imbalance as being due to noisy liquidity trading and to represent a cheap buying (attractive selling) opportunity. Their capacity to absorb order imbalance is given by the threshold function $\kappa(t - t_0)$. When this threshold is crossed, the price correction occurs. The longer the mispricing persists, the smaller is the mass of behavioral traders who remain confident that the "price is right." Hence we assume that the function $\kappa(\cdot)$ is declining.

Note that the critical mass requirement both introduces a coordination element and prevents other arbitrageurs from perfectly inferring the single random variable t_0 from price changes as long as the trading pressure is smaller than $\kappa(\cdot)$. We view our reduced form modeling as reflecting the many uncertainties which, in reality, make it difficult to perfectly infer all variables from the current price level. These extensions are discussed in Section 5.

In order to exploit the arbitrage opportunity, each arbitrageur has to build up a position, thereby incurring explicit and implicit holding costs. In our model, each arbitrageur pays holding or carrying costs of $cp_t |x_i|$ per unit of time for some constant $c > 0$. Hence, the shorter the horizon over which the arbitrageur has to hold this position prior to the price correction, the larger is the overall payoff. There are numerous explicit sources for these holding costs. As pointed out earlier, the short-sale of shares involves some unit-time costs. A short-seller has to borrow the shares from someone who charges an indirect lending fee. Selling a stock short is costly because one has to keep the short-sale proceeds in a margin account that pays low or even negative interest if the stock is on special. D'Avolio (2002) and Duffie, Gârleanu, and Pedersen (2002)

[5] Alternatively, we could assume that the hazard rate of an exogenous price correction increases over time after t_0. For example, the headline news might occur at $\bar{t} = t_0 + \bar{\tau} + \xi$, where $\bar{\tau}$ is a constant and ξ has a Weibull distribution. That is, $\xi \sim F(\xi) = 1 - \exp\{-\frac{\mu}{\alpha+1}\xi^{(\alpha+1)}\}$ with density $f(\xi) = \mu\xi^{\alpha}\exp\{-\frac{\mu}{\alpha+1}\xi^{(\alpha+1)}\}$. This implies an increasing hazard rate for an exogenous correction (conditional on knowing t_0) of $\frac{f(\xi)}{1-F(\xi)} = \mu\xi^{\alpha}$. In such a setting, arbitrageurs can infer more information about the distribution of t_0 since the probability of an exogenous price correction at t is higher for low t_0. This makes this formulation much less tractable.

[6] This possibility does not arise in equilibrium until the extended model of Section 5. At the cost of some complication, we could make ψ a function of the accumulated orders prior to the price correction, and of the order at the point of the correction.

describe institutional details of the U.S. stock lending market. Note that even when an asset is underpriced, the optimal arbitrage strategy involves short-selling. In this case, the arbitrageur buys the undervalued asset, but tries to short the best imperfect substitute in order to minimize the exposure to unrelated risks.

Arbitrageurs also incur *implicit* holding costs. For example, they have to deviate from their diversified portfolio in order to exploit an arbitrage opportunity. Therefore, holding costs can also be viewed as a short-cut formulation for the risk premium required for taking on this risky arbitrage position. The higher the volatility of the mispriced asset and the more difficult it is to hedge accompanied risk with another asset, the higher are these implicit holding costs. Another form of implicit holding costs arises when arbitrageurs have to hold collateral assets in order to execute their arbitrage strategy. This prevents them from investing their capital in more profitable alternatives. Arbitrageurs must also hold some capital to be prepared for margin calls.

Both explicit and implicit holding costs are amplified by the fact that arbitrageurs might be forced to terminate their arbitrage strategy before it pays off. In the simplest case, a liquidity shock occurs with a constant exogenous arrival rate. However, it is more likely that such a liquidity shock will occur when it can do the most harm. For example, an arbitrageur who borrowed shares to sell them short faces a recall risk or "buy in" risk. As soon as the lender of the shares recalls those shares, short-sellers have to cover their positions by buying the assets in the market at unfavorable terms. Finally, as Shleifer and Vishny (1997) point out, fund managers who rely on their clients' money to exploit the trading strategy may face an intermediate liquidation risk. If the price departs even further from the fundamentals, clients might attribute this temporary bad performance to poor management skills. Hence, they will withdraw their money, thereby forcing the fund manager to partially terminate the arbitrage strategy exactly when the arbitrage opportunity is most profitable.

All these implicit or explicit costs are summarized by c in our model. These costs make arbitrageurs cautious and give them an incentive to postpone their trades.

4 Analysis

4.1 Market timing and delayed arbitrage

Each arbitrageur has to decide when to trade based on her information. A trading strategy for arbitrageur $\hat{t}_i$ specifies trades as a function of $\tau_i = t - t_i$, the time elapsed since arbitrageur $\hat{t}_i$ became aware of the mispricing. We focus on trigger strategies, according to which arbitrageur $\hat{t}_i$ trades only once at $t_i + \tau_i^*$. Until this point in time, the arbitrageur holds the ex ante optimal (neutral) portfolio position. Recall that we have normalized this to zero. From $t_i + \tau_i^*$ onwards, the arbitrageur maintains the new position until the price correction occurs.

An arbitrageur who trades just prior to the price correction achieves the highest payoff. By postponing the trade, the arbitrageur reduces holding costs but risks missing the arbitrage opportunity. The aim of each sophisticated trader is to "beat the gun," using Keynes' (1936) words. Optimal market timing depends on each arbitrageur's individual evaluation of the likelihood that the price correction will occur in the next instant. Let $h\left(t|\hat{t}_i\right)$ be arbitrageur $\hat{t}_i$'s perceived hazard rate that the price correction occurs in the next instant t. Consequently, arbitrageur $\hat{t}_i$'s probability estimate that the correction occurs within the small interval $[t, t+\Delta]$ is approximated by $h\left(t|\hat{t}_i\right)\Delta$. In this case, the benefit of trading prior to the correction is βp_t. If the price correction does not occur, then the arbitrageur has to pay holding costs $cp_t\Delta$. In short, an arbitrageur will only trade based on her information if the benefits $\Delta h\left(t|t_i,\cdot\right)\beta$ outweigh the costs of holding a nonoptimally diversified portfolio $(1-\Delta h)\, c\Delta$ for some short time period Δ. Lemma 1 formally states the market timing condition for $\Delta \to 0$.

Lemma 1 *An arbitrageur trades to the maximum short (long) position as soon as the subjective hazard rate $h\left(t|\hat{t}_i\right)$ times the size of the over-(under-)pricing β equals the holding costs c.*

Note that the risk neutrality of the arbitrageurs implies that they always go to their maximum long or short position. Given our parameterization, the aggregate trading imbalance of all arbitrageurs coincides with the number of arbitrageurs buying/selling the asset.

The hazard rate at which the price correction will occur is not exogenous but depends on the other arbitrageurs' trading. The longer other arbitrageurs wait until they trade based on their information, the lower is the probability that the correction will occur in the near future.

We restrict attention to symmetric strategy equilibria, wherein each trader adopts the same trading strategy. Abreu and Brunnermeier (2001) show in a related setting that any trading equilibrium is symmetric and involves only trigger strategies. By symmetry, it follows directly that from each trader's point of view the price correction never occurs with strictly positive probability at any specific instant, that the aggregate selling pressure is exactly equal to $\kappa\left(t-t_0\right)$ when the price correction occurs for endogenous reasons, and that the hazard rate is always well defined.

If all arbitrageurs trade only with a delay of τ' periods, then the price correction only occurs at $t_0+\varphi\left(\tau'\right)$. That is, $\varphi\left(\tau'\right)$ is implicitly defined as $\varphi\left(\tau'\right)=\tau'+\eta\kappa\left(\varphi\left(\tau'\right)\right)$. In the linear case where $\kappa\left(\cdot\right)=\kappa_0\left[1-\frac{1}{\bar{\tau}}\left(t-t_0\right)\right]$, $\varphi\left(\tau'\right)$ simplifies to $\varphi\left(\tau'\right)=\bar{\tau}\frac{\tau'+\eta\kappa_0}{\bar{\tau}+\eta\kappa_0}$ and hence the price correction occurs at $t_0+\bar{\tau}\frac{\tau'+\eta\kappa_0}{\bar{\tau}+\eta\kappa_0}$.

Arbitrageur $\hat{t}_i$ knows that, in equilibrium, the price correction will occur no later than $t_i+\varphi\left(\tau'\right)$ but after $t_i+\varphi\left(\tau'\right)-\eta$. Given that the prior distribution of t_0, $F\left(t_0\right)=1-e^{-\lambda t_0}$, is exponential, so is the distribution of the price correction date $t_0+\varphi\left(\tau'\right)$. Conditioning on $t_0+\varphi\left(\tau'\right)\in\left[t_i+\varphi\left(\tau'\right)-\eta, t_i+\varphi\left(\tau'\right)\right]$ yields arbitrageur $\hat{t}_i$'s posterior

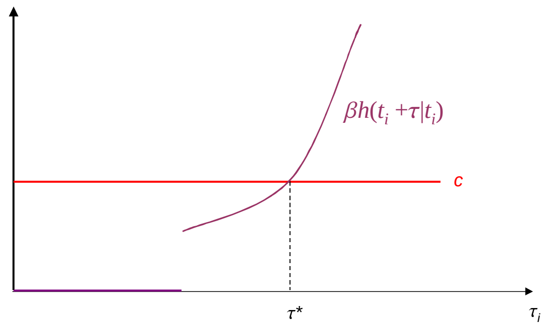

Figure 2: Arbitrageur $\hat{t}_i$'s hazard rate

distribution at t_i of the date of the price correction. If the mispricing persists up to $t > t_i + \varphi(\tau') - \eta$, the support of the possible correction date shrinks to $[t, t_i + \varphi(\tau')]$. The exponential distribution, together with the fact that arbitrageurs become aware of the mispricing in a uniform manner, leads to a tractable posterior at t. Arbitrageur $\hat{t}_i$'s updated density that price correction occurs in the next instant after t given that it persists up to t is given by $\frac{\lambda \exp\{-\lambda(t)\}}{1-\exp\{-\lambda(t_i+\varphi(\tau'))\}-[1-\exp\{-\lambda(t)\}]} = \frac{\lambda}{1-\exp\{-\lambda(t_i+\varphi(\tau')-t)\}}$. That is, arbitrageur $\hat{t}_i$'s hazard rate about the date of the price correction, as illustrated in Figure 2, is[7]

$$h\left(t|\hat{t}_i\right) = \begin{cases} 0 & \text{for} \quad t < t_i + \varphi(\tau') - \eta \\ \frac{\lambda}{1-\exp\{-\lambda(t_i+\varphi(\tau')-t)\}} & \text{for} \quad t \geq t_i + \varphi(\tau') - \eta \end{cases}.$$

In the symmetric equilibrium defined by a common trigger τ^*, arbitrageur $\hat{t}_i$ has to find it optimal to trade at $t = t_i + \tau^*$. By Lemma 1, this is only the case when her subjective hazard rate $h\left(t_i + \tau^*|\hat{t}_i\right) = \frac{c}{\beta}$. A rearrangement of this equation $\frac{\lambda}{1-e^{-\lambda(\varphi(\tau^*)-\tau^*)}} = \frac{c}{\beta}$ reveals that the duration of the mispricing is $\varphi(\tau^*) = \tau^* + \frac{1}{\lambda}\ln\left[\frac{c}{c-\lambda\beta}\right]$. Recall that $\varphi(\tau^*) = \bar{\tau}\frac{\tau^*+\eta\kappa_0}{\bar{\tau}+\eta\kappa_0}$ for the linear case. Solving for τ^* yields the unique symmetric Perfect Bayesian Nash equilibrium, stated in Proposition 1. Note also that the price correction is not immediate ($\tau^* > 0$) but prior to the time it would occur absent trading pressure from rational arbitrageurs, that is at $t_0 + \bar{\tau}$.

Proposition 1 *There exists a unique symmetric trigger strategy equilibrium. In this equilibrium, all arbitrageurs delay their trade by* $\tau^* = \bar{\tau} - \frac{\bar{\tau}+\eta\kappa_0}{\lambda\eta\kappa_0}\ln\left[\frac{c}{c-\lambda\beta}\right] > 0$. *In*

[7] Note that for $\lambda \to 0$, the exponential prior distribution converges to a uniform distribution. Hence for $t \geq t_i + \varphi(\tau') - \eta$, the price correction occurs with a constant harzard rate of $\frac{1}{t_i+\varphi(\tau')-t}$. That is, the time of the price correction is uniformly distributed between the next instant t and $t_i + \varphi(\tau')$.

this equilibrium, the price correction caused by arbitrageurs' trading activity occurs at $t_0 + \bar{\tau}\left(1 - \frac{1}{\lambda\eta\kappa_0}\ln\left[\frac{c}{c-\lambda\beta}\right]\right)$.

The proof of this proposition is presented in the Appendix. If all arbitrageurs postpone their trading, that is, if τ is larger, it takes longer for the price adjustment to occur. This also implies that fewer arbitrageurs are needed for the price correction. Hence, the likelihood that a price correction will occur in the next instant, before arbitrageur $\hat{t}_i$ trades, increases. In other words, the equilibrium hazard rate $h\left(t_i + \tau | t_i, \tau\right)$ is increasing in τ since $\kappa\left(t - t_0\right)$ is monotonically decreasing. The equilibrium delay τ^* for each arbitrageur is given by the intersection between $\beta h\left(t_i + \tau | t_i, \tau\right)$ - viewed as a monotonically increasing function in τ - and the constant per-period holding costs c.

Our model predicts that arbitrage will be delayed. This finding is in sharp contrast to the standard backwards induction argument which predicts that mispricing cannot be sustained over long periods of time. However, classic backwards induction is not applicable in our setting since there is no "commonly known starting point" for the procedure to work. Proposition 2 states that it is never common knowledge among all arbitrageurs, indeed even among any positive mass of arbitrageurs, that the price is not right. This is due to the sequential awareness specification of our model.

Proposition 2 *The mispricing is never common knowledge among a positive mass of arbitrageurs.*

Proof. The arbitrageur who becomes immediately aware of the mispricing at t_0 knows at $t_0 + \eta$ that everybody knows of the mispricing. However, the trader who only becomes aware of the mispricing at $t_0 + \eta$ thinks that she might be the first to hear of it and thus does not know that all traders already know it. Hence, even though everybody knows of the bubble at $t_0 + \eta$, only the first trader knows that everybody knows. At $t_0 + 2\eta$, even the last traders to become aware of the mispricing know that everybody knows, but they do not know that everybody knows that everybody knows of the bubble. As time proceeds the order of mutual knowledge increases, but it will never reach infinity. The argument for any positive mass of arbitrageurs is analogous. ■

This lack of common knowledge about the mispricing preserves the disagreement about the timing of the price correction. A bubble is the most pronounced form of overpricing. Abreu and Brunnermeier (2001) analyze the case in which the price departs further and further from its fundamental value. That paper also draws a connection to the static global games literature, in which the fundamental value is not commonly known. In contrast, the current paper demonstrates that arbitrage is delayed even if the mispricing does not grow over time. Allen, Morris, and Postlewaite (1993) originally define "strong bubbles" as situations in which all traders know with probability one that the price is too high. The authors provide necessary conditions for the existence of bubbles. These conditions include the Pareto interim inefficiency of the initial allocation, a lower bound on the maximum short position, and asymmetric information about the fundamentals. Hence, as they stress in their paper it cannot be the case that it is commonly known among the traders that a bubble exists. They also provide examples of bubbles in three-period economies in which traders have heterogeneous beliefs or state-dependent utility functions. Note that our setting is different because we rely on a coordination element to provide an obstacle to arbitrage. This allows us to show that mispricing can persist without departing from the common prior assumption or state independent utility formulations. In contrast to their paper, our analysis is also closer in spirit to the literature on limits to arbitrage.

4.2 Comparative statics

The next proposition lists some comparative statics results that are proved in the Appendix. These results are discussed following the statement of the proposition.

Proposition 3 *The duration of the mispricing is*
(i) increasing in the length of the awareness window η, initial absorption capacity κ_0, and holding costs c, and
(ii) decreasing in arrival rate λ and the size of the mispricing β.

Recall that λ reflects the arrival rate of t_0 given the prior exponential distribution of t_0. As λ increases, the prior distribution carries more and more information about the starting point of the mispricing. For $\lambda \to 0$, the "sequential awareness" effect is most pronounced and the distribution converges to the uniform distribution. As the arrival rate $\lambda \to \infty$, arbitrageurs know that the mispricing starts immediately at $t_0 = 0$ even before they receive a signal. Therefore, there is no dispersion of opinion among arbitrageurs. The second parameter that determines the dispersion of opinion is the length of the awareness window η. For larger η, it takes longer until sufficiently many arbitrageurs who could correct the mispricing know of the mispricing. Thus, the duration of mispricing is lengthened due to an amplification effect. Given that everybody trades only after a certain initial delay of τ periods, it takes $\bar{\tau}\frac{\tau+\eta\kappa_0}{\bar{\tau}+\eta\kappa_0}$ periods until the price correction occurs. A larger η increases $\bar{\tau}\frac{\tau+\eta\kappa_0}{\bar{\tau}+\eta\kappa_0}$ since $\tau < \bar{\tau}$. Hence, it makes it more attractive to wait even longer to save the holding costs c. This, in turn, induces everybody to increase their delay τ. This postpones the date of the price adjustment even further, which makes waiting even more attractive. A similar reasoning can be used to explain why a larger κ_0 extends the duration of the mispricing. Increasing κ_0 shifts all $\kappa(t - t_0)$-values, thereby exacerbating the synchronization problem.

The larger the magnitude of the mispricing β, the shorter it lasts. In other words, very large mispricings cannot be sustained for very long, while small mispricings can persist. Thus, the fundamental value serves as an anchor for share prices, and behavioral trades cannot drive the price too far away from the asset's fundamental value.

This result conforms with Black's (1986) suggestion that the current fundamental value of an individual stock is between half and double the current price level.[8]

5 Further discussion and possible extensions

5.1 Asymmetric holding costs

One could extend the model to allow for asymmetric holding costs. This extension is interesting in that holding costs are typically higher for short positions than for long positions.[9] In our model, this implies that overpricing persists longer than underpricing. Thus, in effect, "bad news travels slowly." This conforms with the empirical findings of Hong, Lim, and Stein (2000), who discover that stocks with low analyst coverage seem to react more sluggishly to bad news than to good news.

5.2 Fads in the acquisition and use of information

In the 1980s, much ink was spilled on the impact of foreign exchange rates on the profitability of domestic companies. More recently, the growth potential of a company and the Fed's interest rate policy were particularly salient news, while the cash flow of the company was of secondary importance. Arguably, such shifts in emphasis have not always faithfully tracked fundamentals. Frequently, more relevant factors have been ignored in favor of the currently fashionable ones.

In our model, it is futile for well-informed rational arbitrageurs to act on some piece of information unless a mass of other arbitrageurs will do so also (or unless an exogenous price correction is viewed to be imminent). This coordination element creates a strong incentive for an individual arbitrageur to act only on certain types of information, such as information that other arbitrageurs are acting upon, and ignore information that is being generally ignored, even when such information is more important in terms of fundamentals. Thus, fads and fashions in the use of information can easily arise within our framework. If our model were extended such that behavioral traders

[8] One can easily envision a more general setting where arbitrageurs might receive more than one β-signal about the fundamental value of the asset. Furthermore, their estimate of the size of the mispricing β might differ. A very general formulation is given by $v_t = \beta_t M_t$, where M_t follows a commonly observable martingale process, such as a geometric Brownian motion, while β_t is a point process with repeated sequential news of mispricings.

[9] The only way to exploit an overpriced asset is to sell it short. In contrast, if the asset is undervalued, the arbitrageur acquires a long position with lower holding costs. Admittedly, even a risk-averse arbitrageur who buys an undervalued asset might want to short close substitute assets to hedge the risk. Thus, shorting costs can also arise if a security is underpriced. In this case, shorting costs are incurred to reduce the overall "implicit" direct holding costs due to fundamental and noise trader risk. On the other hand, in the case of overpriced securities, shorting costs arise in addition to the "implicit" holding costs.

were (eventually) more likely to be responsive to the currently fashionable kind of information, and very slow to understand and respond to other kinds of information, the coordination effects referred to above would be reinforced. Similarly, information acquisition costs strengthen these effects.

The notion that professional traders try to forecast the forecasts of others, rather than the fundamentals, was originally proposed by Keynes (1936) in his famous comparison of the stock market with a beauty contest. Brennan (1990) notes that an individual trader has no incentive to collect fundamental information if that information will not be reflected in the price before she has to unwind the acquired arbitrage position. In Dow and Gorton's (1994) overlapping generations model, it is less desirable for arbitrageurs to trade on long-run information because (i) they are short-lived and have to unwind their acquired positions in the subsequent trading round; and (ii) this news will only be reflected in the price if next period's arbitrageur also trades on it. That is, in equilibrium, an arbitrageur will only trade on short-run information if the whole chain of future arbitrageurs is willing to collect and trade on the same piece of information. In Froot, Scharfstein, and Stein (1992), traders have to choose between two pieces of information. They demonstrate that traders with short horizons will herd to collect the same piece of information. A more detailed discussion of these and other papers on investigative herding can be found in Brunnermeier (2001, Sections 6.2). In all these models, the interdependence of traders' information acquisition arises from their short horizons. This prevents an explicit analysis of market timing. In our model, arbitrageurs are infinitely long-lived, yet the decision about which information to act upon is interdependent because of holding costs.

5.3 Credible signaling

Since differences of opinion are a key element of our model, it is important to identify potential forces that could reduce or even eliminate such differences. We discuss two such mechanisms: arbitrageurs' announcements and arbitrageurs' trading activities.

An arbitrageur's announcement could make the mispricing commonly known and hence, it could accelerate the price correction. In our setting, this would indeed be optimal for an arbitrageur who has already traded because it reduces holding costs. However, in a richer model it is questionable whether such announcements would be credible. The basic difficulty is that all traders would like to make recommendations that move the price, regardless of whether they have received any information. Such announcements would allow them to front-run their stock recommendations and unload their acquired positions at a profit. Even if traders cared about their reputation, the incentive to manipulate prices for a profit remains potent. This point is made by Benabou and Laroque (1992) in their analysis of stock market "gurus".[10]

[10] One might argue that George Soros was successful in conveying his information to other traders in at least some of his attacks on Asian currencies. Corsetti, Dasgupta, Morris, and Shin (2001)

5.4 Partial price adjustments, rallies, and cascades

In our model, a full price correction occurs *in one step* at the first instant at which trading pressure by rational arbitrageurs exceeds $\kappa(t - t_0)$. This simplifying assumption guarantees that rational arbitrageurs do not perfectly learn each other's information from prices. An earlier version of the paper, Abreu and Brunnermeier (2002), generalizes the model (albeit in a stylized way) to allow for partial price adjustments. These price changes are driven by changes in the total trading imbalance due not only to rational arbitrageurs' trading pressure but also to random shifts in behavioral traders' investor sentiment. Hence, rational arbitrageurs can never be sure whether a partial correction is due to rational traders' trading pressure or just a sentiment shift. This uncertainty ensures that rational arbitrageurs cannot infer "too much" from prices.

The sentiment shocks might be of only a temporary nature since the behavioral traders' mood might swing back. In this case, behavioral traders will consider the lower (higher) price as an attractive buying (selling) opportunity in the subsequent trading round. The resulting rally among behavioral traders will then push the price back to its original level. In addition to a price reversal after a partial price correction, arbitrageurs have to take two additional possible scenarios into account: a partial price correction can be followed by a further price correction (cascade) or the price can simply stay the same. Immediately after the partial correction, rational traders do not know which of the three possible outcomes will prevail. Their estimate of the likelihood of the three outcomes depends on when they became aware of the mispricing. If they have been aware of the mispricing for a long time, they think it more likely that the partial price correction will be followed by a further price move towards the fundamentals. Consequently, these arbitrageurs will trade based on their fundamental information. On the other hand, arbitrageurs who only recently became aware of the mispricing will bet on a price reversal. That is, they buy (sell) the asset, even though they know that it is overvalued (undervalued). In equilibrium, arbitrageurs will trade against each other.[11] Note that arbitrageurs' trades after a partial correction can (by going against the fundamentals) even be instrumental in the occurrence of a price reversal. Thus, the actual price dynamics after a partial correction depend on t_0, which in turn determines the lengths of time traders have been aware of the mispricing. When no price cascade arises, since either the price reverts or it stays at its original level, arbitrageurs typically continue to face the synchronization problem - emphasized in this paper - and hence

show within a static global games environment that the mere presence of traders like George Soros makes currency pegs less stable, since it reduces the coordination problem for the small speculators and makes them more aggressive. The authors also extend their model to a two-period setting where the large trader is assumed to move in the first period. His trading activity not only reduces the coordination problem but also provides an additional public signal to the small speculators, who are then more likely to attack the currency in the second period.

[11] A formal analysis and characterization of arbitrageurs' equilibrium behavior for a generalized setting with partial price adjustments is available from the authors.

mispricing can persist.

The possibility of partial corrections has competing effects on how long rational arbitrageurs delay action on their information. On the one hand, knowing that the full correction will typically be preceded by a partial correction, rational arbitrageurs are even more reluctant to exploit the mispricing as long as the price does not move towards the fundamentals. They know that it is likely that they will have a second chance to trade on their information even after the first partial price correction. Therefore, they wait even longer before they trade. A larger τ^* prolongs the duration of the mispricing. On the other hand, traders might also mistakenly interpret a price move triggered by a random sentiment shock as being caused by rational arbitrageurs. If this occurs sufficiently many periods after t_0, the random shock might trigger a full correction.

6 Conclusion

This paper presents a new explanation - synchronization risk - for why arbitrage might be limited and behavioral biases in prices persist. The distinctive feature of synchronization risk compared to noise trader risk is that it is not directly caused by behavioral traders. It is each arbitrageur's uncertainty about the timing of other arbitrageurs' actions, combined with the desire to minimize holding costs, that causes each trader to delay an arbitrage trade. As a consequence, and contrary to the prediction of the efficient markets hypothesis, there can be significant and long-lasting departures from efficient prices. Nevertheless, our analysis suggests that the price of an asset is (loosely) anchored to its fundamental value since in our model mispricing is corrected more quickly the larger it is. Apart from the basic result of delayed arbitrage, the model also provides an explanation for fads and fashion in the acquisition and use of information and for why "bad news travels slowly."

Many extensions of the basic model could be usefully pursued. Perhaps the most challenging is to model the price process more explicitly than we have done. Another task that merits future research is to model the behavioral traders' decision-making based explicitly on well-documented psychological biases. In that setting, arbitrageurs who want to "read the mind of the market" would not only have to predict the behavior of other rational arbitrageurs, but would also have to take account of the evolving behavior of irrational traders. Such models would combine synchronization risk with endogenous noise trader risk.

A Appendix

A.1 Proof of Proposition 1

We first prove the result for any continuously decreasing function $\kappa(t - t_0)$ before we provide the explicit closed-form solution for linearly declining $\kappa(\cdot)$ functions. Let us first consider the case where $t_0 > \eta$. Conjecture a symmetric trigger strategy profile whereby all rational arbitrageurs trade τ^* periods after they become aware of the mispricing. Hence, the price correction due to arbitrageurs' trading activity occurs at $t = t_0 + \varphi(\tau^*)$, where $\varphi(\tau^*) := \tau^* + \eta\kappa(\varphi(\tau^*))$. Since this is known to all arbitrageurs, they also know that in equilibrium $\kappa(\varphi(\tau^*)) =: \kappa^*$ arbitrageurs are needed to correct the mispricing at $t_0 + \varphi(\tau^*)$. Hence, at the time when arbitrageur $\hat{t}_i$ trades, she knows from the existence of the mispricing that $t_0 \geq t_i + \tau^* - \varphi(\tau^*)$. Otherwise, the price correction would have occurred already. In equilibrium, trader $\hat{t}_i$ attacks the mispricing at $t_i + \tau^*$. Given the equilibrium hazard rate, as discussed in Section 4, the market timing condition is given by

$$\begin{aligned}
\left[\frac{\lambda}{1 - e^{-\lambda(t_i + \tau^* + \eta\kappa(\varphi(\tau^*)) - (t_i + \tau^*))}}\right]\beta &= c \\
\frac{\lambda}{1 - e^{-\lambda\eta\kappa(\varphi(\tau^*))}} &= \frac{c}{\beta} \\
\kappa(\varphi(\tau^*)) &= \frac{1}{\lambda\eta}\ln\left[\frac{c}{c - \lambda\beta}\right] =: \kappa^*.
\end{aligned}$$

Hence,

$$\begin{aligned}
\varphi(\tau^*) &= \kappa^{-1}\left(\frac{1}{\lambda\eta}\ln\left[\frac{c}{c - \lambda\beta}\right]\right) \\
\tau^* &= \varphi^{-1}\left(\kappa^{-1}\left(\frac{1}{\lambda\eta}\ln\left[\frac{c}{c - \lambda\beta}\right]\right)\right).
\end{aligned}$$

For $\tau_i > (<)\ \tau^*$, any arbitrageur $\hat{t}_i$ has a strict (dis-)incentive to trade based on information. By inspection of the market timing condition $\beta h\left(t|\hat{t}_i\right) > c$, one can easily see that at $t = t_0 + \varphi(\tau^*)$, it is optimal for all arbitrageurs who became aware of the mispricing before (after) $\hat{t}_i = t_0 + \eta\kappa(\varphi(\tau^*))$ to depart from (be at) the initial zero position.

For the special case where $t_0 < \eta$, the traders who become aware of the mispricing before η know from the prior distribution that they are more likely to be informed earlier than others. To see that a realization of t_0 prior to η does not affect our analysis, let us conduct the following thought experiment. Consider the realization of $0 \leq t_0' < \eta$ and assume that the prior distribution of t_0' is exponentially distributed with support $[-\eta, \infty)$. Then the analysis described above applies and the price correction occurs at $t_0' + \varphi(\tau^*)$. A single trader $\hat{t}_i \leq t_0' + \eta\kappa(\varphi(\tau^*))$ who receives an additional signal that $t_0 \geq 0$ will start attacking at a later point in time, but before the endogenous price correction will occur. Similarly, if all traders $\hat{t}_i \in [t_0', t_0' + \eta\kappa(\varphi(\tau^*))]$ receive the additional signal $t_0 \geq 0$, the time of the bursting of the bubble remains $t_0' + \eta\kappa(\varphi(\tau^*)) + \tau^*$. Thus, the results described above are also validated for realizations of $t_0 \leq \eta$.

For the special case where $\kappa(t - t_0) = \kappa_0 \left[1 - \frac{1}{\bar{\tau}}(t - t_0)\right]$, $\varphi(\tau^*) = \bar{\tau}\frac{\tau^* + \eta\kappa_0}{\bar{\tau} + \eta\kappa_0}$. Hence, the inverse functions are given by $\kappa^{-1}(\kappa^*) : \ t - t_0 = \bar{\tau}\left(1 - \frac{\kappa^*}{\kappa_0}\right)$ and $\varphi^{-1}(\varphi) : \tau^* = \varphi\frac{\bar{\tau} + \eta\kappa_0}{\bar{\tau}} - \eta\kappa_0$. It is easy to check that in this case

$$\tau^* = \bar{\tau} - \frac{\bar{\tau} + \eta\kappa_0}{\kappa_0}\kappa^*.$$

Consequently, the duration of the mispricing is $\varphi = \bar{\tau}\left(1 - \frac{1}{\lambda\eta\kappa_0}\ln\left[\frac{c}{c - \lambda\beta}\right]\right)$ and the waiting time is $\tau^* = \bar{\tau} - \frac{\bar{\tau} + \eta\kappa_0}{\lambda\eta\kappa_0}\ln\left[\frac{c}{c - \lambda\beta}\right]$, which also has to equal $\varphi - \eta\kappa^*$.

Note that $\tau^* > 0$, since we assume $\bar{\tau} > \max\{\eta, \frac{1}{\lambda}[\frac{1}{\ln\frac{c}{c - \lambda\beta}} - \frac{1}{\lambda\eta\kappa_0}]^{-1}\}$.■

A.2 Proof of Proposition 3

Comparative statics of $\bar{\tau}\left(1 - \frac{1}{\lambda\eta\kappa_0}\ln\left[\frac{c}{c - \beta\lambda}\right]\right)$:

W.r.t. β.

The derivative of $\frac{c}{c - \beta\lambda}$ w.r.t. β is given by $\lambda c > 0$. Hence, the duration is decreasing in β.

W.r.t. $\bar{\tau}$, the derivative is $1 - \frac{1}{\lambda\eta\kappa_0}\ln\left[\frac{c}{c - \beta\lambda}\right] > 0$.

It is obvious that the duration of the mispricing is increasing in κ_0 and η.

Since $-\frac{1}{\lambda\eta\kappa_0}\ln\left[\frac{c}{c - \beta\lambda}\right] = \frac{1}{\lambda\eta\kappa_0}\ln\left[\frac{c - \beta\lambda}{c}\right]$, it is easy to see that both factors are decreasing in λ. Hence, the duration of the mispricing is decreasing in λ.■

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