Internet Appendix to "Blockholder Voting"*

In this appendix we provide proofs to PROPOSITION 5 (Active Blockholder), PROPOSITION 6 (Active Blockholder, symmetric shareholder strategies), PROPOSITION 7 (Possibly biased blockholder), and PROPOSITION 8 (Asymmetric priors) stated in the main text of *Blockholder Voting*.

1 Proof of Proposition 5 (Active Blockholder)

We begin by proving that equilibrium type (i) and equilibrium type (ii) exist. We then prove they are the most informative equilibria for given parameters.

Equilibrium type (i): If $b \geq b^*$, we need to check that there is an equilibrium in which the blockholder votes sincerely with $2b^*$ shares and all shareholders vote sincerely. Because this is sequential, we first take it as given that the blockholder voted $2b^*$ shares for choice X.

An individual shareholder's vote is consequential when the vote is otherwise tied. This may arise when the blockholder's signal is accurate and occurs with probability $q\frac{2n!}{(n-b^*)!(n+b^*)!}p^{n-b^*}(1-p)^{n+b^*}$ or when the blockholder's signal is inaccurate and occurs with probability $(1-q)\frac{2n!}{(n+b^*)!(n-b^*)!}p^{n+b^*}(1-p)^{n-b^*}$.

It follows that the shareholder prefers voting sincerely in opposition to the blockholder rather than voting in line with the blockholder in opposition to its signal when:

$$p\left[(1-q)\frac{2n!}{(n+b^*)!(n-b^*)!}p^{n+b^*}(1-p)^{n-b^*}\right] \geq (1-p)\left[q\frac{2n!}{(n-b^*)!(n+b^*)!}p^{n-b^*}(1-p)^{n+b^*}\right],$$

or, equivalently, $\left(\frac{p}{1-p}\right)^{2b^*+1} > \frac{q}{1-q.}$, which is true from the definition of b^* in Proposition 1.[1]

Finally, given the subsequent behavior of shareholders, the blockholder prefers to vote sincerely with $2b^*$ shares, as this maximizes the common payoff of the game (as in Nitzan and Paroush (1982)).

Equilibrium type (ii): We must prove two things.

First, we must demonstrate that if a number $2b^* - 2b$ of shareholders are voting with the blockholder, and all other shareholders are voting sincerely, then a shareholder will vote sincerely.

Again, to establish this, it is useful to write down the probability that the vote is tied and the blockholder is correct. This involves $n + b$ votes with the

[1]It is simple to show that this also implies that the shareholder prefers voting sincerely to abstention. Similarly, this will be the case for related arguments in this proof.

Of course, this analysis depends on inferences that shareholders draw from the blockholder's communication of voting intentions. Note that the blockholder might make an off-equilibrium statement that she intends to abstain with all her votes; in this case, we assume passive beliefs (i.e., the blockholder's signal is equally likely to be of either type).

blockholder and $n+b$ against. The $n+b$ votes with the blockholder consist of $n-b$ votes from shareholders and $2b$ votes from the blockholder. The $n+b$ votes against the blockholder must come from those shareholders who vote sincerely, and there are $2n-(2b^*-2b)$ of these.[2] Thus, this probability is $q\frac{(2n+2b-2b^*)!}{(n+b-2b^*)!(n+b)!}p^{n+b-2b^*}(1-p)^{n+b}$. Similarly, the probability that the vote is tied and the blockholder is wrong is $(1-q)\frac{(2n+2b-2b^*)!}{(n+b)!(n+b-2b^*)!}p^{n+b}(1-p)^{n+b-2b^*}$. It follows that the shareholder prefers voting sincerely and in opposition to the blockholder to voting in line with the blockholder when:

$$\begin{aligned} & p\left[(1-q)\frac{(2n+2b-2b^*)!}{(n+b)!(n+b-2b^*)!}p^{n+b}(1-p)^{n+b-2b^*}\right] \\ \geq \; & (1-p)\left[q\frac{(2n+2b-2b^*)!}{(n+b-2b^*)!(n+b)!}p^{n+b-2b^*}(1-p)^{n+b}\right], \end{aligned}$$

or equivalently, $\left(\frac{p}{1-p}\right)^{2b^*+1} \geq \frac{q}{1-q}$, which follows from the definition of b^* in Proposition 1.

Second, we must demonstrate that if $2b^*-2b-1$ other shareholders are voting with the blockholder and all other shareholders are voting sincerely, then a shareholder will vote with the blockholder. Once again, we can write the probability that the vote is tied without this shareholder and the blockholder is correct. Here again, this requires $n+b$ shareholders who vote sincerely to vote in opposition to the blockholder, but in this case, there are $2n-(2b^*-2b)+1$ of these. This allows us to write this probability as $q\frac{(2n+2b-2b^*+1)!}{(n+b-2b^*+1)!(n+b)!}p^{n+b-2b^*+1}(1-p)^{n+b}$. Similarly, the probability that the vote is tied and the blockholder is wrong is $(1-q)\frac{(2n+2b-2b^*+1)!}{(n+b)!(n+b-2b^*+1)!}p^{n+b}(1-p)^{n+b-2b^*+1}$. It follows that the shareholder prefers voting in line with the blockholder over voting sincerely when:

$$\begin{aligned} & (1-p)\left[q\frac{(2n+2b-2b^*+1)!}{(n+b-2b^*+1)!(n+b)!}p^{n+b-2b^*+1}(1-p)^{n+b}\right] \\ \geq \; & p\left[(1-q)\frac{(2n+2b-2b^*+1)!}{(n+b)!(n+b-2b^*+1)!}p^{n+b}(1-p)^{n+b-2b^*+1}\right], \end{aligned}$$

[2] Here, we suppose that $n > b^*-b$ for convenience, the case in which $n \leq b^*-b$ involves all agents voting with the blockholder, and it can be established by similar arguments.

or equivalently, $\frac{q}{1-q} \geq \left(\frac{p}{1-p}\right)^{2b^*}$. This again follows from the definition of b^* in Proposition 1.

Finally, given the subsequent behavior of shareholders, the blockholder prefers to vote sincerely her $2b$ shares, as this maximizes the common payoff of the game (as in Nitzan and Paroush (1982)).

Unique most informative equilibrium:

When $b > b^*$, following the arguments in the proof of Proposition 1, it is immediate that the equilibrium in this proposition is the most informative equilibrium.

When $b < b^*$, the arguments above about equilibrium type (ii) make it clear that there is no equilibrium in which the blockholder votes all of her shares sincerely, fewer than $2b^* - 2b$ shareholders ignore their signal and vote with the blockholder, and the rest of the shareholders vote sincerely (since a shareholder who is voting sincerely would prefer to switch to vote with the blockholder).

Next, we rule out conjectured equilibria in which the blockholder does not vote sincerely or abstains. The most informationally efficient scenario one could imagine is where all shareholders vote sincerely. Following the logic of Proposition 1, if all shareholders vote sincerely, then it would be more informationally efficient if the blockholder would prefer to vote sincerely with all $2b$ of her shares (whether this were an equilibrium or not). Following the logic above in demonstrating that the behavior characterized in (ii) is an equilibrium, such an outcome is informationally dominated by an equilibrium of type (ii).

The only possible candidates remaining are an equilibrium that takes the form of (a) the blockholder voting sincerely z shares and fewer than z shareholders voting (sincerely or otherwise) or (b) shareholders voting regardless of their signal $z > 2b$ net[3] shares for proposal $J = \{M, A\}$ and fewer than $z - 2b$ other shares are cast sincerely.

Equilibria that take the form of (a) are informationally equivalent to the equilibrium in which only the blockholder votes. While in the non-active (passive) blockholder case there were conditions under which this was the most informative equilibrium, when there is an active blockholder, this is not the case. This is because, here, the blockholder's information will be

[3] Net means that if x shareholders are voting for M regardless of their signal and y shareholders are voting A regardless of their signal, $z = x - y > 0$ are voting for M regardless of their signal.

reflected in $2b^*$ votes despite the fact that she only has $2b$ votes, e.g., when q approaches 1, all of the shareholders will be voting with the blockholder.

Equilibria that take the form of (b) are informationally inferior to the equilibrium in part (ii) of this proposition. Therefore, the equilibrium in part (ii) is the most informative equilibrium.

Finally, it is immediate that it is possible to construct equilibria in which the blockholder votes in a directly opposite fashion to her own signal (with a corresponding number of shareholders ignoring their information and always voting in exactly the opposite direction and other shareholders voting sincerely, say). As above, such an outcome can never lead to a vote outcome that reflects the state more accurately than the strategies described in the statement of the proposition.

2 Proof of Proposition 6 (Active Blockholder, symmetric shareholder strategies)

Without loss of generality (given that the two states are equally likely), we suppose that in the conjectured equilibrium (which we prove exists below), the blockholder receives an m signal, announces her signal, and votes $2b$ shares for M.

First, it is trivial given the behavior of shareholders that the blockholder communicates her signal truthfully.

Shareholders observe the announcement (which we assume throughout this proof to be m) and their signal. If they receive an m signal, they vote for M (in line with both their own and the blockholder's signal) with probability 1. If they receive an a signal, they vote A (in line with their own and against the blockholder) with probability y and M with probability $1-y$. Given this strategy, we define the probability that a shareholder votes for M provided that the state is M (and after observing the blockholder announce m):

$$\gamma_M := p + (1-p)(1-y) = 1 - (1-p)y.$$

Similarly, define the probability that a shareholder votes for A provided that the state is A (and after observing the blockholder announce m) as:

$$\gamma_A := py.$$

The shareholder's choice

Define a pivotal probability from the perspective of a shareholder by $\pi(\theta)$, where there are an even number of votes (there are $2n$ other shareholders and $2b$ blockholder votes) split equally between the two choices, and θ represents the state (A or M).

The probability of being pivotal with an equal number of votes for each choice and the state being M (and after observing the blockholder announce m) is:

$$\pi(M) = \left(\begin{array}{c} 2n \\ n-b \end{array} \right) \gamma_M^{n-b}(1-\gamma_M)^{n+b}.$$

The probability of being pivotal with an equal number of votes for each choice and the state being A (and after observing the blockholder announce m) is:

$$\pi(A) = \left(\begin{array}{c} 2n \\ n-b \end{array} \right) (1-\gamma_A)^{n-b}{\gamma_A}^{n+b}.$$

To prove that the conjectured equilibrium is indeed an equilibrium, we consider a shareholder's incentive constraints for each signal.

The shareholder receives an a signal

We begin with the shareholder receiving an a signal. We construct a function $H(y)$ that captures the value of a shareholder voting A rather than voting M (conditioning on the behavior of the blockholder and other shareholders employing the strategies of the conjectured equilibrium):

$$H(y) = \frac{\frac{1}{2}(1-q)p}{\frac{1}{2}(1-q)p + \frac{1}{2}q(1-p)} \left[\pi(A)\right] - \frac{\frac{1}{2}q(1-p)}{\frac{1}{2}(1-q)p + \frac{1}{2}q(1-p)} \left[\pi(M)\right],$$

where $\frac{\frac{1}{2}(1-q)p}{\frac{1}{2}(1-q)p+\frac{1}{2}q(1-p)}$ is the probability that the state is A provided that the shareholder has observed the blockholder's signal and her own signal.

The sign of $H(y)$ is the same as the sign of (where we drop the factors $\frac{1}{(1-q)p+q(1-p)}$ and $\left(\begin{array}{c} 2n \\ n-b \end{array} \right)$ that appear throughout):

$$\begin{aligned} \tilde{H}(y) &= (1-q)p(1-\gamma_A)^{n-b}{\gamma_A}^{n+b} - q(1-p)\gamma_M^{n-b}(1-\gamma_M)^{n+b} \\ &= (1-q)p(1-py)^{n-b}(py)^{n+b} - q(1-p)(1-(1-p)y)^{n-b}((1-p)y)^{n+b} \end{aligned}$$

Note that $\tilde{H}(0) = 0$.

For $y = 1$,

$$\tilde{H}(1) = (1-q)(1-p)^{n-b}p^{n+b+1} - qp^{n-b}(1-p)^{n+b+1},$$

which is negative given that $\frac{p}{1-p}^{2b+1} < \frac{q}{(1-q)}$ for $b < b^*$.

Taking the derivative of $\tilde{H}(y)$ with respect to y:

$$\begin{aligned}
\tilde{H}'(y) &= -(1-q)p^2(n-b)(1-py)^{n-b-1}(py)^{n+b} + (1-q)p^2(n+b)(1-py)^{n-b}(py)^{n+b-1} \\
&\quad +q(1-p)^2(n-b)(1-(1-p)y)^{n-b-1}((1-p)y)^{n+b} \\
&\quad -q(1-p)^2(n+b)(1-(1-p)y)^{n-b}((1-p)y)^{n+b-1} \\
&= y^{n+b-1}\left[\begin{array}{c} (1-q)p^2(1-py)^{n-b-1}p^{n+b-1}(n+b-2npy)+ \\ q(1-p)^2(1-(1-p)y)^{n-b-1}(1-p)^{n+b-1}(2n(1-p)y-n+b)) \end{array}\right]
\end{aligned}$$

The derivative $\tilde{H}'(y)$ has the same sign as the term in square brackets since $y \in [0,1]$.

At $y = 0$, the term in brackets can be written as:

$$\begin{aligned}
&\left[(1-q)p^{n+b+1}(n+b) + q(1-p)^{n+b+1}(-(n+b))\right] \\
= \; &(n+b)((1-q)p^{n+b+1} - q(1-p)^{n+b+1})
\end{aligned}$$

If $(\frac{p}{1-p})^{n+b+1} > \frac{q}{1-q}$, this expression is positive. Therefore, it is positive for a large enough n (formally, $n > 2b^* - b$) given the definition of b^*.

Therefore, for $\varepsilon \to 0$, $\tilde{H}'(\varepsilon) > 0$. We showed above that $\tilde{H}(0) = 0$ and $\tilde{H}(1) < 0$. This implies that for a large enough n, $\tilde{H}(y) = 0$ for at least one $y \in (0,1)$.[4]

The shareholder receives an m signal

We define a function $J(y)$ that represents the value of voting for M rather than A in case of an m signal when the blockholder announces m that is given by:

$$J(y) = \frac{\frac{1}{2}qp}{\frac{1}{2}(1-q)(1-p) + \frac{1}{2}qp}\left[\pi(M)\right] - \frac{\frac{1}{2}(1-q)(1-p)}{\frac{1}{2}(1-q)(1-p) + \frac{1}{2}qp}\left[\pi(A)\right],$$

[4] Note that for n small, it is not surprising that this may not be an equilibrium. Consider the case in which $q \approx 1$, $b = 1$ and $p \approx 0.5$. A very large n would be required to perform better than the blockholder, and instead it may be better to simply abstain or vote for M.

where $\frac{\frac{1}{2}qp}{\frac{1}{2}(1-q)(1-p)+\frac{1}{2}qp}$ represents the probability that the state is M provided that both the blockholder and the shareholder received m signals. We remove the terms $\frac{\frac{1}{2}}{\frac{1}{2}(1-q)(1-p)+\frac{1}{2}qp}$ and $\begin{pmatrix} 2n \\ n-b \end{pmatrix}$ to simplify and write:

$$\tilde{J}(y) = qp\left[\pi(M)\right] - (1-q)(1-p)\left[\pi(A)\right].$$

From above, we are focusing on the y that makes $\tilde{H}(y) = 0$. At this y, the expression $\tilde{H}(y) = 0$ implies that $\pi(A) = \frac{q(1-p)}{(1-q)p}\left[\pi(M)\right]$. Therefore, we can rewrite $\tilde{J}(y)$ as follows:

$$\begin{aligned} \tilde{J}(y) &= qp\left[\pi(M)\right] - (1-q)(1-p)\left[\pi(A)\right] \\ &= \pi(M)\left[qp - (1-q)(1-p)\frac{q(1-p)}{(1-q)p}\right] \\ &= \pi(M)[\frac{1}{p}q\,(2p-1)] \end{aligned}$$

The pivotal probability is positive and so is the expression in square brackets since $p > \frac{1}{2}$. Therefore, the shareholder strictly prefers to vote for M when observing an m signal.

The blockholder

We now prove that the blockholder votes with all of her shares $2b < 2b^*$ holding fixed the symmetric mixed strategy of the shareholders.

Voting all of her shares for the signal received: Provided that the blockholder received an m signal, we first check that she prefers to vote all of her shares for M rather than some other number of shares for M. Suppose that the blockholder votes $2\tilde{b}$ shares; then, her payoff per share owned is:

$$V(n,\tilde{b},p,q) = q\sum_{i=0}^{n+\tilde{b}}\begin{pmatrix} 2n+1 \\ i \end{pmatrix}\gamma_M^{2n+1-i}(1-\gamma_M)^i + (1-q)\sum_{i=0}^{n-\tilde{b}}\begin{pmatrix} 2n+1 \\ i \end{pmatrix}\gamma_A^{2n+1-i}(1-\gamma_A)^i$$

The blockholder prefers to vote 2 more shares for M iff:

$$V(n,\tilde{b},p,q) < V(n,\tilde{b}+1,p,q)$$

$$\iff \begin{bmatrix} q\sum_{i=0}^{n+\tilde{b}}\begin{pmatrix} 2n+1 \\ i \end{pmatrix}\gamma_M^{2n+1-i}(1-\gamma_M)^i + \\ (1-q)\sum_{i=0}^{n-\tilde{b}}\begin{pmatrix} 2n+1 \\ i \end{pmatrix}\gamma_A^{2n+1-i}(1-\gamma_A)^i \end{bmatrix} < \begin{bmatrix} q\sum_{i=0}^{n+\tilde{b}+1}\begin{pmatrix} 2n+1 \\ i \end{pmatrix}\gamma_M^{2n+1-i}(1-\gamma_M)^i + \\ (1-q)\sum_{i=0}^{n-\tilde{b}-1}\begin{pmatrix} 2n+1 \\ i \end{pmatrix}\gamma_A^{2n+1-i}(1-\gamma_A)^i \end{bmatrix}$$

$$\iff (1-q)\gamma_A^{2n+1-(n-\tilde{b})}(1-\gamma_A)^{n-\tilde{b}} < q\gamma_M^{2n+1-(n+\tilde{b}+1)}(1-\gamma_M)^{n+\tilde{b}+1}$$

$$\iff \frac{py^{n+\tilde{b}+1}(1-py)^{n-\tilde{b}}}{(1-(1-p)y)^{n-\tilde{b}}((1-p)y)^{n+\tilde{b}+1}} < \frac{q}{1-q}$$

$$\iff \frac{p^{n+\tilde{b}+1}}{(1-p)^{n+\tilde{b}+1}}\frac{(1-py)^{n-\tilde{b}}}{(1-(1-p)y)^{n-\tilde{b}}} < \frac{q}{1-q}$$

We know from above that $\tilde{H}(y) = 0$ implies that:

$$(1-q)p(1-py)^{n-b}(py)^{n+b} - q(1-p)(1-(1-p)y)^{n-b}((1-p)y)^{n+b} = 0$$

$$\iff \frac{p}{(1-p)}\frac{(1-py)^{n-b}(py)^{n+b}}{(1-(1-p)y)^{n-b}((1-p)y)^{n+b}} = \frac{q}{1-q}$$

Using this to substitute for $\frac{q}{1-q}$ in the inequality above yields:

$$\frac{p^{n+\tilde{b}}}{(1-p)^{n+\tilde{b}}}\frac{(1-py)^{b-\tilde{b}}}{(1-(1-p)y)^{b-\tilde{b}}} < \frac{(py)^{n+b}}{((1-p)y)^{n+b}}$$

At $b = \tilde{b}$, the LHS and RHS are equal.

Note that $\frac{p^{n+\tilde{b}}}{(1-p)^{n+\tilde{b}}}$ is increasing in $\tilde{b}$.

We now prove that $\frac{(1-py)^{b-\tilde{b}}}{(1-(1-p)y)^{b-\tilde{b}}}$ is increasing in $\tilde{b}$. This proves the LHS is lower than the RHS for $\tilde{b} < b$, which proves that the blockholder would indeed prefer to vote all of her shares (conditional on voting M).

To prove that $\frac{(1-py)^{b-\tilde{b}}}{(1-(1-p)y)^{b-\tilde{b}}}$ is increasing in $\tilde{b}$, we examine its derivative with respect to $\tilde{b}$:

$$\begin{aligned}\frac{d}{d\tilde{b}}\frac{(1-py)^{b-\tilde{b}}}{(1-(1-p)y)^{b-\tilde{b}}} &= \frac{(1-py)^{b-\tilde{b}}\{\ln(1-(1-p)y) - \ln(1-py)\}}{(1-(1-p)y)^{b-\tilde{b}}} \\ &= \frac{(1-py)^{b-\tilde{b}}\{\ln\frac{(1-(1-p)y)}{(1-py)}\}}{(1-(1-p)y)^{b-\tilde{b}}}\end{aligned}$$

The sign of this depends on the size of the ratio $\frac{(1-(1-p)y)}{(1-py)}$. This is positive since $p > \frac{1}{2}$, which makes the derivative positive, which proves that the blockholder wants to vote all of her b shares for M, not just some of them.

Voting shares for the signal NOT received: Provided that the blockholder received an m signal, we next check that she does not prefer to vote any of her shares for A.[5] Suppose that the blockholder votes $2\tilde{b}$ shares for A; her payoff per share owned is:

$$V_A(n,\tilde{b},p,q) = q\sum_{i=0}^{n-\tilde{b}} \binom{2n+1}{i} \gamma_M^{2n+1-i}(1-\gamma_M)^i + (1-q)\sum_{i=0}^{n+\tilde{b}} \binom{2n+1}{i} \gamma_A^{2n+1-i}(1-\gamma_A)^i$$

The blockholder prefers to vote 2 fewer shares for A iff:

$$V_A(n,\tilde{b},p,q) > V_A(n,\tilde{b}+1,p,q)$$

$$\iff \begin{bmatrix} q\sum_{i=0}^{n-\tilde{b}} \binom{2n+1}{i} \gamma_M^{2n+1-i}(1-\gamma_M)^i + \\ (1-q)\sum_{i=0}^{n+\tilde{b}} \binom{2n+1}{i} \gamma_A^{2n+1-i}(1-\gamma_A)^i \end{bmatrix} > \begin{bmatrix} q\sum_{i=0}^{n-\tilde{b}-1} \binom{2n+1}{i} \gamma_M^{2n+1-i}(1-\gamma_M)^i \\ +(1-q)\sum_{i=0}^{n+\tilde{b}+1} \binom{2n+1}{i} \gamma_A^{2n+1-i}(1-\gamma_A)^i \end{bmatrix}$$

$$\iff q\gamma_M^{n+\tilde{b}+1}(1-\gamma_M)^{n-\tilde{b}} > (1-q)\gamma_A^{n-\tilde{b}}(1-\gamma_A)^{n+\tilde{b}+1}$$

$$\iff \frac{q}{1-q} > \frac{(py)^{n-\tilde{b}}(1-py)^{n+\tilde{b}+1}}{(1-(1-p)y)^{n+\tilde{b}+1}((1-p)y)^{n-\tilde{b}}}$$

$$\iff \frac{q}{1-q} > \frac{p^{n-\tilde{b}}}{(1-p)^{n-\tilde{b}}}\frac{(1-py)^{n+\tilde{b}+1}}{(1-(1-p)y)^{n+\tilde{b}+1}}$$

Given that $1-py < 1-(1-p)y$, the second fraction of the RHS of the inequality is smaller than 1. We also know that $\frac{q}{1-q} > \frac{p^{n-\tilde{b}}}{(1-p)^{n-\tilde{b}}}$. This implies that the blockholder wants to vote as few votes for A as possible when she has an m signal, i.e., zero.

[5] While we write this as if the blockholder is only voting some of her shares for A and abstaining on the remainder, she could also vote an equal number of M and A on the remainder of the shares. Thus, our analysis applies to the net shares voted for A.

3 Proof of Proposition 7 (Possibly biased blockholder)

The behavior of the biased blockholder is trivial.

It remains to consider the other optimality conditions and show that there are values b_a, b_m, μ that satisfy the relevant system of incentive constraints.

Shareholder who receives the m signal.

The probability that the last vote is pivotal provided that the state is M is given by:

$$\begin{aligned}\pi(M) &= \beta \binom{2n}{n-b} (p\mu)^{n-b}(1-p\mu)^{n+b} + (1-\beta)q \binom{2n}{n-b_m} (p\mu)^{n-b_m}(1-p\mu)^{n+b_m} \\ &\quad +(1-\beta)(1-q) \binom{2n}{n+b_a} (p\mu)^{n+b_a}(1-p\mu)^{n-b_a}\end{aligned}$$

where the three terms correspond to the possibilities that the blockholder is biased, is unbiased and received an m signal, and is unbiased and received an a signal.

Similarly, the probability that the last vote is pivotal provided that the state is A is given by:

$$\begin{aligned}\pi(A) &= \beta \binom{2n}{n-b} ((1-p)\mu)^{n-b}(1-(1-p)\mu)^{n+b} \\ &\quad +(1-\beta)(1-q) \binom{2n}{n-b_m} ((1-p)\mu)^{n-b_m}(1-(1-p)\mu)^{n+b_m} \\ &\quad +(1-\beta)q \binom{2n}{n+b_a} ((1-p)\mu)^{n+b_a}(1-(1-p)\mu)^{n-b_a}.\end{aligned}$$

We construct a function $Z_m(\mu)$ that captures the value of a shareholder voting M rather than voting A when receiving the m signal (conditioning on the behavior of the blockholder and other shareholders employing the strategies of the conjectured equilibrium). We leave out a factor of $\frac{1}{2}$ for simplicity:

$$\begin{aligned} Z_m(\mu) &= p\pi(M) - (1-p)\pi(A) \\ &= \beta\mu^{n-b}\left(\begin{array}{c} 2n \\ n-b \end{array}\right)\left[p^{n-b+1}(1-p\mu)^{n+b} - (1-p)^{n-b+1}(1-(1-p)\mu)^{n+b}\right] \\ &\quad +(1-\beta)\mu^{n-b_m}\left(\begin{array}{c} 2n \\ n-b_m \end{array}\right)\left[qp^{n-b_m+1}(1-p\mu)^{n+b_m} - (1-q)(1-p)^{n-b_m+1}(1-(1-p)\mu)^{n+b_m}\right] \\ &\quad +(1-\beta)\mu^{n+b_a}\left(\begin{array}{c} 2n \\ n+b_a \end{array}\right)\left[(1-q)p^{n+b_a+1}(1-p\mu)^{n-b_a} - q(1-p)^{n+b_a+1}(1-(1-p)\mu)^{n-b_a}\right]. \end{aligned}$$

Shareholder indifference when observing an m signal requires that

$$Z_m(\mu) = 0 \tag{IA1}$$

We note that $Z_m(0) = 0$. Furthermore,

$$\begin{aligned} Z_m(1) &= \beta\left(\begin{array}{c} 2n \\ n-b \end{array}\right)\left[p^{n-b+1}(1-p)^{n+b} - (1-p)^{n-b+1}p^{n+b}\right] \\ &\quad +(1-\beta)\left(\begin{array}{c} 2n \\ n-b_m \end{array}\right)\left[qp^{n-b_m+1}(1-p)^{n+b_m} - (1-q)(1-p)^{n-b_m+1}p^{n+b_m}\right] \\ &\quad +(1-\beta)\left(\begin{array}{c} 2n \\ n+b_a \end{array}\right)\left[(1-q)p^{n+b_a+1}(1-p)^{n-b_a} - q(1-p)^{n+b_a+1}p^{n-b_a}\right]. \end{aligned}$$

The first term is strictly negative given that $p > \frac{1}{2}$. The second term is negative given that in the conjectured equilibrium, $b_m \geq b^*$. The third term is negative given that $b_a < b^*$. Therefore, $Z_m(1) < 0$.

Next, we examine $Z'_m(\varepsilon)$ when ε is close to zero. We begin by finding

$Z'_m(\mu)$.

$$\begin{aligned}
Z'_m(\mu) &= \mu^{n-b-1}\{\beta(n-b)\binom{2n}{n-b}\left[p^{n-b+1}(1-p\mu)^{n+b}-(1-p)^{n-b+1}(1-(1-p)\mu)^{n+b}\right] \\
&+\beta\mu\binom{2n}{n-b}\left[-(n+b)p^{n-b+2}(1-p\mu)^{n+b-1}+(n+b)(1-p)^{n-b+2}(1-(1-p)\mu)^{n+b-1}\right] \\
&+(1-\beta)(n-b_m)\mu^{b-b_m}\binom{2n}{n-b_m}\left[\begin{array}{c} qp^{n-b_m+1}(1-p\mu)^{n+b_m} \\ -(1-q)(1-p)^{n-b_m+1}(1-(1-p)\mu)^{n+b_m}\end{array}\right] \\
&+(1-\beta)\mu^{b-b_m+1}\binom{2n}{n-b_m}\left[\begin{array}{c} -(n+b_m)qp^{n-b_m+2}(1-p\mu)^{n+b_m-1} \\ +(n+b_m)(1-q)(1-p)^{n-b_m+2}(1-(1-p)\mu)^{n+b_m-1}\end{array}\right] \\
&+(1-\beta)(n+b_a)\mu^{b+b_a}\binom{2n}{n+b_a}\left[\begin{array}{c} (1-q)p^{n+b_a+1}(1-p\mu)^{n-b_a} \\ -q(1-p)^{n+b_a+1}(1-(1-p)\mu)^{n-b_a}\end{array}\right] \\
&+(1-\beta)\mu^{b+b_a+1}\binom{2n}{n+b_a}\left[\begin{array}{c} -(n-b_a)(1-q)p^{n+b_a+2}(1-p\mu)^{n-b_a-1} \\ -(n-b_a)q(1-p)^{n+b_a+2}(1-(1-p)\mu)^{n-b_a-1}\end{array}\right]\}.
\end{aligned}$$

Now, when ε is close to 0, $Z'_m(\varepsilon)$ has the same sign as:

$$\left[p^{n-b+1}-(1-p)^{n-b+1}\right]>0$$

As this is positive, we have proven that there is an interior $\mu\in(0,1)$ such that $Z_m(\mu)=0$.

Shareholder who receives the a signal.

Note that if $Z_m(\mu)=0$, then the gain from voting A rather than M when observing an a signal is positive, since it is given by $p\pi(A)-(1-p)\pi(M)=\pi(A)\left(p-\frac{(1-p)^2}{p}\right)>0$, where the equality follows from substituting for $\pi(M)$ from $Z_m(\mu)=0$ and the inequality from noting that $p>1-p$.

Unbiased Blockholder with m signal:

We now prove that the blockholder votes with a number of shares $2b_m>2b^*$ holding fixed the symmetric mixed strategy of the shareholders. We define the probability that a shareholder votes for M provided that the state is M:

$$\gamma_M:=p\mu.$$

Similarly, define the probability that a shareholder votes for A provided that the state is A as:

$$\gamma_A:=p+(1-p)(1-\mu).$$

Suppose that the blockholder votes $2\tilde{b}$ shares; then, her payoff per share owned is:

$$V_m(n,\tilde{b},p,q) = q\sum_{i=0}^{n+\tilde{b}} \binom{2n+1}{i} \gamma_M^{2n+1-i}(1-\gamma_M)^i + (1-q)\sum_{i=0}^{n-\tilde{b}} \binom{2n+1}{i} \gamma_A^{2n+1-i}(1-\gamma_A)^i.$$

We define

$$b_m := \arg\max_{\tilde{b}} V_m(n,\tilde{b},p,q) \tag{IA2}$$

The blockholder prefers to vote 2 more shares for M iff:

$$V_m(n,\tilde{b},p,q) < V_m(n,\tilde{b}+1,p,q)$$

$$\iff \left[\begin{array}{c} q\sum_{i=0}^{n+\tilde{b}} \binom{2n+1}{i} \gamma_M^{2n+1-i}(1-\gamma_M)^i + \\ (1-q)\sum_{i=0}^{n-\tilde{b}} \binom{2n+1}{i} \gamma_A^{2n+1-i}(1-\gamma_A)^i \end{array} \right] < \left[\begin{array}{c} q\sum_{i=0}^{n+\tilde{b}+1} \binom{2n+1}{i} \gamma_M^{2n+1-i}(1-\gamma_M)^i + \\ (1-q)\sum_{i=0}^{n-\tilde{b}-1} \binom{2n+1}{i} \gamma_A^{2n+1-i}(1-\gamma_A)^i \end{array} \right]$$

$$\iff (1-q)\gamma_A^{2n+1-(n-\tilde{b})}(1-\gamma_A)^{n-\tilde{b}} < q\gamma_M^{2n+1-(n+\tilde{b}+1)}(1-\gamma_M)^{n+\tilde{b}+1}$$

$$\iff \frac{(p+(1-p)(1-\mu))^{n+\tilde{b}+1}((1-p)\mu)^{n-\tilde{b}}}{(p\mu)^{n-\tilde{b}}(1-p\mu)^{n+\tilde{b}+1}} < \frac{q}{1-q}$$

$$\iff \frac{(1-p)^{n-\tilde{b}}}{p^{n-\tilde{b}}} \frac{(p+(1-p)(1-\mu))^{n+\tilde{b}+1}}{(1-p\mu)^{n+\tilde{b}+1}} < \frac{q}{1-q}$$

The term $\frac{(p+(1-p)(1-\mu))^{n+\tilde{b}+1}}{(1-p\mu)^{n+\tilde{b}+1}}$ is increasing in μ. Setting $\mu = 1$, the LHS is $\frac{(p)^{2\tilde{b}+1}}{(1-p)^{2\tilde{b}+1}}$. Therefore, the condition holds for $\tilde{b} < b^*$.

Unbiased Blockholder with $a-$signal:

We now prove that the blockholder votes with a number of shares $2b_a < 2b^*$ holding fixed the symmetric mixed strategy of the shareholders. We define the probability that a shareholder votes for M provided that the state is M:

$$\gamma_M := p\mu.$$

Similarly, define the probability that a shareholder votes for A provided that the state is A as:

$$\gamma_A := p + (1-p)(1-\mu) = 1-(1-p)\mu.$$

Suppose that the blockholder votes $2\tilde{b}$ shares; then, her payoff per share owned is:

$$V_a(n,\tilde{b},p,q) = q\sum_{i=0}^{n+\tilde{b}} \binom{2n+1}{i} \gamma_A^{2n+1-i}(1-\gamma_A)^i + (1-q)\sum_{i=0}^{n-\tilde{b}} \binom{2n+1}{i} \gamma_M^{2n+1-i}(1-\gamma_M)^i$$

Given this definition, we can define:

$$b_a := \arg\max_{\tilde{b}} V_a(n,\tilde{b},p,q) \tag{IA3}$$

The blockholder prefers to vote 2 fewer shares for A iff:

$$V_a(n,\tilde{b},p,q) > V_a(n,\tilde{b}+1,p,q)$$

$$\iff \left[\begin{array}{c} q\sum_{i=0}^{n+\tilde{b}} \binom{2n+1}{i} \gamma_A^{2n+1-i}(1-\gamma_A)^i + \\ (1-q)\sum_{i=0}^{n-\tilde{b}} \binom{2n+1}{i} \gamma_M^{2n+1-i}(1-\gamma_M)^i \end{array}\right] > \left[\begin{array}{c} q\sum_{i=0}^{n+\tilde{b}+1} \binom{2n+1}{i} \gamma_A^{2n+1-i}(1-\gamma_A)^i + \\ (1-q)\sum_{i=0}^{n-\tilde{b}-1} \binom{2n+1}{i} \gamma_M^{2n+1-i}(1-\gamma_M)^i \end{array}\right]$$

$$\iff (1-q)\gamma_M^{2n+1-(n-\tilde{b})}(1-\gamma_M)^{n-\tilde{b}} > q\gamma_A^{2n+1-(n+\tilde{b}+1)}(1-\gamma_A)^{n+\tilde{b}+1}$$

$$\iff \frac{(p\mu)^{n+\tilde{b}+1}(1-p\mu)^{n-\tilde{b}}}{(1-(1-p)\mu)^{n-\tilde{b}}((1-p)\mu)^{n+\tilde{b}+1}} > \frac{q}{1-q}$$

$$\iff \left(\frac{p}{1-p}\right)^{n+\tilde{b}+1} \frac{(1-p\mu)^{n-\tilde{b}}}{(1-(1-p)\mu)^{n-\tilde{b}}} > \frac{q}{1-q}$$

The term $\frac{(1-p\mu)^{n-\tilde{b}}}{(1-(1-p)\mu)^{n-\tilde{b}}}$ is decreasing in μ. Setting $\mu = 1$, the LHS is $(\frac{p}{1-p})^{2\tilde{b}+1}$. This satisfies the condition, which implies that $b_a < b^*$.

4 Proof of Proposition 8 (Asymmetric priors)

The analysis is clearly related to the proof of Proposition 6 and we abuse notation slightly by adopting the notation of the proof directly. We draw this analogy and reproduce a key step in the proof.

Given the hypothesized strategy, we define the probability that a shareholder votes for M given the state is M:

$$\gamma_M := \mu + (1-\mu)p$$

Similarly, define the probability that a shareholder votes for A given the state is A as:

$$\gamma_A := (1-\mu)p$$

The probability of being pivotal with an equal number of votes for each choice and the state being M is:

$$\pi(M) = q\begin{pmatrix} 2n \\ n-b \end{pmatrix}\gamma_M^{n-b}(1-\gamma_M)^{n+b} + (1-q)\begin{pmatrix} 2n \\ n-b \end{pmatrix}\gamma_M^{n+b}(1-\gamma_M)^{n-b}.$$

Similarly, the probability of being pivotal with an equal number of votes for each choice and the state being A is:

$$\pi(A) = q\begin{pmatrix} 2n \\ n-b \end{pmatrix}\gamma_A^{n-b}(1-\gamma_A)^{n+b} + (1-q)\begin{pmatrix} 2n \\ n-b \end{pmatrix}\gamma_A^{n+b}(1-\gamma_A)^{n-b}$$

Shareholder who receives an $a-$signal:

Consider the shareholder who receives an a-signal. We construct a function $G(\mu)$ that captures the value of a shareholder voting A rather than voting M (conditioning on the behavior of the blockholder and other shareholders employing the strategies of the conjectured equilibrium):

$$G(\mu) = \frac{(1-r)p}{(1-r)p + r(1-p)}[\pi(A)] - \frac{r(1-p)}{(1-r)p + r(1-p)}[\pi(M)],$$

where $\frac{(1-r)p}{(1-r)p+r(1-p)}$ is the probability that the state is A given the shareholder has observed her own a-signal and that the probability that state is M is r.

The sign of $G(\mu)$ has the same as the sign of $\widetilde{G}(\mu)$ as defined below (where we drop the factors of $\frac{1}{(1-r)p+r(1-p)}$ and $\begin{pmatrix} 2n \\ n-b \end{pmatrix}$ that appear throughout.):

$$\begin{aligned}
\widetilde{G}(\mu) &= (1-r)pq\gamma_A^{n-b}(1-\gamma_A)^{n+b} + (1-r)p(1-q)\gamma_A^{n+b}(1-\gamma_A)^{n-b} \\
&\quad -r(1-p)q\gamma_M^{n-b}(1-\gamma_M)^{n+b} - r(1-p)(1-q)\gamma_M^{n+b}(1-\gamma_M)^{n-b}. \\
&= (1-r)p\gamma_A^{n-b}(1-\gamma_A)^{n-b}\left[q(1-\gamma_A)^{2b} + (1-q)(\gamma_A)^{2b}\right] \\
&\quad -r(1-p)\gamma_M^{n-b}(1-\gamma_M)^{n-b}\left[q(1-\gamma_M)^{2b} + (1-q)(\gamma_M)^{2b}\right].
\end{aligned}$$

It remains to show that $\widetilde{G}(\mu) = 0$ has an interior solution.

At $\mu = 0$, $\gamma_M = \gamma_A = p$ and

$$\widetilde{G}(0) = [(1-r)p - r(1-p)]\left[qp^{n-b}(1-p)^{n+b} + (1-q)p^{n+b}(1-p)^{n-b}\right] < 0$$

where the inequality follows on noting that the first square bracket is negative since $r > p$.

At $\mu = 1$, $\gamma_M = 1$ and $\gamma_A = 0$, $\widetilde{G}(1) = 0$. We take an approach that is analogous to the proof of Proposition 6; we demonstrate that for small enough ε, $\widetilde{G}'(1-\varepsilon) < 0$ and consequently there must be an interior solution for $G(\mu) = 0$.

We first take the derivative of $G(\mu)$ and substitute for γ_A and γ_M (where we simplify using $1 - \gamma_M = (1-\mu)(1-p)$):

$$\begin{aligned}
\widetilde{G}'(\mu) \;=\; & -(1-r)p^2\{[(n-b)(1-\mu)^{n-b-1}p^{n-b-1}(1-(1-\mu)p)^{n-b} \\
& -(n-b)(1-\mu)^{n-b}p^{n-b}(1-(1-\mu)p)^{n-b-1}]\left[q\left(1-(1-\mu)p\right)^{2b} + (1-q)(1-\mu)^{2b}p^{2b}\right] \\
& +(1-\mu)^{n-b}p^{n-b}(1-(1-\mu)p)^{n-b}\left[\begin{array}{c} -2bq\left(1-(1-\mu)p\right)^{2b-1} \\ +2b(1-q)(1-\mu)^{2b-1}p^{2b-1} \end{array}\right]\} \\
& -r(1-p)^2\{[(n-b)(1-(1-\mu)(1-p))^{n-b-1}(1-\mu)^{n-b}(1-p)^{n-b} \\
& -(n-b)(1-(1-\mu)(1-p))^{n-b}(1-\mu)^{n-b-1}(1-p)^{n-b-1}] \\
& *\left[\begin{array}{c} q(1-\mu)^{2b}(1-p)^{2b} \\ +(1-q)\left(1-(1-\mu)(1-p)\right)^{2b} \end{array}\right] \\
& +(1-(1-\mu)(1-p))^{n-b}(1-\mu)^{n-b}(1-p)^{n-b}\left[\begin{array}{c} -2bq(1-\mu)^{2b-1}(1-p)^{2b-1} \\ +2b(1-q)\left(1-(1-\mu)(1-p)\right)^{2b-1} \end{array}\right]\}
\end{aligned}$$

Rewriting:

$$\begin{aligned}
\widetilde{G}'(\mu) \;=\; & (1-\mu)^{n-b-1}[-(1-r)p^2\{[(n-b)p^{n-b-1}(1-(1-\mu)p)^{n-b} \\
& -(n-b)(1-\mu)p^{n-b}(1-(1-\mu)p)^{n-b-1}]\left[q\left(1-(1-\mu)p\right)^{2b} + (1-q)(1-\mu)^{2b}p^{2b}\right] \\
& +(1-\mu)p^{n-b}(1-(1-\mu)p)^{n-b}\left[-2bq\left(1-(1-\mu)p\right)^{2b-1} + 2b(1-q)(1-\mu)^{2b-1}p^{2b-1}\right]\} \\
& -r(1-p)^2\{[(n-b)(1-(1-\mu)(1-p))^{n-b-1}(1-\mu)(1-p)^{n-b} \\
& -(n-b)(1-(1-\mu)(1-p))^{n-b}(1-p)^{n-b-1}]\left[\begin{array}{c} q(1-\mu)^{2b}(1-p)^{2b} \\ +(1-q)\left(1-(1-\mu)(1-p)\right)^{2b} \end{array}\right] \\
& +(1-(1-\mu)(1-p))^{n-b}(1-\mu)(1-p)^{n-b}\left[\begin{array}{c} -2bq(1-\mu)^{2b-1}(1-p)^{2b-1} \\ +2b(1-q)\left(1-(1-\mu)(1-p)\right)^{2b-1} \end{array}\right]\}]
\end{aligned}$$

The derivative $\widetilde{G}'(\mu)$ has the same sign as the terms excluding the $(1-\mu)^{n-b-1}$, for $\mu \in [0,1)$. At $\mu = 1$, the terms can be written as:

$$(n-b)[-(1-r)p^{n-b+1}q + r(1-p)^{n-b+1}(1-q)]$$

This expression is negative iff $\frac{r}{1-r} < (\frac{p}{1-p})^{n-b+1}(\frac{q}{1-q})$, which is true given that $q > r, \frac{p}{1-p} > 1$, and $n > b$. Therefore $\widetilde{G}'(1-\varepsilon)$ is negative and there is an interior solution for the mixing probability; that is, $\mu \in (0,1)$.

Shareholder who receives an $m-$signal:

To verify that a shareholder with an m-signal should vote for M, note that the value of a shareholder voting M rather than voting A (conditioning on the behavior of the blockholder and other shareholders employing the strategies of the conjectured equilibrium) is:

$$\frac{rp}{rp+(1-r)(1-p)}\left[\pi(M)\right] - \frac{(1-r)(1-p)}{rp+(1-r)(1-p)}\left[\pi(A)\right],$$

Furthermore, note that from before, $G(\mu) = \frac{(1-r)p}{(1-r)p+r(1-p)}\left[\pi(A)\right] - \frac{r(1-p)}{(1-r)p+r(1-p)}\left[\pi(M)\right] = 0$, so that $\pi(A) = \frac{r(1-p)}{(1-r)p}\left[\pi(M)\right]$. Substituting, the value of a shareholder with an m-signal voting for M rather than A becomes:

$$\frac{rp}{rp+(1-r)(1-p)}\left[\pi(M)\right] - \frac{(1-r)(1-p)}{rp+(1-r)(1-p)}\frac{r(1-p)}{(1-r)p}\left[\pi(M)\right],$$

which is positive iff $rp - (1-r)(1-p)\frac{r(1-p)}{(1-r)p} > 0$. Simplifying, this expression is positive iff $p - \frac{(1-p)^2}{p} > 0$, which is immediate since $p > \frac{1}{2}$. Consequently, a shareholder with an m-signal strictly prefers to vote for M, which is consistent with the hypothesized equilibrium.

The blockholder

We now prove that the blockholder votes with all of her $2b$ shares, recalling that $2b < 2b^*$ and holding fixed the symmetric mixed strategy of the shareholders.

Voting all of her shares for M when receiving an m signal: Given the blockholder received an m signal, we first check she prefers to vote all of her shares for M rather than some other amount of shares for M. Suppose the blockholder votes $2\tilde{b}$ shares, then her payoff per share owned is:

$$\begin{aligned} V_M(n,\tilde{b},p,q) &= \frac{rq}{rq+(1-r)(1-q)}\sum_{i=0}^{n+\tilde{b}}\left(\begin{array}{c}2n+1\\ i\end{array}\right)\gamma_M^{2n+1-i}(1-\gamma_M)^i \\ &\quad +\frac{(1-r)(1-q)}{rq+(1-r)(1-q)}\sum_{i=0}^{n-\tilde{b}}\left(\begin{array}{c}2n+1\\ i\end{array}\right)\gamma_A^{2n+1-i}(1-\gamma_A)^i. \end{aligned}$$

The blockholder prefers to vote 2 more shares for M iff:

$$V_M(n,\tilde{b},p,q) < V_M(n,\tilde{b}+1,p,q)$$

$$\Longleftrightarrow \left[\begin{array}{c} rq\sum_{i=0}^{n+\tilde{b}}\left(\begin{array}{c}2n+1\\ i\end{array}\right)\gamma_M^{2n+1-i}(1-\gamma_M)^i+ \\ (1-r)(1-q)\sum_{i=0}^{n-\tilde{b}}\left(\begin{array}{c}2n+1\\ i\end{array}\right)\gamma_A^{2n+1-i}(1-\gamma_A)^i \end{array}\right] < \left[\begin{array}{c} rq\sum_{i=0}^{n+\tilde{b}+1}\left(\begin{array}{c}2n+1\\ i\end{array}\right)\gamma_M^{2n+1-i}(1-\gamma_M)^i+ \\ (1-r)(1-q)\sum_{i=0}^{n-\tilde{b}-1}\left(\begin{array}{c}2n+1\\ i\end{array}\right)\gamma_A^{2n+1-i}(1-\gamma_A)^i \end{array}\right]$$

$$\Longleftrightarrow (1-r)(1-q)\gamma_A^{2n+1-(n-\tilde{b})}(1-\gamma_A)^{n-\tilde{b}} < rq\gamma_M^{2n+1-(n+\tilde{b}+1)}(1-\gamma_M)^{n+\tilde{b}+1}$$

$$\Longleftrightarrow \frac{\gamma_A^{2n+1-(n-\tilde{b})}(1-\gamma_A)^{n-\tilde{b}}}{\gamma_M^{2n+1-(n+\tilde{b}+1)}(1-\gamma_M)^{n+\tilde{b}+1}} < \frac{rq}{(1-r)(1-q)}$$

$$\Longleftrightarrow \frac{((1-\mu)p)^{n+1+\tilde{b}}(1-(1-\mu)p)^{n-\tilde{b}}}{(\mu+(1-\mu)p)^{n-\tilde{b}}((1-\mu)(1-p))^{n+\tilde{b}+1}} < \frac{rq}{(1-r)(1-q)}$$

$$\Longleftrightarrow \frac{p^{n+\tilde{b}+1}}{(1-p)^{n+\tilde{b}+1}}\frac{(1-(1-\mu)p)^{n-\tilde{b}}}{(\mu+(1-\mu)p)^{n-\tilde{b}}} < \frac{r}{(1-r)}\frac{q}{1-q}$$

It is straightforward to demonstrate that $\frac{1-(1-\mu)p}{\mu+(1-\mu)p}$ is less than one since $p > \frac{1}{2}$. In addition since $\frac{p}{1-p} > 1$, this condition is implied by $\frac{r}{1-r} > \left(\frac{p}{1-p}\right)^{n+b+1}\frac{1-q}{q}$.

Voting all of her shares for A when receiving an a signal: Given the blockholder received an a signal, we check she prefers to vote all of her shares for A. Suppose the blockholder votes $2\tilde{b}$ shares, then her payoff per share owned is:

$$\begin{aligned} V_A(n,\tilde{b},p,q) &= \frac{(1-r)q}{(1-r)q+r(1-q)}\sum_{i=0}^{n+\tilde{b}}\binom{2n+1}{i}\gamma_A^{2n+1-i}(1-\gamma_A)^i \\ &\quad +\frac{r(1-q)}{(1-r)q+r(1-q)}\sum_{i=0}^{n-\tilde{b}}\binom{2n+1}{i}\gamma_M^{2n+1-i}(1-\gamma_M)^i \end{aligned}$$

The blockholder prefers to vote 2 more shares for A iff:

$$V_A(n,\tilde{b},p,q) < V_A(n,\tilde{b}+1,p,q)$$

$$\iff \begin{bmatrix} (1-r)q\sum_{i=0}^{n+\tilde{b}}\binom{2n+1}{i}\gamma_A^{2n+1-i}(1-\gamma_A)^i+ \\ r(1-q)\sum_{i=0}^{n-\tilde{b}}\binom{2n+1}{i}\gamma_M^{2n+1-i}(1-\gamma_M)^i \end{bmatrix} < \begin{bmatrix} (1-r)q\sum_{i=0}^{n+\tilde{b}+1}\binom{2n+1}{i}\gamma_A^{2n+1-i}(1-\gamma_A)^i+ \\ r(1-q)\sum_{i=0}^{n-\tilde{b}-1}\binom{2n+1}{i}\gamma_M^{2n+1-i}(1-\gamma_M)^i \end{bmatrix}$$

$$\iff r(1-q)\gamma_M^{2n+1-(n-\tilde{b})}(1-\gamma_M)^{n-\tilde{b}} < (1-r)q\gamma_A^{2n+1-(n+\tilde{b}+1)}(1-\gamma_A)^{n+\tilde{b}+1}$$

$$\iff \frac{\gamma_M^{2n+1-(n-\tilde{b})}(1-\gamma_M)^{n-\tilde{b}}}{\gamma_A^{2n+1-(n+\tilde{b}+1)}(1-\gamma_A)^{n+\tilde{b}+1}} < \frac{(1-r)q}{r(1-q)}$$

$$\iff \frac{(\mu+(1-\mu)p)^{n+1+\tilde{b}}((1-\mu)(1-p))^{n-\tilde{b}}}{((1-\mu)p)^{n-\tilde{b}}(1-(1-\mu)p)^{n+\tilde{b}+1}} < \frac{(1-r)q}{r(1-q)}$$

$$\iff \frac{(1-p)^{n-\tilde{b}}}{p^{n-\tilde{b}}}\frac{(\mu+(1-\mu)p)^{n+1+\tilde{b}}}{(1-(1-\mu)p)^{n+\tilde{b}+1}} < \frac{(1-r)q}{r(1-q)}$$

$$\iff \frac{r}{1-r}\frac{p^{2\tilde{b}+1}}{(1-p)^{2\tilde{b}+1}}\frac{((1-p)\mu+(1-\mu)(1-p)p)^{n+1+\tilde{b}}}{(p-(1-\mu)p^2)^{n+\tilde{b}+1}} < \frac{q}{(1-q)}.$$

Note that $\frac{d}{d\mu}(\frac{((1-p)\mu+(1-\mu)(1-p)p)}{(p-(1-\mu)p^2)}) = (2p-1)\frac{p-1}{p(-p+p\mu+1)^2} < 0$ so the expression $(\frac{((1-p)\mu+(1-\mu)(1-p)p)}{(p-(1-\mu)p^2)})$ is maximized at $\mu = 0$, where it equals 1. In this case, the left hand side takes the value $\frac{r}{1-r}\frac{p^{2\tilde{b}+1}}{(1-p)^{2\tilde{b}+1}}$. So the condition becomes $\frac{q}{1-q} > \frac{r}{1-r}\frac{p^{2\tilde{b}+1}}{(1-p)^{2\tilde{b}+1}}$. A sufficient condition is therefore:

$$\frac{q}{1-q} > \frac{r}{1-r}\frac{p^{2b+1}}{(1-p)^{2b+1}}$$

Bringing all the conditions together, we obtain

$$\frac{q}{1-q}\left(\frac{1-p}{p}\right)^{2b+1} > \frac{r}{1-r} > \left(\frac{p}{1-p}\right)^{n+b+1}\frac{1-q}{q}.$$

Note that there is a non-empty set of parameters for which these inequalities hold; for example, when $q > r > \frac{1}{2}$ and p is close enough to $\frac{1}{2}$, they hold trivially.