Sovereign credit risk and exchange rates: Evidence from CDS quanto spreads Online Appendix*

Patrick Augustin,† Mikhail Chernov‡ and Dongho Song§

Abstract

Sovereign credit default swap quanto spreads tell us how financial markets view the interaction between a country's likelihood of default and associated currency devaluations (the Twin Ds). A no-arbitrage model applied to the term structure of eurozone quanto spreads can isolate the Twin Ds and can gauge the associated risk premiums. Conditional on the occurrence of default, the true and risk-adjusted one-week probabilities of devaluation are 42% (2%) and 90% (55%) for the core (periphery) countries. The weekly risk premium for euro devaluation in case of default for the core (periphery) exceeds the regular currency premium by up to 18 (13) basis points.

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†Desautels Faculty of Management, McGill University; 1001 Sherbrooke Street West, Montréal, QC H3A 1G5, Canada; patrick.augustin@mcgill.ca.

‡Corresponding author. Anderson School of Management, UCLA, NBER, and CEPR; 101 Westwood Plaza, Los Angeles, CA 90095, USA; mikhail.chernov@anderson.ucla.edu.

§Carey Business School, Johns Hopkins University; 100 International Drive, Baltimore, MD 21202, USA; dongho.song@jhu.edu.

Appendix A. Institutional background

CDS contracts are controlled by three documents: the Credit Derivatives Definitions ("the Definitions"), the ISDA Credit Derivatives Physical Settlement Matrix ("the Physical Settlement Matrix"), and the Confirmation Letter ("the Confirmation").

The Physical Settlement Matrix is the most important document because the push for standardization has created specific transaction types that are by convention applicable to certain types of a sovereign reference entity, e.g., Standard Western European Sovereign (SWES) or Standard Emerging European Sovereign (SEES) single-name contracts. In total, there are nine transaction types listed in the sovereign section of the Physical Settlement Matrix that contain details about the main contractual provisions for transactions in CDS referencing sovereign entities.

Given the over-the-counter (OTC) nature of CDS contracts, parties are free to combine features from different transaction types, which would be recognized in the Confirmation, i.e., the letter that designates the appropriate terms for a CDS contract. The Confirmation, which is mutually negotiated and drafted between two counterparties, can thereby amend legal clauses attributed to conventional contract characteristics. Hence, there may be slight variations from standard transaction types if counterparties agree to alter the terms of conventional CDS contracts. Such changes introduce legal risk, and potentially make the contracts less liquid, given the customization required for efficient central clearing.

The terms used in the documentation of most credit derivatives transactions are defined in the Definitions. On 22 September 2014, ISDA introduced the 2014 Credit Derivatives Definitions, which update the 2003 Credit Derivatives Definitions. We use contracts governed by the 2003 Definitions to guarantee internal consistency throughout our sample period.

It is important to distinguish between the circumstances under which a CDS payout/credit event could be triggered, and the restrictions on obligations eligible for delivery in the settlement process upon the occurrence of a qualifying credit event. The Physical Settlement matrix lays out the credit events that trigger CDS payment, which follows the ruling by a determinations committee of the occurrence of a credit event and a credit event auction. SWES transaction types recognize three sovereign credit events, namely Failure to Pay, Repudiation/Moratorium, and Restructuring. SEES contracts further list Obligation Acceleration as a valid event that could trigger the CDS payout.

The most disputed among all credit events is the Restructuring credit event clause related to a change to the reference obligation that is binding on all holders of the obligation. The most controversial among such changes is the redenomination of the principal or interest payment into a new currency. For the credit event to be triggered under the 2014 Definitions, this new currency must be any currency other than the lawful currency of Canada, Japan, Switzerland, the United Kingdom, the United States of America, and the Euro (or any successor currency to any of the currencies listed; in the case of the Euro, the new currency must replace the Euro in whole). In the 2003 Definitions, permitted currencies were defined as those of G7 countries and AAA-rated OECD economies.

Another important dimension to consider is the obligation category and the associated obligation characteristics which may trigger a credit event. For SWES contracts, the Obligation category is defined broadly as "borrowed money," which includes deposits and reimbursement obligations arising from a letter of credit or qualifying guarantees. Such contracts also feature no restrictions on the characteristics of obligations relevant for the triggers of default payment. For SEESs, however, markets have agreed on more specificity for the reference obligations, which encompass only "bonds," which are not allowed to be subordinated, denominated in domestic currency, issued domestically or under domestic law, as indicated by the restrictions of the obligation characteristics.

One non-trivial aspect relates to the deliverable obligation categories and the associated characteristics. While in the presence of credit events for SWESs, bonds or loans are deliverable during the auction settlement process, SEES contracts allow only for bonds to be delivered. Several restrictions apply to the deliverable obligations, such that for SWESs they have to be denominated in a specified currency (i.e., the Euro or the currencies of Canada, Japan, Switzerland, the UK, or the USA), they must be non-contingent, non-bearer and transferable, limited to a maximum maturity of 30 years, and loans must be assignable and consent is required. SEES contracts exclude these restrictions on loans and the maximum deliverable maturity, but impose the additional constraints that the obligation cannot be subordinated, and issued domestically or under domestic law.

A final institutional detail to consider is the determination of the settlement amount in the instance of a credit event. All CDS contracts during the time frame of our paper are covered by the so-called 'Big Bang protocol', formally called the 2009 ISDA Credit Derivatives Determinations Committees and Auction Settlement CDS Protocol. The Big Bang protocol, which supplements the 2003 Credit Derivatives Definitions, stipulates that all CDS contracts are settled via an auction procedure. The auction determines the value of all deliverable obligations. As a result of the auction, all deliverable obligations are assigned the same price as a percentage of face value. That same price is also used for cash settlement for those parties that prefer cash rather than bonds. The main implication here is that, regardless of who gets cash and who gets bonds, the economic value of settlement is identical.

Article XII ("Terms Relating to Auction Settlement"), Section 12.4, of the 2003 ISDA Credit Derivatives Definitions, thus indicates that the auction settlement amount is determined by a percentage of the notional face value of the contract. This percentage is equal to the greater of the Reference Price minus the Auction Final Price and zero. Lastly, point 2 in the Credit Derivatives Auction Settlement Terms stipulates that the auction administrators will, on the auction currency fixing date, determine the rate of conversion between the relevant auction currency and the currency of denomination of each deliverable obligation.

Appendix B. Term structure of quanto spreads in the iid case

We show that the term structure of quanto spreads is flat when hazard and depreciation rates are iid, and the risk-free interest rate is constant. To achieve analytical tractability, we consider that CDS contracts are fully settled upfront.

Assuming that both the default intensity, h_t^*, and depreciation rate are iid and using the law of iterated expectations, the upfront premium for the EUR CDS is given by

$$\begin{aligned}
C_0^{€} &= L \cdot E_0^* \left[e^{-r(\tau \wedge T)} \cdot S_{\tau \wedge T}/S_0 \cdot I(\tau \leq T) \right] \\
&= L \cdot E_0^* \left[e^{-r(\tau \wedge T)} \cdot S_{\tau \wedge T}/S_0 \cdot (1 - I(\tau > T)) \right] \\
&= L \cdot E_0^* \left[e^{-rT + s_T - s_0} \right] - L \cdot E_0^* \left[e^{-rT + s_T - s_0 - \sum_{j=1}^T h_j^*} \right] \\
&= L \cdot E_0^* \left[e^{-rT} \right] \cdot E_0^* \left[e^{s_T - s_0} \right] \cdot \left[1 - E_0^* e^{-\sum_{j=1}^T h_j^*} \right],
\end{aligned}$$

where $s_t = \log S_t$.

Similarly, the upfront premium for the USD CDS is given by

$$C_0^{\$} = L \cdot E_0^* \left[e^{-rT} \right] \cdot \left[1 - E_0^* e^{-\sum_{j=1}^T h_j^*} \right].$$

Therefore, the quanto spread for any maturity T, is given by

$$q_0^{\$,€} = -T^{-1}(\log C_0^{\$} - \log C_0^{€}) = T^{-1} \log E_0^* e^{s_T - s_0} = T^{-1} \log E_0^* e^{T(s_1 - s_0)} = \log E_0^* e^{s_1 - s_0}.$$

It follows that the difference in quanto spreads of any two maturities is zero, implying a flat term structure of quanto spreads.

Appendix C. Credit risk with contagion

The risk-adjusted default hazard rate of each country $k = 1, \cdots, M_c$ is

$$H_t^{*k} = Prob^*(\tau^k = t | \tau^k \geq t; \mathcal{F}_t),$$

where τ^k is the time of the credit event in country k, and M_c is the number of countries. We posit that the hazard rate is determined by the default intensity h_t^{*k} as follows:

$$H_t^{*k} = 1 - e^{-h_t^{*k}}, \quad h_t^{*k} = \bar{h}^{*k} + \delta_w^{*k\top} w_t + \delta_d^{*k\top} d_{t-1}, \tag{C.1}$$

such that the default intensity is affine in the credit variables w_t and contagion variables d_t that are elements of the state vector x_t.

The credit variables w_t are exactly the same as in the main text. To gain intuition about how our model of contagion works, consider the Poisson arrival of credit events at a conditional rate of d_t. We would like the realization from this process to affect the conditional rate in the subsequent period. Denote the realization by $\mathcal{P} : \mathcal{P}|d_t \sim Poisson(d_t)$.

In our application to eurozone sovereigns, we expect d_t to be small, implying that most of the realizations of $\mathcal{P}$ will be equal to zero (the probability of such an event is e^{-d_t}). The probability of one credit event is equal to $d_t e^{-d_t}$. Theoretically, it is possible that $\mathcal{P} > 1$ with the probability $1 - e^{-d_t} - d_t e^{-d_t}$. However, for a small d_t, such an outcome is unlikely.

In this respect, such a Poisson process can be viewed as an analytically tractable approximation to a Bernoulli distribution that is more appropriate for a credit event in a single country. For reasons of parsimony, we use this process to count all contemporaneous events across the countries in our sample. Thus, a Poisson model is a better fit for our framework.

The next step in the contagion model is to determine how the value of $\mathcal{P}$ affects the subsequent arrival rate d_{t+1}. First, this value has to be non-negative, so we choose a distribution with a non-negative support. Second, we would like to achieve analytical tractability for valuation purposes, so we choose a Gamma distribution whose shape parameter is controlled by $\mathcal{P} : d_{t+1} \sim Gamma(\mathcal{P}, 1)$. The idea is that the more credit events we have at time t, the larger the impact on d_{t+1}. If $\mathcal{P}|d_t = 0$, then $d_{t+1} = 0$, by convention.

The resulting distribution of d_{t+1} is

$$\phi\left(d_{t+1} \mid d_t\right) = \sum_{k=1}^{\infty} \left[\frac{d_t^k}{k!} e^{-d_t} \times \frac{d_{t+1}^{k-1} e^{-d_{t+1}}}{\Gamma\left(k\right)} \right] \mathbf{1}_{[d_{t+1}>0]} + e^{-d_t} \mathbf{1}_{[d_{t+1}=0]}.$$

This expression, representing the description in words above, makes explicit what is missing. We need to replace d_t in this expression with $\bar{d} + \phi d_t$. The constant is needed to preclude $d_t = 0$ from becoming an absorbing state. The coefficient $0 < \phi < 1$ is needed to ensure the stationarity of d_t.

In our model, the contagion factor d_t interacts with other factors that control credit risk, as described below when we specify all of the state variables explicitly. Such a model happens to be autoregressive gamma-zero, ARG_0, a process introduced by Monfort et al. (2017b) for the purpose of modeling interest rates at the zero lower bound. Monfort et al. (2017a) use ARG_0 to model the credit contagion of banks.

Formally, the factor d_t is a multivariate autoregressive gamma-zero process of size M_d. Each component $k = 1, \cdots, M_d$ follows an autoregressive gamma-zero process, $d_{t+1}^k \mid w_{t+1} \sim ARG_0(\bar{h}^{*k} + \delta_w^{*k\top} w_{t+1} + \delta_d^{*k\top} d_t^k, \rho^{*k})$. We add two more features to the description in Section 3.3. First, the contagion factor is affected by conventional credit factors w_t in addition to its own value from the previous period. Second, we allow for a scale parameter, ρ^{*k}, that could be different from unity in the Gamma distribution.

Besides the explicit distribution, an ARG_0 process can be described as

$$d_{t+1}^k = \bar{h}^{*k} + \delta_w^{*k\top} w_{t+1} + \delta_d^{*k\top} d_t^k + \eta_{t+1}^k, \tag{C.2}$$

where $\eta_{k,t+1}$ is a martingale difference sequence (mean zero shock), with conditional variance given by

$$\mathrm{var}_t \eta_{t+1}^k = 2\rho^{*k} \left(\bar{h}^{*k} + \delta_w^{*k\top} \left[\nu_w \odot c_w^* + \phi_w^{*\top} w_t \right] + \delta_d^{*k\top} d_t \right),$$

where $\odot$ denotes the Hadamard product. As for the credit factors, we impose parameter restrictions on the matrices δ_w^{*k} and δ_d^{*k} to guarantee stationarity. Specifically, $\delta_w^{*k} \geq 0$ and the eigenvalues of δ_d^{*k} have a modulus smaller than one. Comparing expressions (C.1) and (C.2) makes it clear that the default hazard rate and the arrival rate of Poisson events in the contagion factors are the same process.

For parsimony, we assume the existence of one common credit event variable that may induce contagion across the different countries and regions. This is conceptually similar to the suggestion of Benzoni et al. (2015), that a shock to a hidden factor may lead to an updating of the beliefs about the default probabilities of all countries. Thus, given such a restriction, the contagion factor is a scalar, $d_{t+1} \mid w_{t+1} \sim ARG_0(\bar{h}^* + \delta_w^{*\top} w_{t+1} + \delta_d^{*\top} d_t, \rho^*)$ with appropriate restrictions on the loadings:

$$\bar{h}^* = \sum_k \bar{h}^{*k}, \quad \delta_w^* = \sum_k \delta_w^{*k}, \quad \delta_d^* = \sum_k \delta_d^{*k}.$$

As a result, we may have more than one credit event per period.

Table G3 reports the estimated model with contagion. The parameters that are contagion-specific, δ_d^{*k}, δ_d, and ρ^* are statistically significant. The question is whether the extra degrees of freedom associated with a larger model are justified from the statistical and economic perspectives. While we find some credence for the contagion mechanism in our sample, the improvement in the model's fit does not justify the associated increase in statistical uncertainty.

Specifically, a table in Online Appendix G reports the distributions of the likelihoods of both models, and the associated BICs (the negative of the likelihood plus the penalty for the number of parameters). Both statistics indicate that the difference between the two models is insignificant. Table G4 reports various measures of pricing errors for the model without contagion. The same metrics for the model with contagion are similar and, therefore, are not reported for brevity.

Appendix D. Details of asset valuation

Appendix D.1. Bonds

To derive closed-form solutions for the term structure of interest rates, we conjecture that zero-coupon bond prices $Q_{t,T}$ are exponentially affine in the state vector u_t

$$q_{t,T} = \log Q_{t,T} = -\tilde{A}_{T-t} - \tilde{B}_{T-t}^{\top} u_t.$$

Because the interest rate is an affine function of the state, $r_t = \bar{r} + \delta_r^{\top} u_t$, log-bond prices are fully characterized by the cumulant-generating function of u_t. The law of iterated expectations implies that $Q_{t,T}$ satisfies the recursion

$$Q_{t,T} = e^{-r_t} E_t^* Q_{t+1,T-1},$$

It can be shown that for all $n = 0, 1 \dots, T-t$, the scalar $\tilde{A}_n$ and components of the column vector $\tilde{B}_n$ are given by

$$\begin{aligned}
\tilde{A}_n &= \tilde{A}_{n-1} + \bar{r} + \tilde{B}_{n-1}^{\top} \mu_u - \frac{1}{2} \tilde{B}_{n-1}^{\top} \Sigma_u \Sigma_u^{\top} \tilde{B}_{n-1} \\
\tilde{B}_n^{\top} &= \delta_r^{\top} + \tilde{B}_{n-1}^{\top} \Phi_u,
\end{aligned}$$

with initial conditions $\tilde{A}_0 = 0$ and $\tilde{B}_0 = 0$.

Then the yields are

$$y_{t,T} = -(T-t)^{-1} \log Q_{t,T} = A_{T-t} + B_{T-t}^{\top} u_t$$

with $A_{T-t} = -(T-t)^{-1} \tilde{A}_{T-t}$ and $B_{T-t} = -(T-t)^{-1} \tilde{B}_{T-t}$.

Appendix D.2. Forward exchange rates

According to covered interest rate parity, a forward exchange rate with settlement date T is equal to the expected future value of the exchange rate, i.e., $F_{t,T} = E_t^* [S_T]$. Dividing each side of this equation by S_t, we can solve for the log ratio of the forward to the spot exchange rate, $\log (F_{t,T}/S_t) = \log E_t^* \left[e^{\Delta s_{t+T}}\right]$. Thus, deriving closed-form solutions for the log ratio of the forward to the spot exchange rates is akin to solving for the cumulant-generating function of the depreciation rate.

We can use recursion techniques to derive analytical solutions for these expressions by solving for $\log F_{t,j} = \log E_t^* [S_{t+j}/S_t]$. To evaluate the expressions for $F_{t,j}$, we conjecture that the expression is (in the model without contagion) an affine function of the state vector x_t, i.e., $\log F_{t,j} = \tilde{A}_j + \tilde{B}_j^{\top} x_t$. The law of iterated expectations implies that $F_{t,j}$ satisfies the recursions $F_{t,j} = E_t^* [(S_{t+1}/S_t) F_{t+1,j-1}]$, where the recursions start at $j = 0$ for $F_{t,j}$. It can be shown that for all $j = 1, 2, \ldots, T-t$, the scalar $\tilde{A}_j$ and components of the column vectors $\tilde{B}_j = \left[\tilde{B}_{u,j}^\top, \tilde{B}_{g,j}^\top\right]^\top$ are given by

$$\begin{aligned}
\tilde{A}_j &= \tilde{A}_{j-1} + \bar{s}^* + \frac{1}{2}\bar{v}^* - \bar{h}^* \left[\frac{\theta^*}{1+\theta^*}\right] + \left(\tilde{B}_{u,j-1} + \delta_{su}^*\right)^\top \mu_u \\
&\quad + \frac{1}{2}\left(\tilde{B}_{u,j-1} + \delta_{su}^*\right)^\top \Sigma_u \left(\tilde{B}_{u,j-1} + \delta_{su}^*\right) \\
&\quad - \sum_{l=1}^{M_g} \nu_l \log\left(1 - \left(\tilde{B}_{g_l,j-1} + \delta_{sg_l}^* - \delta_{w_l}^* \left[\frac{\theta^*}{1+\theta^*}\right]\right) c_l^*\right) \\
\tilde{B}_{u,j} &= \Phi_u^\top \left[\tilde{B}_{u,j-1} + \delta_{su}^*\right] \\
\tilde{B}_{g_i,j} &= \sum_{l=1}^{M_g} \frac{\left(\tilde{B}_{g_l,j-1} + \delta_{sg_l}^* - \delta_{w_i}^* \left[\frac{\theta^*}{1+\theta^*}\right]\right) \phi_{il}^*}{1 - \left(\tilde{B}_{g_l,j-1} + \delta_{sg_l}^* - \delta_{w_i}^* \left[\frac{\theta^*}{1+\theta^*}\right]\right) c_l^*} + \frac{1}{2}\delta_{v_i}^*.
\end{aligned} \tag{D.1}$$

where we have indexed the sub-components of $\tilde{B}_{g,j}$ using an i subscript. The initial condition for F is given by $\log F_{t,0} = \tilde{A}_0 + \tilde{B}_0^\top x_t$, where the scalar $\tilde{A}_0 = 0$ and the column vector $\tilde{B}_0 = 0$.

Appendix D.3. CDS

We rewrite the CDS spread presented in Eq. (5) using the risk-adjusted probability as follows:

$$C_{t,T}^{€} = L \cdot \frac{\sum_{j=1}^{T-t} E_t^*[B_{t,t+j-1}(P_{t+j-1}^* - P_{t+j}^*)S_{t+j}]}{\sum_{j=1}^{(T-t)/\Delta} E_t^*[B_{t,t+j\Delta-1} P_{t+j\Delta}^* S_{t+j\Delta}]}. \tag{D.1}$$

We can use recursion techniques to derive analytical solutions for CDS premiums by solving for the following two objects:

$$\tilde{\Psi}_{j,t} = E_t^*\left[B_{t,t+j-1} \frac{P_{t+j-1}^*}{P_t^*} \frac{S_{t+j}}{S_t}\right] \quad \text{and} \quad \Psi_{j,t} = E_t^*\left[B_{t,t+j-1} \frac{P_{t+j}^*}{P_t^*} \frac{S_{t+j}}{S_t}\right]. \tag{D.2}$$

These expressions jointly yield the solution for the CDS premium after dividing the numerator and the denominator of Eq. (D.1) by the time-t survival probability P_t^* and exchange rate S_t.

$$C_{t,T}^{€} = L \cdot \frac{\sum\limits_{j=1}^{T-t} (\tilde{\Psi}_{j,t} - \Psi_{j,t})}{\sum\limits_{j=1}^{(T-t)/\Delta} \Psi_{j\Delta,t}}. \tag{D.3}$$

The true implementation of CDS valuation extends Eq. (5) by accounting for accrual payments. Express the valuation expression in terms of the risk-adjusted probability

$$C_{t,T}^{€} = L \cdot \frac{\sum\limits_{j=1}^{T-t} (\tilde{\Psi}_{j,t} - \Psi_{j,t})}{\sum\limits_{j=1}^{(T-t)/\Delta} \Psi_{j\Delta,t} + \sum\limits_{j=1}^{T-t} \left(\frac{j}{\Delta} - \lfloor \frac{j}{\Delta} \rfloor \right) (\tilde{\Psi}_{j,t} - \Psi_{j,t})},$$

where the floor function $\lfloor \cdot \rfloor$ rounds to the nearest lower integer. The law of iterated expectations implies that $\tilde{\Psi}_{j,t}$ and $\Psi_{j,t}$ satisfy the recursions

$$\tilde{\Psi}_{j,t} = E_t^* \left[B_{t,t} \left(1 - H_{t+1}\right) \frac{S_{t+1}}{S_t} \tilde{\Psi}_{j-1,t+1} \right], \ \Psi_{j,t} = E_t^* \left[B_{t,t} \left(1 - H_{t+1}\right) \frac{S_{t+1}}{S_t} \Psi_{j-1,t+1} \right],$$

starting at $j = 1$ for $\tilde{\Psi}_{j,t}$ and at $j = 0$ for $\Psi_{j,t}$. To evaluate the expressions for $\tilde{\Psi}$ and Ψ, we conjecture that the expressions in Eq. (D.2) are exponentially affine functions of the state vector x_t:

$$\tilde{\Psi}_{j,t} = e^{\tilde{A}_j + \tilde{B}_j^\top x_t} \quad \text{and} \quad \Psi_{j,t} = e^{A_j + B_j^\top x_t}. \tag{D.4}$$

Based on the more general model with contagion, we define the column vectors $\tilde{B}_j = \left[\tilde{B}_{u,j}^\top, \tilde{B}_{g,j}^\top, \tilde{B}_{d,j}^\top \right]^\top$, with $u = 1, 2, \ldots, M_u$, $g = 1, 2, \ldots, M_g$, and $d = 1, 2, \ldots, M_d$. Next, the column vectors of ones with length M_u, M_g, and M_d are denoted by $\mathbb{1}_{M_u}$, $\mathbb{1}_{M_g}$, and $\mathbb{1}_{M_d}$, respectively. Define the matrices $\Delta_w^* = \left[\delta_w^{*1}, \delta_w^{*2}, \ldots, \delta_w^{*M_c}\right]$ and $\Delta_d^* = \left[\delta_d^{*1}, \delta_d^{*2}, \ldots, \delta_d^{*M_c}\right]$. Finally, we subdivide the vector δ_s^* into sub-matrices $\delta_s^{*\top} = \left[\delta_{su}^{*\top}, \delta_{sg}^{*\top}, \delta_{sd}^{*\top}\right]$, for $u = 1, 2, \ldots, M_u$, $g = 1, 2, \ldots, M_g$, and $d = 1, 2, \ldots, M_d$. It can be shown that for each country $k = 1, 2, \ldots, M_c$ and for all $j = 1, 2, \ldots, T - t$, the scalar $\tilde{A}_j$ and components of the column vectors $\tilde{B}_j$ are given by

$$\begin{aligned}
\tilde{A}_j &= \tilde{A}_{j-1} - \bar{r} - \bar{h}^{*k} + \bar{s}^* + \frac{1}{2}\bar{v}^* - \bar{h}^* \left[\frac{\theta^*}{1+\theta^*}\right] + \left(\tilde{B}_{u,j-1} + \delta^*_{su}\right)^\top \mu_u \\
&\quad + \frac{1}{2}\left(\tilde{B}_{u,j-1} + \delta^*_{su}\right)^\top \Sigma_u \Sigma_u^\top \left(\tilde{B}_{u,j-1} + \delta^*_{su}\right) + \left[\frac{\tilde{B}_{d,j-1} \odot \bar{H}^*}{\mathbb{1}_{M_d} - \tilde{B}_{d,j-1} \odot \rho^*}\right]^\top \cdot \mathbb{1}_{M_d} \\
&\quad - \left(\nu \odot log \left[\mathbb{1}_{M_g} - \left(\tilde{B}_{g,j-1} + \delta^*_{sg} - \delta^{*k}_w - \delta^*_w \odot \left[\frac{\theta^*}{1+\theta^*}\right]\right.\right.\right. \\
&\quad \left.\left.\left. + \left[\Delta^*_w \left(\frac{\tilde{B}_{d,j-1}}{\mathbb{1}_{M_d} - \tilde{B}_{d,j-1} \odot \rho^*}\right)\right]\right) \odot c^*\right]\right)^\top \cdot \mathbb{1}_{M_g} \\
\tilde{B}_{u,j} &= \Phi_u^\top \left[\tilde{B}_{u,j-1} + \delta^*_{su}\right] - \delta_r \\
\tilde{B}_{g,j} &= \Phi_g^{*\top} \left(\frac{\tilde{B}_{g,j-1} + \delta^*_{sg} - \delta^{*k}_w - \delta^*_w \odot \left[\frac{\theta^*}{1+\theta^*}\right] + \left[\Delta^*_w \left(\frac{\tilde{B}_{d,j-1}}{\mathbb{1}_{M_d} - \tilde{B}_{d,j-1} \odot \rho^*}\right)\right]}{\mathbb{1}_{M_g} - \left[\tilde{B}_{g,j-1} + \delta^*_{sg} - \delta^{*k}_w - \delta^*_w \odot \left[\frac{\theta^*}{1+\theta^*}\right] + \left[\Delta^*_w \left(\frac{\tilde{B}_{d,j-1}}{\mathbb{1}_{M_d} - \tilde{B}_{d,j-1} \odot \rho^*}\right)\right]\right] \odot c^*} \right) \\
&\quad + \frac{1}{2}\delta^*_v \\
\tilde{B}_{d,j} &= \Delta_d^{*\top} \left(\frac{\tilde{B}_{d,j-1}}{\mathbb{1}_{M_d} - \tilde{B}_{d,j-1} \odot \rho^*}\right) - \delta^{*k}_d - \delta^*_d \odot \left[\frac{\theta}{1+\theta}\right],
\end{aligned} \tag{D.5}$$

where $\odot$ defines the Hadamard product for element-wise multiplication, and where we slightly abuse notation as the division and log operators work element-by-element when applied to vectors or matrices. The recursions for Ψ are identical. It is sufficient to replace $\tilde{A}$. and $\tilde{B}$. by A. and B., respectively.

The initial condition for $\tilde{\Psi}$ is given by

$$\tilde{\Psi}_{1,t} = e^{\tilde{A}_1 + \tilde{B}_1^\top x_t}, \tag{D.6}$$

where the scalar $\tilde{A}_1$ and components of the column vectors $\tilde{B}_1 = \left[\tilde{B}_{u,1}^\top, \tilde{B}_{g,1}^\top, \tilde{B}_{d,1}^\top\right]^\top$ are given by

$$\begin{aligned}
\tilde{A}_1 &= \bar{s}^* - \bar{r} + \delta_{su}^{*\top} u_u + \frac{1}{2}\delta_{su}^{*\top}\Sigma_u\Sigma_u^\top\delta_{su}^* + \frac{1}{2}\bar{v}^* \\
&\quad - \left[\nu \odot \log\left(\mathbb{1}_{M_g} - \left(\delta_{sg}^* - \delta_w^* \odot \left[\frac{\theta^*}{1+\theta^*}\right]\right)\odot c^*\right)\right]^\top \cdot \mathbb{1}_{M_g} - \bar{h}^*\left[\frac{\theta^*}{1+\theta^*}\right] \\
\tilde{B}_{u,1} &= \Phi_u^\top \delta_{su}^* - \delta_r \\
\tilde{B}_{g,1} &= \Phi_g^{*\top}\left(\frac{\delta_{sg}^* - \delta_w^* \odot \left[\frac{\theta^*}{1+\theta^*}\right]}{\mathbb{1}_{M_g} - \left(\delta_{sg}^* - \delta_w^* \odot \left[\frac{\theta^*}{1+\theta^*}\right]\right)\odot c^*}\right) + \frac{1}{2}\delta_v^* \\
\tilde{B}_{d,1} &= -\delta_d^* \odot \left[\frac{\theta^*}{1+\theta^*}\right].
\end{aligned} \tag{D.7}$$

The initial condition for Ψ is given by

$$\Psi_{0,t} = e^{A_0 + B_0^\top x_t}, \tag{D.8}$$

where the scalar $A_0 = 0$ and the column vector $B_0 = 0$.

The pricing equation for the USD-CDS spread is obtained in closed form by setting $S_{t+j} = 1$ in all recursions.

Appendix E. The affine pricing kernel

A multiperiod pricing kernel is a product of one-period ones: $M_{t,t+n} = M_{t,t+1} \cdot M_{t+1,t+2} \cdot \ldots \cdot M_{t+n-1,t+n}$. In this section, we specify the pricing kernel and show how the associated prices of risk modify the distribution of state x_t. To simplify notation, we are considering only one Poisson jump here.

The (log) pricing kernel is:

$$m_{t,t+1} = -r_t - k_t(-\gamma_{x,t}; \varepsilon_{x,t+1}) - k_t(-1; z_{m,t+1}) - \gamma_{x,t}^\top \varepsilon_{x,t+1} - z_{m,t+1},$$

where $k_t(s; e_{t+1}) = \log E_t e^{s \cdot e_{t+1}}$ is the cumulant-generating function (cgf), and $z_{m,t+1}$ is a jump process with intensity λ_{t+1}. The behavior of risk premiums is determined by $\gamma_{x,t}$ and the jump magnitude $z_{m,t+1}|j_{t+1}$.

In the case of factor u_t, the risk premium is $\gamma_{u,t} = \Sigma_u^{-1}(\bar{\gamma}_u + \delta_u u_t)$ implying

$$\Phi_u^* = \Phi_u - \delta_u, \quad \mu_u^* = \mu_u - \bar{\gamma}_u$$

with $k_t(-\gamma_{u,t}; \varepsilon_{u,t+1}) = \gamma_{u,t}^\top \gamma_{u,t}/2$. In the case of factor g_t, the risk premium is $\gamma_{g,t} = \bar{\gamma}_g$ implying

$$\phi_{ij}^* = \phi_{ij}(1 - \bar{\gamma}_{gi} c_i)^{-2}, \quad c_i^* = c_i(1 - \bar{\gamma}_{gi} c_i)^{-1}$$

with

$$k_t(-\gamma_{g,t};\eta_{g,t+1}) = -\sum_{i=1}^{M_g}\left(\nu_i[\log(1+\gamma_{gi,t})-\gamma_{gi,t}c_i]+\gamma_{gi,t}[(1+\gamma_{gi,t}c_i)^{-1}-1]\phi_i^\top g_t\right).$$

See Le et al. (2010).

In the case of jumps, the risk premium is

$$z_{m,t+1}|j_{t+1} = \sum_k -(h^k_{t+1}-h^{*k}_{t+1})-j^k_{t+1}\log h^{*k}_{t+1}/h^k_{t+1}+j^k_{t+1}\log\theta^*/\theta+(\theta^{*-1}-\theta^{-1})\cdot z_{s,t+1}|j^k_{t+1}$$

implying a risk-adjusted jump $z_{s,t+1}$ with Poisson arrival rate of $\lambda^*_{t+1}=\sum_k h^{*k}_{t+1}$ and magnitude $z_{s,t+1}|j_{t+1}\sim Gamma\,(j_{t+1},\theta^*)$. The corresponding cgf is $k_t(-1;z_{m,t+1})=0$.

Indeed, a Poisson mixture of gammas distribution implies arbitrary forms of risk premiums without violating no-arbitrage conditions. To see this, first observe that

$$p_{t+1}(z_s|j) = \frac{e^{-\lambda_{t+1}}\lambda^j_{t+1}}{j!}\frac{1}{\Gamma(j)\theta^j}z_s^{j-1}e^{-z_s/\theta}.$$

Second, assume that the risk-adjusted distribution features an arbitrary arrival rate λ^*_{t+1} and jump size mean θ^* (this does not have to be a constant). Then

$$p^*_{t+1}(z_s|j) = \frac{e^{-\lambda^*_{t+1}}\lambda^{*j}_{t+1}}{j!}\frac{1}{\Gamma(j)\theta^{*j}}z_s^{j-1}e^{-z_s/\theta^*}.$$

We characterize the ratio $p^*_{t+1}(z)/p_{t+1}(z)$ via the moment-generating function (mgf) of its log. First, we compute the expectation with respect to the jump-size distribution

$$\begin{aligned}&\tilde{h}_{t+1}(s;\log p^*_{t+1}(z_s)/p_{t+1}(z_s)) = E_{t+1}e^{s\log p^*_{t+1}(z_s)/p_{t+1}(z_s)}\\ =\;&\sum_{j=0}^{\infty}\frac{e^{-\lambda_{t+1}}\lambda^j_{t+1}}{j!}e^{(\lambda_{t+1}-\lambda^*_{t+1})+j\log\lambda^*_{t+1}/\lambda_{t+1}-j\log\theta^*/\theta}(1-s\theta(\theta^{-1}-\theta^{*-1}))^{-j}.\end{aligned}$$

This functional form of the mgf reflects a Poisson mixture with intensity λ_{t+1} and magnitude $z_m|j=(\lambda_{t+1}-\lambda^*_{t+1})+j_{t+1}\log\lambda^*_{t+1}/\lambda_{t+1}-j_{t+1}\log\theta^*/\theta-(\theta^{*-1}-\theta^{-1})\cdot z_s|j$. The expression could be simplified further:

$$\begin{aligned}\tilde{h}_{t+1}(s;\log p^*_{t+1}(z_s)/p_{t+1}(z_s)) &= \sum_{j=0}^{\infty}\frac{e^{-\lambda^*_{t+1}}}{j!}[\lambda^*_{t+1}\theta/\theta^*(1-s\theta(\theta^{-1}-\theta^{*-1}))^{-1}]^j\\ &= e^{\lambda^*_{t+1}(\theta/\theta^*(1-s\theta(\theta^{-1}-\theta^{*-1}))^{-1}-1)}.\end{aligned}$$

Second, we obtain the mgf by computing the expectation with respect to the distribution of the jump intensity:

$$h_t(s;\log p^*_{t+1}(z_s)/p_{t+1}(z_s)) = E_t\tilde{h}_{t+1}(s;\log p^*_{t+1}(z_s)/p_{t+1}(z_s)) \equiv E_t e^{\lambda^*_{t+1}f(s,\theta,\theta^*)}.$$

The cgf is

$$k_t(s;\log p^*_{t+1}(z_s)/p_{t+1}(z_s)) = \log E_t e^{\lambda^*_{t+1}f(s,\theta,\theta^*)} = k_t(f(s,\theta,\theta^*),\lambda^*_{t+1}).$$

Note that $k_t(-1;z_{m,t+1})$ corresponds to $k_t(1;\log p^*_{t+1}(z_s)/p_{t+1}(z_s))$, and $f(1,\theta,\theta^*)=0$, so $k_t(-1;z_{m,t+1})=0$.

Appendix F. Details of the estimation

Appendix F.1. State-space representation

State transition equation

We consider three interest rate factors $u_{1,t},u_{2,t},u_{3,t}$, three credit factors $g_{1,t},g_{2,t},g_{3,t}$, one volatility factor $g_{4,t}=v_t$, and one contagion factor d_t

$$\underbrace{\begin{bmatrix} u_{1,t+1} \\ u_{2,t+1} \\ u_{3,t+1} \end{bmatrix}}_{u_{t+1}} = \underbrace{\begin{bmatrix} \mu_{u1} \\ \mu_{u2} \\ \mu_{u3} \end{bmatrix}}_{\mu_u} + \underbrace{\begin{bmatrix} \phi_{u11} & \phi_{u12} & \phi_{u13} \\ \phi_{u21} & \phi_{u22} & \phi_{u23} \\ \phi_{u31} & \phi_{u32} & \phi_{u33} \end{bmatrix}}_{\Phi_u} \underbrace{\begin{bmatrix} u_{1,t} \\ u_{2,t} \\ u_{3,t} \end{bmatrix}}_{u_t} + \underbrace{\begin{bmatrix} \eta_{u1,t+1} \\ \eta_{u2,t+1} \\ \eta_{u3,t+1} \end{bmatrix}}_{\eta_{u,t+1}}$$

$$\underbrace{\begin{bmatrix} g_{1,t+1} \\ g_{2,t+1} \\ g_{3,t+1} \\ g_{4,t+1} \end{bmatrix}}_{g_{t+1}} = \underbrace{\begin{bmatrix} \nu_{g1}c_{g1} \\ \nu_{g2}c_{g2} \\ \nu_{g3}c_{g3} \\ \nu_{g4}c_{g4} \end{bmatrix}}_{\mu_g} + \underbrace{\begin{bmatrix} \phi_{g11} & \phi_{g12} & \phi_{g13} & \phi_{g14} \\ \phi_{g21} & \phi_{g22} & \phi_{g23} & \phi_{g24} \\ \phi_{g31} & \phi_{g32} & \phi_{g33} & \phi_{g34} \\ \phi_{g41} & \phi_{g42} & \phi_{g43} & \phi_{g44} \end{bmatrix}}_{\Phi_g} \underbrace{\begin{bmatrix} g_{1,t} \\ g_{2,t} \\ g_{3,t} \\ g_{4,t} \end{bmatrix}}_{g_t} + \underbrace{\begin{bmatrix} \eta_{g1,t+1} \\ \eta_{g2,t+1} \\ \eta_{g3,t+1} \\ \eta_{g4,t+1} \end{bmatrix}}_{\eta_{g,t+1}}$$

$$d_{t+1} = \mu_d + \delta_{d,g}\underbrace{\left(\mu_g + \Phi_g g_t + \eta_{g,t+1}\right)}_{g_{t+1}} + \Phi_d d_t + \eta_{d,t+1}$$

where $\eta_{u,t} \sim N(0,\Sigma_u\Sigma_u')$, $\eta_{g,t},\eta_{d,t},\eta_{\lambda,t}$ represent a martingale difference sequence (mean zero shock).

In vector notation, the joint dynamics are

$$\underbrace{\begin{bmatrix} u_{t+1} \\ g_{t+1} \\ d_{t+1} \end{bmatrix}}_{x_{t+1}} = \underbrace{\begin{bmatrix} \mu_u \\ \mu_g \\ \mu_d + \delta_{d,g}\mu_g \end{bmatrix}}_{\mu_x} + \underbrace{\begin{bmatrix} \Phi_u & 0 & 0 \\ 0 & \Phi_g & 0 \\ 0 & \delta_{d,g}\Phi_g & \Phi_d \end{bmatrix}}_{\Phi_x} \underbrace{\begin{bmatrix} u_t \\ g_t \\ d_t \end{bmatrix}}_{x_t} + \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \delta_{d,g} & 1 \end{bmatrix}}_{\Omega_x} \underbrace{\begin{bmatrix} \eta_{u,t+1} \\ \eta_{g,t+1} \\ \eta_{d,t+1} \end{bmatrix}}_{\eta_{x,t+1}}. \tag{F.1}$$

Here, we are assuming that we observe the sequence $u_{1,1:T}$ and $u_{2,1:T}$. Note that while $\mu_d, \delta_{d,g}, \Phi_d$, which govern the true dynamics, are estimated freely, we impose the following restrictions in the risk neutral dynamics.

$$\mu_d^* = \sum_{k=1}^{M_c} \bar{h}^{*,k}, \quad \delta_{d,g}^* = \begin{bmatrix} \sum_{k=1}^{M_c} \delta_{h,g1}^{*,k} & \sum_{k=1}^{M_c} \delta_{h,g2}^{*,k} & \sum_{k=1}^{M_c} \delta_{h,g3}^{*,k} & 0 \end{bmatrix}, \quad \Phi_d^* = \sum_{k=1}^{M_c} \delta_{h,d}^{*,k}.$$

Measurement equations

There are two forms of measurement equations. Denote observables in the first measurement equation and the second measurement equation by $y_{1,t}$ and $y_{2,t}$, respectively. Define $y_t = \{y_{1,t}, y_{2,t}\}$ and $Y_{1:t-1} = \{y_1, ..., y_{t-1}\}$.

The first measurement equation consists of quanto spreads of six different maturities for each country k

$$qs_t^k = \left\{ qs_{t,1y}^k, qs_{t,3y}^k, qs_{t,5y}^k, qs_{t,7y}^k, qs_{t,10y}^k, qs_{t,15y}^k \right\}^\top,$$

and the log ratio of the forward to the spot exchange rate

$$fs_t = \left\{ fs_{t,1w}, fs_{t,1m} \right\},$$

and the log depreciation USD/EUR rate. To ease exposition, define

$$A_{1:T}^k = \{A_1^k, ..., A_T^k\}, \quad B_{1:T}^k = \{B_1^k, ..., B_T^k\}$$

and $\tilde{A}_{1:T}^k, \tilde{B}_{1:T}^k$ are defined similarly. The model-implied quanto spread is a nonlinear function of the solution coefficients and the current and lagged states, which we express as

$$qs_{t,T}^k = \Xi(A_{1:T}^k, B_{1:T}^k, \tilde{A}_{1:T}^k, \tilde{B}_{1:T}^k, x_t).$$

Similarly, the model-implied log ratio of the forward to the spot exchange rate can be expressed as

$$fs_{t,T} = \Xi(A_{1:T}, B_{1:T}, \tilde{A}_{1:T}, \tilde{B}_{1:T}, x_t)$$

where the solution coefficients associated with the forward exchange rate valuation are expressed without any superscript.

Put together, the first measurement equation becomes

$$y_{1,t} = \begin{bmatrix} qs_t^1 \\ \vdots \\ qs_t^k \\ \vdots \\ qs_t^{M_c} \\ fs_t \\ \Delta s_t \end{bmatrix} = \begin{bmatrix} \Xi(A^1_{1:15y}, B^1_{1:15y}, \tilde{A}^1_{1:15y}, \tilde{B}^1_{1:15y}, x_t) \\ \vdots \\ \Xi(A^k_{1:15y}, B^k_{1:15y}, \tilde{A}^k_{1:15y}, \tilde{B}^k_{1:15y}, x_t) \\ \vdots \\ \Xi(A^{M_c}_{1:15y}, B^{M_c}_{1:15y}, \tilde{A}^{M_c}_{1:15y}, \tilde{B}^{M_c}_{1:15y}, x_t) \\ \Xi(A_{1w:1m}, B_{1w:1m}, \tilde{A}_{1w:1m}, \tilde{B}_{1w:1m}, x_t) \\ \bar{s} + \delta_s^\top x_t + \sqrt{v_{t-1}}\epsilon_{s,t} - z_{1,t} - z_{2,t} \end{bmatrix}. \tag{F.2}$$

The second measurement equation consists of credit events for each country $e_{k,t}$

$$y_{2,t} = \Big\{ e_{1,t}, ..., e_{M_c,t} \Big\}.$$

Instead of providing its measurement equation form, we directly express the likelihood function below.

Appendix F.2. Implementation

Likelihood function

We exploit the conditional independence between $y_{1,t}$ and $y_{2,t}$. We express $P(y_{1,t}, y_{2,t}|Y_{1:t-1}, \Theta)$

$$\begin{aligned} &= \int P(y_{1,t}, y_{2,t}|x_t, Y_{1:t-1}, \Theta) P(x_t|x_{t-1}, Y_{1:t-1}, \Theta) P(x_{t-1}|Y_{1:t-1}, \Theta) dx_{t-1} \qquad \text{(F.3)} \\ &= \int \underbrace{P(y_{2,t}|x_t, Y_{1:t-1}, \Theta)}_{(A)} \underbrace{P(y_{1,t}|x_t, Y_{1:t-1}, \Theta)}_{(B)} \underbrace{P(x_t|x_{t-1}, Y_{1:t-1}, \Theta)}_{(C)} P(x_t|Y_{1:t-1}, \Theta) dx_{t-1}, \end{aligned}$$

where (C) can be deduced from (F.1).

The likelihood function corresponding to (A) in (F.3) can be written as

$$P(y_{1,t}|x_t, Y_{1:t-1}, \Theta) = (2\pi)^{-n_1/2} |V_1|^{-1/2} \exp\bigg\{ -\frac{1}{2}(y_{1,t} - \hat{y}_{1,t})^\top V_1^{-1} (y_{1,t} - \hat{y}_{1,t}) \bigg\} \tag{F.4}$$

where n_1 is the dimensionality of the vector space, V_1 is a measurement error variance matrix, and $\hat{y}_{1,t}$ is from (F.2).

The likelihood function corresponding to (B) can be expressed as

$$P(y_{2,t}|x_t, Y_{1:t-1}, \Theta) = \exp\left(-M_c\lambda_t\right)\prod_{i=k}^{M_c}\left\{e_{k,t}\lambda_t + (1-e_{k,t})\right\},$$

following (Das et al., 2007).

Bayesian inference

For convenience, the parameters associated with the factors, hazard rates, exchange rate, and defaults are collected in Θ_g, Θ_h, Θ_s, Θ_d, Θ_l, Θ_u, respectively.

$$\begin{aligned}
\Theta_g &= \Big\{\{\phi^*_{g11}, \phi^*_{g21}, \phi^*_{g22}, \phi^*_{g31}, \phi^*_{g33}, \phi^*_{g44}\}, \{\phi_{g11}, \phi_{g21}, \phi_{g22}, \phi_{g31}, \phi_{g33}, \phi_{g44}\}, \ldots \\
&\quad \{\nu^*_{g1}, \nu^*_{g2}, \nu^*_{g3}, \nu^*_{g4}\}, \{c^*_{g1}, c^*_{g2}, c^*_{g3}, c^*_{g4}\}\Big\}, \\
\Theta_h &= \Big\{\{\bar{h}^{*,1}, \delta^{*,1}_{h,g1}, \delta^{*,1}_{h,d}\}, \{\bar{h}^{*,2}, \delta^{*,2}_{h,g1}, \delta^{*,2}_{h,g2}, \delta^{*,2}_{h,d}\}, \{\bar{h}^{*,3}, \delta^{*,3}_{h,g1}, \delta^{*,3}_{h,g2}, \delta^{*,3}_{h,d}\}, \ldots \\
&\quad \{\bar{h}^{*,4}, \delta^{*,4}_{h,g1}, \delta^{*,4}_{h,g3}, \delta^{*,4}_{h,d}\}, \{\bar{h}^{*,5}, \delta^{*,5}_{h,g1}, \delta^{*,5}_{h,g3}, \delta^{*,5}_{h,d}\}, \{\bar{h}^{*,6}, \delta^{*,6}_{h,g1}, \delta^{*,6}_{h,g3}, \delta^{*,6}_{h,d}\}, \ldots \\
&\quad \{\bar{h}^{*,7}, \delta^{*,7}_{h,g1}, \delta^{*,7}_{h,g3}, \delta^{*,7}_{h,d}\}\Big\}, \\
\Theta_s &= \Big\{\{\bar{s}^*, \delta^*_{s,3}, \delta^*_{s,7}, \theta^*_1, \frac{\theta^*_2}{\theta^*_1}\}, \{\bar{s}, \delta_{s,3}, \delta_{s,7}, \bar{v}, \delta_v, \theta_1\}\Big\}, \\
\Theta_d &= \Big\{\{\mu_d, \delta_{d,g1}\}, \{\Phi_d, \rho^*_d\}\Big\}, \\
\Theta_l &= \Big\{L\Big\}, \\
\Theta_u &= \Big\{\{\mu^*_{u3}, \phi^*_{u33}\}, \{\phi_{u33}\}\Big\}.
\end{aligned}$$

The number of parameters are as follows:

- The model with contagion has a total of 66 parameters $\#\Theta_g = 20, \#\Theta_h = 27, \#\Theta_s = 11, \#\Theta_d = 4, \#\Theta_l = 1, \#\Theta_u = 3$.
- The model without contagion has a total of 57 parameters $\#\Theta_g = 20, \#\Theta_h = 20, \#\Theta_s = 11, \#\Theta_d = 2, \#\Theta_l = 1, \#\Theta_u = 3$. Here, we are removing $\delta^{*,k}_{h,d}$ for $k \in \{1, \ldots, 7\}$ and $\{\Phi_d, \rho^*_d\}$.

It is important to mention that the following parameters associated with the interest rate factors

$$\Theta_u = \{\mu^*_{u1}, \mu^*_{u2}, \phi^*_{u11}, \phi^*_{u21}, \phi^*_{u22}, \bar{r}, \delta_{u1}, \delta_{u2}\}$$

are not estimated and provided from the first stage interest rate estimation. We use a Bayesian approach to make joint inference about parameters $\Theta = \{\Theta_g, \Theta_h, \Theta_s, \Theta_d, \Theta_l, \Theta_u\}$ and the latent state vector x_t in Equation (F.1). Bayesian inference requires the specification of a prior distribution $p(\Theta)$ and the evaluation of the likelihood function $p(Y|\Theta)$. Most of our priors are noninformative. We use MCMC methods to generate a sequence of draws $\{\Theta^{(j)}\}_{j=1}^{n_{sim}}$ from the posterior distribution $p(\Theta|Y) = \frac{p(Y|\Theta)p(\Theta)}{p(Y)}$. The numerical evaluation of the prior density and the likelihood function $p(Y|\Theta)$ is done with the particle filter.

Given (A), (B), (C), we use a particle-filter approximation of the likelihood function (F.3) and embed this approximation into a fairly standard random walk Metropolis algorithm. See Herbst and Schorfheide (2016) for a review of the particle filter.

In the subsequent exposition we omit the dependence of all densities on the parameter vector Θ. The particle filter approximates the sequence of distributions $\{p(x_t|Y_{1:t})\}_{t=1}^{T}$ by a set of pairs $\left\{x_t^{(i)}, \pi_t^{(i)}\right\}_{i=1}^{N}$, where $x_t^{(i)}$ is the ith particle vector, $\pi_t^{(i)}$ is its weight, and N is the number of particles. As a by-product, the filter produces a sequence of likelihood approximations $\hat{p}(y_t|Y_{1:t-1})$, $t = 1, \ldots, T$.

- Initialization: We generate the particle values $x_0^{(i)}$ from the unconditional distribution. We set $\pi_0^{(i)} = 1/N$ for each i.
- Propagation of particles: We simulate (F.1) forward to generate $x_t^{(i)}$ conditional on $x_{t-1}^{(i)}$. We use $q(x_t|x_{t-1}^{(i)}, y_t)$ to represent the distribution from which we draw $x_t^{(i)}$.
- Correction of particle weights: Define the unnormalized particle weights for period t as

$$\tilde{\pi}_t^{(i)} \quad = \quad \pi_{t-1}^{(i)} \times \frac{p(y_t|x_t^{(i)})p(x_t^{(i)}|x_{t-1}^{(i)})}{q(x_t^{(i)}|x_{t-1}^{(i)}, y_t)}.$$

The term $\pi_{t-1}^{(i)}$ is the initial particle weight and the ratio $\frac{p(y_t|x_t^{(i)})p(x_t^{(i)}|x_{t-1}^{(i)})}{q(x_t^{(i)}|x_{t-1}^{(i)}, y_t)}$ is the importance weight of the particle. The last equality follows from the fact that we chose $q(x_t^{(i)}|x_{t-1}^{(i)}, y_t) = p(x_t^{(i)}|x_{t-1}^{(i)})$.

The log likelihood function approximation is given by

$$\log \hat{p}(y_t|Y_{1:t-1}) = \log \hat{p}(y_{t-1}|Y_{1:t-2}) + \log\left(\sum_{i=1}^{N} \tilde{\pi}_t^{(i)}\right).$$

- Resampling: Define the normalized weights

$$\pi_t^{(i)} = \frac{\tilde{\pi}_t^{(i)}}{\sum_{j=1}^{N} \tilde{\pi}_t^{(j)}}$$

and generate N draws from the distribution $\{x_t^{(i)}, \pi_t^{(i)}\}_{i=1}^{N}$ using multinomial resampling. In slight abuse of notation, we denote the resampled particles and their weights also by $x_t^{(i)}$ and $\pi_t^{(i)}$, where $\pi_t^{(i)} = 1/N$.

Appendix G. Tables

Table G1. Parameter estimates: Model of the OIS term structure

In this table, we report the parameter estimates for the OIS term structure. The model is estimated using Bayesian MCMC. We report the posterior medians, as well as the 5th and 95th percentiles of the posterior distribution. The sample period is August 25, 2010 to September 26, 2018. We deal with missing data in the estimation procedure because not all OIS rates exist at the beginning of our sample period. For example, the 7-year maturity OIS rates are available after May 11, 2012 and the 15-year maturity OIS rates are available after September 27, 2011. The data frequency is weekly, based on Wednesday rates.

	5%	50%	95%		5%	50%	95%
μ^*_{u1}	0.1859	0.5422	0.8651	$\bar{r}$	0.0000	0.0001	0.0001
μ^*_{u2}	–0.6540	–0.3636	0.0464	δ_{u1}	0.0012	0.0015	0.0019
μ^*_{u3}	–0.1693	–0.1400	–0.1223	δ_{u2}	0.0014	0.0021	0.0025
ϕ^*_{u11}	0.9995	0.9999	0.9999	ϕ_{u11}	0.9561	0.9796	0.9921
ϕ^*_{u12}	–	–	–	ϕ_{u12}	–0.0045	–0.0013	0.0010
ϕ^*_{u21}	–0.0029	–0.0023	–0.0020	ϕ_{u21}	0.0102	0.0136	0.0156
ϕ^*_{u22}	0.9911	0.9918	0.9925	ϕ_{u22}	0.9980	0.9990	0.9999
ϕ^*_{u33}	0.9812	0.9900	0.9990	ϕ_{u33}	0.8660	0.9011	0.9575

Table G2. Parameter estimates: Model without contagion

We report the parameter estimates for the CDS quanto model without contagion. We report the posterior medians, as well as the 5th and 95th percentiles of the posterior distribution. In Panel A, we report estimates for the credit and volatility factors. In Panel B, we report estimates for the hazard rates. The superscripts in the default intensity parameters refer to countries in the following order: Germany, Belgium, France, Ireland, Italy, Spain, and Greece. In Panel C, we report the estimate for loss given default. In Panel D, we report estimates for the exchange rate dynamics. In Panel E, we report estimates for the aggregate physical default intensity. In Panel F, we provide the implied estimates and model simulation results.

	5%	50%	95%		5%	50%	95%
(A) factor dynamics							
ϕ^*_{11}	0.9987	0.9994	0.9999	ϕ_{11}	0.9522	0.9880	0.9964
ϕ^*_{21}	0.0045	0.0061	0.0066	ϕ_{21}	–0.0496	–0.0287	0.0021
ϕ^*_{22}	0.9971	0.9976	0.9992	ϕ_{22}	0.9167	0.9901	0.9995
ϕ^*_{31}	–0.0036	–0.0029	–0.0021	ϕ_{31}	0.0136	0.0540	0.1152
ϕ^*_{33}	0.9971	0.9974	0.9987	ϕ_{33}	0.8901	0.9411	0.9639
ϕ^*_{44}	0.9936	0.9942	0.9950	ϕ_{44}	0.9477	0.9694	0.9937
c^*_1	0.0033	0.0045	0.0066	ν_1	1.5432	1.7488	1.9234
c^*_2	0.0119	0.0138	0.0159	ν_2	1.8833	1.9902	2.1219
c^*_3	0.0079	0.0097	0.0115	ν_3	0.8282	0.8413	0.8633
c^*_4	0.0061	0.0067	0.0075	ν_4	2.5834	2.7045	2.9222
(B) hazard rates							
$10000 \times \bar{h}^{*1}$	0.3011	0.3585	0.4066	$10000 \times \bar{h}^{*2}$	0.0745	0.1236	0.1743
δ^{*1}_{w0}	0.0012	0.0016	0.0017	δ^{*2}_{w0}	0.0016	0.0018	0.0021
–	–	–	–	δ^{*2}_{w1}	0.0012	0.0013	0.0015
$10000 \times \bar{h}^{*3}$	0.4741	0.5919	0.7029	$10000 \times \bar{h}^{*4}$	0.1434	0.2087	0.2789
δ^{*3}_{w0}	0.0007	0.0008	0.0008	δ^{*4}_{w0}	0.0017	0.0018	0.0021
δ^{*3}_{w1}	0.0010	0.0012	0.0014	δ^{*4}_{w2}	0.0040	0.0043	0.0047
$10000 \times \bar{h}^{*5}$	0.3994	0.4890	0.5444	$10000 \times \bar{h}^{*6}$	0.4812	0.5957	0.6714
δ^{*5}_{w0}	0.0020	0.0022	0.0027	δ^{*6}_{w0}	0.0033	0.0036	0.0039
δ^{*5}_{w2}	0.0028	0.0030	0.0032	δ^{*6}_{w2}	0.0039	0.0043	0.0045
$10000 \times \bar{h}^{*7}$	0.2011	0.3022	0.4914	–	–	–	–
δ^{*7}_{w0}	0.0018	0.0029	0.0051	–	–	–	–
δ^{*7}_{w2}	0.0023	0.0057	0.0079	–	–	–	–
(C) loss given default							
L	0.4804	0.5925	0.7281				
(D) exchange rates							
$\bar{s}^*$	0.0086	0.0094	0.0106	$\bar{s}$	–0.0053	–0.0049	–0.0039
δ^*_{s3}	–0.0043	–0.0035	–0.0031	δ_{s3}	–0.0001	0.0004	0.0008
δ^*_{s7}	–0.0078	–0.0071	–0.0059	δ_{s7}	0.0010	0.0014	0.0015
–	–	–	–	$\bar{v}$	0.0000	0.0001	0.0002
–	–	–	–	δ_v	0.0000	0.0001	0.0002
θ^*_1	0.2113	0.2603	0.3031	θ_1	0.0210	0.0293	0.0334
(E) default intensity							
$10000 \times \bar{h}$	1.2020	4.2004	8.3729				
δ_{w0}	0.0012	0.0016	0.0021				
(F) derived quantities							
θ^*_2	0.0444	0.0724	0.1613	θ_2	0.0044	0.0082	0.0178
1w-$Prob^*_1$ twin Ds	0.8798	0.8988	0.9115	1w-$Prob_1$ twin Ds	0.3549	0.4180	0.4608
1w-$Prob^*_2$ twin Ds	0.4898	0.5497	0.5917	1w-$Prob_2$ twin Ds	0.0145	0.0227	0.0308
1y-$Prob^*_1$ twin Ds	0.0025	0.0120	0.0214	1y-$Prob_1$ twin Ds	0.0000	0.0001	0.0005
1y-$Prob^*_2$ twin Ds	0.0018	0.0100	0.0187	1y-$Prob_2$ twin Ds	0.0000	0.0001	0.0003

Table G3. Parameter estimates: Model with contagion

In this table, we report the parameter estimates for the CDS quanto model with contagion. The model is estimated using Bayesian MCMC. We report the posterior medians, as well as the 5th and 95th percentiles of the posterior distribution. In Panel A, we report estimates for the credit and volatility factors. In Panel B, we report estimates for the hazard rates. The superscripts in the default intensity parameters refer to countries in the following order: Germany, Belgium, France, Ireland, Italy, Spain, and Greece. In Panel C, we report the estimate for loss given default. In Panel D, we report estimates for the exchange rate dynamics. In Panel E, we report estimates for the aggregate physical default intensity. In Panel F, we provide the implied estimates and model simulation results.

	5%	50%	95%		5%	50%	95%
			(A) factor dynamics				
ϕ^*_{11}	0.9990	0.9995	0.9997	ϕ_{11}	0.9672	0.9860	0.9949
ϕ^*_{21}	0.0048	0.0050	0.0058	ϕ_{21}	–0.0174	0.0387	0.0680
ϕ^*_{22}	0.9975	0.9978	0.9983	ϕ_{22}	0.8885	0.9670	0.9834
ϕ^*_{31}	–0.0036	–0.0031	–0.0027	ϕ_{31}	–0.0179	0.0367	0.0668
ϕ^*_{33}	0.9972	0.9977	0.9984	ϕ_{33}	0.8645	0.9298	0.9721
ϕ^*_{44}	0.9965	0.9973	0.9991	ϕ_{44}	0.9032	0.9642	0.9818
c^*_1	0.0024	0.0046	0.0071	ν_1	1.5750	1.7353	1.8813
c^*_2	0.0091	0.0115	0.0137	ν_2	1.8624	2.0072	2.1751
c^*_3	0.0081	0.0091	0.0097	ν_3	0.7774	0.8537	0.9423
c^*_4	0.0064	0.0070	0.0076	ν_4	2.4123	2.6558	2.9251
			(B) hazard rates				
$10000 \times \bar{h}^{*1}$	0.2575	0.3161	0.4170	$10000 \times \bar{h}^{*2}$	0.1830	0.2003	0.2434
δ^{*1}_{w0}	0.0013	0.0014	0.0016	δ^{*2}_{w0}	0.0008	0.0011	0.0012
–	–	–	–	δ^{*2}_{w1}	0.0010	0.0016	0.0019
δ^{*1}_{d}	0.0011	0.0013	0.0023	δ^{*2}_{d}	0.0075	0.0102	0.0132
$10000 \times \bar{h}^{*3}$	0.3729	0.4478	0.5121	$10000 \times \bar{h}^{*4}$	0.1753	0.2019	0.2611
δ^{*3}_{w0}	0.0004	0.0007	0.0011	δ^{*4}_{w0}	0.0017	0.0021	0.0026
δ^{*3}_{w1}	0.0007	0.0009	0.0012	δ^{*4}_{w2}	0.0037	0.0040	0.0047
δ^{*3}_{d}	0.0001	0.0027	0.0108	δ^{*4}_{d}	0.0094	0.0122	0.0151
$10000 \times \bar{h}^{*5}$	0.4023	0.4523	0.5132	$10000 \times \bar{h}^{*6}$	0.3922	0.4239	0.4784
δ^{*5}_{w0}	0.0017	0.0021	0.0025	δ^{*6}_{w0}	0.0032	0.0038	0.0045
δ^{*5}_{w2}	0.0017	0.0022	0.0024	δ^{*6}_{w2}	0.0020	0.0030	0.0037
δ^{*5}_{d}	0.0100	0.0144	0.0184	δ^{*6}_{d}	0.0187	0.0249	0.0308
$10000 \times \bar{h}^{*7}$	0.0987	0.1872	0.2574				
δ^{*7}_{w0}	0.0019	0.0025	0.0040				
δ^{*7}_{w2}	0.0054	0.0065	0.0079				
δ^{*7}_{d}	0.0164	0.0201	0.0233				
			(C) loss given default				
L	0.5278	0.6352	0.8284				
			(D) exchange rates				
$\bar{s}^*$	0.0092	0.0097	0.0103	$\bar{s}$	–0.0055	–0.0049	–0.0046
δ^*_{s3}	–0.0043	–0.0041	-0.0039	δ_{s3}	-0.0005	-0.0000	0.0002
δ^*_{s7}	–0.0085	–0.0077	–0.0069	δ_{s7}	0.0007	0.0012	0.0016
–	–	–	–	$\bar{v}$	0.0000	0.0001	0.0002
–	–	–	–	δ_v	0.0001	0.0001	0.0002
θ^*_1	0.1226	0.1912	0.2372	θ_1	0.0190	0.0202	0.0255
			(E) default intensity				
$10000 \times \bar{h}$	4.7892	8.0172	10.7534	ρ^*	1.0619	1.4039	2.0572
δ_{w0}	0.0010	0.0012	0.0016	δ_d	0.0000	0.0002	0.0004
			(F) derived quantities				
θ^*_2	0.0465	0.0795	0.1075	θ_2	0.0072	0.0084	0.0116
1w-$Prob^*_1$ twin Ds	0.9034	0.9237	0.9475	1w-$Prob_1$ twin Ds	0.3320	0.3567	0.3975
1w-$Prob^*_2$ twin Ds	0.4451	0.4976	0.5405	1w-$Prob_2$ twin Ds	0.0212	0.0245	0.0287
1y-$Prob^*_1$ twin Ds	0.0062	0.0092	0.0147	1y-$Prob_1$ twin Ds	0.0000	0.0001	0.0003
1y-$Prob^*_2$ twin Ds	0.0054	0.0085	0.0102	1y-$Prob_2$ twin Ds	0.0000	0.0001	0.0002

Table G4. Model comparison

In this table, we report the distributions of the likelihoods of both models, and the associated Bayesian Information Criteria (negative of the likelihood plus penalty for the number of parameters). The model is estimated using Bayesian MCMC. We report the posterior medians, as well as the 5th and 95th percentiles of the posterior distribution. The model with the lowest Bayesian Information Criterion (BIC) is preferred.

	With contagion			Without contagion		
	5%	50%	95%	5%	50%	95%
$\ln p(Y\|\Theta)$	110510	111100	111650	110423	111039	111619
BIC	–111450	–110900	–110310	–111455	–110873	–110250

Appendix H. Figures

Fig. H1. Time series of credit events

These figures depict the time series of credit events for 16 eurozone countries that have a minimum of 365 days of non-zero information on USD-EUR quanto CDS spreads. Greece is omitted from this figure. In the absence of true credit events, we define them as occurrences when a 5-year quanto spread is above the 99th percentile of the country-specific distribution of quanto spread changes. The sample period is August 25, 2010 to September 26, 2018.

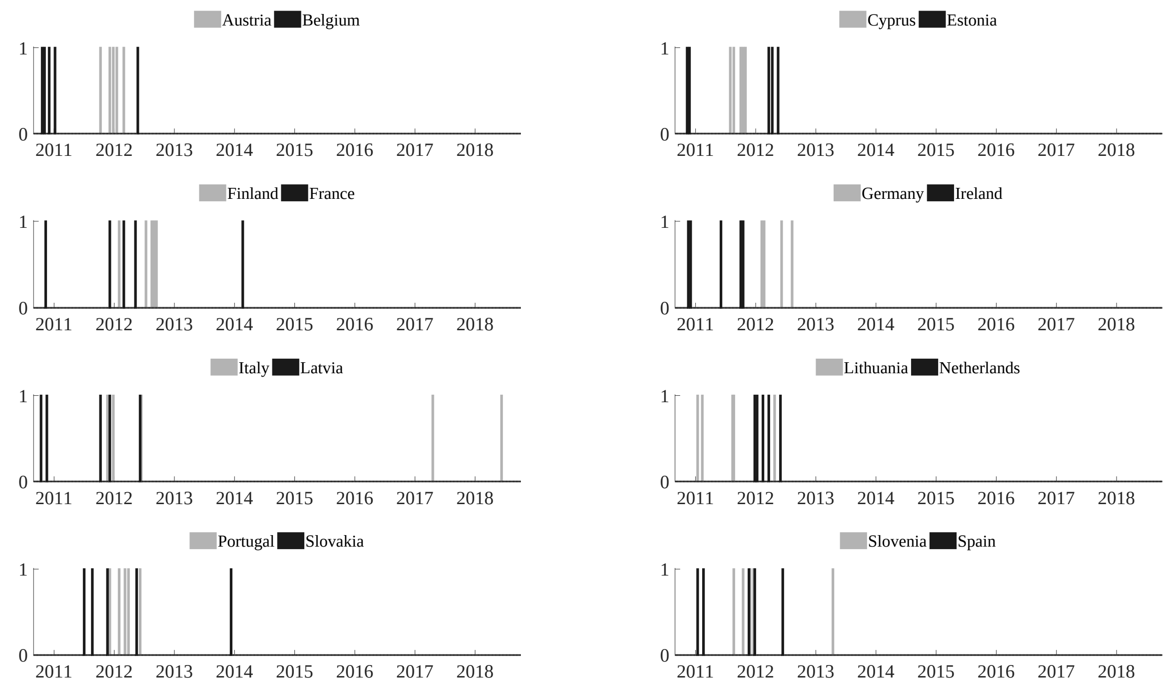
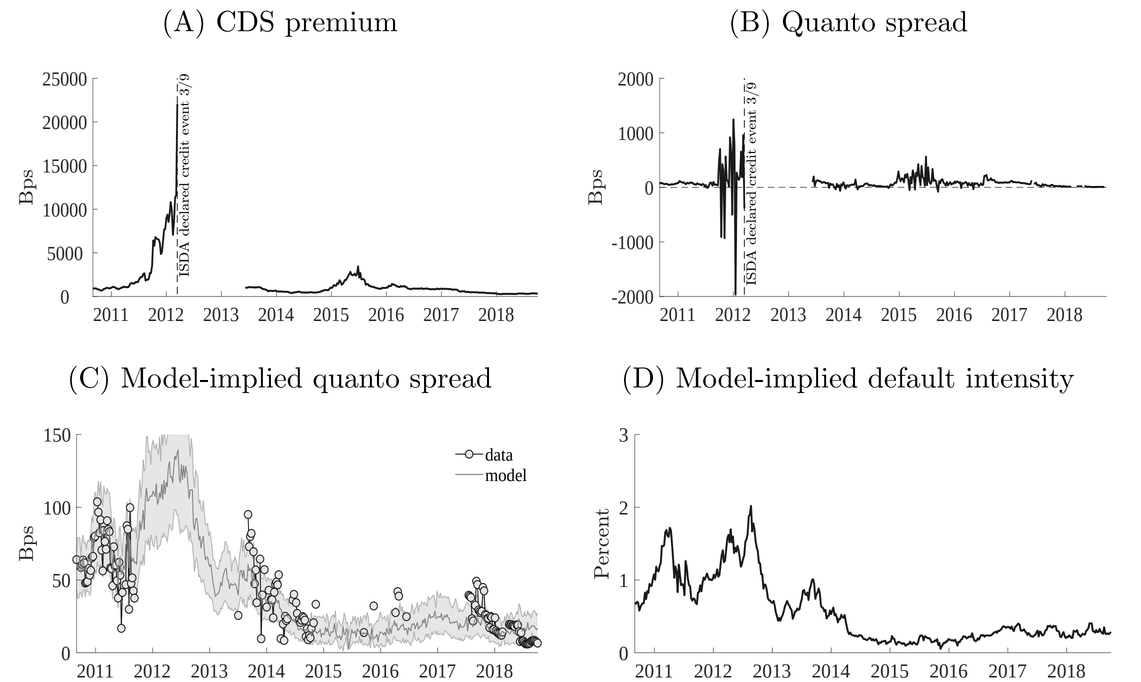


Fig. H2. Greece

In these figures, we plot the observed USD denominated CDS premium for Greece (A), the observed and model-implied USD/EUR quanto spreads for Greece (B and C). We report values for the 5-year maturity. Gray lines represent posterior medians of quanto spreads and gray-shaded areas correspond to 90% credible intervals. The true quanto spreads are plotted with black-circled lines. In Panel D, we provide an illustration of the model-implied default intensity for Greece. The sample period is August 25, 2010 to September 26, 2018.



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