Appendices Containing Supplemental Details for: The Dark Side of Financial Innovation: A Case Study of the Pricing of a Retail Financial Product

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Appendix A Post-Issuance Returns of the SPARQS

This section contains supplemental details for the Cumulative Abnormal Residual Returns (CARRs) in Section 3.2 of The Dark Side of Financial Innovation: A Case Study of the Pricing of a Retail Financial Product.

Examining the secondary market returns requires a return metric for the SPARQS. We construct a total return measure, defined as the change in the invoice or "full" price, divided by the invoice price. Using the fact that the invoice price is the sum of the traded or "clean" price and accrued interest, the daily return from date $t-1$ to date t is:

$$R_{i,t} = \begin{cases} \frac{AC_{i,t}-AC_{i,t-1}+P_{i,t}-P_{i,t-1}}{AC_{i,t-1}+P_{i,t-1}} & \text{if } AC_{i,t} \geq AC_{i,t-1}, \\ \\ \frac{CPN+AC_{i,t}-AC_{i,t-1}+P_{i,t}-P_{i,t-1}}{AC_{i,t-1}+P_{i,t-1}} & \text{if } AC_{i,t} < AC_{i,t-1}, \end{cases} \tag{A.1}$$

where $P_{i,t}$ and $P_{i,t-1}$ are the closing market prices on trading dates t and $t-1$ respectively, CPN is the periodic interest payment, and $AC_{i,t}$ and $AC_{i,t-1}$ are the accrued interest on trading dates t and $t-1$ respectively. The accrued interest portion tracks the part of the periodic coupon the seller of a bond owes to the buyer upon settlement. Since the coupon is paid to the bondholder as of the close of business on the record date, which is a specified number of calendar days before the payment date, the accrued interest component resets to zero on the day after the ex-coupon date and the condition $AC_{i,t} < AC_{i,t-1}$ is satisfied only on this date. Thus, an investor who owned the security on the record date will receive the coupon on the distribution date.

The SPARQS derive their value from the underlying stock price and their returns can be described by the model

$$R_{i,t} - r_t \approx \Omega_{i,t}\left(r_{i,t} - r_t\right) + \eta_{i,t}, \tag{A.2}$$

where $i = 1, \ldots, N$ indexes the SPARQS and their underlying stocks, $R_{i,t}$ is the return on the ith SPARQS on date t, $r_{i,t}$ is the return on the common stock underlying the ith SPARQS, r_t is the riskless rate on date t, and $\Omega_{i,t} \equiv \frac{\partial V_i(S_{i,t},t)}{\partial S_{i,t}} \frac{S_{i,t}}{V_i(S_{i,t},t)}$, where $V_i(S_{i,t}, t)$ is the value of the ith SPARQS given by the pricing model in Section 3.2. With this definition $\Omega_{i,t}$ is the elasticity of the SPARQS price with respect to the stock price. While equation (A.2) holds only approximately because it treats the elasticity $\Omega_{i,t}$ as constant within each day, we expect the approximation to be excellent.

If the SPARQS are priced correctly relative to the stock without any markup or premium then the residual $\eta_{i,t}$ in (A.2) would be close to zero, where the difference from zero is due to possible miss-specification of the pricing model, inherent approximation errors in its numerical solution, the assumption that the elasticity $\Omega_{i,t}$ is constant within each day, and features of the market microstructure such as the discreteness of price quotations. If the SPARQS are over-priced relative to the stock then cumulative residuals of the form $\sum_t \eta_{i,t}$ should be negative.

We also consider the risk-adjusted performance of the SPARQS relative to the market index. Applying the market model to the underlying stock, the return for the underlying stock is

$$r_{i,t} - r_t = \beta_i \left(r_{M,t} - r_t\right) + \varepsilon_{i,t}, \tag{A.3}$$

where $r_{i,t} - r_t$ is the excess return of the underlying stock i on date t over the risk-free rate of return r_t and $r_{M,t} - r_t$ is the excess return on the value-weighted CRSP market portfolio on date t. Combining equations (A.2) and (A.3), the market model for the SPARQS based on the index returns is

$$\begin{aligned} R_{i,t} - r_t &= \Omega_{i,t}\left[\beta_i \left(r_{M,t} - r_t\right) + \varepsilon_{i,t}\right] + \eta_{i,t} \\ &= \Omega_{i,t}\beta_i \left(r_{M,t} - r_t\right) + \Omega_{i,t}\varepsilon_{i,t} + \eta_{i,t}. \end{aligned} \tag{A.4}$$

Referring to Table 2, the reference equities for SPARQS appear to consist primarily of large, growth stocks. For this reason, the market model of equation (A.3) may not explain adequately the returns to the underlying stocks. Applying the three-factor model of Fama and French (1992), the return for the underlying stock is

$$r_{i,t} - r_t = \beta_i \left(r_{M,t} - r_t\right) + s_i \left(SMB_t\right) + h_i \left(HML_t\right) + \varepsilon_{i,t}, \tag{A.5}$$

where SMB_t are the excess returns to the portfolio of small stocks over large stocks on date t, and HML_t are the excess returns to the portfolio of high book-to-market firms over low book-to-market firms.[1] Combining equations (A.2) and (A.5), the three-factor model for the SPARQS is

$$\begin{aligned} R_{i,t} - r_t &= \Omega_{i,t}\left[\beta_i \left(r_{M,t} - r_t\right) + s_i \left(SMB_t\right) + h_i \left(HML_t\right) + \varepsilon_{i,t}\right] + \eta_{i,t} \\ &= \Omega_{i,t}\left(\beta_i \left(r_{M,t} - r_t\right) + s_i \left(SMB_t\right) + h_i \left(HML_t\right)\right) + \Omega_{i,t}\varepsilon_{i,t} + \eta_{i,t}. \end{aligned} \tag{A.6}$$

If the SPARQS underperform on a risk-adjusted basis, the accumulated (over event-time) averages across all SPARQS issues of the residuals in equations (A.2), (A.4), and (A.6) should be negative. Similarly, if the underlying stocks underperform their risk-adjusted benchmarks, equations (A.3) and (A.5) will have negative cumulative residuals. Testing these hypotheses requires averaging the residuals across issues in addition to accumulating them over time. A complication arises because the numbers of issues in the averages differ on different dates because some SPARQS

[1] The time series of factor portfolio returns are made available by Ken French at the website http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

drop out of the sample due to calls and the price acceleration event that occurred on one issue. This complication is handled by first computing the average returns of the available SPARQS (the inside sums over i in equations (A.7), (A.8), and (A.9) below), and then cumulating the average residual returns over event time (the outside sums over t).

Some notation is required to formulate the test statistics. Let t, and later u, index event time, i.e. $t = 0$ is the pricing date, $t = 1$ is the first date of secondary market trading, etc. Let τ_i be the calendar-time pricing date of the ith SPARQS. For example, 1 January 1995 might be calendar day 0, and the ith SPARQS might have been priced on calendar day $\tau_i = 1,321$ (which is event day 0 for this issue) and start trading on calendar day $1,322$ (event day 1). Thus $\tau_i + t$ is the calendar date t days following the pricing date of the ith SPARQS. Let A_t denote the set of the indices of the SPARQS that are available for trading on event day t. Thus, A_t is the set of SPARQS indices that have not been called or had a price acceleration event on the tth date of secondary market trading. Initially (at event date 1) the set A_1 has N elements $A_1 = \{1, 2, 3, \ldots, N\}$, where $N = 52$ is the total number of SPARQS for which we have sufficient secondary market price data. As some SPARQS are called, or disappear due to a price acceleration event, some indices disappear from A_t. Let $\mathrm{card}(A_t)$ denote the cardinality of A_t, that is $\mathrm{card}(A_t)$ is the number of SPARQS that are still available for trading on event-date t. We are interested in testing the hypotheses:

$$\sum_{t=1}^{L}\left(\frac{1}{\mathrm{card}(A_t)}\sum_{i\in A_t}\eta_{i,\tau_i+t}\right) = 0, \tag{A.7}$$

$$\sum_{t=1}^{L}\left(\frac{1}{\mathrm{card}(A_t)}\sum_{i\in A_t}\epsilon_{i,\tau_i+t}\right) = 0, \tag{A.8}$$

$$\sum_{t=1}^{L}\left(\frac{1}{\mathrm{card}(A_t)}\sum_{i\in A_t}(\Omega_{it}\epsilon_{i,\tau_i+t}+\eta_{i,\tau_i+t})\right) = 0, \tag{A.9}$$

where the residuals being summed are defined in equations (A.2)–(A.4) or (A.2), (A.5) and (A.6), depending upon whether the market or three-factor model is being used. Appendix C describes the computation of standard errors for the test statistics above.

Appendix B Calendar Time Regressions

This section contains supplemental details pertaining to the calendar time regressions in Section 3.2.

Recalling that $\Omega_{i,t}$ is the elasticity of the value of the ith SPARQS with respect to the stock price, the (approximately) correct benchmark for the SPARQS' excess return $R_{i,t} - r_t$ is the product

$\Omega_{i,t}(r_{i,t} - r_t)$. This benchmark is only approximately correct because for each month for each SPARQS the elasticity is fixed at its value as of the end of the preceding month. Letting B_t be the set of SPARQS that were issued during the six months preceding month t, if none of the SPARQS in B_t are called or terminated due to a price acceleration event during the month the excess returns on the equally-weighted portfolio of SPARQS and the elasticity-weighted benchmark return are $R_{\Pi,t} - r_t = (1/\text{card}(B_t)) \sum_{i \in B_t} R_{i,t} - r_t$ and $r_{\Omega,t} - \bar{\omega} r_t = (1/\text{card}(B_t)) \sum_{i \in B_t} \Omega_{i,t}(r_{i,t} - r_t)$, respectively, where $\bar{\omega} \equiv (1/\text{card}(B_t)) \sum_{i \in B_t} \Omega_{i,t}$ so that $\bar{\omega} r_t$ is the omega-weighted risk-free return corresponding to $r_{\Omega,t}$.

The calendar-time regression specification for the SPARQSs' underlying stock benchamrk is:

$$R_{\Pi,t} - r_t = \alpha + \beta(r_{\Omega,t} - \bar{\omega} r_t) + \epsilon_t. \tag{B.1}$$

Next, the analysis uses a cruder benchmark, the returns $r_{\text{equal},t} = (1/\text{card}(B_t)) \sum_{i \in B_t} r_{i,t}$ of an equally-weighted portfolio of the underlying stocks. The disadvantage of the equally-weighted stock portfolio is that the portfolio construction does not account for differing sensitivities of the SPARQS to the returns of their reference equities.

Turning to the underlying stocks, the next pair of regressions are for calendar-time returns of equally-weighted portfolios of the underlying stocks. Employing both the market model and the three-factor model, the regression equations are

$$r_{\text{equal},t} - r_t = \alpha + \beta\,(r_{M,t} - r_t) + \epsilon_t, \tag{B.2}$$

$$r_{\text{equal},t} - r_t = \alpha + \beta\,(r_{M,t} - r_t) + s\,(SMB_t) + h\,(HML_t) + \epsilon_t, \tag{B.3}$$

where all variables are as defined previously.

The final pair of results are for the regression equations

$$R_{\Pi,t} - r_t = \alpha + \beta\,(r_{M,t} - r_t) + \epsilon_t, \tag{B.4}$$

$$R_{\Pi,t} - r_t = \alpha + \beta\,(r_{M,t} - r_t) + s\,(SMB_t) + h\,(HML_t) + \epsilon_t, \tag{B.5}$$

that explain the SPARQS calendar-time portfolio returns using the market model and three-factor model of Fama and French (1992).

Appendix C Computation of CARRs Standard Errors

Testing the significance of the CARRs described above involves computing the variance of the following statistic:

$$\sum_{t=1}^{L}\left(\frac{1}{\mathrm{card}(A_t)}\sum_{i\in A_t}\varepsilon_{i,\tau_i+t}\right), \tag{C.1}$$

where $\tau_i + t$ is a calendar date. We make the following assumptions about the residuals:

$$\begin{aligned}
\mathrm{var}(\varepsilon_{i,\tau_i+t}) &= \sigma_i^2, && \text{(C.2)}\\
\mathrm{cov}(\varepsilon_{i,\tau_i+t},\varepsilon_{j,\tau_i+t}) &= \rho_{ij}\sigma_i\sigma_j, && \text{(C.3)}\\
\mathrm{cov}(\varepsilon_{i,\tau_i+t},\varepsilon_{i,\tau_i+u}) &= 0 \quad \text{for } \tau_i+t\neq\tau_i+u, && \text{(C.4)}\\
\mathrm{cov}(\varepsilon_{i,\tau_i+t},\varepsilon_{j,\tau_i+u}) &= 0 \quad \text{for } \tau_j+u\neq\tau_i+t. && \text{(C.5)}
\end{aligned}$$

Comparing (C.2) to (C.4) and (C.3) to (C.5), one can see that the covariance differs depending upon whether the time indices of the ε's are the same. Let $I(v,w)$ be the indicator function taking the value 1 if $w = v$, and 0 otherwise. The variance of the sum (C.1) is

$$\begin{aligned}
&\mathrm{var}\left[\sum_{t=1}^{L}\frac{1}{\mathrm{card}(A_t)}\sum_{i\in A_t}\varepsilon_{i,\tau_i+t}\right]\\
&= \sum_{t=1}^{L}\sum_{u=1}^{L}\frac{1}{\mathrm{card}(A_t)\times\mathrm{card}(A_u)}\sum_{i\in A_t}\sum_{j\in A_u}\mathrm{cov}(\varepsilon_{i,\tau_i+t},\varepsilon_{j,\tau_j+u}) && \text{(C.6)}\\
&= \sum_{t=1}^{L}\sum_{1=0}^{L}\frac{1}{\mathrm{card}(A_t)\times\mathrm{card}(A_u)}\sum_{i\in A_t}\sum_{j\in A_u}\rho_{ij}\sigma_i\sigma_j I(\tau_i+t,\tau_j+u), && \text{(C.7)}
\end{aligned}$$

where if $i = j$ (the same issue) then $\rho_{ij} = 1$. To compute the right-hand side of (C.7) we estimated σ_i, σ_j, and ρ_{ij} from the returns over the first 150 post-event days following the ith and jth issues. This involves computing σ_j from the 150 post-event days following the jth issue, and computing ρ_{ij} from the overlapping days. If there is no overlap in the ith and jth post-event windows, then there is no need to calculate ρ_{ij}. Once we have computed the right-hand side of equation (C.7), the standard error is simply the square root of the variance and the t-statistic is straightforward.

The next case involves testing the significance of the CARRs in equation (A.7), which are based on the SPARQS pricing model benchmark described by equation (A.2). Thus, it is necessary to compute the variance of the sum on the left-hand side of (A.7), which is the variance of the sum

$$\sum_{t=1}^{L}\left(\frac{1}{\mathrm{card}(A_t)}\sum_{i\in A_t}\eta_{i,\tau_i+t}\right). \tag{C.8}$$

Computation of this sum's variance is similar to the procedure for the sum in equation (C.1), where we make assumptions about the η's similar to the assumptions in (C.2) through (C.5) about the ε's. One difference is that we allow for the (negative) serial correlation in the errors at different dates. The source of η_{i,τ_i+t} is the pricing error of a derivative relative to its underlying stock, which seems likely to be (and in fact is) negatively serially correlated. We assume that the pricing errors for SPARQS i follow an AR(1) process with autocorrelation coefficient ζ_i, or specifically that

$$cov(\eta_{i,\tau_i+t}, \eta_{j,\tau_j+u}) = \begin{array}{ll} \gamma_{ij}\theta_i\theta_j & \text{for } \tau_j + u = \tau_i + t, \\ \gamma_{ij}\theta_i\theta_j\zeta_j^{(\tau_j+u)-(\tau_i+t)} & \text{for } \tau_j + u > \tau_i + t, \\ \gamma_{ij}\theta_i\theta_j\zeta_j^{(\tau_i+t)-(\tau_j+u)} & \text{for } \tau_j + u < \tau_i + t, \end{array} \tag{C.9}$$

where $\gamma_{ij} = 1$ for $i = j$. Thus, the variance of the sum (C.8) is

$$\begin{aligned} &\text{var}\left[\sum_{t=1}^{L} \frac{1}{\text{card}(A_t)} \sum_{i \in A_t} \eta_{i,\tau_i+t}\right] \\ &= \sum_{t=1}^{L}\sum_{u=1}^{L} \frac{1}{\text{card}(A_t) \times \text{card}(A_u)} \sum_{i \in A_t}\sum_{j \in A_u} \text{cov}(\eta_{i,\tau_i+t}, \eta_{j,\tau_j+u}) &\text{(C.10)} \\ &= \sum_{t=1}^{L}\sum_{u=1}^{L} \frac{1}{\text{card}(A_t) \times \text{card}(A_u)} \sum_{i \in A_t}\sum_{j \in A_u} \gamma_{ij}\theta_i\theta_j J(i, j, \tau_i + t, \tau_j + u), &\text{(C.11)} \end{aligned}$$

where the function J is the autocorrelation function defined by

$$J(i, j, v, w) = \begin{array}{ll} 1 & \text{for } \tau_j + u = \tau_i + t, \\ \zeta_j^{(\tau_j+u)-(\tau_i+t)} & \text{for } \tau_j + u > \tau_i + t, \\ \zeta_i^{(\tau_i+t)-(\tau_j+u)} & \text{for } \tau_j + u < \tau_i + t. \end{array} \tag{C.12}$$

The final CARRs test statistic for which we derive standard errors are the CARRs for the SPARQSs' market model benchmark as described by equation (A.9). This case is more complicated because the residual is $\Omega_{it}\epsilon_{i,\tau_i+t} + \eta_{i,\tau_i+t}$ and thus has two components. If the residual ε_{i,τ_i+t} is homoskedastic then $\Omega_{i,\tau_i+t}\varepsilon_{i,\tau_i+t}$ and $\Omega_{it}\varepsilon_{i,\tau_i+t} + \eta_{i,\tau_i+t}$ are not, implying that simply to mimic the method for the left-hand side of (A.7) is not correct. For this purpose, it is necessary to consider a procedure that recognizes $\Omega_{it}\varepsilon_{i,\tau_i+t}$ and η_{i,τ_i+t} are distinct errors with different properties, but correlated. To do this we need to specify the correlation between the ε's and the η's. Assume:

$$\text{cov}(\varepsilon_{i,\tau_i+t}, \eta_{i,\tau_i+t}) = c_{ii}, \tag{C.13}$$

$$\text{cov}(\varepsilon_{i,\tau_i+t}, \eta_{j,\tau_i+t}) = c_{ij}, \tag{C.14}$$

$$\text{cov}(\varepsilon_{i,\tau_i+t}, \eta_{i,\tau_i+u}) = 0, \tag{C.15}$$

$$\text{cov}(\varepsilon_{i,\tau_i+t}, \eta_{j,\tau_j+u}) = 0, \tag{C.16}$$

Then, the variance of the sum on the left-hand side of (A.9) is:

$$
\begin{aligned}
&\operatorname{var}\left[\sum_{t=1}^{L}\frac{1}{\operatorname{card}(A_t)}\sum_{i\in A_t}(\Omega_{it}\varepsilon_{i,\tau_i+t}+\eta_{i,\tau_i+t})\right] \\
&= \sum_{t=1}^{L}\sum_{u=1}^{L}\Bigg[\frac{1}{\operatorname{card}(A_t)\times\operatorname{card}(A_u)} \qquad \text{(C.17)}\\
&\quad\times\sum_{i\in A_t}\sum_{j\in A_u}\Omega_{it}\Omega_{ju}\operatorname{cov}(\varepsilon_{i,\tau_i+t},\varepsilon_{j,\tau_j+u})+\Omega_{it}\operatorname{cov}(\varepsilon_{i,\tau_i+t},\eta_{j,\tau_j+u})+\operatorname{cov}(\eta_{i,\tau_i+t},\eta_{j,\tau_j+u})\Bigg] \\
&= \sum_{t=1}^{L}\sum_{u=1}^{L}\Bigg(\frac{1}{\operatorname{card}(A_t)\times\operatorname{card}(A_u)} \qquad \text{(C.18)}\\
&\quad\times\sum_{i\in A_t}\sum_{j\in A_u}\left[\left[\Omega_{it}\Omega_{ju}\rho_{ij}\sigma_i\sigma_j+\Omega_{it}c_{ij}\right]I(\tau_i+t,\tau_j+u)+\gamma_{ij}\theta_i\theta_j J(i,j,\tau_i+t,\tau_j+u)\right]\Bigg).
\end{aligned}
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References

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