Supplementary appendix for "Technical trading revisited: false discoveries, persistence tests, and transaction costs" by P. Bajgrowicz and O. Scaillet:

Six-month rebalancing and mean return results for the empirical study and mean return results for the Monte Carlo study

This appendix presents results of the persistence tests when we rebalance the portfolios every six months instead of monthly. It also gathers results when performance is measured with the mean return instead of the Sharpe ratio. Finally, it reports additional Monte Carlo results when performance is measured with the mean return instead of the Sharpe ratio.

Persistence analysis under the Sharpe ratio updating the rules every six months

[Tables 1 and 2]

Mean return results

We set the rule of a neutral position at all times (zero return, always out of the market) as the benchmark. Hence, the test statistic of rule k is simply its average return. Let y_t be the (arithmetic) period t return on the price series on which the strategies are applied. We use the same notation as STW and denote by $f_{k,t} = \ln(1 + s_{k,t-1}y_t)$ the (logarithmic) period t return generated by rule k, where $s_{k,t-1}$ is the investment signal of rule k. Then, the test statistic for the mean return criterion can be written as $\varphi_k = \bar{f}_k = \frac{1}{N}\sum_{t=L}^{T} f_{k,t+1}$, where $N = T - L + 1$ is the number of prediction periods. Tables 3 to 7 of this appendix correspond respectively to Tables 1 to 5 in the paper. We do not describe the mean return analysis in detail, as the results are qualitatively very similar, and lead to identical conclusions. The remarks in the paper concerning the Sharpe ratio criterion carry over to the mean return criterion. We can only mention that

even fewer rules are selected into the 10%–FDR$^+$ portfolio under the mean return criterion.

[Figure 1]

[Tables 3 to 7]

Monte Carlo results under the mean return criterion

Tables 8 to 10 of this appendix correspond respectively to Tables 7 to 9 in the paper. We have followed the same methodology as the one described in Appendix G of the paper. The only difference is that we set the values used to shift the trajectories of the selected outperforming and underperforming rules such as to match sensible levels of annualized mean returns instead of annualized Sharpe ratios. We choose three specific levels of outperformance, namely an annualized mean return equal to 10, 20, or 30 (in %), and three specific levels of underperformance, namely an annualized mean return equal to -10, -20, or -25 (in %). In our historical sample, we observe annualized mean returns as high as 38% for the outperforming rules, and as low as -57% for the underperforming rules. The positive annualized mean return of 10% corresponds to the 62th percentile of the distribution of observed positive mean returns in our sample. The negative annualized mean return of -10% corresponds to the 49th percentile of the distribution of observed negative mean returns. In order to provide an accurate idea of the alternative hypotheses, Table 8 reports the quartiles of the annualized Sharpe ratios of the outperforming and underperforming rules, for the 9 combinations of annualized mean returns. The remarks in Appendix G of the paper about the Monte Carlo study under the Sharpe ratio criterion carry over to the results under the mean return criterion reported in Tables 9 and 10.

[Tables 8 to 10]

	FDR portfolio			RW portfolio			50 best rules		Best rule	
Sample	IS	OOS	Med. size	IS	OOS	Med. size	IS	OOS	IS	OOS
1: 1897–1914	3.20	0.65	10	-	-	-	3.68	0.55	3.44	0.49
2: 1915–1938	2.62	0.12	9	-	-	-	3.11	0.33	2.89	-0.21
3: 1939–1962	3.37	0.80	15	1.02	0.54	0	3.46	0.65	3.59	1.25
4: 1962–1986	3.45	0.36	12	-	-	-	3.58	0.08	3.47	0.76
5: 1987–1996	2.29	0.55	4	-	-	-	3.06	0.16	2.84	-0.34
6: 1997–2010	2.29	-0.45	6	-	-	-	2.75	-0.15	2.58	0.15

Table 1: Performance persistence analysis under the Sharpe ratio criterion and with no transaction costs, six-month rebalancing. This table displays the in-sample (IS) and the out-of-sample (OOS) annualized Sharpe ratio of trading rules selected according to the following criteria and updated every six months: the 10%–FDR$^+$ portfolio, the RW portfolio, the 50 best rules in-sample, and the best rule in-sample. The table also reports the median size of the FDR and RW portfolios across the different rebalancings.

	FDR portfolio		RW portfolio		50 best rules	Best rule
	TC such that	Median	TC such that	Median	TC such that	TC such that
Sample	OOS perf. = 0	port. size	OOS perf. = 0	port. size	OOS perf. = 0	OOS perf. = 0
1: 1897–1914	30-35 bps	0	-	-	25-30 bps	25-30 bps
2: 1915–1938	5-10 bps	5	-	-	25-30 bps	-
3: 1939–1962	15-20 bps	2	0-5 bps	0	25-30 bps	30-35 bps
4: 1962–1986	5-10 bps	4	-	-	0-5 bps	5-10 bps
5: 1987–1996	5-10 bps	0	-	-	25-30 bps	-
6: 1997–2011	-	-	-	-	-	10-15 bps

Table 2: Transaction costs (TC) such that out-of-sample (OOS) performance disappears under the Sharpe ratio criterion, six-month rebalancing. This table reports the level of one-way transaction costs for which the OOS performance of different portfolios of trading rules becomes zero. As in the previous table, the rules are selected according to the following criteria and updated every six months: the 10%–FDR$^+$ portfolio, the RW portfolio, the 50 best rules in-sample, and the best rule in-sample. The table also displays the median size of the FDR and RW portfolios across the different rebalancings.

	RW portfolio		Best rule		DJIA
Sample	Mean return (%)	Port. size	Mean return (%)	BRC p-value	Mean return (%)
1: 1897–1914	16.3	32	19.0	0.00	2.0
2: 1915–1938	16.7	2	16.8	0.03	4.5
3: 1939–1962	12.0	46	25.3	0.00	5.9
4: 1962–1986	16.8	26	24.1	0.00	5.1
5: 1987–1996	-	0	13.8	0.13	14.2
6: 1997–2010	-	0	10.4	0.89	5.3
1897–1996	7.6	214	15.9	0.00	4.5

Table 3: Performance indicators from existing studies under the mean return criterion and with no transaction costs. This table presents long-term performance results of rules chosen according to the mean return criterion, across the different sample periods. The table reports the annualized mean return and size of the portfolio obtained with the method of RW, the annualized mean return and corresponding BRC p-value of the best rule in the sample, and the annualized mean return of the buy-and-hold strategy (DJIA).

Sample	TC such that $\widehat{\pi}_A^+ = 0$	Corresponding $\widehat{\pi}_A^-$
1: 1897–1914	30-35 bps	0%
2: 1915–1938	40-45 bps	22%
3: 1939–1962	70-75 bps	37%
4: 1962–1986	-	0%*
5: 1987–1996	45-50 bps	29%
6: 1997–2011	-	18%*

Table 4: Transaction costs (TC) such that the long-term in-sample performance disappears under the mean return criterion. This table presents one-way transaction costs in basis points (bps) such that $\widehat{\pi}_A^+$ becomes zero, across the different sample periods. It also displays the corresponding average $\widehat{\pi}_A^-$. An asterisk (*) indicates that π_A^- is estimated with zero transaction costs.

Sample	Costs	Best trading rule
1: 1897–1914	zero	5-day moving average, 5-day holding period*
	12.5 bps	5-day moving average, 5-day holding period
2: 1915–1938	zero	5-day moving average, 0.001 band*
	12.5 bps	30-day on-balance volume, 50-day holding period
3: 1939–1962	zero	2-day on-balance volume*
	12.5 bps	75- and 100-day on-balance volume, 50-day holding period
4: 1962–1986	zero	2-day moving average*
	12.5 bps	Filter rule, 0.01 position initiation
5: 1987–1996	zero	Filter rule, 0.1 position initiation, 25-day holding period
	12.5 bps	Filter rule, 0.1 position initiation, 25-day holding period
6: 1997–2011	zero	Filter rule, 0.01 position initiation, 25-day holding period
	12.5 bps	Filter rule, 0.01 position initiation, 25-day holding period

Table 5: Best in-sample technical trading rules under the mean return criterion. This table reports the historically best-performing trading rule chosen with respect to the mean return criterion, in each sample period, and for either zero or 12.5 bps one-way transaction costs. An asterisk (i.e., *) indicates that, according to the BRC, the performance of the rule remains significant after accounting for data snooping.

	FDR portfolio			RW portfolio			50 best rules		Best rule	
Sample	IS	OOS	Med. size	IS	OOS	Med. size	IS	OOS	IS	OOS
1: 1897–1914	35.5	3.5	0	11.2	-0.1	0	65.0	7.4	85.2	1.9
2: 1915–1938	60.9	-0.6	1	13.0	0.0	0	88.5	2.0	117.6	-3.7
3: 1939–1962	29.2	2.3	0	9.1	0.3	0	48.7	4.3	64.4	8.7
4: 1962–1986	43.5	1.1	1	13.6	-1.6	0	61.1	2.4	81.9	1.7
5: 1987–1996	28.2	-0.2	0	2.3	-0.7	0	51.7	-0.2	68.9	0.9
6: 1997–2010	38.9	-3.4	0	9.2	0.7	0	69.3	-4.9	93.0	-9.5

Table 6: Performance persistence analysis under the mean return criterion and with no transaction costs. This table displays the in-sample (IS) and the out-of-sample (OOS) annualized mean return (in %) of trading rules selected according to the following criteria and updated monthly: the 10%–FDR$^+$ portfolio, the RW portfolio, the 50 best rules in-sample, and the best rule in-sample. The table also reports the median size of the FDR and RW portfolios across the different rebalancings.

	FDR portfolio		RW portfolio		50 best rules	Best rule
Sample	TC such that OOS perf. = 0	Median port. size	TC such that OOS perf. = 0	Median port. size	TC such that OOS perf. = 0	TC such that OOS perf. = 0
1: 1897–1914	45-50 bps	0	-	-	20-25 bps	5-10 bps
2: 1915–1938	-	-	0-5 bps	0	0-5 bps	-
3: 1939–1962	5-10 bps	0	10-15 bps	0	10-15 bps	10-15 bps
4: 1962–1986	0-5 bps	1	-	-	5-10 bps	0-5 bps
5: 1987–1996	-	-	-	-	-	0-5 bps
6: 1997–2011	-	-	20-25 bps	0	-	-

Table 7: Transaction costs (TC) such that out-of-sample (OOS) performance disappears under the mean return criterion. This table reports the level of one-way transaction costs in basis points (bps) for which the OOS performance of different portfolios of trading rules becomes zero. As in the previous table, the rules are selected according to the following criteria and updated monthly: the 10%–FDR$^+$ portfolio, the RW portfolio, the 50 best rules in-sample, and the best rule in-sample. The table also displays the median size of the FDR and RW portfolios across the different rebalancings.

Outperf. mean		Underperf. mean -10%		-20%		-25%	
		Outperf. SR	Underperf. SR	Outperf. SR	Underperf. SR	Outperf. SR	Underperf. SR
10%	1st quartile	0.91 (0.72)	-0.86 (0.58)	0.95 (0.72)	-2.44 (0.66)	0.93 (0.75)	-3.27 (0.71)
	2nd quartile	1.97 (0.72)	-2.15 (0.59)	2.03 (0.78)	-3.88 (0.77)	2.01 (0.78)	-4.76 (0.84)
	3rd quartile	3.19 (0.72)	-3.47 (0.77)	3.25 (0.77)	-5.65 (1.10)	3.22 (0.74)	-6.74 (1.25)
20%	1st quartile	2.55 (0.76)	-0.82 (0.58)	2.53 (0.72)	-2.45 (0.64)	2.55 (0.74)	-3.23 (0.70)
	2nd quartile	3.77 (0.86)	-2.13 (0.62)	3.71 (0.83)	-3.87 (0.74)	3.73 (0.83)	-4.71 (0.89)
	3rd quartile	5.39 (1.05)	-3.47 (0.78)	5.31 (1.04)	-5.61 (1.04)	5.32 (1.02)	-6.70 (1.32)
30%	1st quartile	4.11 (0.77)	-0.80 (0.61)	4.11 (0.76)	-2.40 (0.61)	4.08 (0.79)	-3.19 (0.68)
	2nd quartile	5.45 (0.91)	-2.13 (0.63)	5.39 (0.94)	-3.82 (0.75)	5.38 (0.93)	-4.72 (0.82)
	3rd quartile	7.60 (1.46)	-3.46 (0.77)	7.46 (1.42)	-5.54 (1.06)	7.51 (1.43)	-6.68 (1.19)

Table 8: Distribution of annualized Sharpe ratios corresponding to chosen mean return levels. This table displays the quartiles of the distribution of annualized Sharpe ratios induced by setting the mean return levels in the Monte Carlo simulations. The values correspond to averages over 1,000 Monte Carlo simulations. Numbers in parentheses are standard deviations. The different settings correspond to all the combinations of outperforming rules having a positive annualized mean return equal 10, 20, or 30 (in %), and underperforming rules having a negative annualized mean return equal to -10, -20, or -25 (in %). The proportions of outperforming (π_A^+), underperforming (π_A^-), and zero performance (π_0) rules in the population are set to respectively 20, 30, and 50 (in %).

		Underperf. mean		
Outperf. mean		-10%	-20%	-25%
10%	$\pi_0 = 50$	71.2 (8.4)	61.6 (5.9)	59.0 (6.7)
	$\pi_A^+ = 20$	8.7 (9.2)	8.8 (7.4)	9.2 (7.3)
	$\pi_A^- = 30$	20.1 (11.0)	29.5 (5.8)	31.8 (5.5)
20%	$\pi_0 = 50$	64.7 (7.4)	54.3 (4.9)	52.6 (5.7)
	$\pi_A^+ = 20$	16.7 (6.4)	16.5 (4.7)	16.3 (4.8)
	$\pi_A^- = 30$	18.6 (9.7)	29.3 (5.0)	31.1 (4.7)
30%	$\pi_0 = 50$	62.7 (8.4)	52.6 (5.4)	50.5 (6.2)
	$\pi_A^+ = 20$	19.1 (6.5)	18.6 (4.1)	18.6 (4.2)
	$\pi_A^- = 30$	18.2 (9.2)	28.8 (5.3)	30.9 (4.7)

Table 9: Average estimates of the proportions of null, outperforming, and underperforming rules, under the mean return ratio criterion. This table presents the average over 1,000 Monte Carlo simulations of the FDR estimates of π_0, π_A^+, and π_A^-. The true values are set respectively to 50, 20 and 30 (in %). Numbers in parentheses correspond to standard deviations (in %). The results are provided for the 9 combinations of outperforming rules annualized mean returns set to 10, 20, or 30 (in %), and underperforming rules annualized mean returns set to -10, -20, or -25 (in %).

Outperf. mean		Underperf. mean -10% FDR$^+$	Power	Port. size	-20% FDR$^+$	Power	Port. size	-25% FDR$^+$	Power	Port. size
10%	10%–FDR$^+$ port.	12.9 (9.7)	26.2 (14.2)	494 (318)	12.4 (8.4)	27.6 (15.0)	511 (315)	12.3 (8.8)	28.7 (15.6)	532 (325)
	20%–FDR$^+$ port.	14.0 (10.6)	33.6 (17.3)	649 (410)	13.6 (9.2)	36.1 (18.7)	684 (409)	13.5 (9.5)	37.6 (19.0)	711 (416)
	5%–RW port.	2.2 (9.7)	0.8 (2.0)	14 (41)	1.5 (7.4)	0.7 (1.9)	11 (35)	2.5 (10.7)	0.8 (2.3)	15 (49)
	20%–RW port.	5.2 (12.4)	2.7 (4.3)	52 (93)	4.1 (10.7)	2.7 (4.6)	50 (94)	4.6 (10.8)	2.9 (4.9)	53 (101)
20%	10%–FDR$^+$ port.	8.0 (7.9)	64.8 (15.9)	1,119 (317)	7.7 (7.1)	67.3 (15.4)	1,158 (305)	7.6 (6.6)	67.5 (15.1)	1,157 (295)
	20%–FDR$^+$ port.	10.6 (9.7)	74.5 (14.6)	1,331 (335)	10.1 (8.5)	77.8 (13.7)	1,376 (306)	10.3 (8.2)	77.7 (13.6)	1,376 (302)
	5%–RW port.	0.6 (3.5)	5.0 (6.7)	81 (109)	0.6 (3.0)	5.4 (7.4)	87 (122)	0.4 (2.4)	4.9 (6.6)	79 (106)
	20%–RW port.	1.7 (5.6)	15.9 (12.2)	259 (207)	1.5 (4.7)	15.9 (12.9)	257 (218)	1.2 (3.8)	15.0 (12.2)	242 (202)
30%	10%–FDR$^+$ port.	7.2 (7.6)	90.3 (7.8)	1,540 (200)	6.5 (5.9)	91.8 (7.5)	1,547 (164)	6.9 (6.2)	91.8 (7.6)	1,554 (164)
	20%–FDR$^+$ port.	10.2 (9.5)	94.7 (5.7)	1,676 (231)	9.6 (7.5)	95.9 (5.0)	1,677 (169)	10.0 (7.8)	96.0 (5.3)	1,685 (172)
	5%–RW port.	0.3 (1.8)	28.6 (18.2)	452 (289)	0.1 (1.2)	26.0 (16.9)	409 (267)	0.2 (1.4)	27.2 (17.4)	429 (276)
	20%–RW port.	0.9 (3.1)	61.6 (17.7)	977 (286)	0.5 (1.9)	60.7 (17.8)	958 (284)	0.7 (2.2)	60.4 (17.9)	956 (285)

Table 10: Power and size of the FDR approach and of the method of RW under the mean return criterion. This table examines the composition of the 10%– and 20%–FDR$^+$ portfolio, and of the 5%– and 20%–RW portfolio. It reports average values over 1,000 Monte Carlo simulations for the true false discovery rate (in %), the percentage of true outperforming rules detected (in %), and the portfolio size. Numbers in parentheses correspond to standard deviations (in %). The different settings correspond to all the combinations of outperforming rules having a positive annualized mean return equal to 10, 20, or 30 (in %), and underperforming rules having a negative annualized mean return equal to -10, -20, or -25 (in %).

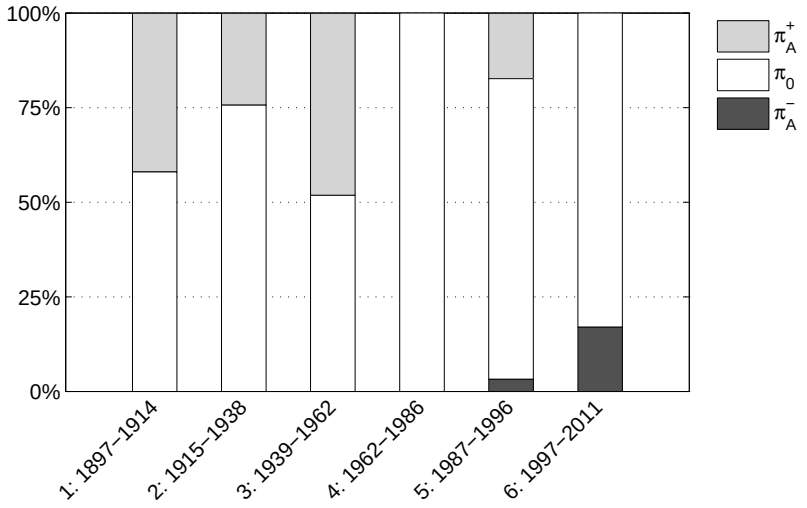

Fig. 1: Proportions of outperforming (π_A^+), null (π_0), and underperforming (π_A^-) rules, under the mean return criterion and with no transaction costs. This figure displays estimates of π_A^+, π_0, and π_A^-, across the different sample periods.