Internet Appendix for "Stocks for the long run? Evidence from a broad sample of developed markets"

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Abstract

This Internet Appendix provides material that is supplemental to the paper "Stocks for the long run? Evidence from a broad sample of developed markets." Section 1 describes the provenance and attributes of the GFDatabase, provides details on data construction, and presents results from tests to validate our approach to constructing country-level returns. Section 2 examines the robustness of our block bootstrap design parameters. Section 3 presents additional empirical results. Section 4 describes the cumulative wealth calculations used to generate the asset allocation results in the paper and presents additional asset allocation results.

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1. Data appendix

Sections 1.1 to 1.4 provide background material on data construction. Section 1.5 discusses empirical tests used to validate our approach to estimating total returns from price indexes and dividend yields for cases in which a total return index is unavailable. Section 1.6 compares our country-level returns data from the GFDatabase with the data used in Jordà et al. (2019).

1.1. GFDatabase from Global Financial Data

The main data source for our study is the GFDatabase from Global Financial Data (GFD). The database provides macroeconomic and financial data with broad coverage of countries and time periods. As shown in Fig. 2 of the paper, our data construction procedure achieves substantial coverage of the periods in which countries in our sample are classified as developed.

The GFDatabase reports stock market data including total return indexes, price indexes, and dividend yields. The computation of these index values by Global Financial Data depends on the country and the time period. For most countries, and especially for more recent time periods, the GFDatabase provides data for indexes that are created and calculated by stock exchanges (e.g., the Tokyo Stock Price Index from the Tokyo Stock Exchange) or well-known index providers (e.g., the S&P 500 Index). In earlier periods for several countries, the GFDatabase includes proprietary total return indexes, price indexes, and dividend yields that are based on data transcribed by Global Financial Data from original historical documents such as newspapers, periodicals, and books. In several cases, GFD has gathered historical data for individual stocks in a country and formed indexes that span early historical periods.

An important issue for ensuring that reported returns reflect stock market performance is the proper handling of capital changes. In private correspondence with Dr. Bryan Taylor and Mike Cerneant from Global Financial Data, we were assured that "GFD handles capital changes in the total return indices that we calculate and that we obtain from other sources. This includes cash dividends, stock dividends and splits, subscription rights, mergers, and exchange offers. We are well aware of how these particular changes impact individual companies and we incorporate these changes into the calculation of returns for individual companies and thus for the index in general."

Return data from the GFDatabase have been used extensively in the finance and economics literature, as the database provides notable coverage of historical periods for a broad set of countries. Recent studies using country-level index data and appearing in premier finance and economics journals include Barro (2006), Berkman et al. (2011), Schularick and Taylor (2012), Colacito and Croce (2013), Nakamura et al. (2013), Rapach et al. (2013), Albuquerque et al. (2015), and Nakamura et al. (2017).

1.2. Data construction

We compute country-level stock market returns inclusive of dividend distributions to ensure that we measure total returns rather than just price returns (i.e., capital gains). For cases in which the GFDatabase contains multiple indexes for a given country, we identify the index with the broadest coverage of firms and with data availability over the longest sample period.

We use total return indexes from GFD for months in which these indexes are available. In these cases, the total nominal return in a given month follows from Eq. (1) in the paper.

If a total return index is not available, we compute total returns from price indexes and dividend yields. Dividend yields in the GFDatabase are recorded either monthly or annually depending on country and period. To estimate country-level dividends, we rely on dividend yields that are reported in December. All annual dividend yield time series from GFD are December observations. For dividend yields available on a monthly basis, we use only the December yields so that cumulative monthly returns accurately reflect the annual dividend yields. The dividend yield data are based on the dividends for the trailing 12 months (i.e., December yields reflect dividends from January to December of the corresponding year). To compute dividends for a given country-year, we multiply the December dividend yield by the price index at the end of December. We then estimate monthly dividends for each of the prior 12 months by dividing the annual dividend amount by 12. This approach relies on the assumption that dividends are issued in 12 equal monthly payments over a given year. The total nominal return in a given month follows from Eq. (3) in the paper.

We use monthly changes in country-level CPIs from GFD to estimate monthly inflation rates. Most of the observations on CPI levels are reported monthly. For cases with missing data (e.g., if the data are reported quarterly or annually over a given period), we estimate the monthly CPI levels through interpolation by assuming a constant monthly inflation rate between reported CPI levels. We compute the gross real return for a given month as the ratio of the gross nominal return and the gross inflation rate following Eq. (5) in the paper.

We use monthly data on local currency exchange rates vis-à-vis the U.S. dollar from GFD to estimate monthly local currency appreciation relative to the dollar. From March 1938 to December 1945, we use German exchange rate data for Austria. The Austrian Schilling was abolished following annexation by Germany, and German Reichsmarks became the local currency during World War II. The GFDatabase reports a constant Austrian Schilling exchange rate vis-à-vis the U.S. dollar over the March 1938 to December 1945 period, so we replace these data to better reflect the currency exchanges that would have been necessary to invest in Austria during this time. We compute the gross real USD return for a given month using the nominal return in local currency, local currency appreciation relative to the U.S. dollar, and U.S. inflation following Eq. (6) in the paper.

Table IA1 reports the total return indexes, the price indexes, and the dividend yields that we use to construct our sample of returns for developed countries. We supplement the GFD data with information from other sources in two cases. First, Luxembourg is missing return data from 2016 to 2019 in the GFDatabase, so we fill in the missing observations using data from the Luxembourg Stock Exchange's official website.[1]

[1] See https://www.bourse.lu/home.

Second, we use annual dividend yields from Jordà et al. (2019) to fill in missing data for Norway from 1914 to 1969 and Portugal from 1934 to 1988. Table IA1 also shows the sample period start and end dates for each country. The data series labels are from GFD, and the subperiods covered by the total return indexes and the subperiods covered by the price indexes and dividend yields combine to form the full sample period for each country (with the exception of Czechoslovakia, which has a multi-month terminal return that extends beyond the GFD sample period as discussed in Section 2.3 of the paper).

1.3. Data adjustments

The following subsections outline additional adjustments required to compute nominal returns and real returns for our developed country sample.

1.3.1. Missing data

For periods in which dividend yield data are missing in the GFDatabase, we estimate yields by examining the reported dividend yields from before and after the data gaps. We apply this correction for missing data in three cases. First, the GFDatabase does not have a dividend yield for Austria from June 1939 to June 1969. The dividend yield for May 1939 is 3.7%, and the yield for July 1969 is 3.5%. We therefore use the average dividend yield of 3.6% to fill in the missing dividend yield data from June 1939 to June 1969. For comparison, the average dividend yield for Austria is 4.3% in the 1930s before the gap and 3.4% in the 1970s after the gap. Second, Chile is missing dividend yield data from 1967 to 1970, which we fill in with a 7.0% yield based on the dividend yield observation in December 1966. Third, Czechoslovakia is missing dividend yield data from April 1938 to March 1943. The dividend yield in Czechoslovakia fluctuates between 1.4% and 2.6% in the three years before the break in the data, so we assume a 2.0% dividend yield for the missing observations.

In a few cases, we use a smoothing procedure to fill gaps in return series. Argentina is missing return data for April, May, November, and December in 1955. We estimate returns for April 1955 and May 1955 using price index data for March 1955 and June 1955 under the assumption of a constant return for April, May, and June of 1955. We make an analogous calculation to fill in the missing data for November 1955 and December 1955. We have either semiannual or annual return data for France from 1915 to 1918 and either quarterly or semiannual data for Switzerland from 1914 to 1920. We treat the intermediate months as missing and smooth returns across the months in each quarterly, semiannual, or annual period.

The price index in Norway is missing for May 1940 and June 1940. We use price index values of 5.779 for May 1940 and 5.930 for June 1940 from Klovland (2004). Finally, as discussed in Section 2.3 of the paper, we estimate a −90% nominal return for the 39-month period from April 1943 to June 1946 in Czechoslovakia.

1.3.2. Hyperinflation in Germany

We use a total return index for Germany from 1917 to 1923 that is denominated in gold marks rather than paper marks. Gold marks were backed by gold, whereas the value of German paper marks effectively went to zero over this period. The CPI data from GFD correspond to the paper marks series. With gold as a base for the index, however, there is presumably negligible inflation. We use the total return series in gold marks for both nominal and real returns. One of the multi-month returns for Germany in Table 2 of the paper spans a 42-month period from August 1914 to January 1918. As such, this multi-month period partially overlaps with the period in which we rely on the gold marks series. We compute inflation for this multi-month return observation using CPI levels from July 1914 and December 1916 and assume a zero inflation rate from January 1917 to January 1918 (i.e., the portion of the multi-month period in which we use returns denominated in gold marks). We also assume zero appreciation of gold marks relative to the U.S. dollar from 1917 to 1923 because both currencies were backed by gold during this period.

The reported data for Germany's total return index indicate a −62.2% return in January 1923 followed by a 333.4% return in February 1923 for a two-month return of 63.7%. We examine alternative sources of information and find that Bittlingmayer (1998) does not show these extreme returns. We treat this sequence of returns as a data error and smooth the two-month return across January 1923 and February 1923.

1.4. Multi-month return periods

Several historical events disrupted stock exchange operations during our sample period. To best reflect the experience of stock market participants during these episodes, we compute multi-month return observations that span the periods of disruption. This approach assumes that an investor could not liquidate her holdings in a given stock market during these times because of market closure or severe restrictions on trading. Table 2 in the paper shows the multi-month return periods in our sample and notes the underlying events.

The events surrounding World War I and World War II impacted stock exchange operations in a number of the countries in our sample. Most of the corresponding multi-month returns listed in Panels A and B of Table 2 cover full periods in which the stock exchange was officially closed. We note four exceptions below. First, although the Swedish exchange was closed from August 1914 to September 1914, the price index values from GFD for July 1914 and October 1914 are identical. Thus, we calculate a four-month return covering August 1914 to November 1914. Second, price controls in Germany started in January 1943 and extended until July 1948, leading us to construct a 67-month return covering this period. Third, Japanese stock market trading stopped in August 1945, and although over-the-counter trading started in May 1946, the stock exchange did not reopen until May 1949. We therefore compute a 45-month return covering September 1945 to May 1949. Fourth, Czechoslovakia's stock exchange was intermittently open during the German occupation. Even when the market was open, it was subject to price controls and limited trading. For this reason, we calculate three multi-month returns associated with the German occupation period in Czechoslovakia: a 16-month return from October 1938 to January 1940, a four-month return from January 1942 to April 1942, and a 39-month return from April 1943 to June 1946.

The other events in our sample that disrupted stock market operations include financial crises, labor strikes, and political revolutions. Both Austria and Germany experienced financial crises in 1931. The Austrian stock exchange closed, and we compute a two-month return from October 1931 to November 1931.

Germany's stock exchange closed in July 1931, reopened for just over two weeks in September 1931, and subsequently closed again until April 1932. We compute a two-month return from August 1931 to September 1931 and a seven-month return covering October 1931 to April 1932. Greece's stock market was closed for five weeks starting from June 29, 2015 because of the country's financial crisis, so we compute a two-month return from July 2015 to August 2015. The exchange in France was closed because of labor strikes in April 1974 and March 1979. In Portugal, trading stopped on April 25, 1974 with the start of a military coup, and stock trading resumed on March 7, 1977. We therefore compute a 35-month return for Portugal that covers May 1974 to March 1977.

1.5. Internal validation of total returns data

If total return indexes are missing in the GFDatabase, we estimate total returns using price indexes and dividend yields. To validate this approach, we compute total returns based on total return indexes and compare them with total returns estimated from price indexes and dividend yields. We make these comparisons over the periods for which total return indexes, price indexes, and dividend yields are all available. Table IA2 lists the total return indexes, price indexes, and dividend yields that we use for our validation tests as well as the testing period for each country.[2]

Table IA3 reports our test results. The table shows correlations of returns calculated using total return indexes and returns computed from price indexes and dividend yields for each country and for the pooled sample. The correlation for the pooled sample is 0.98, and Israel's correlation of 0.86 represents the minimum value across individual countries. In addition, the table compares average monthly return and standard deviation across the two approaches for computing returns. None of the differences in mean return is statistically significant at the 10% level. For the comparisons of standard deviations, only the differences in return volatilities for the Czech Republic and Israel are statistically significant at the 10% level. The difference in mean return for the pooled sample is only one basis point per month, and the difference in standard deviation of returns is only four basis points. For both comparisons in the pooled sample (i.e., means and volatilities), we fail to reject the null hypothesis of equal performance.

1.6. External validation of total returns data

We perform an additional analysis that compares the return data in our sample with another data source with broad coverage of countries and periods. Specifically, we examine returns from the overlapping periods in our sample and the sample of Jordà et al. (2019).[3] Jordà et al. (2019) use a variety of sources for historical return data across countries. Many of these data sources are different from those used by Global Financial Data in the construction of their database. For example, post-war data for Japan in the GFDatabase are

[2] Because there are no comparable indexes for Germany from 1917 to 1923, we exclude the German returns over this period from the validation tests.

[3] The data from Jordà et al. (2019) are available at http://www.macrohistory.net/data/. We thank the authors for making these data available.

available for the Nikkei 225 Index and the Tokyo Stock Price Index, whereas Jordà et al. (2019) use annual data from the Statistical Yearbooks published by the Statistics Bureau of Japan. As such, the comparison in this section provides external validation of the historical return data in both databases.

The return data from Jordà et al. (2019) are annual, so we annualize the returns in our sample to facilitate comparison. We compound returns within each year, and we calculate real returns by compounding returns and CPI separately and then adjusting the annual returns for inflation. If a year ends during a multi-month period (see Section 1.4), we form a multi-year observation in the annual return dataset. During these periods, we also compound the annual returns from Jordà et al. (2019) to calculate a comparable multi-year observation. We use annualized CPI inflation to compute real returns based on the nominal returns from Jordà et al. (2019).

Our sample overlaps with the sample from Jordà et al. (2019) for 16 countries. Jordà et al.'s (2019) data span 1870 to 2015, so the sample start date for each country is the later of 1870 and the start of our developed sample period. As previously noted, we use a return index in gold marks in Germany for the 1917 to 1923 period. Jordà et al. (2019) report returns in paper marks, so we omit the 1917 to 1923 period in Germany from this analysis.

Table IA4 shows summary statistics for the two historical return samples for each country and for the pooled sample. The table reports the number of years and the number of observations (i.e., including multi-year observations) of overlap across the samples. For each sample, we report the arithmetic and geometric means, the standard deviation, and the minimum and maximum return. We also compute the correlation between the GFDatabase returns and the Jordà et al. (2019) returns. Panel A reports statistics for nominal returns, and Panel B provides statistics for real returns.

The summary statistics in Table IA4 show that the return data from the two databases have very similar characteristics. Return correlation exceeds 0.90 for nearly all countries and for the pooled sample. The pooled sample means and standard deviations closely match across the databases, and the two databases agree cross-sectionally about which countries have had relatively high or low average returns and standard deviations during the shared sample periods. The two databases also report similar extreme returns. In particular, the minimum returns reported for each country are very close in magnitude, which is important given our focus on characterizing the left tails of the cumulative return distributions. Based on this analysis, we conclude that the quality of return data from both the GFDatabase and the Jordà et al. (2019) database is supported by the external validation exercise.

2. Bootstrap appendix

Section 2.1 discusses the choice of the mean bootstrap block length parameter for the base case. Section 2.2 evaluates the robustness of our main results to changes in the block length parameter.

2.1. Bootstrap design

The purpose of the block resampling scheme is to reflect the time-series properties of weakly dependent country-level return observations. Serial dependencies in returns that are included in the same return block will be reflected by the block bootstrap draws. Based on prior literature, we anticipate that the data are characterized by multiple types of serial dependencies that could be important to long-term investors. In Section 3 of the paper, we specifically note the potential effects of short-term positive autocorrelation, persistence in volatility, and long-term mean reversion on buy-and-hold returns. Additional forms of dependence may exist in the data, and we do not need to take a stance on these patterns as long as they are often included in the same return blocks in the bootstrap.

The optimal block length parameter is not ex ante obvious. The return blocks should be long enough to allow for longer-term effects like mean reversion to be reflected in the cumulative return distribution but short enough to allow the bootstrap to generate simulated return sequences that are not directly observed in the historical data. We ultimately select 120 months as our base case block length parameter. As described further below, this block length allows for the effects of mean reversion and produces return distributions that are similar to those from other specifications with relatively long block lengths. We also show the robustness of our findings to alternative block lengths in Section 2.2.

To initially examine the effect of the block length parameter, we consider the moments of the distribution of cumulative log returns. Anticipating that the standard deviation of cumulative log returns could decrease with longer block lengths as long blocks of return data can reflect more mean reversion, we examine whether and how the distribution moments stabilize across longer block lengths. Fig. IA1 plots the standard deviations of cumulative log return distributions. The six panels correspond to the six return horizons considered in the paper, and we study standard deviation as a function of the bootstrap block length parameter. The solid blue line corresponds to the developed country sample, and the dashed black line corresponds to the U.S. sample. We consider block length parameters varying from one month (which is equivalent to i.i.d. resampling) to 240 months.

Fig. IA1 reveals that the results exhibit some sensitivity to the block length parameter. The standard deviation of each log return distribution initially increases as the block length parameter increases from one to around ten to 20 months. The increase in risk likely reflects positive short-term autocorrelation and persistence in volatility. Especially with the longer horizons of 20 or 30 years, the standard deviation decreases as the block length parameter further increases. This pattern likely reflects the role of mean reversion, which is a longer-term effect that can be picked up by larger return blocks. The U.S. sample displays a greater reduction in risk as the block length increases compared with the developed sample. For example, at the 30-year horizon for the developed sample, the standard deviation with a block size parameter of 240 months is 1.12, which is comparable in magnitude with the one-month block value of 1.21. In the U.S. sample, the 240-month block length parameter produces a standard deviation of only 0.71 relative to the one-month block standard deviation of 0.94. In an unreported analysis, we find that variance ratios calculated following Poterba and Summers (1988) tend to be low in the U.S. relative to many of the other developed countries, which is consistent with the longer block lengths having a larger effect in the U.S. sample owing to stronger mean reversion in this sample. For our base case of developed countries, the standard deviation at each horizon stabilizes with longer block lengths. Similar patterns exist for skewness and kurtosis of the cumulative log return distributions. As such, we choose a base case block length parameter of 120 months, which allows the bootstrap to reflect mean reversion while still generating sufficient resampling.

2.2. Impact of mean block length

To construct bootstrap distributions of payoffs and continuously compounded returns, we resample with replacement from the sample of returns in developed markets using a stationary block bootstrap approach. Our base case design draws blocks of consecutive returns, where the length of each block has a geometric distribution with a mean of 120 months. A block resampling procedure is appropriate for stationary weakly dependent time series (Politis and Romano, 1994b). There are several reasons to believe that the country-level time series of real returns in our sample are weakly dependent. Notably, the empirical evidence of time-varying volatility in stock market returns is overwhelming (Bollerslev et al., 1994), and several studies document evidence of mean reversion in long-horizon returns (e.g., Poterba and Summers, 1988; Siegel, 2014). The mean block length for the base case design of 120 months accounts for these features of the data, but we also consider the sensitivity of the results to other choices for the block length.

Table IA5 summarizes the distribution of real payoffs at various investment horizons for alternative choices of mean block length in the bootstrap procedure. The panels of the table are organized by horizon, and each panel considers i.i.d. resampling and mean block lengths of 12, 120 (base case), and 240 months. The i.i.d. design in the top row of each panel corresponds to the approach used in Fama and French (2018a,b). The block length parameter does not have a major impact on the results, particularly at short investment horizons. There are some noticeable differences at longer horizons. For the 30-year distributions in Panel F, the mean and standard deviation of real payoff are highest for the bootstrap design with a 12-month mean block length. This design is also associated with the most extreme tail outcomes and the largest loss probability. These features of the bootstrap distribution likely reflect that a 12-month block length incorporates effects from time-varying volatility and short-term autocorrelation but not from long-term mean reversion. In Panel F, the payoff moments, payoff percentiles, and loss probability for our 120-month block base case design are similar to the corresponding values for the i.i.d. resampling design. The decrease in payoff volatility for the 120-month block length relative to the 12-month block length is consistent with longer blocks reflecting mean reversion in the data.

Fig. IA2 shows loss probability as a function of mean block length for each investment horizon. For horizons of five years and beyond, the loss probability tends to spike around a block length of 12 months. The general conclusion, however, is that the block length parameter has a minor impact on the results. The 30-year loss probability for the developed sample ranges from 11.1% to 15.3% across block lengths compared with only 0.6% to 3.8% for the U.S. sample.

3. Additional results appendix

This appendix presents supplementary empirical results. Section 3.1 considers the bootstrap distribution of nominal payoffs. Section 3.2 characterizes the bootstrap distribution of continuously compounded returns at various investment horizons. Section 3.3 presents results for the alternative samples considered in Section 4.2 of the paper. Section 3.4 reproduces the analyses in Section 4.2 of the paper using real USD returns rather than real local currency returns.

3.1. Nominal payoffs

As discussed in Section 2.4 of the paper, our primary analysis focuses on real returns and payoffs because periods of hyperinflation can lead to nominal performance that poorly reflects the true economic experience of investors. For completeness, Table IA6 reports statistics for the bootstrap distributions of nominal payoffs.

3.2. Continuously compounded returns

This section evaluates properties of the bootstrap distributions of continuously compounded returns for the full sample of developed countries and for the U.S. sample. This analysis is motivated by Fama and French's (2018a) finding that the distribution of continuously compounded returns from the U.S. is close to normal for horizons beyond ten years, such that the wealth distribution is approximately lognormal. We are specifically interested in whether or not this result extends to our developed country sample.

Our base case bootstrap simulation procedure described in Section 3 of the paper uses a stationary block bootstrap approach to draw continuously compounded returns according to Eq. (8) in the paper. If the bootstrap procedure were to draw individual single-month observations with replacement, then the continuously compounded returns would be sums of independent and identically distributed (i.i.d.) log returns. In this setting, the central limit theorem states that the distribution of continuously compounded returns converges toward the normal distribution as the return horizon increases.

Our stationary bootstrap approach differs from monthly i.i.d. resampling in two dimensions. First, we resample return observations in blocks of random length rather than individual observations. Second, our bootstrap design incorporates the multi-month observations in Table 2 of the paper. These features of our procedure take us outside the scope of the classical central limit theorem. Politis and Romano (1994a), however, show that a stationary resampling scheme applied to weakly dependent random variables leads to distributions that converge to normal distributions asymptotically. Thus, it seems reasonable to expect our bootstrap distributions of continuously compounded returns from developed markets to converge to normal with horizon.

Table IA7 allows us to assess the rate of this convergence. Panel A of this table reports moments of continuously compounded real returns based on the full sample across the 1,000,000 bootstrap simulations at each horizon. At the one-month horizon, returns are highly non-normal, as indicated by the skewness and kurtosis statistics of -8.14 and 315.99, respectively. Skewness declines in magnitude as the investment horizon increases, but each distribution exhibits considerable negative skewness. For example, skewness is −1.90 at one year, −0.74 at ten years, and −0.44 at 30 years. Similarly, kurtosis decreases with horizon, but even at 30 years the kurtosis statistic of 4.02 well exceeds the value of 3.00 for normally distributed log returns.

Fig. IA3 plots the kernel smoothed density of simulated continuously compounded returns at each horizon. Each panel of the figure also shows the normal density with mean and variance equal to those of the simulated returns. The evidence in Panel A of Table IA7 and Fig. IA3 suggests that the return distribution converges to normal as the horizon increases, but the convergence remains far from complete. The distribution of continuously compounded real returns displays negative skewness and excess kurtosis at the horizons relevant to the majority of investors. Further, the pronounced left tail reflects downside risk that is particularly harmful for long-term investors.

For comparison, we also examine the rate at which the distribution of continuously compounded returns from the U.S. bootstrap simulations converges to normal. In Panel B of Table IA7, one-month log returns are negatively skewed and exhibit excess kurtosis. The magnitude of skewness for the U.S. sample declines toward zero as the holding period increases. The 30-year continuously compounded returns remain negatively skewed, but the skewness estimate of −0.02 is small in magnitude. The kurtosis of 30-year returns for the U.S. sample is 3.04. This estimate is smaller than the kurtosis value of 4.02 for the full developed sample and is also close to the value of 3.00 for normally distributed log returns.

Fig. IA4 presents kernel smoothed densities for the simulated log returns based on the U.S. sample. At 20-year and 30-year horizons, the distributions of investment outcomes appear mostly indistinguishable from their corresponding normal distributions. These results are consistent with those from Fama and French (2018a), who use a post-1963 U.S. sample to demonstrate that bootstrap distributions of continuously compounded nominal returns converge to normal at long horizons.

3.3. Alternative samples

In Section 4.2 of the paper, we compare the bootstrap distribution of real payoffs based on the full sample of developed countries with the corresponding bootstrap distributions from alternative samples. This section provides supplementary information and results for the analysis in the paper.

Table IA8 reports the sample start date for each developed country in the base sample and in the alternative samples with screens for population size and the ratio of aggregate market capitalization to GDP.

Tables 6 and 7 in the paper summarize the distributions of real payoffs at a 30-year investment horizon for alternative underlying samples. For completeness, Table IA9 presents results for the alternative samples corresponding to the six return horizons considered in the paper.

3.4. Real USD payoffs

Fig. 1 in the paper shows histograms of 30-year real payoffs based on the pooled sample of developed countries and on the U.S. sample. These payoffs are from the perspective of a domestic investor in a representative country and are measured in local currency. Fig. IA5 summarizes the distributions of 30-year real USD payoffs from the perspective of a global USD investor.

In Section 4.2 of the paper, we characterize the distributions of real payoffs at a 30-year investment horizon for alternative underlying samples. The empirical results are presented in Tables 6 and 7. Tables IA10 and IA11 report corresponding results based on real USD payoffs.

4. Portfolio choice appendix

In Section 5 of the paper, we examine asset allocation with a two-asset design. The investors use the bootstrap distribution of stock market payoffs to form expectations about stock market performance and allocate their portfolios to a single risky asset (i.e., a country-level equity index) and a risk-free asset. Section 5 of the paper focuses on investors making single lump sum contributions, and this appendix extends these results to investors making contributions as monthly annuities. Section 4.1 provides details on the calculations of cumulative wealth for investors who follow the lump sum and annuity contribution patterns. Section 4.2 presents the empirical results for the annuity investors.

4.1. Cumulative wealth calculations

We begin with the lump sum investors. In the cases with an inflation-protected risk-free asset, an investment in the risk-free asset maintains its value in each period. Given that the portfolio is rebalanced each month to maintain a weight in stocks of w, the cumulative wealth in the portfolio at an H-month horizon in bootstrap draw m is given by

$$W_H^{(m)}(w) = \prod_{s=1}^{H^*} (wR_s^{(m)} + (1-w)), \tag{IA1}$$

where H^* is the smallest number such that $\sum_{s=1}^{H^*} N_s^{(m)} \geq H$. To calculate cumulative wealth in the cases with cash, we need to account for the effects of inflation on the real value of cash. In the bootstrap method described in Section 3 of the paper, we store gross realized inflation $\Pi_{i,t}$ alongside each gross real return observation $R_{i,t}$ while forming draws. Each iteration m of the bootstrap thus produces a bootstrap return vector draw $R^{(m)} = \{R_1^{(m)}, R_2^{(m)}, \ldots, R_{360}^{(m)}\}$ and a bootstrap inflation vector draw $\Pi^{(m)} = \{\Pi_1^{(m)}, \Pi_2^{(m)}, \ldots, \Pi_{360}^{(m)}\}$. If $R_s^{(m)}$ is one of the multi-month return observations listed in Table 2 of the paper, then $\Pi_s^{(m)}$ is a multi-month cumulative inflation observation. Cumulative wealth with the cash alternative is

$$W_H^{(m)}(w) = \prod_{s=1}^{H^*} \left(wR_s^{(m)} + \frac{1-w}{\Pi_s^{(m)}} \right) \tag{IA2}$$

for all draws in which $\Pi_{H^*}^{(m)}$ (i.e., the last realized inflation observation to be included in the bootstrap draw) is a single-month inflation observation. If $\Pi_{H^*}^{(m)}$ is a multi-month observation and $\sum_{s=1}^{H^*} N_s^{(m)} > H$, then Eq. (IA2) would cause the cash balance to continue to be impacted by inflation even after the horizon H has been reached. We require the investor to remain invested in stocks during these periods to mirror an investment in a closed stock market, but we allow the investor to withdraw the cash balance upon reaching the horizon H without incurring any additional inflation. We therefore recalculate the last inflation observation $\Pi_{H^*}^{(m)}$ in these cases such that the new quantity only reflects inflation from the beginning of the multi-month period through the horizon date H. We then calculate cumulative wealth according to Eq. (IA2).

We proceed to the annuity investors. These investors contribute \$1/H at the beginning of each month of the H-month holding period, and they rebalance the portfolio each period to maintain a weight in stocks of w. Cumulative wealth with access to an inflation-protected risk-free asset is recursively calculated for $s = 1, 2, \ldots, H^*$,

$$W_s^{(m)}(w) = \left(W_{s-1}^{(m)} + \frac{1}{H}\right)\left(wR_s^{(m)} + (1-w)\right) + \frac{N_s^{(m)} - 1}{H}, \tag{IA3}$$

where $W_0^{(m)} = 0$ and $W_H^{(m)} = W_{H^*}^{(m)}$. The first term reflects that wealth from the end of last period and this month's annuity contribution are invested with weights w in the stock market and $1 - w$ in the inflation-protected risk-free asset. If $N_s^{(m)} = 1$ such that $R_s^{(m)}$ is a single-month observation, the second term in Eq. (IA3) is equal to zero and the first term fully reflects the evolution of wealth over the month. When $N_s^{(m)} > 1$, the stock market is closed in the intermediate months of the multi-month period. The investor continues to save \$1/H per month in this case, but this entire new investment is placed in the inflation-protected risk-free asset because the stock market is closed. A total of $N_s^{(m)} - 1$ additional contributions take place after the multi-month period has started, which accounts for the second term in Eq. (IA3). After the multi-month period ends, the wealth balance is rebalanced to invest in the stock market with weight w and the inflation-protected risk-free asset with weight $1 - w$. An exception occurs if $\sum_{s=1}^{H^*} N_s^{(m)} > H$ because $N_s^{(m)} - 1$ additional contributions would cause the investor to contribute more than \$1.00 in aggregate during the holding period. In these cases, we replace $N_s^{(m)} - 1$ with $H - \sum_{s=1}^{H^*-1} N_s^{(m)}$ in Eq. (IA3) such that the investor stops new contributions upon reaching the horizon H. For annuity investors with a cash alternative, the wealth calculation is similar but accounts for the effect of inflation on the real value of cash. The wealth calculation is recursive for $s = 1, 2, \ldots, H^*$,

$$W_s^{(m)}(w) = \left(W_{s-1}^{(m)} + \frac{1}{H}\right)\left(wR_s^{(m)} + \frac{1-w}{\Pi_s^{(m)}}\right) + K_s^{(m)}, \tag{IA4}$$

where $K_s^{(m)}$ is an additional cash term that we calculate if $N_s^{(m)} > 1$ (and $K_s^{(m)} = 0$ if $N_s^{(m)} = 1$). The calculation of $K_s^{(m)}$ takes into account that contributions are made each month and uses the monthly inflation observations within the multi-month period to calculate the cumulative real value of any cash that is contributed during the multi-month period. Exceptions occur if $\sum_{s=1}^{H^*} N_s^{(m)} > H$, and we modify both

$\Pi_s^{(m)}$ and $K_s^{(m)}$ in these cases to ensure that contributions stop at the horizon H and that the cash balance is not subject to additional inflation after the horizon H.

In Section 5 of the paper and in Section 4.2 below, we report the maximum annualized fee that the developed country investor would be willing to pay to maintain her optimal weight in stocks rather than adopt the optimal U.S. investor weight. These fees are applied to the entire wealth balance in each period, such that each period in the wealth calculations from Eqs. (IA1) to (IA4) is further multiplied by $(1 - f/12)^{N_s^{(m)}}$. To find the value of f, we first calculate utility using the optimal U.S. weight and no fee for the investor who evaluates expected utility using the developed country sample. We then calculate utility for the developed country investor under the optimal developed country weight and a range of values of f. We report the value of f that equalizes utility across these two cases.

4.2. Annuity investors

The asset allocation results in Table 8 of the paper correspond to investors who contribute \$1.00 as a lump sum at the beginning of the holding period. We find that investors using the developed country sample to form expectations about stock market outcomes invest less in stocks relative to investors relying on the U.S. sample. We also find that the differences in optimal weights are economically significant. Table 8 specifically shows that the maximum annualized fees on wealth that developed country investors would be willing to pay to adopt their own optimal policies rather than those of the U.S. investors are economically large.

The results in Table IA12 indicate that our findings in Table 8 of the paper are robust to an annuity investment pattern rather than a lump sum approach. An annuity investor contributes a total of \$1.00 throughout the holding period as a monthly annuity. That is, an investor with an H-month horizon contributes $\$1/H$ each month. As with the lump sum case, the annuity investor rebalances monthly to ensure that the weight in stocks remains at w. Outcomes from annuity investments in the stock market are generally less volatile compared with lump sum investments as much of the invested capital has less time to grow and investors can more easily recover from crashes that occur early in the holding period. The optimal weights in stocks are higher for each investor type under the annuity plan compared with the lump sum plan, but developed country investors remain less aggressive relative to U.S. investors and the associated fees remain high.

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Table IA1
Data series

The table summarizes the data series used to compute returns for each developed country in the sample. The table reports the sample period start and end dates and lists the total return index, price index, and dividend yield series used to construct returns (along with the corresponding start and end dates). For a given country, we use the total return index over the periods for which it is available, and we otherwise rely on price index and dividend yield data to estimate returns. The data symbols correspond to those in the GFDatabase from Global Financial Data. The total return index for Luxembourg for the period from 2016:12 to 2019:12 is from the Luxembourg Stock Exchange (https://www.bourse.lu/home). The dividend yields for Norway from 1914:02 to 1969:12 and Portugal from 1934:01 to 1988:01 are from Jordà et al. (2019).

			Total return index			Price index and dividend yield			
Country	Sample start	Sample end	Total return index	Start	End	Price index	Dividend yield	Start	End
Argentina	1947:02	1966:12				_IBGD	SYARGYM	1947:02	1966:12
Australia	1901:01	2019:12	_AORDAD	1901:01	2019:12				
Austria	1925:02	2019:12	_ATXTRD	1970:01	2019:12	_WBKID	SYAUTYM	1925:02	1969:12
Belgium	1897:01	2019:12	_BCSHD	1897:01	2019:12				
Canada	1891:01	2019:12	_TRGSPTSE	1891:01	2019:12				
Chile period I	1927:01	1970:12				_IGPAD	SYCHLYM	1927:01	1970:12
Chile period II	2010:01	2019:12	_IPSAD	2010:01	2019:05	_IGPAD	SYCHLYM	2019:06	2019:12
Czechoslovakia	1926:01	1946:06				CZINDXM	SYCZEYM	1926:01	1937:11
						CZINDEXM	SYCZEYM	1937:12	1943:03
Czech Republic	1995:01	2019:12	_PXTRD	1995:01	2019:12				
Denmark	1890:01	2019:12	_OMXCGID	1890:01	2019:12				
Estonia	2010:01	2019:12	_OMXTGID	2010:01	2019:12				
Finland	1969:01	2019:12	_OMXHGID	1969:01	2019:12				
France	1866:01	2019:12	TRSBF250D	1866:01	2019:12				
Germany	1882:01	2019:12	_CDAXD	1882:01	2019:12				
Greece	1977:01	2019:12	_RETMD	1977:01	2019:12				
Hungary	1996:01	2019:12	_BUXD	1996:01	2019:12				
Iceland	2002:01	2019:12	_OMXIGID	2002:07	2019:12	_OMXIPID	SYISLYM	2002:01	2002:06
Ireland	1936:01	2019:12	_IVRTD	1936:01	2019:12				
Israel	2010:01	2019:12	TRISRSTM	2010:01	2019:11	ILTLVGD	SYISRYM	2019:12	2019:12
Italy	1931:01	2019:12	_BCIPRD	1931:01	2019:12				
Japan	1930:01	2019:12	_TOPXDVD	1930:01	2019:12				
Latvia	2016:01	2019:12	_OMXRGID	2016:01	2019:12				
Lithuania	2018:01	2019:12	_OMXVGID	2018:01	2019:12				
Luxembourg	1982:01	2019:12	_LUXXRD	1985:01	2016:11	_LUXXD	SYLUXYM	1982:01	1984:12
			See table caption	2016:12	2019:12				
Mexico	1994:01	2019:12	_IRTD	1994:01	2019:12				

(*continued on next page*)

Table IA1 (*continued*)

			Total return index			Price index and dividend yield			
Country	Sample start	Sample end	Total return index	Start	End	Price index	Dividend yield	Start	End
Netherlands	1914:01	2019:12	_AAXGRD	1914:01	2019:12				
New Zealand	1896:01	2019:12	_NZGID	1896:01	2019:12				
Norway	1914:02	2019:12	_OSEAXD	1970:01	2019:12	_OBXPD	See table caption	1914:02	1969:12
Poland	1996:01	2019:12	_WIGD	1996:01	2019:12				
Portugal	1934:01	2019:12	_BVLGD	1988:02	2019:12	_IBTAD	See table caption	1934:01	1988:01
Singapore	1970:01	2019:12	_TFTFSTD	1970:01	2019:12				
Slovakia	2000:01	2019:12	_SAXD	2000:01	2019:12				
Slovenia	2010:01	2019:12				_SBITOPD	SYSVNYM	2010:01	2019:12
South Korea	1996:01	2019:12	TRKORSTM	1996:01	2019:12				
Spain	1959:01	2019:12	_BCNPR30	1959:01	2019:12				
Sweden	1910:01	2019:12	_OMXSBGI	1910:01	2019:12				
Switzerland	1914:01	2019:12	_SSHID	1914:01	2019:12				
Turkey	1986:02	2019:12	TRRBILED	1986:02	2019:12				
United Kingdom	1841:01	2019:12	_TFTASD	1841:01	2019:12				
United States	1890:01	2019:12	_SPXTRD	1890:01	2019:12				

Table IA2
Internal validation test data series

The table reports the total return index, price index, and dividend yield data used to test the validity of our return construction approach. For each country, we compare returns estimated from two approaches: (i) using the total return index and (ii) using the price index and dividend yield. The final testing sample includes all periods between the sample start date and the sample end date for which the total return index, the price index, and the dividend yield are available. The data symbols correspond to those in the GFDatabase from Global Financial Data.

Country	Testing period start date	Testing period end date	Total return index	Price index	Dividend yield
Australia	1901:01	2019:12	_AORDAD	_AORDD	SYAUSYM
Austria	1970:01	2019:12	_ATXTRD	_WBKID	SYAUTYM
Belgium	1897:01	2019:12	_BCSHD	_BSPTD	SYBELYM
Canada	1891:01	2019:12	_TRGSPTSE	_GSPTSED	SYCANYTM
Chile	2010:01	2019:05	_IPSAD	CLIPSAM	SYCHLYM
Czech Republic	1995:01	2019:12	_PXTRD	_CTXUSDD	SYCZEYM
Denmark	1921:01	2001:06	_OMXCGID	_CSEID	SYDNKYM
	2001:07	2019:12	_OMXCGID	_OMXCPID	SYDNKYM
Finland	1969:01	2019:12	_OMXHGID	_OMXHPID	SYFINYM
France	1866:01	2019:12	TRSBF250D	_CACTD	SYFRAYM
Germany	1882:01	2019:12	_CDAXD	_CXKXD	SYDEUYM
Greece	1977:01	2019:12	_RETMD	_ATGD	SYGRCYM
Iceland	2002:07	2006:12	_OMXIGID	_OMXIPID	SYISLYM
Ireland	1936:01	2019:12	_IVRTD	_ISEQD	SYIRLYM
Israel	2010:01	2019:11	TRISRSTM	ILTLVGD	SYISRYM
Italy	1931:01	2019:12	_BCIPRD	_BCIID	SYITAYM
Japan	1930:01	2019:12	_TOPXDVD	_TOPXD	SYJPNYM
Luxembourg	1985:01	1994:12	_LUXXRD	_LUXXD	SYLUXYM
Mexico	1994:01	2019:12	_IRTD	_MXXD	SYMEXYM
Netherlands	1914:01	2019:12	_AAXGRD	_AAXD	SYNLDYAM
New Zealand	1896:01	2019:12	_NZGID	_NZCID	SYNZLYM
Poland	1996:01	2019:12	_WIGD	_WIG20D	SYPOLYM
Portugal	1988:02	2006:05	_BVLGD	_IBTAD	SYPRTYM
Singapore	1972:01	2019:12	_TFTFSTD	_FTSTID	SYSGPYM
South Korea	1996:01	2019:12	TRKORSTM	_KS11D	SYKORYM
Spain	1959:01	2019:12	_BCNPR30	_SMSID	SYESPYM
Sweden	1910:01	2019:12	_OMXSBGI	_OMXSPID	SYSWEYM
Switzerland	1918:01	2019:12	_SSHID	_SPIXD	SYCHEYM
Turkey	1986:02	2019:12	TRRBILED	_XU100D	SYTURYM
United Kingdom	1933:07	2019:12	_TFTASD	_FTASD	_DFTASD
United States	1890:01	2019:12	_SPXTRD	_SPXD	SYUSAYM

Table IA3
Internal validation test results

The table reports results of tests to assess the validity our approach of computing returns from price index and dividend yield data. For each country, the table shows the number of monthly observations for which the total return index, the price index, and the dividend yield are available and the correlation between nominal returns computed using the total return index (R) and nominal returns computed using the price index and the dividend yield (R^*). The table also compares the arithmetic mean and the standard deviation of the total return index series with the corresponding statistics for the return series based on price index and dividend yield. The p-value for the comparison of means (volatilities) corresponds to a t-test (F-test) for difference in average return (variance).

			Comparison of means			Comparison of volatilities		
Country	Observ	Corr	$\bar{R}_a$ (%)	$\bar{R}^*_a$ (%)	p-value	$\sigma(R)$ (%)	$\sigma(R^*)$ (%)	p-value
Australia	1,424	0.997	0.98	0.99	0.932	3.88	3.86	0.861
Austria	600	0.971	0.74	0.76	0.948	5.54	5.22	0.144
Belgium	1,401	0.974	0.75	0.76	0.989	5.12	5.07	0.711
Canada	1,541	0.995	0.80	0.81	0.961	4.24	4.22	0.902
Chile	113	0.994	0.37	0.68	0.558	4.08	3.80	0.464
Czech Republic	300	0.942	1.09	0.87	0.732	7.47	8.39	0.045
Denmark	816	0.931	0.90	0.87	0.887	4.35	4.26	0.542
Finland	612	0.983	1.34	1.32	0.948	6.31	6.21	0.706
France	1,771	0.976	0.85	0.86	0.940	4.76	4.63	0.235
Germany	1,450	0.945	0.83	0.83	0.966	4.59	4.71	0.311
Greece	514	0.977	1.43	1.31	0.837	9.92	9.24	0.109
Iceland	54	0.943	3.21	2.97	0.818	5.69	5.18	0.501
Ireland	1,008	0.999	0.98	0.98	0.999	4.67	4.68	0.940
Israel	119	0.863	0.02	0.49	0.387	4.76	3.56	0.002
Italy	1,056	0.997	1.24	1.25	0.963	7.55	7.55	0.985
Japan	1,032	0.999	1.05	1.06	0.965	5.81	5.81	0.981
Luxembourg	120	0.993	1.41	1.68	0.640	4.35	4.68	0.435
Mexico	300	0.995	1.28	1.31	0.948	6.32	6.54	0.553
Netherlands	1,242	0.997	0.82	0.81	0.975	5.05	5.06	0.951
New Zealand	1,483	0.985	0.86	0.84	0.843	3.63	3.65	0.825
Poland	288	0.974	0.94	0.83	0.856	6.82	7.41	0.160
Portugal	220	0.932	0.63	0.70	0.886	5.59	5.53	0.870
Singapore	576	0.944	0.93	0.99	0.890	7.52	7.13	0.209
South Korea	288	0.978	0.96	0.76	0.772	8.27	7.96	0.517
Spain	732	0.955	0.99	1.07	0.776	5.44	5.39	0.807
Sweden	1,320	0.992	0.89	0.91	0.890	4.76	4.71	0.708
Switzerland	1,224	0.993	0.68	0.68	0.994	4.38	4.33	0.696
Turkey	407	0.984	4.26	4.24	0.982	15.75	15.72	0.979
United Kingdom	1,038	0.998	0.99	1.00	0.972	4.87	4.86	0.948
United States	1,560	0.999	0.87	0.87	0.977	4.95	4.95	0.979
Full sample	24,609	0.980	0.97	0.98	0.956	5.65	5.61	0.232

Table IA4
External validation test results

The table reports summary statistics for annual returns for each developed country with a return sample that overlaps with the sample from Jordà et al. (2019). For each country, the table shows the number of sample years and the number of sample observations (i.e., including multi-year observations). The table also shows the following summary statistics for our sample and for Jordà et al.'s (2019) sample: the arithmetic average return ($\bar{R}_a$), the geometric average return ($\bar{R}_g$), the standard deviation of return (SD), the minimum (Min) and the maximum (Max) return, and the correlation between the return samples (Corr). Statistics for the pooled sample of all observations are also reported. Panel A (Panel B) shows results for nominal returns (real returns).

	Sample size		Summary statistics for returns										
			Global Financial Data					Jordà et al. (2019)					
Country	Years	Observ	$\bar{R}_a$ (%)	$\bar{R}_g$ (%)	SD (%)	Min (%)	Max (%)	$\bar{R}_a$ (%)	$\bar{R}_g$ (%)	SD (%)	Min (%)	Max (%)	Corr
Panel A: Nominal returns													
Australia	115	114	12.70	11.30	17.60	−40.38	66.80	11.93	10.57	17.21	−40.38	63.70	0.99
Belgium	118	113	10.03	7.75	23.41	−47.56	117.31	10.67	8.07	25.04	−56.06	125.80	0.93
Denmark	126	126	9.46	7.63	21.14	−47.96	105.86	11.10	9.62	18.65	−57.96	81.64	0.72
Finland	47	47	20.51	15.21	37.64	−51.31	167.04	19.47	14.17	37.46	−51.31	167.04	0.99
France	146	146	12.30	9.76	27.16	−40.89	211.62	8.38	6.26	23.13	−40.89	125.38	0.90
Germany	124	118	10.83	7.20	27.94	−86.20	137.27	11.24	7.16	29.83	−89.43	137.76	0.97
Italy	85	85	15.94	11.71	33.32	−46.47	132.22	15.88	11.32	35.35	−46.84	160.20	0.97
Japan	86	82	16.61	11.99	40.31	−40.62	257.57	13.74	10.61	32.31	−27.82	226.33	0.81
Netherlands	100	98	10.75	8.44	22.47	−49.46	73.19	10.90	8.39	24.60	−49.84	130.07	0.82
Norway	100	100	11.88	8.49	30.02	−52.59	179.47	10.17	7.74	23.27	−54.06	92.80	0.88
Portugal	81	78	15.54	8.02	44.47	−89.21	225.14	11.82	6.43	34.28	−86.70	136.88	0.85
Spain	57	57	14.14	11.42	25.32	−39.38	91.51	14.15	11.55	25.36	−36.95	113.26	0.91
Sweden	106	106	11.99	9.68	22.55	−38.60	71.23	12.11	9.80	22.55	−39.27	69.76	1.00
Switzerland	99	99	9.17	7.13	21.05	−34.05	72.82	9.21	7.40	19.91	−34.05	61.36	0.97
United Kingdom	145	144	9.32	7.82	19.55	−51.75	152.12	10.14	8.64	19.52	−50.20	149.61	0.96
United States	126	126	11.00	9.19	19.32	−43.86	52.89	10.91	9.26	18.47	−40.30	52.64	0.99
Full sample	1,661	1,637	12.02	9.17	26.81	−89.21	257.57	11.44	8.84	24.95	−89.43	226.33	0.90

(*continued on next page*)

Table IA4 (*continued*)

	Sample size		Summary statistics for returns										
			Global Financial Data					Jordà et al. (2019)					
Country	Years	Observ	$\bar{R}_a$ (%)	$\bar{R}_g$ (%)	SD (%)	Min (%)	Max (%)	$\bar{R}_a$ (%)	$\bar{R}_g$ (%)	SD (%)	Min (%)	Max (%)	Corr
Panel B: Real returns													
Australia	115	114	8.58	7.10	17.46	−42.50	53.53	7.82	6.39	17.10	−42.50	50.67	0.99
Belgium	118	113	5.20	2.68	22.80	−63.88	81.80	5.63	2.98	23.50	−60.28	88.90	0.94
Denmark	126	126	5.76	3.91	20.47	−49.17	92.27	7.20	5.83	17.04	−49.17	71.30	0.73
Finland	47	47	15.35	10.00	37.17	−52.92	161.38	14.37	9.02	37.06	−52.92	161.38	0.99
France	146	146	6.31	3.97	23.10	−42.54	126.59	3.09	0.65	22.76	−50.55	111.39	0.91
Germany	124	118	9.21	5.33	28.08	−90.61	157.17	9.58	5.29	29.63	−92.80	156.69	0.97
Italy	85	85	5.73	1.89	27.86	−70.76	89.81	5.52	1.54	28.12	−72.78	96.89	0.97
Japan	86	82	8.94	3.57	31.34	−93.15	143.11	6.39	2.28	23.84	−93.75	81.82	0.71
Netherlands	100	98	7.36	5.08	21.78	−50.43	66.07	7.41	5.03	23.02	−50.43	99.72	0.82
Norway	100	100	7.72	4.42	28.65	−53.58	166.92	6.10	3.70	22.55	−55.02	80.07	0.89
Portugal	81	78	8.52	1.09	38.31	−95.18	168.71	5.41	−0.40	31.03	−94.07	102.75	0.85
Spain	57	57	7.38	4.49	25.12	−40.24	76.91	7.41	4.60	25.19	−43.64	97.00	0.92
Sweden	106	106	8.09	5.72	22.15	−40.01	68.99	8.20	5.83	22.17	−40.00	67.53	1.00
Switzerland	99	99	7.08	4.93	21.36	−36.39	69.24	7.11	5.20	20.14	−37.83	56.30	0.98
United Kingdom	145	144	6.24	4.73	18.07	−59.54	101.69	7.02	5.52	17.82	−58.23	99.69	0.95
United States	126	126	8.08	6.22	19.51	−38.09	53.43	7.99	6.29	18.64	−38.81	50.72	0.99
Full sample	1,661	1,637	7.51	4.60	24.65	−95.18	168.71	6.99	4.28	23.22	−94.07	161.38	0.91

Table IA5
Bootstrap distributions of payoffs for alternative block sampling lengths

The table summarizes the distribution of real payoffs from a $1.00 buy-and-hold investment across 1,000,000 bootstrap simulations at various return horizons for alternative mean block sampling lengths. The real payoffs are from the perspective of a domestic investor in a representative country. Each panel of the table corresponds to a specific return horizon. Each panel presents results for i.i.d. sampling and sampling with mean block lengths of 12 months, 120 months (base case), and 240 months. The underlying sample is the pooled sample of all developed countries. The real payoff for bootstrap iteration m at the H-month horizon is $W_H^{(m)}$. For each horizon and block sampling length, the table reports the mean, standard deviation, and distribution percentiles of real payoffs. The last column in the table shows the proportion of payoff draws that are less than one [$\mathbb{P}(W_H^{(m)} < 1)$]. The bootstrap sampling procedure is based on the stationary bootstrap approach of Politis and Romano (1994b) as described in the text. For the designs other than i.i.d. sampling, we sample blocks of random length, where the length of each block has a geometric distribution with the indicated mean block length.

	Moments		Percentiles									
Block length	Mean	SD	1%	5%	10%	25%	50%	75%	90%	95%	99%	$\mathbb{P}(W_H^{(m)} < 1)$
Panel A: 1 month												
1 month (i.i.d.)	1.01	0.06	0.85	0.92	0.95	0.98	1.01	1.03	1.06	1.09	1.16	0.432
12 months	1.01	0.06	0.85	0.92	0.95	0.98	1.01	1.03	1.06	1.09	1.16	0.432
120 months	1.01	0.06	0.85	0.92	0.95	0.98	1.01	1.03	1.06	1.09	1.16	0.432
240 months	1.01	0.06	0.85	0.92	0.95	0.98	1.01	1.03	1.06	1.09	1.16	0.432
Panel B: 1 year												
1 month (i.i.d.)	1.07	0.22	0.62	0.76	0.83	0.93	1.05	1.19	1.33	1.43	1.70	0.385
12 months	1.08	0.27	0.54	0.73	0.81	0.93	1.06	1.19	1.34	1.48	1.87	0.372
120 months	1.08	0.28	0.52	0.72	0.80	0.93	1.06	1.19	1.35	1.49	1.90	0.368
240 months	1.08	0.28	0.52	0.72	0.80	0.93	1.06	1.19	1.35	1.49	1.90	0.368
Panel C: 5 years												
1 month (i.i.d.)	1.40	0.66	0.33	0.59	0.72	0.95	1.28	1.70	2.21	2.60	3.61	0.288
12 months	1.46	0.94	0.20	0.52	0.66	0.93	1.29	1.77	2.39	2.92	4.54	0.300
120 months	1.45	0.93	0.19	0.54	0.69	0.95	1.29	1.72	2.34	2.86	4.41	0.283
240 months	1.45	0.94	0.19	0.54	0.70	0.96	1.29	1.71	2.33	2.86	4.41	0.280
Panel D: 10 years												
1 month (i.i.d.)	1.95	1.38	0.16	0.52	0.70	1.05	1.61	2.45	3.56	4.47	6.96	0.226
12 months	2.13	2.11	0.12	0.42	0.61	1.00	1.64	2.62	4.04	5.34	9.47	0.249
120 months	2.02	1.76	0.13	0.47	0.69	1.08	1.64	2.44	3.60	4.63	8.75	0.215
240 months	1.99	1.65	0.14	0.48	0.71	1.10	1.65	2.41	3.50	4.44	8.62	0.209
Panel E: 20 years												
1 month (i.i.d.)	3.79	4.27	0.14	0.47	0.74	1.38	2.56	4.68	8.00	11.04	20.36	0.159
12 months	4.54	7.44	0.10	0.35	0.61	1.27	2.62	5.22	9.77	14.36	30.94	0.189
120 months	3.88	5.40	0.14	0.43	0.73	1.43	2.63	4.57	7.76	10.89	22.97	0.155
240 months	3.70	4.59	0.16	0.45	0.77	1.47	2.64	4.39	7.25	10.05	20.97	0.147
Panel F: 30 years												
1 month (i.i.d.)	7.35	11.32	0.13	0.48	0.85	1.86	4.05	8.53	16.51	24.52	51.78	0.123
12 months	9.69	27.53	0.09	0.35	0.67	1.68	4.16	9.88	21.43	34.34	85.44	0.152
120 months	7.38	13.76	0.14	0.47	0.85	1.94	4.16	8.28	15.58	23.30	53.45	0.121
240 months	6.73	11.24	0.17	0.53	0.91	2.04	4.20	7.86	13.95	20.29	43.79	0.111

Table IA6
Bootstrap distributions of nominal payoffs

The table summarizes the distribution of nominal payoffs from a $1.00 buy-and-hold investment across 1,000,000 bootstrap simulations at various return horizons. The underlying sample is the pooled sample of all developed countries. The nominal payoffs are from the perspective of a domestic investor in a representative country. The nominal payoff for bootstrap iteration m at the H-month horizon is $W_H^{(m)}$. For each horizon, the table reports the mean, standard deviation, and distribution percentiles of nominal payoffs. The last column in the table shows the proportion of payoff draws that are less than one [$\mathbb{P}(W_H^{(m)} < 1)$]. The bootstrap sampling procedure is based on the stationary bootstrap approach of Politis and Romano (1994b) as described in the text. We sample blocks of random length, where the length of each block has a geometric distribution with a mean of 120 months.

	Moments		Percentiles									
Horizon	Mean	SD	1%	5%	10%	25%	50%	75%	90%	95%	99%	$\mathbb{P}(W_H^{(m)} < 1)$
1 month	1.01	0.07	0.86	0.93	0.95	0.98	1.01	1.03	1.06	1.09	1.17	0.396
1 year	1.13	0.35	0.58	0.77	0.85	0.98	1.09	1.23	1.41	1.58	2.18	0.291
5 years	1.98	2.87	0.40	0.71	0.89	1.17	1.54	2.10	3.05	4.01	8.32	0.147
10 years	5.00	31.78	0.33	0.82	1.09	1.60	2.37	3.73	6.22	9.40	33.36	0.080
20 years	77.92	3,001.12	0.45	1.21	1.81	3.17	5.74	11.18	23.64	42.95	269.57	0.036
30 years	1,027.13	340,838.32	0.72	2.00	3.23	6.58	14.16	32.76	83.23	175.81	1,637.38	0.017

Table IA7
Bootstrap distributions of continuously compounded returns

The table summarizes the distribution of continuously compounded returns across 1,000,000 bootstrap simulations at various return horizons. The underlying sample in Panel A (Panel B) is the pooled sample of all developed countries (the United States over the period from 1890 to 2019). The real returns are from the perspective of a domestic investor in a representative country. For each horizon and sample, the table reports the mean, standard deviation, skewness, and kurtosis of continuously compounded real returns. The bootstrap sampling procedure is based on the stationary bootstrap approach of Politis and Romano (1994b) as described in the text. We sample blocks of random length, where the length of each block has a geometric distribution with a mean of 120 months.

Horizon	Mean	SD	Skew	Kurt
Panel A: Full sample				
1 month	0.0038	0.0640	−8.141	315.986
1 year	0.0453	0.2583	−1.899	23.464
5 years	0.2293	0.5573	−0.911	8.021
10 years	0.4522	0.7396	−0.743	5.875
20 years	0.8982	0.9946	−0.525	4.497
30 years	1.3448	1.1912	−0.445	4.022
Panel B: United States sample				
1 month	0.0052	0.0496	−0.380	10.996
1 year	0.0615	0.1905	−0.584	5.115
5 years	0.3105	0.3931	−0.114	3.041
10 years	0.6207	0.5197	−0.069	2.842
20 years	1.2407	0.6902	−0.035	2.887
30 years	1.8619	0.8079	−0.025	3.043

Table IA8
Alternative sample periods

The table shows the sample start dates for developed countries in the alternative samples with screens based on population or equity market size. The alternative samples are described in Table 5 of the paper.

Country	Base sample start date	POP 0.2% start date	POP 0.5% start date	M/GDP 0.5 start date	M/GDP 1.0 start date
United Kingdom	1841:01	1841:01	1841:01	1846:01	1880:01
Netherlands	1914:01	1914:01	—	1914:01	1914:01
Belgium	1897:01	1897:01	—	1897:01	1908:01
France	1866:01	1866:01	1866:01	1881:01	1999:01
Norway	1914:02	—	—	2005:01	—
Germany	1882:01	1882:01	1882:01	1999:01	—
Denmark	1890:01	—	—	1997:01	2015:01
Switzerland	1914:01	1914:01	—	1927:01	1928:01
United States	1890:01	1890:01	1890:01	1927:01	1998:01
Canada	1891:01	1891:01	1943:01	1901:01	1928:01
Argentina	1947:02	1947:02	1947:02	—	—
New Zealand	1896:01	—	—	1899:01	1931:01
Australia	1901:01	1901:01	—	1931:01	1950:01
Sweden	1910:01	1910:01	—	1910:01	1997:01
Austria	1925:02	1925:02	—	2006:01	—
Chile period I	1927:01	1927:01	—	1927:01	1928:01
Greece	1977:01	1977:01	—	1998:01	1999:01
Czechoslovakia	1926:01	1926:01	1926:01	—	—
Japan	1930:01	1930:01	1930:01	1937:01	1988:01
Portugal	1934:01	1934:01	—	1998:01	—
Italy	1931:01	1931:01	1931:01	1999:01	—
Ireland	1936:01	—	—	1944:01	1964:01
Singapore	1970:01	—	—	1970:01	1970:01
Luxembourg	1982:01	—	—	1984:01	1988:01
Turkey	1986:02	1986:02	1986:02	1999:01	—
Spain	1959:01	1959:01	1959:01	1997:01	2007:01
Finland	1969:01	—	—	1997:01	1998:01
Iceland	2002:01	—	—	2002:01	2004:01
Mexico	1994:01	1994:01	1994:01	—	—
Czech Republic	1995:01	—	—	—	—
Hungary	1996:01	—	—	—	—
Poland	1996:01	1996:01	1996:01	—	—
South Korea	1996:01	1996:01	1996:01	1999:01	2007:01
Slovakia	2000:01	—	—	—	—
Chile period II	2010:01	2010:01	—	2010:01	2010:01
Estonia	2010:01	—	—	—	—
Israel	2010:01	—	—	2010:01	—
Slovenia	2010:01	—	—	—	—
Latvia	2016:01	—	—	—	—
Lithuania	2018:01	—	—	—	—

Table IA9
Bootstrap distributions of payoffs for alternative samples

The table summarizes the distribution of real payoffs from a $1.00 buy-and-hold investment across 1,000,000 bootstrap simulations at various return horizons for alternative underlying samples. The real payoffs are from the perspective of a domestic investor in a representative country. Each panel of the table corresponds to a specific sample, and the samples are summarized in Table 5 of the paper. The real payoff for bootstrap iteration m at the H-month horizon is $W_H^{(m)}$. For each horizon and sample, the table reports the mean, standard deviation, and distribution percentiles of real payoffs. The last column in the table shows the proportion of payoff draws that are less than one [$\mathbb{P}(W_H^{(m)} < 1)$]. The bootstrap sampling procedure is based on the stationary bootstrap approach of Politis and Romano (1994b) as described in the text. We sample blocks of random length, where the length of each block has a geometric distribution with a mean of 120 months.

	Moments		Percentiles									
Horizon	Mean	SD	1%	5%	10%	25%	50%	75%	90%	95%	99%	$\mathbb{P}(W_H^{(m)} < 1)$
Panel A: U.S. sample												
1 month	1.01	0.05	0.87	0.93	0.95	0.98	1.01	1.03	1.06	1.08	1.12	0.407
1 year	1.08	0.20	0.62	0.76	0.84	0.96	1.08	1.20	1.32	1.40	1.56	0.320
5 years	1.47	0.59	0.54	0.71	0.82	1.04	1.40	1.77	2.24	2.57	3.32	0.217
10 years	2.12	1.15	0.59	0.77	0.92	1.31	1.86	2.68	3.68	4.26	5.85	0.127
20 years	4.38	3.32	0.70	1.11	1.41	2.16	3.47	5.55	8.54	10.65	16.37	0.036
30 years	8.91	8.49	0.96	1.69	2.29	3.76	6.46	11.03	18.08	24.30	41.90	0.012
Panel B: U.K. sample												
1 month	1.00	0.04	0.89	0.94	0.97	0.99	1.00	1.02	1.04	1.06	1.09	0.398
1 year	1.06	0.15	0.71	0.83	0.88	0.99	1.05	1.13	1.23	1.32	1.51	0.293
5 years	1.34	0.43	0.50	0.71	0.86	1.05	1.29	1.55	1.90	2.10	2.71	0.197
10 years	1.76	0.79	0.52	0.76	0.95	1.24	1.61	2.08	2.85	3.29	4.34	0.129
20 years	3.06	1.98	0.56	0.95	1.23	1.81	2.60	3.72	5.41	6.94	10.38	0.058
30 years	5.28	4.25	0.69	1.24	1.68	2.68	4.21	6.48	9.95	12.97	21.70	0.030
Panel C: Survival sample												
1 month	1.01	0.06	0.85	0.92	0.95	0.98	1.01	1.03	1.06	1.08	1.16	0.430
1 year	1.08	0.28	0.53	0.72	0.81	0.94	1.06	1.19	1.35	1.48	1.90	0.365
5 years	1.47	0.96	0.22	0.54	0.69	0.96	1.30	1.73	2.36	2.90	4.52	0.278
10 years	2.05	1.81	0.16	0.48	0.70	1.09	1.66	2.48	3.66	4.72	8.94	0.211
20 years	3.98	5.37	0.16	0.46	0.76	1.47	2.70	4.68	7.99	11.23	23.62	0.147
30 years	7.68	14.05	0.17	0.53	0.92	2.05	4.33	8.59	16.24	24.41	55.89	0.111
Panel D: Continuous sample												
1 month	1.01	0.06	0.85	0.92	0.95	0.98	1.01	1.03	1.06	1.09	1.16	0.427
1 year	1.09	0.27	0.55	0.73	0.80	0.93	1.07	1.21	1.37	1.50	1.90	0.362
5 years	1.51	0.87	0.36	0.59	0.73	0.98	1.34	1.81	2.49	3.04	4.46	0.262
10 years	2.21	1.79	0.30	0.62	0.80	1.18	1.78	2.67	3.94	5.08	9.16	0.173
20 years	4.66	5.77	0.33	0.73	1.05	1.82	3.16	5.47	9.35	13.16	26.67	0.091
30 years	9.78	16.71	0.40	0.97	1.49	2.86	5.60	10.87	20.63	30.93	69.01	0.052
Panel E: Post-1880 sample												
1 month	1.01	0.06	0.85	0.92	0.95	0.98	1.01	1.03	1.06	1.09	1.16	0.434
1 year	1.08	0.28	0.52	0.72	0.80	0.93	1.06	1.19	1.35	1.49	1.91	0.373
5 years	1.46	0.94	0.19	0.53	0.69	0.95	1.29	1.73	2.35	2.88	4.47	0.286
10 years	2.02	1.78	0.13	0.46	0.68	1.07	1.64	2.45	3.62	4.67	8.79	0.219
20 years	3.89	5.55	0.13	0.42	0.72	1.41	2.62	4.58	7.81	10.96	22.94	0.158
30 years	7.41	15.45	0.14	0.46	0.83	1.91	4.13	8.30	15.71	23.57	54.41	0.124

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Table IA9 (*continued*)

	Moments		Percentiles									
Horizon	Mean	SD	1%	5%	10%	25%	50%	75%	90%	95%	99%	$\mathbb{P}(W_H^{(m)} < 1)$
Panel F: Post-1920 sample												
1 month	1.01	0.06	0.85	0.92	0.95	0.98	1.01	1.03	1.06	1.09	1.17	0.434
1 year	1.09	0.29	0.52	0.71	0.80	0.93	1.07	1.21	1.37	1.52	1.95	0.376
5 years	1.50	0.99	0.18	0.54	0.69	0.95	1.31	1.79	2.45	3.01	4.70	0.286
10 years	2.13	1.95	0.13	0.48	0.71	1.10	1.70	2.59	3.84	4.99	9.47	0.208
20 years	4.27	6.10	0.14	0.45	0.78	1.52	2.81	5.03	8.67	12.21	25.65	0.141
30 years	8.50	20.04	0.15	0.52	0.95	2.16	4.65	9.43	18.08	27.29	63.15	0.106
Panel G: Post-1960 sample												
1 month	1.01	0.06	0.84	0.92	0.94	0.98	1.01	1.04	1.07	1.09	1.17	0.440
1 year	1.09	0.30	0.52	0.70	0.79	0.92	1.07	1.21	1.38	1.53	1.99	0.382
5 years	1.50	1.03	0.28	0.56	0.69	0.94	1.29	1.77	2.49	3.08	4.98	0.289
10 years	2.14	2.02	0.21	0.54	0.73	1.08	1.68	2.58	3.84	4.97	9.75	0.211
20 years	4.34	6.48	0.20	0.56	0.86	1.54	2.79	5.01	8.79	12.50	26.96	0.129
30 years	8.73	17.73	0.23	0.66	1.09	2.25	4.66	9.49	18.46	28.05	66.19	0.089
Panel H: Post-2000 sample												
1 month	1.00	0.06	0.85	0.91	0.94	0.98	1.01	1.04	1.06	1.08	1.14	0.430
1 year	1.06	0.23	0.48	0.67	0.76	0.92	1.07	1.20	1.33	1.43	1.66	0.373
5 years	1.33	0.77	0.23	0.48	0.62	0.87	1.20	1.61	2.10	2.56	4.27	0.347
10 years	1.67	1.33	0.18	0.45	0.62	0.93	1.38	2.01	2.91	3.74	7.07	0.288
20 years	2.69	3.23	0.15	0.40	0.62	1.09	1.86	3.17	5.30	7.50	15.11	0.220
30 years	4.33	7.09	0.14	0.39	0.65	1.29	2.53	4.88	9.09	13.52	30.11	0.180
Panel I: POP 0.2% sample												
1 month	1.01	0.06	0.85	0.92	0.95	0.98	1.01	1.03	1.06	1.08	1.16	0.435
1 year	1.08	0.29	0.52	0.71	0.80	0.93	1.06	1.19	1.34	1.48	1.90	0.369
5 years	1.44	0.96	0.17	0.52	0.67	0.94	1.28	1.70	2.31	2.83	4.29	0.294
10 years	1.98	1.83	0.11	0.42	0.65	1.04	1.62	2.38	3.52	4.54	8.86	0.231
20 years	3.69	5.40	0.11	0.36	0.64	1.32	2.50	4.35	7.37	10.40	22.41	0.177
30 years	6.83	14.13	0.11	0.39	0.72	1.73	3.82	7.65	14.40	21.63	50.53	0.143
Panel J: POP 0.5% sample												
1 month	1.01	0.07	0.84	0.92	0.94	0.98	1.01	1.03	1.06	1.09	1.17	0.430
1 year	1.08	0.31	0.50	0.71	0.80	0.94	1.06	1.19	1.35	1.48	1.93	0.359
5 years	1.44	0.90	0.10	0.52	0.68	0.97	1.29	1.70	2.28	2.83	4.16	0.271
10 years	1.98	1.69	0.09	0.35	0.65	1.08	1.65	2.39	3.52	4.51	8.17	0.220
20 years	3.66	4.68	0.10	0.29	0.60	1.37	2.58	4.37	7.40	10.32	20.81	0.174
30 years	6.74	12.03	0.09	0.33	0.66	1.77	3.95	7.74	14.35	21.28	47.51	0.147
Panel K: M/GDP 0.5 sample												
1 month	1.01	0.05	0.86	0.92	0.95	0.98	1.01	1.03	1.06	1.08	1.14	0.424
1 year	1.07	0.23	0.54	0.73	0.81	0.94	1.07	1.19	1.33	1.44	1.75	0.354
5 years	1.41	0.68	0.33	0.59	0.72	0.97	1.30	1.71	2.23	2.67	3.66	0.271
10 years	1.94	1.29	0.20	0.57	0.75	1.12	1.66	2.41	3.44	4.20	6.41	0.196
20 years	3.62	3.52	0.20	0.58	0.88	1.55	2.70	4.49	7.21	9.64	17.10	0.124
30 years	6.72	8.71	0.22	0.68	1.10	2.21	4.37	8.15	14.28	20.20	39.41	0.087

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Table IA9 (*continued*)

Horizon	Moments		Percentiles									$\mathbb{P}(W_H^{(m)} < 1)$
	Mean	SD	1%	5%	10%	25%	50%	75%	90%	95%	99%	
Panel L: M/GDP 1.0 sample												
1 month	1.01	0.05	0.86	0.92	0.95	0.98	1.01	1.03	1.06	1.08	1.14	0.427
1 year	1.07	0.22	0.53	0.72	0.80	0.94	1.06	1.19	1.33	1.44	1.71	0.363
5 years	1.37	0.61	0.36	0.57	0.71	0.95	1.29	1.68	2.12	2.47	3.35	0.289
10 years	1.82	1.10	0.23	0.56	0.73	1.07	1.59	2.30	3.19	3.85	5.53	0.213
20 years	3.23	2.88	0.23	0.55	0.82	1.45	2.48	4.08	6.34	8.36	14.15	0.139
30 years	5.69	6.50	0.24	0.62	0.99	1.98	3.87	7.09	12.00	16.61	30.93	0.102

Table IA10
Bootstrap distributions of 30-year USD payoffs for biased samples

The table summarizes the distribution of payoffs from a \$1.00 buy-and-hold investment across 1,000,000 bootstrap simulations at the 30-year horizon for alternative samples. The real payoffs are from the perspective of a global USD investor. The underlying sample in Panel A is the pooled sample of all developed countries. The underlying samples in Panel B are the United States over the period from 1890 to 2019 (U.S.) and the United Kingdom over the period from 1841 to 2019 (U.K.). The underlying samples in Panel C are the sample conditioned on current membership in the OECD (Survival) and the sample conditioned on current membership in the OECD and continuous data (Continuous). The real payoff for bootstrap iteration m at the H-month horizon is $W_H^{(m)}$. For each horizon, the table reports the mean, standard deviation, and distribution percentiles of real payoffs. The last column in the table shows the proportion of payoff draws that are less than one [$\mathbb{P}(W_H^{(m)} < 1)$]. The bootstrap sampling procedure is based on the stationary bootstrap approach of Politis and Romano (1994b) as described in the text. We sample blocks of random length, where the length of each block has a geometric distribution with a mean of 120 months.

Sample	Moments		Percentiles									$\mathbb{P}(W_H^{(m)} < 1)$
	Mean	SD	1%	5%	10%	25%	50%	75%	90%	95%	99%	
Panel A: Base case												
Full sample	8.66	22.92	0.06	0.34	0.71	1.82	4.21	9.00	18.22	28.51	71.96	0.140
Panel B: Single country												
U.S.	8.91	8.49	0.96	1.69	2.29	3.76	6.46	11.03	18.08	24.30	41.90	0.012
U.K.	4.90	4.89	0.50	0.94	1.29	2.13	3.50	5.89	9.86	13.49	24.54	0.058
Panel C: Survival and easy data												
Survival	8.94	21.54	0.06	0.39	0.77	1.92	4.35	9.22	18.83	29.69	75.52	0.130
Continuous	11.02	24.09	0.32	0.84	1.33	2.68	5.56	11.59	23.46	36.74	88.19	0.065

Table IA11
Bootstrap distributions of 30-year USD payoffs with additional sample screens

The table summarizes the distribution of real payoffs from a \$1.00 buy-and-hold investment across 1,000,000 bootstrap simulations at the 30-year horizon for alternative samples. The real payoffs are from the perspective of a global USD investor. The underlying sample in Panel A is the pooled sample of all developed countries. Panel B (Panel C) [Panel D] presents results for the full developed sample with additional sample screens based on sample start date (population) [ratio of market capitalization to GDP]. The underlying samples in Panels B to D are described in Table 5 of the paper. The real payoff for bootstrap iteration m at the H-month horizon is $W_H^{(m)}$. For each sample, the table reports the mean, standard deviation, and distribution percentiles of real payoffs. The last column in the table shows the proportion of payoff draws that are less than one [$\mathbb{P}(W_H^{(m)} < 1)$]. The bootstrap sampling procedure is based on the stationary bootstrap approach of Politis and Romano (1994b) as described in the text. We sample blocks of random length, where the length of each block has a geometric distribution with a mean of 120 months.

	Moments		Percentiles									
Sample	Mean	SD	1%	5%	10%	25%	50%	75%	90%	95%	99%	$\mathbb{P}(W_H^{(m)} < 1)$
Panel A: Base case												
Full sample	8.66	22.92	0.06	0.34	0.71	1.82	4.21	9.00	18.22	28.51	71.96	0.140
Panel B: Sample period												
Post-1880	8.76	32.19	0.06	0.34	0.70	1.80	4.20	9.07	18.50	29.10	74.25	0.143
Post-1920	10.35	27.08	0.06	0.38	0.81	2.11	4.92	10.67	21.92	34.52	88.32	0.122
Post-1960	10.68	27.37	0.22	0.66	1.11	2.37	5.06	10.78	22.12	35.13	91.21	0.088
Post-2000	5.72	17.87	0.11	0.33	0.56	1.18	2.51	5.40	11.48	19.04	53.19	0.207
Panel C: Population												
POP 0.2%	7.87	20.74	0.04	0.26	0.58	1.61	3.85	8.21	16.50	25.93	66.36	0.164
POP 0.5%	7.86	18.13	0.02	0.18	0.50	1.60	3.88	8.34	17.05	26.82	65.51	0.170
Panel D: Equity market size												
M/GDP 0.5	7.11	10.80	0.15	0.58	0.98	2.05	4.23	8.31	15.35	22.41	47.09	0.103
M/GDP 1.0	5.85	7.34	0.18	0.52	0.87	1.84	3.76	7.17	12.66	17.82	34.62	0.120

Table IA12
Asset allocation for annuity investment

The table shows results from asset allocation tests. For each return horizon, the table reports the optimal weight in stocks for an investor who relies on the developed country sample to form expectations about stock market performance (w_d) and the optimal weight in stocks for an investor who relies on the U.S. sample to form expectations about stock market performance (w_{us}). The investors have exponential utility with a risk aversion parameter of three and make contributions as a monthly annuity. Each investor allocates across the domestic stock market and a risk-free asset, where the risk-free asset is either the inflation-protected risk-free asset or cash. The table also reports the maximum annualized fee that the investor relying on the developed country sample would be willing to pay to use her optimal weight rather than adopt the optimal weight based on the U.S. sample.

	Inflation-protected risk-free asset			Cash		
	Weight in stocks			Weight in stocks		
Horizon	Developed country sample, w_d	United States sample, w_{us}	Fee (%)	Developed country sample, w_d	United States sample, w_{us}	Fee (%)
1 month	0.62	0.89	0.41	1.03	1.19	0.15
1 year	0.67	1.09	0.88	1.27	1.50	0.20
5 years	0.68	1.11	0.82	1.27	1.53	0.16
10 years	0.68	1.10	0.80	1.36	1.51	0.07
20 years	0.65	1.05	0.76	1.08	1.49	0.39
30 years	0.62	1.03	0.79	1.04	1.41	0.91

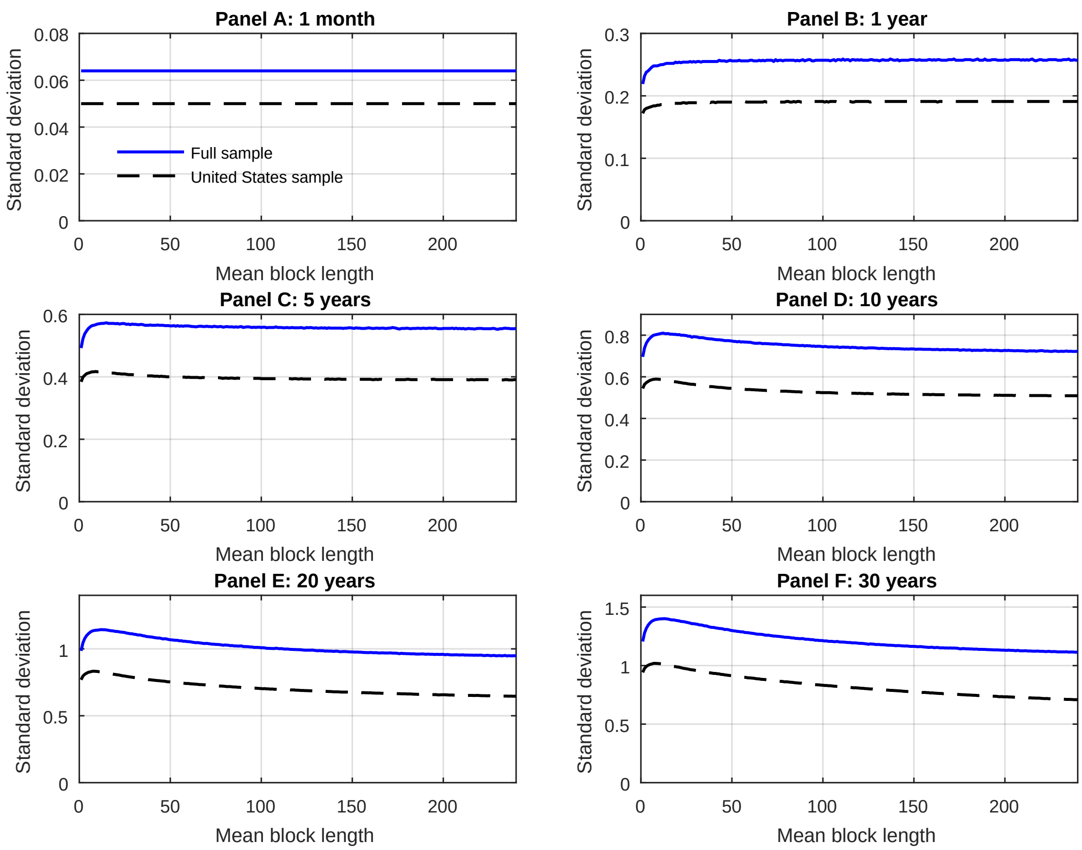

Fig. IA1. Standard deviation of continuously compounded returns for alternative block sampling lengths. The figure shows the standard deviation of continuously compounded returns across 1,000,000 bootstrap simulations at various return horizons for alternative mean block sampling lengths. The real returns are from the perspective of a domestic investor in a representative country. Each panel of the figure corresponds to a specific return horizon. The underlying sample for the simulated returns is the pooled sample of all developed countries (solid line) or the United States over the period from 1890 to 2019 (dashed line). The bootstrap sampling procedure is based on the stationary bootstrap approach of Politis and Romano (1994b) as described in the text.

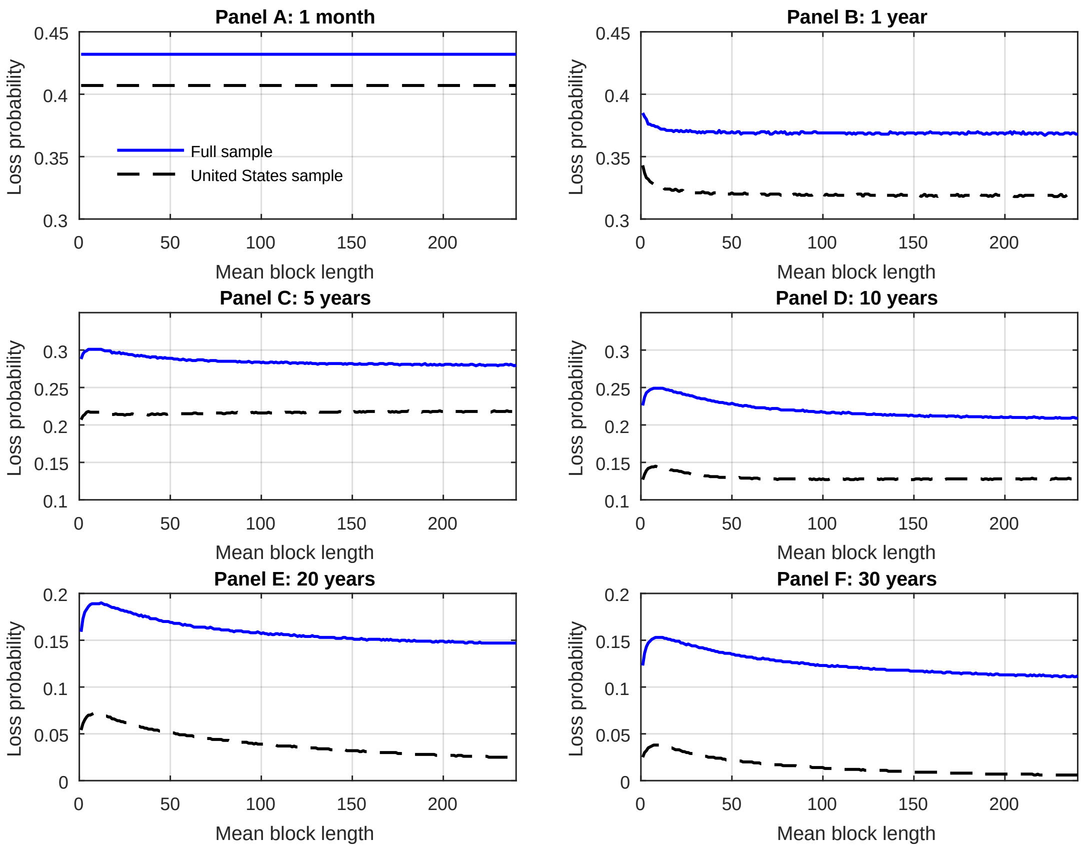

Fig. IA2. Loss probabilities for alternative block sampling lengths. The figure shows the proportion of real payoffs that are less than the initial investment across 1,000,000 bootstrap simulations at various return horizons for alternative mean block sampling lengths. The real payoffs are from the perspective of a domestic investor in a representative country. Each panel of the figure corresponds to a specific return horizon. The underlying sample for the simulated returns is the pooled sample of all developed countries (solid line) or the United States over the period from 1890 to 2019 (dashed line). The bootstrap sampling procedure is based on the stationary bootstrap approach of Politis and Romano (1994b) as described in the text.

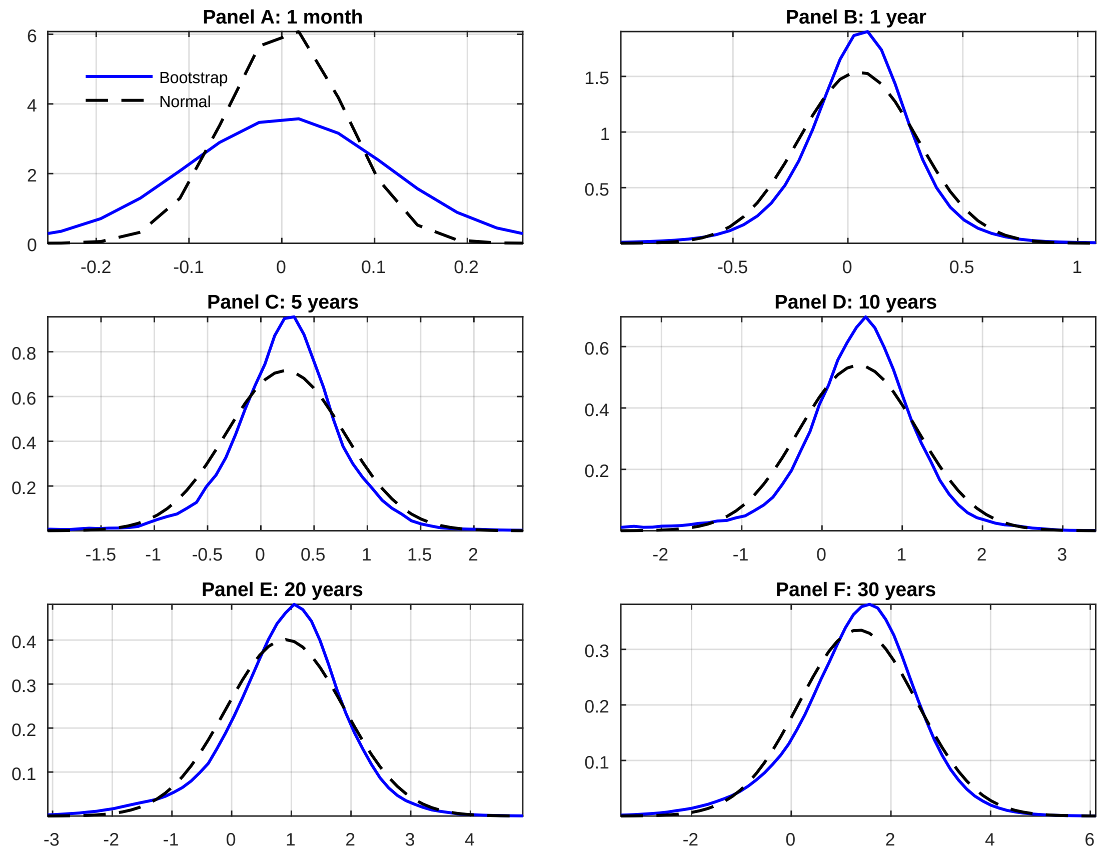

Fig. IA3. Continuously compounded returns. The figure shows distributions of continuously compounded real returns across 1,000,000 bootstrap simulations at various return horizons. The underlying sample for the simulated returns is the pooled sample of all developed countries. The real returns are from the perspective of a domestic investor in a representative country. In each panel, the solid line is the kernel smoothed density of simulated returns, and the dashed line is a normal density with mean and variance equal to those of the simulated returns. The bootstrap sampling procedure is based on the stationary bootstrap approach of Politis and Romano (1994b) as described in the text. We sample blocks of random length, where the length of each block has a geometric distribution with a mean of 120 months.

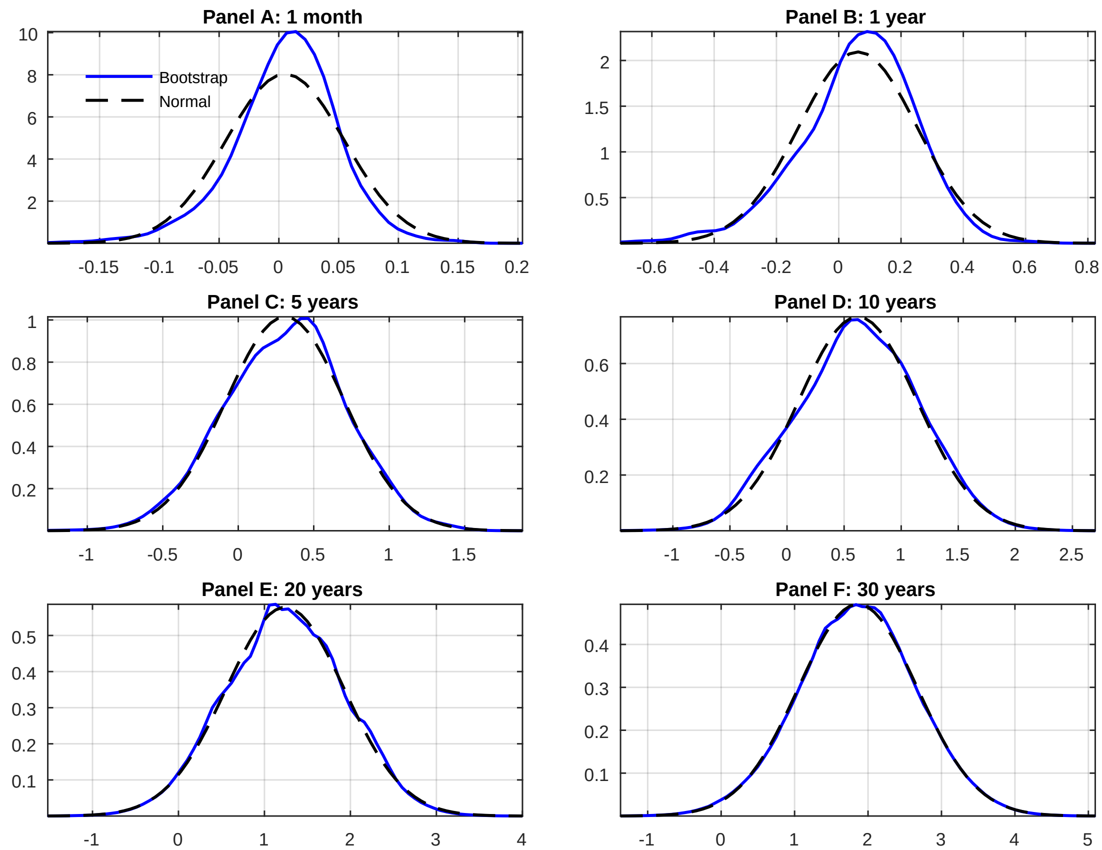

Fig. IA4. Continuously compounded returns for the United States sample. The figure shows distributions of continuously compounded real returns across 1,000,000 bootstrap simulations at various return horizons. The underlying sample for the simulated returns is the United States over the period from 1890 to 2019. In each panel, the solid line is the kernel smoothed density of simulated returns, and the dashed line is a normal density with mean and variance equal to those of the simulated returns. The bootstrap sampling procedure is based on the stationary bootstrap approach of Politis and Romano (1994b) as described in the text. We sample blocks of random length, where the length of each block has a geometric distribution with a mean of 120 months.

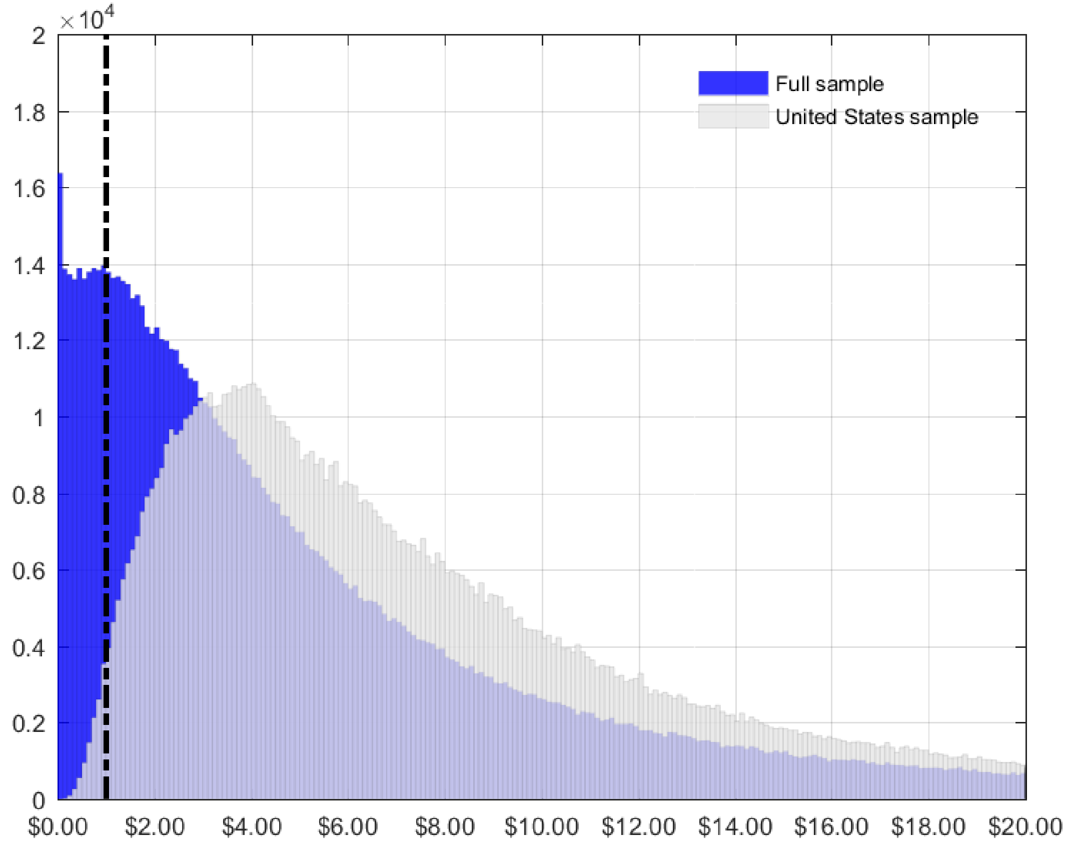

Fig. IA5. Cumulative 30-year payoffs for global USD investors. The figure shows histograms of real payoffs across 1,000,000 bootstrap simulations at a return horizon of 30 years. The real payoffs are from the perspective of a global USD investor. The underlying sample for the simulated returns is the pooled sample of all developed countries (blue) or the United States sample (gray). The dashed line separates the regions of real loss and gain on a $1.00 initial investment. The bootstrap sampling procedure is based on the stationary bootstrap approach of Politis and Romano (1994b) as described in the text. We sample blocks of random length, where the length of each block has a geometric distribution with a mean of 120 months.