Spillover Effects in Empirical Corporate Finance

Online Appendix (Not for Publication)

The Online Appendix consists of two parts. In Section 1, we study the structure of spillover effects in workhorse models of imperfect competition (i.e., oligopoly models), and in Section 2, we analyze spillover effects in models of spatial interaction.

1 Imperfect Competition

We analyze three different settings of imperfect competition. We first consider classic Cournot competition, that is, firms compete in quantities (Section 1.1). We then study price competition between firms selling differentiated products, using the model of Salop (1979) (Section 1.2). Finally, we analyze a model with a representative consumer, thereby allowing for both quantity and price competition, and for product differentiation in both settings (Section 1.3). For each setting, we consider two different types of shocks: a shock on the marginal costs of treated firms and a shock on the capacity of treated firms.

1.1 Cournot Competition

Consider a simple model of Cournot competition (i.e., quantity competition) with n firms. The firms face a linear inverse demand function of $p = 1 - \sum_{i=1}^{n} y_i = 1 - Y$, where p denotes the price, y_i the quantity produced by firm i, and Y the aggregate quantity produced by all firms. Following the notation of Section 3, we denote by d_i the treatment indicator. A proportion $\bar{d} = \frac{1}{n} \sum_{i=1}^{n} d_i$ of firms is treated, while firms in the control group represent a proportion $1 - \bar{d}$ of all firms. We first consider the case in which treated firms face a shock on their marginal costs and then analyze the case in which treated firms face a shock on their capacity.

Shock on marginal costs

Firms' marginal costs are constant but differ between firms in the control and the treatment group. Firms in the control group have marginal costs of $c^C = c$, whereas firms in the treatment group have marginal costs of $c^T = c + \gamma$, with $\gamma > 0$ (i.e., the treatment is a negative cost shock, for instance, as a result of higher funding costs or a shock on the input price).

The profit function of firm i, denoted by π_i and the resulting first-order conditions are:

$$\pi_i = y_i\left(p - c_i\right) = y_i\left[1 - \left(\sum_{j\neq i}^{n} y_j + y_i\right) - c_i\right], \quad i = 1, ..., n,$$

and:

$$\frac{\partial \pi_i}{\partial y_i} = 1 - Y - c_i - y_i = 0, \quad i = 1, ..., n,$$

respectively. Deducting the first-order condition of a firm in the control group from the first-order condition of a firm in the treatment group provides the relationship between the quantities of the firms in the different groups:

$$y_i^T - y_i^C = -\left(c_i^T - c_i^C\right) = -\gamma \quad \leftrightarrow \quad y_i^T = y_i^C - \gamma. \tag{1}$$

Using (1), we can write $Y = n\left(1 - \bar{d}\right) y_i^C + n\bar{d} y_i^T = n\left(1 - \bar{d}\right) y_i^C + n\bar{d}\left(y_i^C - \gamma\right)$. Plugging this into the first-order condition of a firm in the control group yields:

$$1 - n\left(1 - \bar{d}\right) y_i^C - n\bar{d}\left(y_i^C - \gamma\right) - c - y_i^C \quad \leftrightarrow y_i^C = \frac{1 - c + \gamma n\bar{d}}{n+1}. \tag{2}$$

Combining (1) and (2) yields a simple linear equation for the equilibrium quantities denoted by y_i^*:

$$y_i^* = \underbrace{\frac{1-c}{n+1}}_{f_1} - \underbrace{\gamma d_i}_{f_2(d_i)} + \underbrace{\gamma \frac{n}{n+1}\bar{d}}_{f_3(\bar{d})}. \tag{3}$$

It is evident from (3) that firms suffering from a negative marginal cost shock produce an output that is below the output of firms in the control group by an amount γ. This is reflected in the second term of (3), which is denoted by $f_2\left(d_i\right)$. This result is intuitive and is based on the effect that the margin of treated firms is below the one of firms in the control group due to the higher marginal costs of treated firms. However, the spillover effect is the same regardless of whether a firm belongs to the treatment or the control group, and is represented by the third term in (3), denoted by $f_3\left(\bar{d}\right)$. The intuition for the homogeneous spillover effect is as follows: spillovers occur because a change in the quantity of competitors affects each firm through the change in the market price. If the portion of treated firms increases, these firms sell less output, which leads to an increase in the market price. As the same price prevails for all firms, the effect of this price increase is the same for both types of firms. The increase in each firm's margin is therefore homogeneous among firms, which implies that treated and non-treated firms increase their quantities by the same extent.[1]

Shock on the capacity constraint

Consider now a (negative) shock on the capacity instead of a (negative) shock on marginal costs. Specifically, firms in the treatment group are only able to produce an amount of K, whereas firms in the control group are not capacity constrained. As above, a proportion $\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$ of firms is treated. Apart from the negative shock on the capacity constraint, firms in the treatment and the control group are homogeneous (i.e., marginal costs of all firms are c). For the shock of the capacity constraint to have economic consequences, we assume that:

$$K < \frac{1-c}{1+n}. \tag{4}$$

The equilibrium quantity of each firm in case without a shock is $(1-c)/(1+n)$; hence, if (4) does not hold, the capacity constraint does not bind, which implies that all firms produce a quantity of $(1-c)/(1+n)$. In what follows, we denote by Δ the difference between the equilibrium quantity in a model without a shock on the capacity constraint and the capacity constraint, that is:

$$\Delta \equiv \frac{1-c}{1+n} - K = \frac{1-c-K(n+1)}{n+1}.$$

The parameter Δ serves a similar role as γ in the previous case, as it represents the extent of the shock.

In the equilibrium with shock, each firm in the treatment group produces at the capacity constraint and sells a quantity of K. The profit function of firm i in the control

[1] The profit increase is nevertheless larger for firms in the control group as they sell a larger output than treated firms.

group can therefore be written as:

$$\pi_i = y_i \left[1 - n\bar{d}K - \left(\sum_{j \neq i}^{n} (1 - d_j) y_j + y_i \right) - c \right].$$

Taking the first-order condition and solving for the symmetric equilibrium (i.e., all firms in the control group sell the same quantity) yields:

$$y^C = \frac{1 - c - n\bar{d}K}{1 + n(1 - \bar{d})}.$$

Therefore, the equilibrium quantity y_i^* of each firm i is:

$$y_i^* = \underbrace{\frac{1-c}{1+n}}_{f_1} - \underbrace{\Delta d_i}_{f_2(d_i)} + \underbrace{\Delta \frac{n\bar{d}}{1 + n(1 - \bar{d})} (1 - d_i)}_{f_3\left(\bar{d}, d_i \bar{d}\right)}. \tag{5}$$

In contrast to the case of a shock on the marginal costs, the spillover effect is now only relevant for the control group but not for the treatment group. This can be seen in (5) because the last term, denoted by $f_3\left(\bar{d}, d_i \bar{d}\right)$, depends on $1 - d_i$. This result is intuitive: as a firm in the treatment group produces at the capacity constraint, is is not affected by spillover effects. In addition, it is easy to check that $f_3\left(\bar{d}, d_i \bar{d}\right)$ is increasing in $\bar{d}$, that is, the more firms face a capacity constraint, the higher the quantity produced by firms in the control group. The intuition is again that the reduction in output of the treated firms leads to a higher market price, and thereby to a higher margin. If more firms are treated, this margin increase is higher, which implies that non-treated firms sell more.

1.2 Circular Competition (Salop, 1979)

Consider next a model of circular competition between firms, which was developed by Salop (1979). There are n firms producing differentiated products. The firms are evenly distributed on a circle with circumference 1. Consumers are uniformly distributed on this circle and incur a transport cost of t per unit of distance. That is, if a consumer located at point x on the circle purchases from firm i located on point x_i, her net utility is $v - p_i - t\,|x - x_i|$, where v is the benefit from the product and p_i is firm i's price.

Consumers wish to buy one unit of the good and buy from the firm that offers them the highest net utility. To simplify the exposition, we assume that v is sufficiently large, so that all consumers buy one unit of the product. Firms simultaneously set prices.

Shock on marginal costs

We first consider a marginal-cost shock as in the previous section. Firms have constant marginal costs, and firms in the treatment group face a negative cost shock—i.e., firms in the control group have marginal costs of $c^C = c$ and firms in the treatment group have marginal costs of $c^T = c + \gamma$. As above, a proportion $\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$ of firms is treated, while firms in the control group represent a proportion $1 - \bar{d}$ of all firms. Following e.g. Raith (2003) and Aghion and Schankerman (2004), we assume that firms do not know the cost characteristics of neighboring firms, and thus base their pricing decisions on the 'average' costs of their neighbors.

We first derive the demand for each firm i. Denoting the prices by firm i's neighboring firms by p_{i-1} and p_{i+1}, the marginal consumer between i and $i-1$ (i.e., the consumer indifferent between buying from firm i and from firm $i-1$) is given by:

$$x_{i,i-1} = \frac{1}{2n} + \frac{p_{i-1} - p_i}{2t}$$

and the marginal consumer between i and $i+1$ is given by:

$$x_{i,i-1} = \frac{1}{2n} + \frac{p_{i+1} - p_i}{2t}.$$

Therefore, firm i's demand is:

$$D_i(p_i, p_{i-1}, p_{i+1}) = \frac{1}{n} + \frac{p_{i-1} + p_{i+1} - 2p_i}{2t}.$$

When setting its price p_i, firm i does not observe prices p_{i-1} and p_{i+1} charged by its neighbors. However, it anticipates that all treated (non-treated) firms will charge the same price p^T (p^C) in equilibrium. Given that a proportion $\bar{d}$ of firms is treated, a firm in the control group faces an expected price of its neighbors given by:

$$\frac{n\bar{d}p^T + \left(n(1-\bar{d})-1\right)p^C}{n-1},$$

whereas a firm in the treatment group faces an expected price of its neighbors given by:

$$\frac{\left(n\bar{d}-1\right)p^T + n(1-\bar{d})p^C}{n-1}.$$

The maximization problem of firm j belonging to the control group is therefore:

$$\max_{p_j} \quad \pi_j = (p_j - c)\left(\frac{1}{n} + \frac{\frac{n\bar{d}p^T + \left(n(1-\bar{d})-1\right)p^C}{n-1} - p_j}{t}\right)$$

and the maximization problem of firm k belonging to the treatment group is:

$$\max_{p_k} \quad \pi_k = (p_k - c - \gamma)\left(\frac{1}{n} + \frac{\frac{\left(n\bar{d}-1\right)p^T + n(1-\bar{d})p^C}{n-1} - p_k}{t}\right)$$

Taking the first-order conditions and solving for the symmetric Nash equilibrium ($p_j = p^C$ and $p_k = p^T$), we obtain that the equilibrium prices of the two types of firms are:

$$p^C = c + \frac{t}{n} + \frac{\gamma n\bar{d}}{2n-1} \quad \text{and} \quad p^T = c + \frac{t}{n} + \frac{\gamma(n-1)}{2n-1} + \frac{\gamma n\bar{d}}{2n-1}.$$

This implies that $p^T = p^C + \gamma(n-1)/(2n-1)$, that is, the price difference between the two types of firms is smaller than the cost difference. In other words, treated firms do not shift the cost shock γ fully into the consumer price. The intuition is that, due to the fact that firms in the control group face lower costs and therefore set a lower price, shifting the cost increase fully onto consumers would reduce the demand of a treated firm by a very large amount. Solving for the equilibrium quantities y_i^*, we obtain:

$$y_i^* = \underbrace{\frac{1}{n}}_{f_1} \underbrace{- \gamma\frac{n}{t(2n-1)}d_i}_{f_2(d_i)} + \underbrace{\gamma\frac{n}{t(2n-1)}\bar{d}}_{f_3\left(\bar{d}\right)}. \tag{6}$$

It follows from (6) that firms in the treatment group produce an output that is $\gamma n/(t(2n-1))$ below the one of firms in the control group. This is reflected in the term denoted by $f_2(d_i)$. However, as in the Cournot model above, the spillover effect is the same regardless of whether a firm belongs to the treatment or the control group, and is represented by the spillover effect $f_3\left(\bar{d}\right)$. The homogeneous spillover effect occurs because each firm benefits from the expected price increase resulting from a larger portion of treated firms due to the higher expected price charged by its neighboring firms. As both treated and non-treated firms face the same expectation with respect to the type of its neighboring firms, the spillover effect is homogeneous. We also note that the relation between shock and quantity is a linear one, as in the Cournot model.

Shock on the capacity constraint

We consider next a shock on the capacity constraint, that is, firms in the treatment group are only able to produce a quantity of K. As above, a proportion $\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$ of firms is treated. To ensure that K is a real constraint, we assume:

$$K < \frac{1}{n},$$

which implies that the capacity constraint is binding in equilibrium (i.e., the constraint is below the quantity that a firm would have produced in the equilibrium without shock). As above, we denote by $\Delta \equiv 1/n - K$ the difference between the equilibrium in a model without shock and the capacity constraint.

Anticipating that all treated (non-treated) firms will charge the same price p^T (p^C), the maximization problem of firm j in the control group is:

$$\max_{p_j} \quad \pi_j = (p_j - c)\left(\frac{1}{n} + \frac{\frac{n\bar{d}p^T + \left(n(1-\bar{d})-1\right)p^C}{n-1} - p_j}{t}\right),$$

yielding a first-order condition of:

$$\frac{1}{n} + \frac{\frac{n\bar{d}p^T + \left(n(1-\bar{d})-1\right)p^C}{n-1} - p_j}{t} - \frac{p_j - c}{t} = 0.$$

In equilibrium, $p_j = p^C$. Inserting this into the the first-order condition and solving for p^C, we obtain:

$$p^C = \frac{(t + cn)(n-1) + p^T n^2 \bar{d}}{n\left(n(1+\bar{d}) - 1\right)}. \tag{7}$$

Instead, a firm belonging to the treatment group will set its price in such a way that it will sell its capacity K, regardless of the type of its neighboring firms. Specifically, if its two neighboring firms are firms which are not treated—and therefore face no capacity constraint—the firm will set its equilibrium price so that it sells to $K/2$ consumers on each side. This implies that the relationship between p^C and p^T is such that:

$$K = \frac{1}{n} + \frac{p^C - p^T}{t}. \tag{8}$$

Setting its price according to (8) also ensures that the firm sells its entire capacity if its neighbors do not only consist of non-treated firms: as firms in the treatment group sell a lower quantity, the firm can then also serve $K/2$ consumers on each side. However, some consumers do not get served in equilibrium if two firms of the treatment group are neighbors.

Solving (7) and (8) for the equilibrium prices yields:

$$p^C = c + \frac{t}{n} + \frac{nt\bar{d}(1-Kn)}{n(n-1)} \quad \text{and} \quad p^T = c + \frac{2t}{n} - Kt + \frac{nt\bar{d}(1-Kn)}{n(n-1)}.$$

Inserting these prices into the expressions for the quantities yields that the equilibrium quantity for each firm i is:

$$y_i^* = \underbrace{\frac{1}{n}}_{f_1} - \underbrace{\Delta d_i}_{f_2(d_i)} + \underbrace{\Delta \frac{n}{n-1} \bar{d}\,(1-d_i)}_{f_3\left(\bar{d}, d_i \bar{d}\right)}. \tag{9}$$

As a consequence, we obtain a linear relationship between the shock and the quantity of each firm. As firms in the treatment group produce at the capacity constraint, the spillover effect is only relevant for firms in the control group, which can be seen in the last term of (9), as this term involves $(1 - d_i)$.

1.3 Representative-Consumer Model

A third widely-used demand model is one with a representative consumer. Its linear version was first developed by Bowley (1924) and popularized in a highly-cited paper by Singh and Vives (1984). We first describe how to derive the linear demand function from the representative consumer's utility function and then solve for the equilibrium, both with quantity and price competition.

The utility function of the representative consumer is:

$$U(y_1, ..., y_n) = \alpha \sum_{i=1}^{n} y_i - \frac{1}{2} \left(\beta \sum_{i=1}^{n} y_i^2 - \delta \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} y_i y_j \right),$$

The consumer maximizes her utility subject to the budget constraint, which is given by $\sum_{i=1}^{n} p_i y_i \leq M$, where M is the consumer's income. Because the consumer optimally spends her entire income on the goods, the maximization problem is $U(y_1, ..., y_n) - \sum_{i=1}^{n} p_i y_i$. This leads to an inverse demand function of product i given by:

$$p_i = \alpha - \beta y_i - \delta \sum_{j=1, j \neq i}^{n} y_j, \quad i = 1, ..., n. \tag{10}$$

Therefore, β measures the effect of firm i's quantity on its own price—i.e., it determines firm i's price elasticity of its own demand—whereas δ measures the effect of another firm's quantity on firm i's price—i.e., it determines the cross elasticity of demand. The inverse of δ represents the degree of differentiation between firms' products. Specifically, if $\delta = 0$, products are independent and each firm is a monopolist. Instead, if $\delta \rightarrow \beta$, products become perfect substitutes.

Shock on the marginal costs

As in the examples above, firms have constant marginal costs. Firms in the treatment group face a negative cost shock and have marginal costs of $c^T = c + \gamma$ whereas firms in the control group have marginal costs of $c^C = c$. A proportion $\bar{d} = \frac{1}{n} \sum_{i=1}^{n} d_i$ of firms is treated, while firms in the control group represent a proportion $1 - \bar{d}$ of all firms.

(i) Quantity competition

If firms compete in quantities, each firm i's profit function is:

$$\pi_i = y_i \left[\alpha - \left(\beta y_i + \delta \sum_{j \neq i}^{n} y_j \right) - c_i \right].$$

It is evident that this is a generalized version of the first setting (i.e., Cournot competition). In the latter case $\alpha = \beta = \delta = 1$, which implies that firms' products are homogeneous.

Following the same procedure as above, we can solve for the equilibrium quantities to get:

$$y_i^* = \underbrace{\frac{\alpha - c}{2\beta + \delta(n-1)}}_{f_1} - \underbrace{\gamma \frac{1}{2\beta - \delta} d_i}_{f_2(d_i)} + \underbrace{\gamma \frac{n\delta}{(2\beta - \delta)(2\beta + \delta(n-1))} \bar{d}}_{f_3(\bar{d})}.$$

It follows that the output of the treatment group is lower by an amount of $\gamma/(2\beta - \delta)$ compared to the control group but the spillover effect is again homogeneous among firms in the treatment and firms in the control group. The intuition is the same as the one given in Section 1.1.

(ii) Price competition

To analyze price competition, we first need to invert the demand system given by (10) to obtain the quantity as a function of the prices. This can be done following the technique introduced by Hackner (2000). Specifically, summing (10) over all n firms yields:

$$n\alpha - \beta \sum_{j=1}^{n} y_j - \delta(n-1) \sum_{j=1}^{n} y_j - \sum_{j=1}^{n} p_j = 0.$$

Using that $\sum_{j=1}^{n} y_j = y_i + \sum_{j=1, j \neq i}^{n} y_j$ and solving (10) for $\sum_{j=1, j \neq i}^{n} y_j$, allows us to derive firm i's quantity as a function of prices:

$$y_i(p_i, p_{-i}) = \frac{\alpha}{\beta + \delta(n-1)} - \frac{p_i}{\beta - \delta} + \frac{\delta \sum_{j=1, j \neq i}^{n} p_j}{(\beta - \delta)(\beta + \delta(n-1))}, \tag{11}$$

where, following standard notation, p_{-i} denotes the set of prices of all firms but firm i.

The respective profit function of firm i is:

$$(p_i - c_i)\, y_i(p_i, p_{-i}). \tag{12}$$

We can solve for the equilibrium prices in the same way as above. Plugging the equilibrium prices into the quantities yields that the equilibrium quantities are given by:

$$y_i^* = \underbrace{\frac{\alpha(\beta-\delta)-\beta c}{(\beta-\delta)\,(\beta+\delta(n-1))}}_{f_1} \underbrace{-\gamma\frac{\beta+n\delta}{(\beta-\delta)\,(2\beta+\delta(2n-1))}d_i}_{f_2(d_i)}$$

$$\underbrace{+\gamma\frac{n\delta\,(\beta+\delta(n-1))}{(\beta-\delta)\,(2\beta+\delta(2n-1))\,(\beta+\delta(2n-1))}\bar{d}}_{f_3(\bar{d})}.$$

It is evident from the term $f_2\,(d_i)$ that a treated firm sells a lower quantity than a firm in the control group. However, the spillover effect is the same on both types of firms, as can be seen from the term $f_3\left(\bar{d}\right)$, which does not depend in d_i. The intuition is that a price increase by competitors (positively) affects the residual demand of each firm in the same way, regardless of whether the firm is treated or not (as can be seen from (11)). The equation determining y_i^* is also linear, both for quantity and price competition.

Shock on the capacity constraint

As above, we consider next a shock on the capacity constraint. Firms in the treatment group, of which there is a proportion $\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$, are then only able to produce an amount of K, whereas firms in the control group are not capacity constrained.

(i) Quantity competition

We again start with quantity competition. To ensure that K is a real constraint, we assume:

$$K < \frac{\alpha - c}{2\beta + \delta(n-1)},$$

and denote by Δ the difference between the equilibrium quantity in a model without a shock on the capacity constraint and the capacity constraint, that is:

$$\Delta \equiv \frac{\alpha - c}{2\beta + \delta(n-1)} - K = \frac{\alpha - c + K\left(2\beta + \delta(n-1)\right)}{2\beta + \delta(n-1)}.$$

Solving for the equilibrium quantities in the same way as in the Cournot model yields:

$$y_i^* = \underbrace{\frac{\alpha - c}{2\beta + \delta(n-1)}}_{f_1} - \underbrace{\Delta d_i}_{f_2(d_i)} + \underbrace{\Delta \frac{n\bar{d}}{2\beta + \delta\left(n(1-\bar{d}) - 1\right)}\left(1 - d_i\right)}_{f_3\left(\bar{d}, d_i\bar{d}\right)}.$$

It is evident that the same result as in the Cournot model is obtained, that is, the spillover effect only affects firms in the control group but not those in the treatment group.

(ii) Price competition

Turning to price competition, the assumption that guarantees that K is below the equilibrium quantity that a firm in the treatment group produces without shock is:

$$K < \frac{\alpha}{2\beta - \delta(n-1)} - \frac{c\beta}{(\beta - \delta)\left(2\beta - \delta(n-1)\right)}.$$

The maximization problem of a firm in the treatment group is, as above, given by (12), with $y_i(p_i, p_{-i})$ given by (11), whereas a firm in the control group sets its price such that is sells its capacity K. Solving for the equilibrium then yields that the equilibrium quantity of each firm i is:

$$y_i^* = \underbrace{\frac{\alpha}{2\beta - \delta(n-1)} - \frac{c\beta}{(\beta - \delta)\left(2\beta - \delta(n-1)\right)}}_{f_1} - \underbrace{\Delta d_i}_{f_2(d_i)}$$

$$\underbrace{+\Delta \frac{\delta n\left(\beta + \delta(n-1)\right)\bar{d}}{2\beta^2 - \delta^2 n(n-1)(1-\bar{d}) + \beta\delta\left(n(3-\bar{d}) - 1\right)}\left(1 - d_i\right)}_{f_3\left(\bar{d}, d_i\bar{d}\right)}.$$

Therefore, the same result as above is obtained: the spillover effect only affects the control group but not the treatment group, as the last term, which involves $\bar{d}$, is multiplied by $1 - d_i$.

2 Spatial Interaction

We next turn to firm interdependencies via spatial interaction. As described in Section 3 of the main text, we consider two settings. First, we study a model of demand spillovers (Section 2.1); second, we analyze agglomeration effects between firms (Section 2.2).

2.1 Demand Spillovers

In this section, we consider a simple model of demand spillovers and determine how such spillovers affect treated and control firms. In contrast to the oligopoly models, there is no classic model of demand spillovers. However, the most influential papers considering demand spillovers are Shleifer and Vishny (1988) and Murphy, Shleifer, and Vishny (1989). Our model is an adaptation of these models, allowing us to analyze the different effects on output and employment of treated and control firms. Specifically, these papers analyze demand spillovers in a general equilibrium model in which demand in each market is a function of labor income and profits. To achieve this, they make some simplifying assumptions, such as inelastic demand and the existence of a competitive fringe, which puts a cap on prices. In contrast to these papers, we focus on the case in which local demand only depends on labor income but we allow for elastic demand and put no constraint on prices.

The description of the model is as follows: There are n goods, where n is considered to be a very large number (in the limit, a continuum). Each good is produced by one firm, which is a local monopolist for the good it produces. The local demand in each market i is denoted by q_i and is given by $q_i = \alpha(L)(1 - p_i)$, where p_i is the price charged by firm i and L is the sum of the labor income over all n markets. The function $\alpha(L)$ is strictly increasing and is assumed to take the form $\alpha(L) = \sqrt{L}$. This represents the demand spillover effect. The economic reasoning behind this function is that the employees of each firm consume the local goods and spend a portion of their labor income to buy these goods. The concave shape of $\alpha(L)$ is due to decreasing marginal utility of each good.

For simplicity, we assume that the relationship between employment and the aggregate output sold by the n firms (i.e., $\sum_{i=1}^{n} q_i$) is one-to-one, and that the wage is normalized to 1. This implies that $L = \sum_{i=1}^{n} q_i$. Inserting this into $\alpha(L)$ yields

$$\alpha(L) = \sqrt{L} = \sqrt{\sum_{i=1}^{n} q_i} = \sqrt{\sum_{i=1}^{n} \alpha(1-p_i)}.$$

Shock on marginal costs

As in the previous section, each firm has constant marginal costs. They are equal to c for non-treated firms and $c+\gamma$ for treated firms (i.e., a negative cost shock). A proportion $\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$ of firms is treated.

First, consider the maximization problem of a firm in the control group. As n is very large, the firm cannot influence α and takes it as given. The profit function of a firm j in this group can therefore be written as:

$$\pi_j = \alpha\left(p_j - c\right)\left(1 - p_j\right).$$

Maximizing with respect to p_j leads to an optimal price of $(1+c)/2$, which implies that the optimal quantity is $\alpha(1-c)/2$. Similarly, the profit function of a firm k in the treatment group is:

$$\pi_k = \alpha\left(p_j - c - \gamma\right)\left(1 - p_k\right),$$

leading to an optimal price of $(1+c+\gamma)/2$ and an optimal quantity of $\alpha(1-c-\gamma)/2$.

Because the wage is normalized to 1 and the aggregate output equals the aggregate employment, we can write:

$$L = n(1-\bar{d})\frac{\alpha(1-c)}{2} + n\bar{d}\frac{\alpha(1-c-\gamma)}{2}.$$

Inserting this into $\alpha = \sqrt{L}$ and solving for α yields:

$$\alpha = \frac{n(1-c-\gamma\bar{d})}{2}.$$

As a consequence, the equilibrium quantity of firm i can be written as:

$$y_i^* = \frac{n}{4}\left(\underbrace{(1-c)^2}_{f_1} - \underbrace{\gamma(1-c)d_i}_{f_2(d_i)} - \underbrace{\gamma(1-c)\bar{d}}_{f_3\left(\bar{d}\right)} + \underbrace{\gamma^2\bar{d}d_i}_{f_4\left(d_i\bar{d}\right)}\right) \tag{13}$$

As the assumption of the model is that the relation between employment and quantity in a sector is one-to-one, quantities also reflect employment.

The quantity (and therefore the employment) of a treated firm is smaller than the one of a non-treated firm, which can be seen from the term $f_2(d_i)$ of the right-hand side in (13). This result is the same as in the models with imperfect competition. It is again driven by the fact that a higher marginal cost induce a treated firm to reduce its output. In contrast to the case of imperfect competition, however, the spillover effect is negative in case of demand spillovers. This can be seen from the term $f_3\left(\bar{d}\right)$. The reason is that the shock not only lowers quantities but, through the reduced employment, also the disposable income of consumers, which implies that demand falls. As a consequence, due to the spillover, employment of each firm is lower than in the absence of a shock. In addition, the spillover effect now affects treated and non-treated firms heterogeneously. This can be seen from the term $f_4\left(d_i\bar{d}\right)$, which enters the expression with a positive sign. Therefore, the spillover effect is less dramatic for treated firms as compared to firms in the control group.[2] The intuition is that treated firms produce a lower quantity and are therefore less affected by the demand reduction. It follows that a lower employment level, and thus a reduction in disposable income, affects larger firms more negatively than smaller firms.

We also note that the relationship between output (or employment) and the marginal cost shock/spillover effect is again linear in this simple model.

Shock on the capacity constraint

We next consider a shock on the capacity of the treated firms. Suppose that the proportion $\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$ of firms can only sell an amount K (whereas firms in the control group are not capacity constrained). Solving the model without such a shock yields that

[2]The overall spillover effect on treated firms is, however, always negative, as $\gamma < 1-c$, due to the fact that all firms produce a strictly positive quantity.

each firm sells a quantity of $n(1-c)^2/4$ in equilibrium. Hence, to allow the capacity constraint to have bite, we assume:

$$K < \frac{n(1-c)^2}{4}.$$

We again denote by Δ the difference between $n(1-c)^2/4$ and K.

The maximization problem of a firm j in the control group is the same as in case of a shock on marginal costs—i.e., it is given by $\alpha\,(p_j - c)\,(1-p_j)$—which implies that the optimal quantity is again $\alpha(1-c-\gamma)/2$. Instead, a firm in the treatment group sells its entire capacity K. As a consequence, the aggregate output is:

$$n(1-\bar{d})\frac{\alpha(1-c)}{2} + n\bar{d}K. \tag{14}$$

Due to the fact that the wage is normalized to 1 and the relationship between output and employment is one-to-one, labor income is given by (14). Using this in $\alpha = \sqrt{L}$ and solving for α yields:

$$\alpha = \frac{n(1-c)(1-\bar{d}) + \sqrt{n\left(n(1-c)^2(1-\bar{d})^2 + 16K\bar{d}\right)}}{4}.$$

As a consequence, the equilibrium quantity for each firm i can be written in a concise form as:

$$y_i^* = \underbrace{\frac{n(1-c)^2}{4}}_{f_1} - \underbrace{\Delta d_i}_{f_2(d_i)} - \underbrace{\frac{(1-c)\left(n(1+\bar{d})(1-c) - \sqrt{n\left(n(1-\bar{d})^2(1-c)^2 - 16\bar{d}\Delta\right)}\right)}{8}(1-d_i)}_{f_3\left(\bar{d},d_i\bar{d}\right)}, \tag{15}$$

and the equilibrium employment in sector i is also given by this expression. It follows from the last term of (15) that there is no spillover effect for firms in the treatment group, as these firms produce at the capacity constraint. Instead, for firms in the control group, the term $f_3\left(\bar{d}, d_i\bar{d}\right)$ shows that their quantity is negatively affected by Δ, which implies that the spillover effect is negative for them. As $\bar{d}$ is present in the square root of the term, the effect is non-linear.

An important case is the one in which the shock is so severe that firms in the treatment group go bankrupt, and therefore need to exit the market. This is equivalent to a capacity constraint of $K = 0$. In this case, $\Delta = n(1-c)^2/4$; using this in (15), we obtain that a firm in the control group sells an output of:

$$y_C^* = \underbrace{\frac{n(1-c)^2}{4}}_{f_1} - \underbrace{\Delta \bar{d}\,(1-d_i)}_{f_3\left(\bar{d},d_i\bar{d}\right)} .$$

As a consequence, the spillover effect is linear in this case.

2.2 Agglomeration

Finally, we analyze a simple model in which spillovers occur due to agglomeration effects. Dating back to Marshall (1890), agglomeration economies are broadly considered as factors that allow clustered firms, or firms that are present in the same location, to obtain higher profits than isolated firms. Among the various reasons for this, Marshall (1890), Hoover (1948), and Krugman (1991), among many others, emphasize that agglomerated firms, first, benefit from information spillovers, allowing them to operate with a cheaper cost function than segmented firms, and, second, offer a pooled market for workers with industry-specific skills, leading to a higher probability to get skilled labor, which again allows for cheaper production. Taking these reasons into account, we formulate a simple model in which firms benefit from spillovers of other firms via a reduction in their marginal costs—i.e., marginal costs are the lower, the more firms are present and the larger is each firm's investment in cost reduction. The model follows d'Aspremont and Jacquemin (1988) in the way spillovers between firms are modeled.[3]

The description of the model is as follows: there are n firms, each one operating in a separate market, that is, there are no competition externalities between firms. The inverse demand function in each market i is $p_i(y_i) = \alpha - y_i$. The marginal cost of firm i depends on its own investment and of the investment of all other firms. To capture this

[3]To simplify the exposition, we abstract from competition between firms, which is also considered by d'Aspremont and Jacquemin (1988). However, we extend their model by allowing for n firms instead of only 2 and, naturally, consider firm heterogeneity.

in a simple way, marginal costs of firm i are:

$$c_i - x_i - \beta \sum_{j=1, j \neq i}^{n} x_j, \tag{16}$$

where x_j is the investment level of firm $j \neq i$. Therefore, own investment reduces marginal costs at a one-to-one relation, whereas agglomeration effects due to spillovers are measured by the parameter $\beta \in [0, 1]$. If $\beta = 0$, agglomeration effects are absent; instead, if $\beta = 1$, there are full spillovers, which implies that each firm benefits to same extent from investment of other firms in the cluster as from own investment. Investment cost is quadratic and given by $\kappa x_i^2/2$, which reflects diminishing returns from investment. To ensure an interior solution, we assume $\kappa > (1 + \beta(n-1))/2$, that is, investment costs are sufficiently convex.

The profit function of each firm i is therefore given by:

$$\pi_i(y_i, y_{-i}, x_i, x_{-i}) = (\alpha - y_i)\, y_i - \left(c_i - x_i - \beta \sum_{j=1, j \neq i}^{n} x_j \right) y_i - \kappa \frac{x_i^2}{2}.$$

The maximization variables are y_i and x_i.

Shock on the marginal costs

As in the previous examples, we first consider a shock on the firms' marginal costs. Following the above examples, a firm in the control group is characterized by $c_i = c$, whereas a firm in the treatment group is characterized by marginal costs $c_i = c + \gamma$, with $\gamma > 0$ (i.e., the treatment is again a negative cost shock). A proportion $\bar{d} = \frac{1}{n} \sum_{i=1}^{n} d_i$ of firms is treated, while the remaining proportion $1 - \bar{d}$ are firms is in the control group.

The first-order conditions for profit maximization are:[4]

$$\alpha - c_i + x_i + \beta \sum_{j=1, j \neq i}^{n} x_j - 2y_i = 0 \quad \text{and} \quad y_i - \kappa x_i = 0, \quad \forall i = 1, ..., n.$$

In a symmetric equilibrium, all firms of the same type set the same variables in equilibrium, that is, all firms in the control group set $y = y^C$ and $x = x^C$ in equilibrium, whereas

[4]Due to the assumption $\kappa > (1 + \beta(n-1))/2$, the Hessian is strictly positive definite, which implies that all maximization problems are strictly concave.

all firms in the treatment group set $y = y^T$ and $x = x^T$. The respective first-order conditions for firms in the control and firms in the treatment group can then be written as follows:

$$\alpha - c + x^C + \beta\left((n-1-k)x^C + kx^T\right) - 2y^C = 0, \quad y^C - \kappa x^C = 0,$$

and:

$$\alpha - c - \gamma + x^T + \beta\left((n-k)x^C + (k-1)x^T\right) - 2y^T = 0, \quad y^T - \kappa x^T = 0.$$

Solving these four equations for the equilibrium investment levels and quantities of treated firms and control firms yields:

$$y_i^* = \frac{1}{2\kappa - 1 - \beta(n-1)}\left(\underbrace{\kappa(\alpha - c)}_{f_1} - \underbrace{\kappa\gamma d_i}_{f_2(d_i)} - \underbrace{\frac{n\kappa\gamma\beta}{2\gamma - 1 + \beta}\bar{d}}_{f_3(\bar{d})}\right) \quad \text{and} \quad x_i^* = \frac{y_i^*}{\kappa}. \tag{17}$$

As can be seen from (17), both investment level and quantity are affected in a similar way from the shock. The term $f_2(d_i)$ shows that firms in the treatment group invest less and sell a smaller quantity than firms in the control group. The term $f_3\left(\bar{d}\right)$ shows that firms are negatively affected from spillover effects and that this effect is homogeneous for treated and control firms. The intuition for these results is as follows: first, as treated firms have higher marginal costs, they produce less, which in turn renders investment in cost reduction less profitable for them. Therefore, a larger portion of treated firms leads to less aggregate investment, which implies that the spillover effect is negative. Second, investments by other firms reduce marginal costs of treated and non-treated firms in the same (as can be seen from (16)). As a consequence, the spillover lowers the output and investment of both types of firms to the same extent.[5]

Shock on the capacity constraint

Finally, we consider a shock on the capacity of firms in the treatment group. Suppose

[5]We note, however, that the spillover affects the profit of firms in the control group more than firms in the treatment group, as the former sell a larger quantity.

that the proportion $\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$ of firms can only sell an amount K. In a model without such a shock, each firm sells a quantity of $\kappa\,(\alpha - c)\,/\,(2\kappa - 1 - \beta(n-1))$ in equilibrium. Hence, to allow the capacity constraint to have bite, we assume:

$$K < \frac{\kappa\,(\alpha - c)}{2\kappa - 1 - \beta(n-1)},$$

and denote by Δ the difference between the equilibrium quantity without shock and the capacity constraint.

Solving the model in the same way as with a shock on the marginal costs but setting $\gamma = 0$ and instead inserting $y^T = K$ yields that the equilibrium quantities and investment levels are:

$$y_i^* = \underbrace{\frac{\kappa\,(\alpha - c)}{2\kappa - 1 - \beta(n-1)}}_{f_1} - \underbrace{\Delta d_i}_{f_2(d_i)} - \underbrace{\Delta \frac{\beta n \bar{d}}{2\kappa - 1 - \beta\left(n(1-\bar{d}) - 1\right)}(1 - d_i)}_{f_3\left(\bar{d}, d_i \bar{d}\right)} \quad \text{and} \quad x_i^* = \frac{y_i^*}{\kappa}. \tag{18}$$

From (18), it is easy to see that the spillover effect resulting from a shock on the capacity is again only relevant for the control group but not for the treatment group. For a firm in the control group, the effect is negative (and non-linear). In addition, as the term denoted by $f_3\left(\bar{d}, d_i \bar{d}\right)$ is increasing in $\bar{d}$, the more firms face a capacity constraint, the lower the quantity and the investment level of a firm in the control group.

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