Asset pricing with beliefs-dependent risk aversion and learning

ONLINE APPENDIX

May 3, 2017

This online appendix presents additional results with regard to the performance of the model and discusses the model specification tests used in the paper. Section 1 provides basic evidence of performance based on ROC curves. Section 2 illustrates differences between three specifications of interest, namely constant relative risk aversion (CRRA), constant aversion to state uncertainty (CASU) and beliefs-dependent risk aversion (BDRA), in their ability to reproduce empirical properties of the equity premium and the yield curve. Section 3 examines the robustness of parameter estimates with respect to moment conditions employed in the estimation procedure. Section 4 carries out specification tests for the three alternatives under consideration as well as for a unified specification containing all alternatives as special cases.

1 Model as predictor of recessions: ROC curve

The first element suggesting a good fit of the model with BDRA to moments that were not targeted in the GMM estimation is the evolution of the conditional probability of the bad state (p_{2t}). The trajectory of this conditional probability depends on the estimates of the transition matrix Λ, the covariance matrix and the drifts of the three state variables, namely consumption, dividend and unemployment rate. Figure 1 suggests that p_{2t} correctly identifies the recession periods measured by the NBER.

To formally gauge the ability of the model to identify the recession phases, we use a standard tool for diagnostic test called the Relative Operating Characteristic (ROC) curve.[1] The ROC curve

[1] ROC curves are widely used to evaluate diagnostic decision making in medicine, radiology, signal detection, data mining and psychology.

measures the sensitivity with respect to a diagnostic threshold, of the True Positive Rate (TPR), which is the rate of correct diagnostic, versus the False Positive Rate (FPR), which is the rate of incorrect diagnostic. It plots power (one minus the probability of a type II error) against the probability of a type I error. The type I error is to accept H_1 when H_0 is true. The type II error is to accept H_0 when H_1 is true. In our context, the two rates pertain to recession diagnostic and are calculated as follows. Fix a threshold level $c \in [0, 1]$. The TPR is the number of quarters in which p_{2t} is above the threshold c and the economy is in recession divided by the total number of quarters in which the economy is in recession (rate of correct recession diagnostic). The FPR is the number of quarters in which the recession probability is above the threshold c and the economy is not in recession divided by the total number of quarters in which the economy is not in recession (rate of incorrect recession diagnostic).

We perform these calculations for all values of $c \in [0, 1]$ and plot the resulting ROC curve in Figure 1. This two-dimensional figure shows the relation between the TPR (y-axis) and the FPR (x-axis) for all levels of $c \in [0, 1]$. The ROC curve is seen to diverge significantly from the 45 degree line in the north-west direction. This divergence indicates that the model is a good recession diagnostic tool for all threshold levels c.

2 Model comparison: equity premium and yields

This section provides further evidence of the incremental performance of the model with beliefs-dependent risk aversion (BDRA) in comparison to (i) the model with constant relative risk aversion and constant aversion to state uncertainty (CASU) and (ii) the model with constant relative risk aversion (CRRA).

Figure 2 shows the trajectories of the equity premium for the three specifications examined. In the model with BDRA, the equity premium displays the countercyclical behavior found in the data (see Campbell and Cochrane (1999) and Melino and Yang (2003)). With CASU, it peaks during booms and also takes negative values. With CRRA, the equity premium remains nearly flat and close to zero. Among the three specification, BDRA is best at capturing empirical properties of the equity premium. Figure 3 shows the time series of the 10-year yield, calculated as described in Section 4.1.4, and the corresponding trajectories for the three model specifications. The models

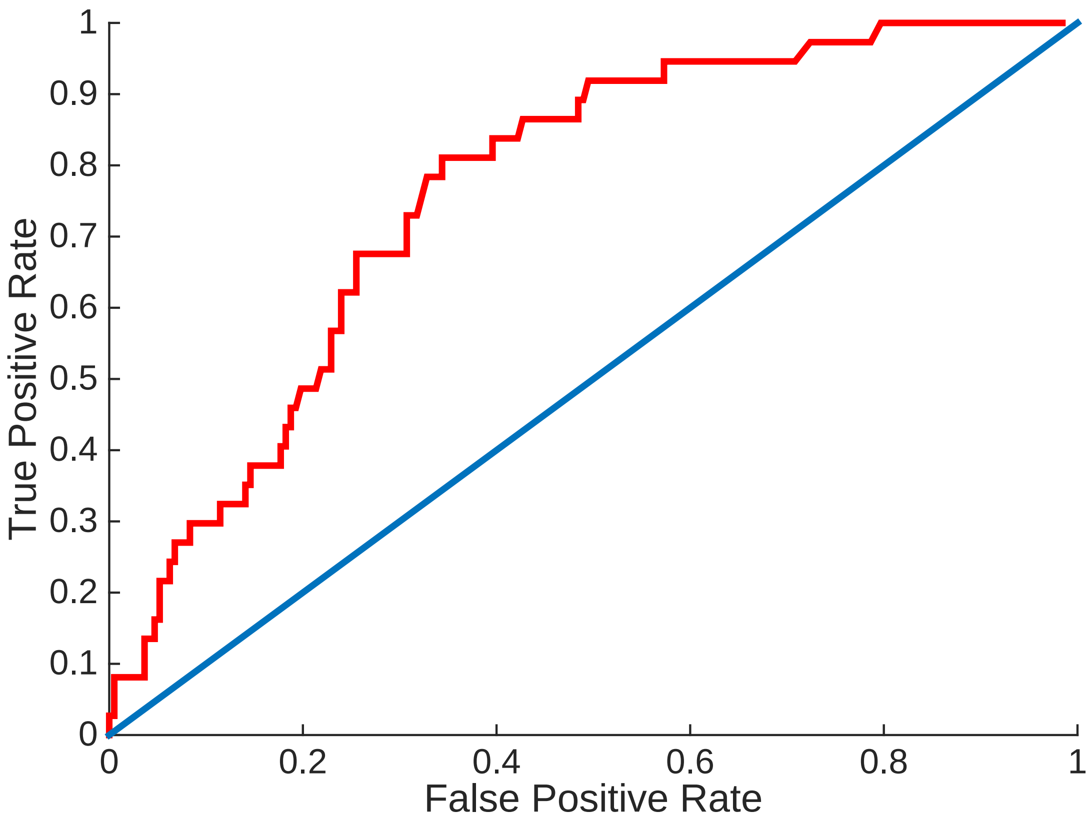

Figure 1: Relative Operating Characteristic (ROC) curve for recession diagnostic. The model implied conditional probability of the bad state is used as a diagnostic for a recession quarter as identified by the NBER.

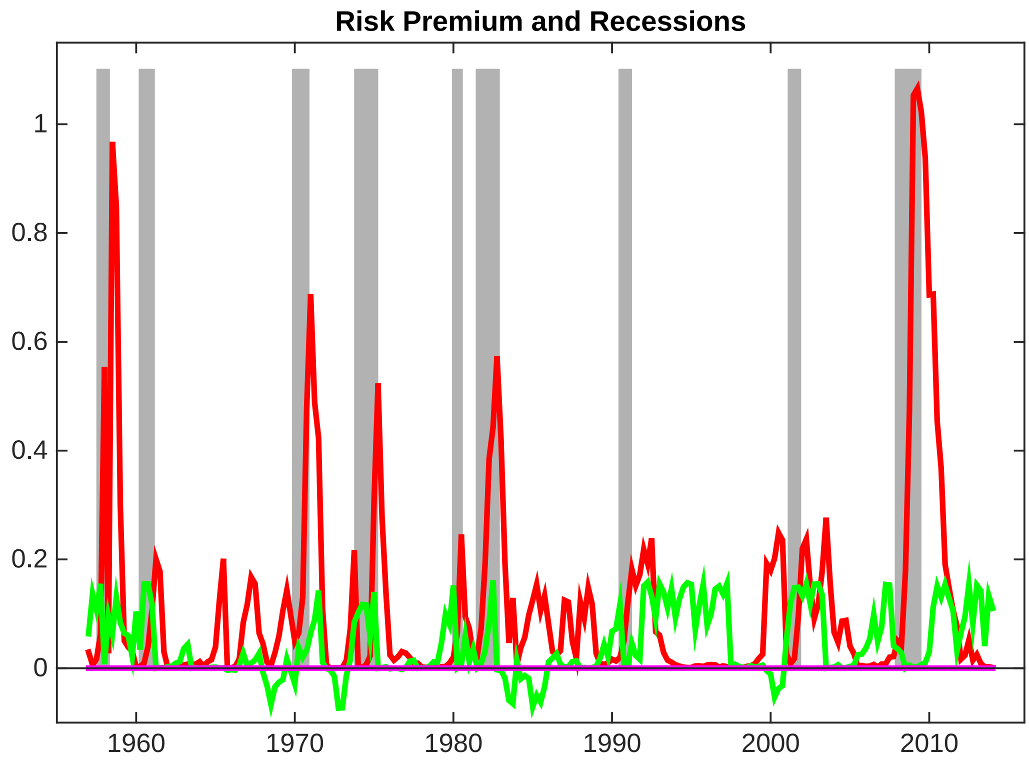

Figure 2: The plots compares the risk premium trajectories for the BDRA (red), CRRA (magenta) and CASU (green) model. Parameters estimates are given in Table 6 for CRRA and Table 7 for CASU. BDRA parameters are given in the paper in Table 2. Conditional probabilities are updated using innovations from Consumption, Dividend, and Unemployment time series.

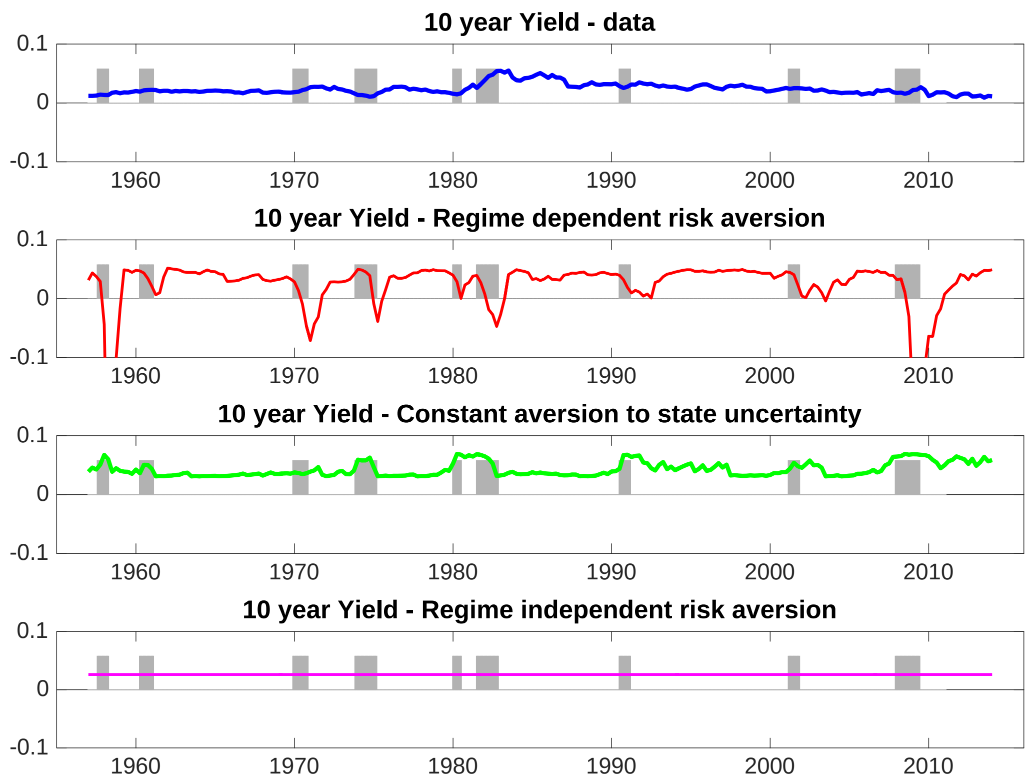

Figure 3: The plots compares the 10-year real yield trajectory for the data (blue), BDRA (red), CASU (green) and CRRA (magenta). Parameters estimates are given in Table 2 in the paper (BDRA), Table 6 (CRRA) and Table 7 (CASU). Conditional probabilities are updated using innovations from Consumption, Dividend, and Unemployment time series. The 10-year yield data series is computed using the methodology described in Section 4.1 in the paper.

with BDRA and CASU produce large fluctuations and fail to capture empirical properties of the yield. The large amplitude of the yield is due to the larger variability of the marginal utility of consumption in these settings, which is essential for capturing the dynamic behavior of the volatility and the equity premium. The model with CRRA fares best, in the sense that it produces a more stable yield, precisely because it cannot generate enough variability to explain the behavior of the other two components of returns.

Figure 4 shows the time series of the volatility for the models with BDRA and CASU along with the realized rolling volatility of the stock index. The BDRA specification captures the empirical dynamics well, with sharp increases during recessions, whereas the CASU specification display large increases in volatility also during expansion periods, in particular during the 90s and in the aftermath of the global financial crisis.

We run simple regressions to test the relationship between the realized rolling volatility and the model implied volatilities. We find that only the BDRA specification produces a positive and statistically significant slope coefficient. The estimation results are provided in Table 1

Table 1: This table provides regression estimates of the realized rolling volatility against model implied volatility for the three different model specifications.

	α	t-stat	β	t-stat	adj. R-squared
BDRA	0.1054	9.2422	0.114	2.069	0.1067
CASU	0.1365	7.7801	-0.023	-0.348	-0.003
CRRA	0.1339	12.115	-0.01397	-0.17959	-0.0043

Figure 6 shows the yield curve for the CASU model. In contrast to BDRA (Figure 5), the real yield curve lies outside the 95% confidence interval around the average real yield curve estimate. In contrast to the data and the BDRA specification, the real yield curve is slightly downward sloping. Short term yields are too large and significantly different from the data. Figure 7 shows the yield curve for the CRRA model. The yield curve in this specification is flat, and for short maturities close to the boundary of the confidence interval. The BDRA model, in contrast, is able to capture the level and the slope of the average yield curve.

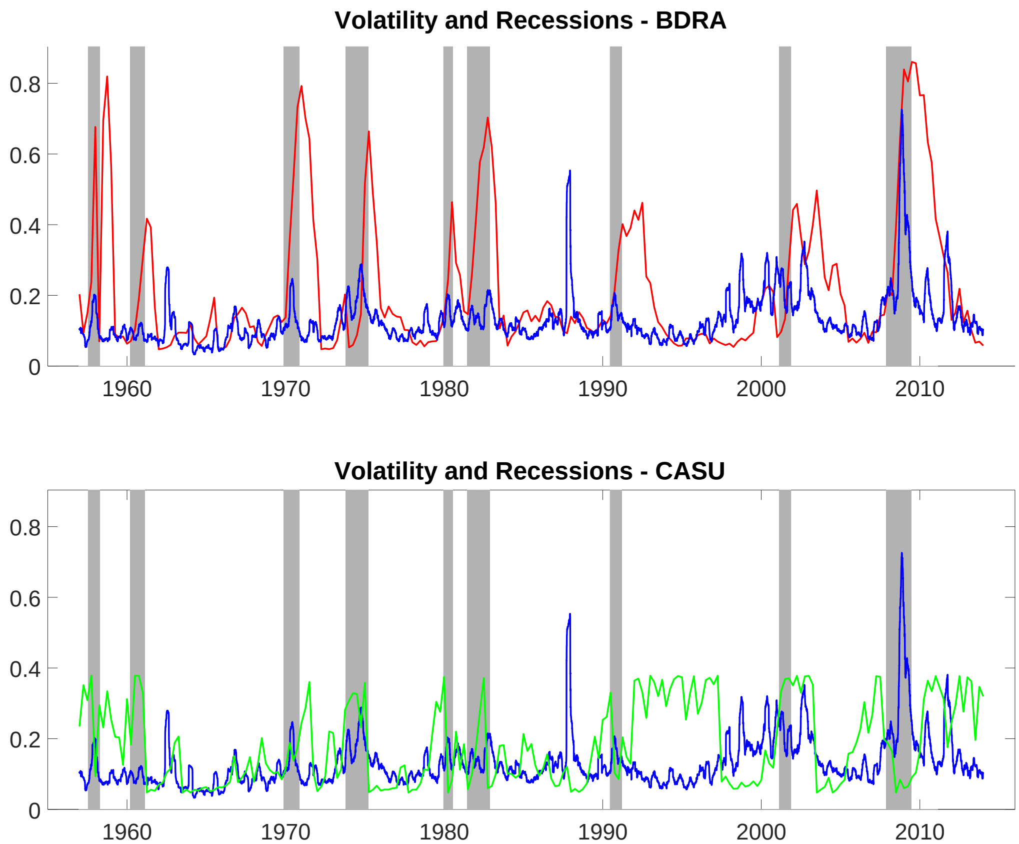

Figure 4: The plots compares the volatility trajectories for the data (blue), BDRA (red) and CASU state uncertainty (green). Parameter estimates are given in Table 2 in the paper (BDRA) and Table 7 (CASU). Conditional probabilities are updated using innovations from Consumption, Dividend, and Unemployment time series. The 10-year yield data series is computed using the methodology described in Section 4.1 in the paper.

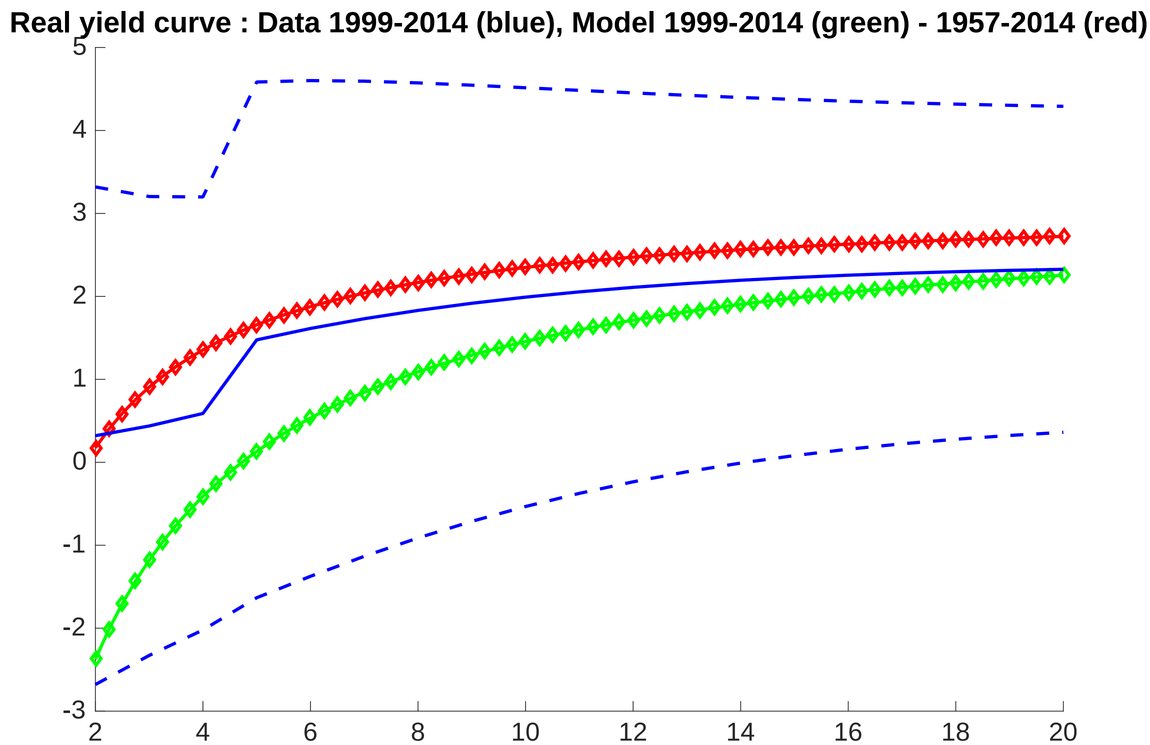

Figure 5: The plot shows the average yield curve in the model over the period 01/1999 to 01/2014 (red), over the period 01/1957 to 01/2014 (green) and in the data (blue) over the period 01/1999 to 01/2014. y-axis is in percentage term. Dashed lines are 95% confidence bounds. Yield data cover maturities from 2 to 20 years. Risk aversion parameters are $R_1 = 1.4384$, $R_2 = 1.9251$ and $R_3 = 1.5938$. Conditional probabilities are updated using innovations from Consumption, Dividend, and Unemployment time series.

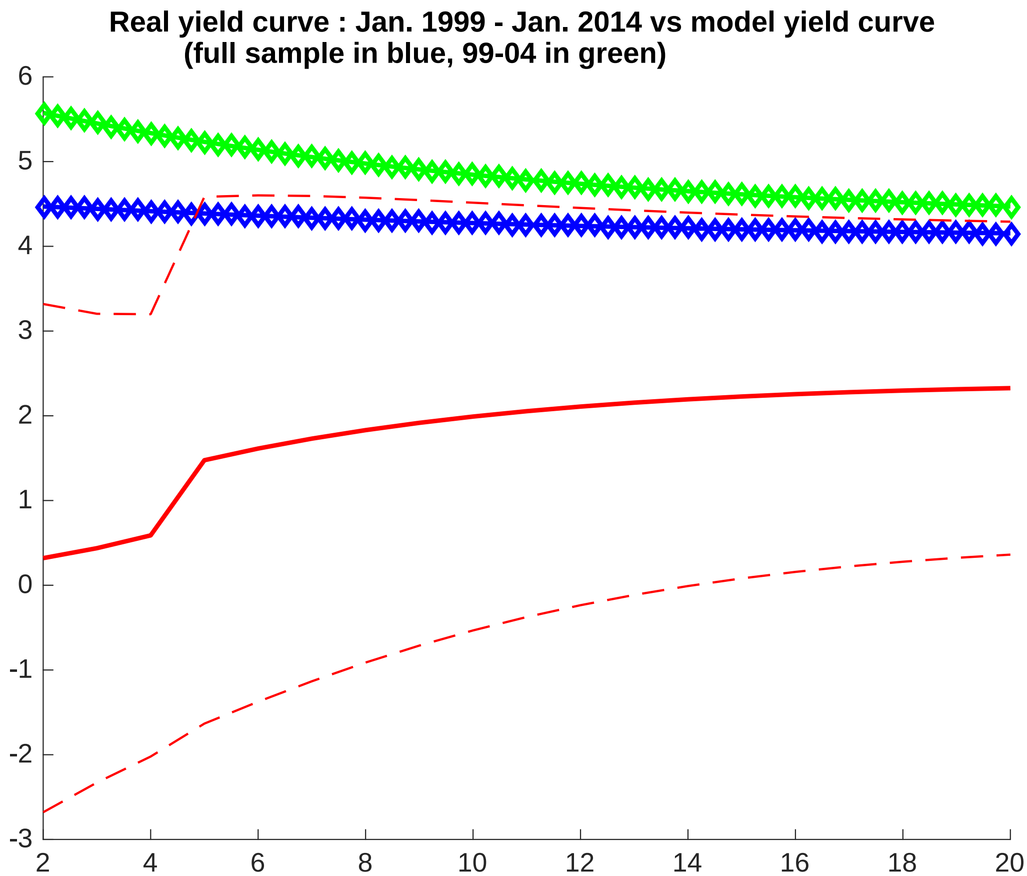

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Figure 6: The plot shows the real yield curve in the CASU model compared to the average yield curve and 95% confidence bounds. The plot shows the average yield curve in the data (red) with 95% confidence intervals (red-dotted) and the model implied yield curve (full sample (blue), later sample (green diamonds)). Parameter estimates are given in Table 7. Conditional probabilities are updated using innovations from Consumption, Dividend, and Unemployment time series. The 10-year yield data series is computed using the methodology described in Section 4.1 in the paper.

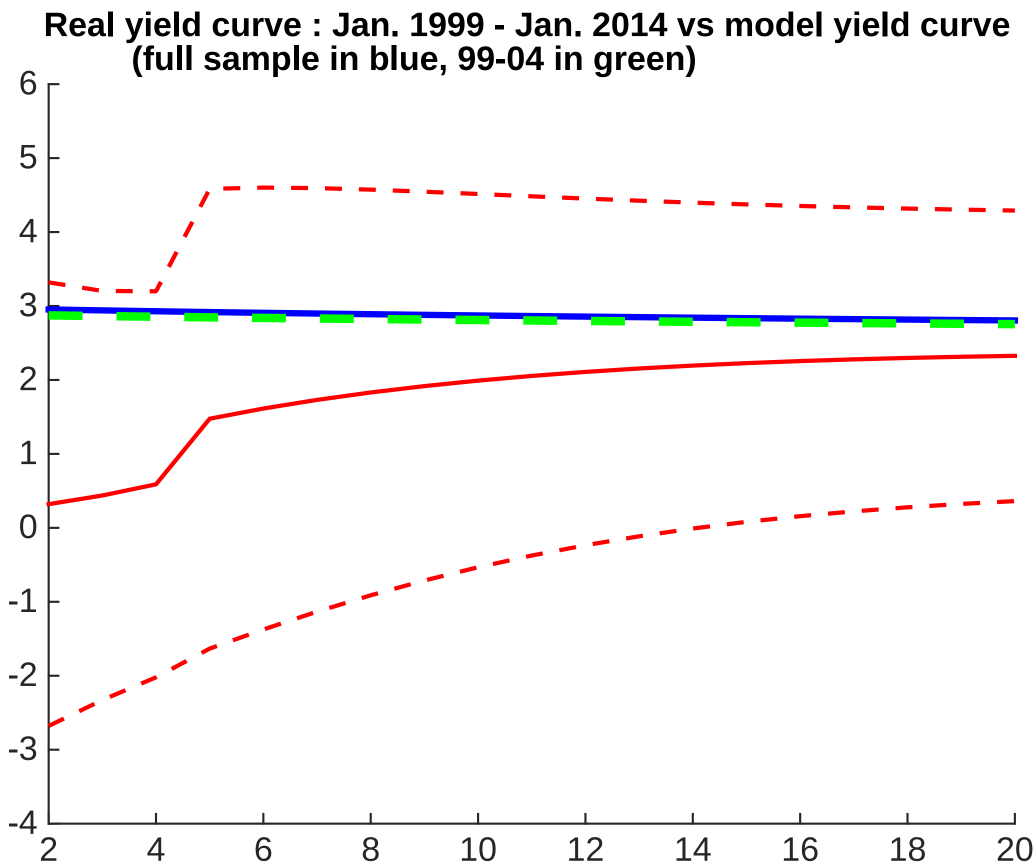

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Figure 7: The plot shows the real yield curve in the CRRA model compared to the average yield curve and 95% confidence bounds. The plot shows the average yield curve in the data (red) with 95% confidence intervals (red-dotted) and the model implied yield curve (full sample (blue), later sample (green diamonds)). Parameter estimates are given in Table 6. Conditional probabilities are updated using innovations from Consumption, Dividend, and Unemployment time series. The 10-year yield data series is computed using the methodology described in Section 4.1 in the paper.

3 Robustness: alternative moment conditions

The risk aversion parameters R_1, R_3 are estimated using the correlations (contemporaneous and lagged one quarter) between log simple returns and changes in the log-PDR (M1). This section shows that these risk aversion parameters estimates are robust with regard to alternative sets of moment conditions given by first and second order return autocorrelations (M2) and by correlations between log simple returns and one and two quarter lagged changes log-PDR (M3). The risk aversion parameters estimates outside the minimal regime, (R_2, R_3), are (1.9251, 1.5938) for moment conditions M1, (1.7895, 1.5823) for M2, and (1.9396,1.775) for M3. Parameter values change slightly but the $\bigcap$-shape of risk aversion parameters across regimes is preserved. This finding establishes that the estimates of parameters in Θ_3 are robust with regard to moment selection.

4 Alternative models: specification tests

Equilibrium formulas for CASU and CRRA are provided in Sections 4.1 and 4.2. Salient differences between the various model specifications are discussed in Section 4.3. A unified framework containing CASU, CRRA and BDRA as subcases is developed in Section 4.4. Model specification tests are carried out in Section 4.5.

4.1 CASU (Veronesi (2004))

Let $\overline{m}' \equiv \left[\exp\left(-\varrho\mu_1^C\right), \cdots, \exp\left(-\varrho\mu_K^C\right)\right]$ where ϱ denotes the constant aversion to state uncertainty parameter. With CASU, marginal utility is $u_c\left(t, c_t, p_t\right) = \exp\left(\varrho g_n - \beta t\right) C_t^{-R} \overline{m}' p_t$. The state price density (SPD) is, $\xi_t = u_c\left(t, c_t, p_t\right) / u_c\left(0, c_0, p_0\right)$.

4.1.1 *MPR*

Market prices of risk are obtained from the covariation of the SPD with the innovations $\left(\nu^C, \nu^G, \nu^Y\right)$. This gives,

$$\theta_t^C = R\sigma^C - \frac{\overline{m}' \operatorname{diag}\left[\frac{\mu_k^C - \widehat{\mu}_t^C}{\sigma^C}\right] p_t}{\overline{m}' p_t} \tag{1}$$

$$\theta_t^G = -\frac{\overline{m}' \operatorname{diag}\left[\frac{\mu_k^G - \widehat{\mu}_t^G}{\sigma^G}\right] p_t}{\overline{m}' p_t} \tag{2}$$

$$\theta_t^Y = -\frac{\overline{m}' \operatorname{diag}\left[\frac{\mu_k^Y - \widehat{\mu}_t^Y}{\sigma^Y}\right] p_t}{\overline{m}' p_t}. \tag{3}$$

4.1.2 *Price-dividend ratio*

The price-dividend ratio (PDR) is,

$$\frac{S_t}{D_t} = \frac{\overline{m}' E_t\left[\int_t^T e^{-\beta_v} C_v^{\kappa - R} G_v p_v dv\right]}{\overline{m}' e^{-\beta t} C_t^{\kappa - R} G_t p_t}.$$

Letting $\overline{\Phi} \equiv -\left(\left(\frac{(\sigma^C)^2}{2}(\kappa - R)(\kappa - R - 1) - \beta\right) I_K + \Lambda' + \text{diag}\left[(\kappa - R)\mu_k^C - \mu_k^G\right]\right)$ and noticing that,

$$d\left(e^{-\beta v} C_v^{\kappa - R} G_v p_v\right) = -\overline{\Phi} e^{-\beta v} C_v^{\kappa - R} G_v p_v dv + dM_v$$

where M_v is a martingale, it follows that $E_t\left[e^{-\beta_v} C_v^{\kappa - R} G_v p_v\right] = \exp\left(-\overline{\Phi}(v - t)\right) e^{-\beta t} C_t^{\kappa - R} G_t p_t$. As long as the real part of the largest eigenvalue of $-\overline{\Phi}$ is negative, we have,

$$\int_t^\infty E_t\left[e^{-\beta v} C_v^{\kappa - R} G_v p_v\right] dv = \overline{\Phi}^{-1} e^{-\beta t} C_t^{\kappa - R} G_t p_t$$

and the PDR becomes,

$$\frac{S_t}{D_t} = \frac{\overline{m}' \overline{\Phi}^{-1} p_t}{\overline{m}' p_t}. \tag{4}$$

4.1.3 *Stock price volatility*

Given (4) for the PDR, the stock volatility coefficients are obtained from the covariations of the log-PDR with the innovations in $\left(\mu^C, \mu^G, \mu^Y\right)$ as follows,

$$\begin{aligned}
\sigma_t^{SC} &= \kappa\sigma^C + \frac{\overline{m}'\Phi^{-1}\text{diag}\left[\frac{\mu_k^C - \widehat{\mu}_t^C}{\sigma^C}\right] p_t}{\overline{m}'\Phi^{-1} p_t} - \frac{\overline{m}'\text{diag}\left[\frac{\mu_k^C - \widehat{\mu}_t^C}{\sigma^C}\right] p_t}{\overline{m}' p_t} \\
&= \kappa\sigma^C + \frac{\overline{m}'\left(\left(\frac{S_t}{D_t}\right)^{-1}\Phi^{-1} - I_K\right)\text{diag}\left[\frac{\mu_k^C - \widehat{\mu}_t^C}{\sigma^C}\right] p_t}{\overline{m}' p_t} &(5) \\
\sigma_t^{SG} &= \sigma^G + \frac{\overline{m}'\Phi^{-1}\text{diag}\left[\frac{\mu_k^G - \widehat{\mu}_t^G}{\sigma^G}\right] p_t}{\overline{m}'\Phi^{-1} p_t} - \frac{\overline{m}'\text{diag}\left[\frac{\mu_k^G - \widehat{\mu}_t^C}{\sigma^G}\right] p_t}{\overline{m}' p_t} \\
&= \sigma^G + \frac{\overline{m}'\left(\left(\frac{S_t}{D_t}\right)^{-1}\Phi^{-1} - I_K\right)\text{diag}\left[\frac{\mu_k^G - \widehat{\mu}_t^G}{\sigma^G}\right] p_t}{\overline{m}' p_t} &(6) \\
\sigma_t^{SY} &= \frac{\overline{m}'\Phi^{-1}\text{diag}\left[\frac{\mu_k^Y - \widehat{\mu}_t^Y}{\sigma^Y}\right] p_t}{\overline{m}'\Phi^{-1} p_t} - \frac{\overline{m}'\text{diag}\left[\frac{\mu_k^Y - \widehat{\mu}_t^Y}{\sigma^Y}\right] p_t}{\overline{m}' p_t} \\
&= \frac{\overline{m}'\left(\left(\frac{S_t}{D_t}\right)^{-1}\Phi^{-1} - I_K\right)\text{diag}\left[\frac{\mu_k^Y - \widehat{\mu}_t^Y}{\sigma^Y}\right] p_t}{\overline{m}' p_t}. &(7)
\end{aligned}$$

4.1.4 *Bond prices*

The bond price is,

$$B_t^{t+\tau} = \exp\left(-\beta\left(T-t\right)\right)\frac{\overline{m}' E_t\left[C_T^{-R} p_T\right]}{\overline{m}' C_t^{-R} p_t}.$$

Letting $\overline{\Phi}^B \equiv \left(\frac{\left(\sigma^C\right)^2}{2} R\left(1+R\right) - \beta\right) I_K + \Lambda' - R\mathrm{diag}\left[\mu_k^C\right]$ and noticing that,

$$d\left(e^{-\beta v} C_v^{-R} p_v\right) = \overline{\Phi}^B e^{-\beta v} C_v^{-R} p_v dv + dM_v^B$$

for some martingale M^B, it follows that $E_t\left[e^{-\beta T} C_T^{-R} p_T\right] = \exp\left(\overline{\Phi}^B\left(T-t\right)\right) e^{-\beta t} C_t^{-R} p_t$ and

$$B_t^{t+\tau} = \frac{\overline{m}' \exp\left(\overline{\Phi}^B \tau\right) p_t}{\overline{m}' p_t}. \tag{8}$$

4.1.5 *Bond yields and short rate*

Given the bond prices (8), the bond yields and short rate are,

$$Y_t^{t+\tau} = -\frac{1}{T-t}\log\left(\frac{\overline{m}' \exp\left(\overline{\Phi}^B\left(T-t\right)\right) p_t}{\overline{m}' p_t}\right)$$

$$r_t = \lim_{t \downarrow T}\left(-\partial_T \log B_t^{t+\tau}\right) = -\frac{\overline{m}' \overline{\Phi}^B p_t}{\overline{m}' p_t}.$$

4.1.6 *Bond and short rate volatilities*

Bond volatility Using the expression for the bond price with $\alpha \in \{C, G, Y\}$ leads to,

$$\begin{aligned}
\sigma^{B,\alpha}\left(t,\tau\right) &= \frac{\overline{m}' \exp\left(\overline{\Phi}^B\left(T-t\right)\right)\mathrm{diag}\left[\frac{\mu_k^\alpha - \hat{\mu}_t^\alpha}{\sigma^\alpha}\right] p_t}{\overline{m}' \exp\left(\Phi^B\left(T-t\right)\right) p_t} - \frac{\overline{m}' \mathrm{diag}\left[\frac{\mu_k^\alpha - \hat{\mu}_t^\alpha}{\sigma^\alpha}\right] p_t}{\overline{m}' p_t} \\
&= \frac{\overline{m}' \left(\left(B_t^{t+\tau}\right)^{-1} \exp\left(\overline{\Phi}^B\left(T-t\right)\right) - I_K\right)\mathrm{diag}\left[\frac{\mu_k^\alpha - \hat{\mu}_t^\alpha}{\sigma^\alpha}\right] p_t}{\overline{m}' p_t}.
\end{aligned}$$

Short rate volatility Using the expression for the short rate for $\alpha \in \{C, G, Y\}$ leads to,

$$\begin{aligned}
\sigma_t^{r,\alpha} &= -\frac{\overline{m}'\overline{\Phi}^B \mathrm{diag}\left[\frac{\mu_k^\alpha + \widehat{\mu}_t^\alpha}{\sigma^\alpha}\right] p_t}{\overline{m}' p_t} + r_t \frac{\overline{m}' \mathrm{diag}\left[\frac{\mu_k^\alpha - \widehat{\mu}_t^\alpha}{\sigma^\alpha}\right] p_t}{\overline{m}' p_t} \\
&= -\frac{\overline{m}'\left(\overline{\Phi}^B - r_t I_K\right) \mathrm{diag}\left[\frac{\mu_k^\alpha - \widehat{\mu}_t^\alpha}{\sigma^\alpha}\right] p_t}{\overline{m}' p_t}.
\end{aligned}$$

4.2 CRRA

The equilibrium coefficients in the CRRA model are obtained by setting $\varrho = 0$, therefore $\overline{m}' = 1_K'$, in the CASU model. Table 2 summarizes the equilibrium quantities for all models considered.

4.3 Model comparison

4.3.1 *CASU*

The functional form of equilibrium in the CASU model is similar to that in the BDRA model as $\left(Z_t, \Upsilon, A(\tau), H, H^{\tau,B}, H^r\right)$ are replaced by $\left(\overline{Z}_t, \overline{\Upsilon}, \overline{A}(\tau), \overline{H}, \overline{H}^{\tau,B}, \overline{H}_t^r\right)$. BDRA generates additional terms in the stock and bond volatility components that are tied to consumption, namely $-Z_t' \mathrm{diag}[R_k] H_t p_t \sigma^C$ and $Z_t' H_t^{\tau,B} p_t \sigma^C$. These terms emerge as Z_t depends on both C_t and p_t, whereas with CASU, $\overline{Z}_t$ only depends on p_t. These additional volatility terms vanish asymptotically as consumption becomes large.

If $\arg\min_j \mu_j^C = \arg\min_j R_j$ and $(\varrho, C_t) \to (\infty, \infty)$, then the asymptotic equilibrium coefficients with CASU and BDRA are identical.

4.3.2 *CRRA*

With CRRA, learning only affects the drift of the state dynamics. MPRs are $\theta_t^C = R\sigma^C$, $\theta_t^G = \theta_t^Y = 0$. The risk premium is therefore the same as in the i.i.d. setting of Mehra and Prescott (1982). Similarly, the short rate becomes $r_t = \beta + R\widehat{\mu}_t^C - \frac{1}{2}R(1+R)\left(\sigma^C\right)^2$ and only differs from the interest rate in the i.i.d. model of Mehra and Prescott by the stochastic consumption growth rate $\widehat{\mu}_t^C$. Empirically, as shown in the original articles on this topic, the i.i.d. model produces a poor fit for the equity premium and the risk free rate for moderate levels of risk aversion. Market volatility and the equity premium are both small. Furthermore, a correlation puzzle emerges as

Table 2: This table shows the equilibrium MPRs θ_t^{α}, PDRs S_t/D_t, stock volatilities $\sigma_t^{S\alpha}$, bond prices $B_t^{t+\tau}$, yields $Y_t^{t+\tau}$, short rates r_t, bond and short rate volatilities $\sigma^{B\alpha}(t,\tau)$ and $\sigma_t^{r\alpha}$ for the BDRA, CASU and CRRA model specifications.

	BDRA	CASU	CRRA
		MPRs	
θ_t^C	$Z_t'\mathrm{diag}\left[R_k\right] p_t\sigma^C - Z_t'\sigma_t^{p,C}p_t$	$R\sigma^C - \overline{Z}_t'\sigma_t^{p,C}p_t$	$R\sigma^C - 1_K'\sigma_t^{p,C}p_t = R\sigma^C$
θ_t^G	$-Z_t'\sigma_t^{p,G}p_t$	$-\overline{Z}_t'\sigma_t^{p,G}p_t$	$1_K'\sigma_t^{p,G}p_t = 0$
θ_t^Y	$-Z_t'\sigma_t^{p,C}p_t$	$-\overline{Z}_t'\sigma_t^{p,Y}p_t$	$1_K'\sigma_t^{p,Y}p_t = 0$
		PDR	
$\frac{S_t}{D_t}$	$Z_t'\Upsilon p_t$	$\overline{Z}_t'\overline{\Upsilon}p_t$	$1_K'\overline{\Upsilon}p_t$
		Stock volatility	
σ_t^{SC}	$\rho^{DC}\sigma^D + Z_t'H_t\sigma_t^{p,C}p_t$ $-Z_t'\mathrm{diag}\left[R_k\right]H_tp_t\sigma^C$	$\rho^{DC}\sigma^D + \overline{Z}_t'\overline{H}_t\sigma_t^{p,C}p_t$	$\rho^{DC}\sigma^C + 1_K'\overline{H}_t\sigma_t^{p,C}p_t$
σ_t^{SG}	$\sigma^D\sqrt{1-(\rho^{DC})^2} + Z_t'H_t\sigma_t^{p,G}p_t$	$\sigma^D\sqrt{1-(\rho^{DC})^2} + \overline{Z}_t'\overline{H}_t\sigma_t^{p,G}p_t$	$\sigma^D\sqrt{1-(\rho^{DC})^2} + 1_K'\overline{H}_t\sigma_t^{p,G}p_t$
σ_t^{SY}	$Z_t'H_t\sigma_t^{p,Y}p_t$	$\overline{Z}_t'\overline{H}_t\sigma_t^{p,Y}p_t$	$1_K'\overline{H}_t\sigma_t^{p,Y}p_t$
		Bond price, yields, and short rate	
$B_t^{t+\tau}$	$Z_t'A(T-t)\,p_t$	$\overline{Z}_t'\overline{A}(T-t)\,p_t$	$1_K'\overline{A}(T-t)\,p_t$
$Y_t^{t+\tau}$	$-\frac{\log\left(Z_t'A(T-t)p_t\right)}{T-t}$	$-\frac{\log\left(\overline{Z}_t'\overline{A}(T-t)p_t\right)}{T-t}$	$-\frac{\log\left(1_K'\overline{A}(T-t)p_t\right)}{T-t}$
r_t	$-Z_t'\dot{A}(0)\,p_t$	$-\overline{Z}_t'\dot{\overline{A}}(0)\,p_t$	$-1_K'\dot{\overline{A}}(0)\,p_t = \beta + R\widehat{\mu}_t^C - \frac{1}{2}R(1+R)\left(\sigma^C\right)^2$
		Bond and short rate volatility	
$\sigma^{BC}(t,\tau)$	$Z_t'H_t^{\tau,B}\sigma_t^{p,C}p_t$ $-Z_t'\mathrm{diag}\left[R_k\right]H_T^{\tau,B}p_t\sigma^C$	$\overline{Z}_t'\overline{H}_t^{\tau,B}\sigma_t^{p,C}p_t$	$1_K'\overline{H}_t^{\tau,B}\sigma_t^{p,C}p_t$
$\sigma^{B\alpha}(t,\tau)$	$Z_t'H_t^{\tau,B}\sigma_t^{p,\alpha}p_t$	$\overline{Z}_t'\overline{H}_t^{\tau,B}\sigma_t^{p,\alpha}p_t$	$1_K'\overline{H}_t^{\tau,B}\sigma_t^{p,\alpha}p_t$
$\sigma_t^{r\alpha}$	$-Z_t'H_t^r\sigma_t^{p,\alpha}p_t$	$-\overline{Z}_t'\overline{H}_t^r\sigma_t^{p,\alpha}p_t$	$-1_K'\overline{H}_t^r\sigma_t^{p,\alpha}p_t = -RCOV_t\left(\mu^C(s_t),\mu^{\alpha}(s_t)\right)$

$$\sigma_t^{p,\alpha} \equiv \mathrm{diag}\left[\frac{\mu_k^Y - \widehat{\mu}_t^Y}{\sigma^Y}\right]; \quad \overline{Z}_t \equiv \frac{\overline{m}}{\overline{m}'p_t}; \quad Z_t = \frac{M_t}{M_t'p_t}$$

$$\overline{m}' \equiv \left[\exp\left(-\varrho\mu_1^C\right),\cdots,\exp\left(-\varrho\mu_K^C\right)\right]; \quad M_t' \equiv \left[C_t^{-R_1},\ldots,C_t^{-R_k}\right],$$

$$\overline{\Upsilon} \equiv \overline{\Phi}^{-1}; \quad \overline{\Phi} \equiv -\left(\left(\frac{\left(\sigma^C\right)^2}{2}(\kappa-R)(\kappa-R-1)-\beta\right)I_K + \Lambda' + \mathrm{diag}\left[(\kappa-R)\,\mu_k^C - \mu_k^G\right]\right)$$

$$\Upsilon_{ij} = e_i'\Phi_i^{-1}e_j; \quad \Phi_i \equiv -\left(\left(\frac{\left(\sigma^C\right)^2}{2}(\kappa-R_i)(\kappa-R_i-1)-\beta\right)I_K + \Lambda' + \mathrm{diag}\left[(\kappa-R_i)\,\mu_k^C - \mu_k^G\right]\right)$$

$$H_t \equiv \left(\frac{S_t}{D_t}\right)^{-1}\Upsilon - I_K; \quad \overline{H}_t \equiv \left(\frac{S_t}{D_t}\right)^{-1}\overline{\Upsilon} - I_K;$$

$$\overline{A}(T-t) \equiv \exp\left(\overline{\Phi}^B(T-t)\right); \quad \overline{\Phi}^B \equiv \left(\frac{\left(\sigma^C\right)^2}{2}R(1+R)-\beta\right)I_K + \Lambda' - R\,\mathrm{diag}\left[\mu_k^C\right]$$

$$A_{ij}(T-t) = e_i'\exp\left(\Phi_i^B(T-t)\right)e_j; \quad \Phi_i^B \equiv \left(\frac{\left(\sigma^C\right)^2}{2}R_i(1+R_i)-\beta\right)I_K + \Lambda' - \mathrm{diag}\left[R_i\mu_k^C\right]$$

$$\dot{A}_{ij}(0) \equiv e_i'\Phi_i^B e_j; \quad \dot{\overline{A}}(0) \equiv \overline{\Phi}^B$$

$$H_t^{\tau,B} \equiv \left(B_t^{t+\tau}\right)^{-1}A(\tau) - I_K; \quad \overline{H}_t^{\tau,B} \equiv \left(B_t^{t+\tau}\right)^{-1}\overline{A}(\tau) - I_K$$

$$H_t^r \equiv \dot{A}(0) - r_tI_K; \quad \overline{H}_t^r \equiv \dot{\overline{A}}(0) - r_tI_K$$

only one risk factor is priced.

4.3.3 *Steady state*

Steady state values in the model with BDRA, if there is a single minimal risk aversion regime, are obtained by taking $Z_\infty = e_1'/p_{1\infty}$ and $p_t = p_\infty$. In the model with CASU, they are found by setting $\overline{Z}_t$ equal to $Z_\infty = m/m'p_\infty$. In the model with CRRA, they are obtained by taking $p_t = p_\infty$. Note that in the steady state, the additional volatility coefficients of the BDRA model, i.e., $-Z_t'\text{diag}\left[R_k\right] H_t^{\tau,B} p_t \sigma^C$ and $-Z_t'\text{diag}\left[R_k\right] H_t p_t \sigma^C$, vanish. These coefficients contribute to the short run dynamics of the model.

4.4 Unified model

To perform specification tests, the models with CASU and BDRA are embedded in a unified framework where the marginal utility of the representative agent is $\sum_{k=1}^K e^{-\beta t} e^{-\varrho \mu_k^C} C_t^{-R_k} p_{kt}$. Equilibrium formulas are the same as with BDRA except that Z_t is replaced by,

$$\widetilde{Z}_{kt} = \frac{\exp\left(-\varrho \mu_k^C\right) C_t^{-R_k}}{\sum_{k=1}^K \exp\left(-\varrho \mu_k^C\right) C_t^{-R_k} p_{kt}}.$$

In the steady state, when $C_t \to \infty$, $\widetilde{Z}_t - Z_t \to 0$. Hence, the steady state equilibrium quantities in the unified model are the same as in the model without state uncertainty aversion or beliefs-dependent risk aversion. As a consequence, a specification test for BDRA versus CASU must rely on dynamic quantities that pin down the parameters in Θ_3. Furthermore, if the minimal risk aversion regime coincides with the minimal consumption growth regime, i.e., if $\arg\min_k R_k = \arg\min_k \mu_k^C$, then the steady state moment conditions with CASU converge to those with BDRA as the state uncertainty aversion parameter $\varrho \to \infty$. Therefore, if the true model has BDRA, inference based on static moment conditions for CASU will rely on parameter estimates on the boundary of the parameter space. Standard specification tests are then invalid. For this reason, dynamic moment conditions must be used. The corresponding specification tests are discussed next.

4.5 Specification tests

Both nested and non-nested specification tests are performed. The various nested model specification tests performed next are based on the D-statistic, $D_T = T\left(\widehat{J}_T - \widetilde{J}_T\right)$ where $\widehat{J}_T$ (resp. $\widetilde{J}_T$) is the unrestricted (resp. restricted) GMM objective function. This statistic has been pioneered for instrumental variable estimation by Gallant and Jorgenson (1979) and studied for method of moment estimators by West and Whitney (1987). Similar to a likelihood ratio test, the statistic corresponds to an appropriately scaled difference between unrestricted and restricted model estimates. Asymptotically, for large T, the statistic is χ^2_r-distributed where r is the number of constraints.[2]

Non-nested specification tests are based on the N-statistic, $N_T = T^{1/2}\left(\widehat{J}_T^{model1} - \widehat{J}_T^{model2}\right)/\widehat{\sigma}_J$ where $\widehat{J}_T^{model1}$, $\widehat{J}_T^{model2}$ are the GMM-objective functions and $\widehat{\sigma}_J$ is an estimator of the asymptotic variance of the numerator of N, $\sigma_J^2 \equiv \lim_{T\to\infty} VAR\left[T^{1/2}\left(\widehat{J}_T^{model1} - \widehat{J}_T^{model2}\right)\right]$. This test statistic for non-nested hypothesis testing was pioneered by Rivers and Vuong (2002). Under the null hypothesis that the two models, i.e., the two sets of moment restrictions, provide an equivalent fit, the N-statistic is asymptotically standard normal if both models are misspecified and, as demonstrated by Hall and Pelletier (2011), it has a non-standard distribution that depends on nuisance parameters if both models are (locally) correctly specified.[3] Rivers and Vuong assume that $\sigma_J^2 \neq 0$. Hall and Pelletier (2011) show that this is only the case if both models are misspecified. The N-test rejects the null hypothesis against the alternative H_{1a} that model 2 provides a better fit if the N-statistic is larger than the $1-\alpha\%$ quantile of the asymptotic limit distribution. It rejects the null hypothesis against the alternative H_{1b} that model 1 dominates if the test statistic is smaller than the $\alpha\%$ quantile of the asymptotic limit distribution. To calculate the N-statistic, the standard deviation in the denominator is obtained from a stationary bootstrap estimator based on Romano and Politis (2004). As the test based on the asymptotic distribution has potentially limited power in finite samples and as the distribution itself depends on auxiliary assumptions about model misspecification, the N-test is also implemented using empirical quantiles and p-values from the empirical cumulative distribution function calculated using the stationary bootstrap.

[2] As shown by West and Whitney (1987), the D statistic used for the test is numerically equivalent to the Wald or LM/score statistics in the just identified case.

[3] They show that if the model is (locally) correctly specified, $\sigma_J = 0$, and therefore, the limit distribution of the the N-statistic is not standard normal but given by the ratio of a quadratic form involving standard normal random variables in the numerator and the square root of a different quadratic form involving the same normal random variables in the denominator.

To consider model specification tests based on a unified model structure has the advantage that, in contrast to specification tests of non-nested models, it does not require additional generic assumptions on the asymptotic variance σ_J^2, which potentially imply non-standard asymptotic distributions of the N-statistic. Furthermore, the statistic for tests of nested models is not sensitive to the weighting of moment conditions. In non-nested specification tests, the test statistic depends on the weighting of moment conditions both through the numerator and the denominator. In contrast to correctly specified models, the choice of the optimal weighting scheme cannot rely on asymptotic efficiency considerations if the models are misspecified. As emphasized by Hall and Pelletier (2011) an optimal choice of instruments and therefore an optimal weighting of moment conditions for misspecified models has yet to be developed. The fact that the asymptotic distribution of the non-nested test relies on auxiliary assumptions about misspecification renders inference difficult. Ideally, using overidentifying restrictions, a statistical test should decide whether σ_J^2 is zero or not, i.e. whether models are misspecified. Hall and Pelletier (2011) show that such a test is not feasible as it depends on the unknown bias of the misspecified model. The non-nested specification test results presented in this paper have been obtained under non-adapted choices of weighting schemes, i.e., the weighting is not model-dependent. To use a fixed weighting for both CASU and BDRA alleviates some of the concerns expressed in Hall and Pelletier (2011).

To estimate the unified model and perform the various specification tests, the correlation between the short and long term yields is used as an additional stationary moment condition. Similarly, the dynamic moment conditions used are the autocorrelation of simple returns for one and two quarters, the cross-correlation between simple stock returns and short term yields lagged by one quarter, as well as the cross-correlation between simple stock returns and short and long term yields lagged by two quarters.

The following nested model specification tests are performed $H_0 : R_k = R$, $\beta_k = \beta$ and $\varrho = 0$ versus $H_1 : R_k \neq R$, $\beta_k = \beta$ and $\varrho = 0$ or $\varrho \neq 0$ (CRRA (A1)) as well as $H_0 : \varrho = 0$, $\beta_k = \beta$ and $R_n \neq R$ some k versus $H_1 : \varrho \neq 0$, $\beta_k = \beta$ and $R_k \neq R$ some k (CASU (A2)). Note that under the alternative hypothesis, as a minimal regime exists, the stationary moment constraints are the same whether or nor the state uncertainty parameter $\varrho = 0$. It is therefore appropriate to base the test of CRRA against BDRA just on stationary moment conditions. Compared to a test based on all moment conditions, this test is conservative, i.e., if the null hypothesis is rejected, it will also be rejected in a test that includes dynamic moment conditions. Similarly, to test whether the state uncertainty parameter is null must be based exclusively on dynamic moment conditions. This follows because the stationary moment conditions do not depend on the aversion to state uncertainty parameter in the presence of a minimal risk aversion regime. As a result, the D-statistic calculated for the test $H_0 : \varrho = 0$ versus $H_1 : \varrho \neq 0$ only depends on the dynamic moment conditions.[4]

Table 3 summarizes the test results. Parameter estimates for the constrained model are given in Table 6 for CRRA and Table 7 for CASU. The null hypothesis of constant risk aversion is rejected (A1, B1) whereas the null hypothesis of no state uncertainty aversion cannot be rejected (A2). These two tests provide evidence that the model with beliefs-dependent risk aversion, i.e., the BDRA model is the best model. Two additional tests provide further evidence for this claim. Within the BDRA model specification ($\varrho = 0$), the null hypothesis $H_0 : \beta_k = \beta$ for all k and $R_k \neq R$ some k vs. $H_1 : \beta_k \neq \beta$ and $R_k \neq R$ some k cannot be rejected (B3). There is no evidence for regime-dependent subjective discount rate parameters. Similarly, the null hypothesis $H_0 : R_1 = R_3$ vs. $H_1 : R_1 \neq R_3$ can be rejected (B2). Restricting the number of risk aversion regimes and forcing a symmetric $\bigcap$-shaped structure of risk aversion parameters does not improve the model fit.

Non-nested specification tests and AIC and BIC model selection criteria strongly favor the BDRA model over CASU. The null hypothesis that BDRA and CASU provide equivalent fits of the moment conditions is strongly rejected against the hypothesis that BDRA provides a better fit. Both model selection criteria strongly favor BDRA even though BDRA has one more parameter. Non-nested specification tests and model selection criteria confirm the results of nested specification tests.

To summarize, all the specification tests suggest that the best model is the BDRA model with three risk aversion parameters, no state uncertainty aversion and constant subjective discount rates.

[4] If only stationary moment conditions are used and the BDRA model is the correct model, the estimated state uncertainty parameter must be infinite. Estimation results obtained from stationary moment conditions alone confirm this, providing further evidence against the CASU model.

Table 3: This table summarizes all the specification tests of the models and the model selection criteria based on the sample period 01/1957 - 01/2014. Both nested (panels (A) and (B)) and non-nested (panel (C)) tests are performed (see online Appendix for more details). GMM-AIC and GMM-BIC model selection criteria are calculated as described in Andrews (1999).

I. Specification tests		
Test-statistic (D_T resp. N_T)	critical value	p-value
(A) Nested model specification tests unified model (D-test, size $\alpha = 5\%$)		
(A1) Test CRRA:(*) $H_0 : R_k = R$, $\beta_k = \beta$ and $\varrho = 0$ vs. $H_1 : R_k \neq R$, $\beta_k = \beta$ and $\varrho = 0$ or $\varrho \neq 0$		
>75.0963	5.9915	0
(A2) Test CASU: $H_0 : \varrho = 0$, $\beta_k = \beta$, $R_k \neq R$ some k vs. $H_1 : \varrho \neq 0, \beta_k = \beta$, $R_k \neq R$ some k		
0.0098	3.8415	0.9210
(A3) Test beta: $H_0 : \beta_k = \beta, \varrho = 0$, and $R_k \neq R$ some k vs. $H_1 : \beta_k \neq \beta, \varrho \neq 0$ and $R_k \neq R$ some k		
0.0113	5.9915	0.9944
(B) Nested model specification test within BDRA ($\varrho = 0$) (D-test, size $\alpha = 5\%$)		
(B1) Test CRRA:(**) $H_0 : R_k = R$ vs. $H_1 : R_k \neq R$		
>75.0963	5.9915	0
(B2) Test # R-regimes: $H_0 : R_1 = R_3$ vs. $H_1 : R_1 \neq R_3$		
10.3081	3.8415	0.0013
(B3) Test beta: $H_0 : \beta_k = \beta$ all k, and $R_k \neq R$ some k vs. $H_1 : \beta_k \neq \beta$ and $R_k \neq R$ some k		
0.0013	5.9915	0.9993
(C) Non-nested model specification tests (Rivers-Vuong test, size $\alpha = 5\%$)		
(C1) Asymptotic, non-nested Test CASU:(***) $H_0 : CASU \sim BDRA$ vs. $H_{1a,b} : BDRA \gtrless CASU$		
6082.14	1.6449	0
(C2) Bootstrapped, non-nested Test CASU: $H_0 : CASU \sim BDRA$ vs. $H_{1a,b} : BDRA \gtrless CASU$		
6082.14	3.4894	0
II. GMM-AIC and GMM-BIC model selection criteria		
Model	GMM-AIC	GMM-BIC
CASU	1368.24	1364.81
BDRA	868.30	868.30

(∗) This test only depends on stationary moment conditions and is therefore conservative.

(∗∗) This test is a conservative test within the BDRA specification.

(∗ ∗ ∗) This test assumes that both CASU and BDRA are misspecified such that the asymptotic distribution is Gaussian.

Table 4 compares stationary moments of various model specifications. The CRRA model generates low asset volatility and low risk premium, consistent with the predictions above. The CASU model provides reasonable unconditional moments, but the correlations are off as well. In fact, as shown in Figure 8, the most important volatility component in the CASU model is the consumption component, σ_t^{SC}. This translates into a strong consumption component in the risk premium and, as a consequence, a correlation between consumption and stock returns that is too high. The same occurs for the correlation between the changes in log-PDR and consumption growth. In the CASU model, moments are affected by both cash-flow and discount factor risks, but the discount factor risk is heavily affected by consumption variations. This is the case despite the presence of state probabilities in the stochastic discount factor, because the consumption component dominates from an empirical point of view.

The tension between matching the equity premium and the stock volatility, and matching correlations between cash-flows and stock returns also affects the parameter estimation in the CRRA model. To match correlations in the CRRA model, the risk aversion coefficient must be low. This results in a risk premium that is too small. Alternatively, if risk aversion is high, the risk premium is larger, but as this risk premium is generated exclusively by consumption risk, a strong correlation between returns and cash flows emerges, i.e., there is a correlation puzzle.

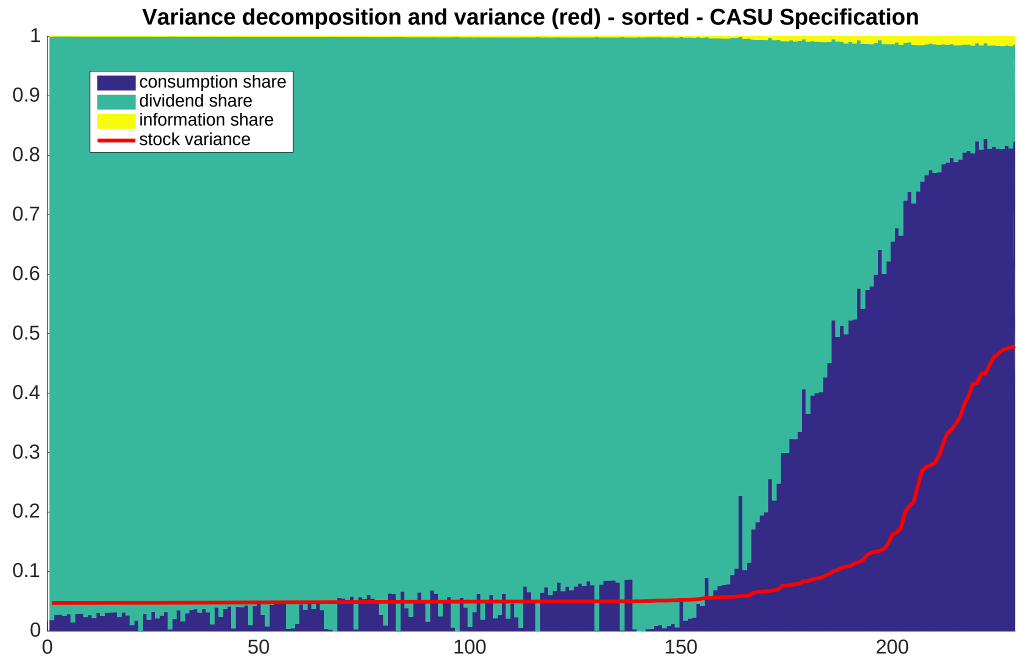

Figure 8: The plot shows the stock return variance decomposition along with the model implied stock variance displayed by increasing level of variance, for the CASU specification. The three components correspond to consumption source (blue), dividend source (green) and information source (yellow).

In contrast, in the BDRA specification, the most important volatility component is σ_t^{SY}, i.e., the information risk factor (see Figure 6 in the paper and Table 5 below). This solves the correlation puzzles: the correlation between stock returns and consumption and between stock returns and dividends are both low. In addition, other moments such as risk premia and volatilities match the data well. This is a direct consequence of the differences in model specifications between CASU and BDRA. Table 5 provides the decompositions of volatilities in their orthogonalized constituents for each model. This decomposition highlights the importance of the information factor for volatility in the BDRA specification. In contrast to CASU and CRRA, the BDRA model can generate a high risk premium through the discount factor risk channel (information premium) and keep correlations between cash-flows (consumption and dividends) and stock returns low and as such resolve the correlation and equity premium puzzles.

Table 4: Model comparison: The following table compares various moments for the different specifications with empirical estimates and confidence intervals.

	BDRA	CASU	CRRA	data	lower bound	upper bound
μ_C	0.019227	0.021328	0.011313	0.019528	0.017107	0.021825
μ_D	0.007995	0.029983	0.022961	0.017837	0.0063248	0.031133
log PDR	3.735323	3.395930	3.6158	3.5828	3.5374	3.6369
yield 10 years	0.030116	0.032893	0.024883	0.023794	0.022747	0.025143
σ_{stock}	0.210065	0.190269	0.05750	0.17323	0.15623	0.19618
σ_{yield}	0.021058	0.016859	0.0009958	0.0089543	0.0078969	0.010165
excess_return	0.048652	0.049889	0.0001554	0.048838	-0.000075296	0.095248
ρ_{SC}	0.108912	0.914189	0.26635	0.27898	0.13665	0.40471
ρ_{SD}	0.249208	0.495474	0.22027	0.13565	-0.0047628	0.25919
ρ_{YC}	0.156844	-0.958982	0.38881	0.22974	0.12735	0.33731
ρ_{YD}	-0.005639	-0.295547	-0.19572	-0.048961	-0.15731	0.067263
μ_{unem}	0.015751	0.022114	-0.023579	0.015664	-0.022489	0.040154
ρ_{rC}	0.270482	-0.954150	0.30545	0.22051	0.09226	0.33557
ρ_{rD}	0.004233	-0.308277	-0.3073	-0.088903	-0.23183	0.042373
$\sigma_{\log(pdr)}$	0.162782	0.142986	0.01022	0.17554	0.15822	0.19816
$\rho_{\log(pdr),C}$	0.291318	0.923253	0.19344	0.2534	0.11371	0.37134
yield 3 months	0.030494	0.031377	0.024906	0.010114	0.0082206	0.012191
$\rho_{r,Y}$	0.999860	0.999840	0.9916	0.72625	0.66436	0.78034

Table 5: Volatility decomposition: $\sigma_t^{S\alpha}/\sigma_t^S$ where $\alpha \in \{C, G, Y\}$.

		lower tier	mid tier	top tier
BDRA	cons (C)	12.7%	37.6%	21.4%
	div (G)	48.4%	11.2%	2.8%
	info (Y)	38.9%	51.2%	75.8%
CASU	cons (C)	3.0%	4.29%	50.25%
	div (G)	97.0%	95.59%	48.82%
	info (Y)	0.0%	0.12%	0.93%

Table 6: Estimated parameters (standard errors) CRRA model: GMM parameter estimates with standard errors obtained from stationary bootstrap (Politis and Romano (1994).

Growth Regime			Growth Regime		
Normal	Low	High	Normal	Low	High
Consumption			Unemployment		
μ_1^C	μ_2^C	μ_3^C	μ_1^{UE}	μ_2^{UE}	μ_3^{UE}
0.00984	0.01451	0.01949	-0.086546	0.14994	0.009994
(0.00224)	(0.00195)	(0.00337)	(0.02741)	(0.03644)	(0.04075)
Dividend			Preferences: risk aversion		
μ_1^D	μ_2^D	μ_3^D		R	
0.03398	-0.01228	0.04672		1.1051	
(0.00702)	(0.00737)	(0.00860)		(0.90457)	

Preferences: subjective discount rate
0.01250
(0.00413)

Standard Deviations and Correlations

	Consumption	Dividend	Unemployment
Consumption	0.0092	0.1664	-0.3914
	(0.0006)	(0.0653)	(0.0594)
Dividend	0.1664	0.0473	-0.3231
	(0.0653)	(0.0079)	(0.0596)
Unemployment	-0.3914	-0.3231	0.1244
	(0.0594)	(0.0596)	(0.0137)

Infinitesimal Generator

	Normal	Low	High	**Steady state probabilities**
Normal	-0.07347	0.07347	3.6528e-07	**0.7197**
	-	(0.01857)	(7.728e-07)	
Low	0.20943	- 0.209430	0.00436	**0.2473**
	(0.02535)	-	(0.00186)	
High	0.032735	1.8917e-06	-0.032735	**0.0330**
	(0.00970)	(3.0869e-06)	-	

Table 7: Estimated parameters (standard errors) CASU model: GMM parameter estimates with standard errors obtained from stationary bootstrap (Politis and Romano (1994).

Growth Regime		
Normal	Low	High
Consumption		
μ_1^C	μ_2^C	μ_3^C
0.02422	0.00126	0.04514
(0.00032)	(0.00562)	(0.008405)
Dividend		
μ_1^D	μ_2^D	μ_3^D
0.03600	-0.003456	0.01720
(0.00463)	(0.00954)	(0.01044)

Growth Regime		
Normal	Low	High
Unemployment		
μ_1^{UE}	μ_2^{UE}	μ_3^{UE}
0.01508	0.07910	-0.09784
(0.02559)	(0.04722)	(0.03945)

Preferences: risk aversion
R
1.7033
(0.01963)

Preferences: subjective discount rate
0.0049583
(0.00124)

Preferences: state uncertainty aversion
25.645
(6.0630)

Standard Deviations and Correlations

	Consumption	Dividend	Unemployment
Consumption	0.0092	0.1664	-0.3914
	(0.0006)	(0.0653)	(0.0594)
Dividend	0.1664	0.0473	-0.3231
	(0.0653)	(0.0079)	(0.0596)
Unemployment	-0.3914	-0.3231	0.1244
	(0.0594)	(0.0596)	(0.0137)

Infinitesimal Generator

	Normal	Low	High	**Steady state probabilities**
Normal	-0.036724	0.036724	1.8975e-07	**0.8375**
	-	(0.00304)	(1.5733e-06)	
Low	0.0054871	-0.214418	0.0046882	**0.1434**
	(0.03084)	-	(0.00354)	
High	0.03531	1.9677e-07	-0.03531	**0.0190**
	(0.00861)	(1.6672e-06)	-	

Table 8: Estimated parameters (standard errors) BDRA model: GMM parameter estimates with standard errors obtained from stationary bootstrap (Politis and Romano (1994)). The sample period is January 1957 - December 1999.

Growth Regime			Growth Regime		
Normal	Low	High	Normal	Low	High
Consumption			Unemployment		
μ_1^C	μ_2^C	μ_3^C	μ_1^{UE}	μ_2^{UE}	μ_3^{UE}
0.01391	0.01238	0.02603	-0.01634	0.10143	0.00992
(0.00776)	(0.00783)	(0.00867)	(0.03768)	(0.03689)	(0.03489)
Dividend			Preferences: risk aversion		
μ_1^D	μ_2^D	μ_3^D	R_1	R_2	R_3
0.00672	0.00144	0.04651	2.3981	2.7097	2.4221
(0.00865)	(0.01135)	(0.00916)	(0.74061)	(0.06445)	(0.09766)

Preferences: subjective discount rate
β
0.00019
(0.0035609)

Standard Deviations and Correlations			
	Consumption	Dividend	Unemployment
Consumption	0.0092	0.2520	-0.3421
	(0.0618)	(0.2110)	(0.2239)
Dividend	0.2520	0.0292	-0.3770
	(0.2110)	(0.0221)	(0.1801)
Unemployment	-0.3421	-0.3770	0.1274
	(0.2239)	(0.1801)	(0.0773)

Infinitesimal Generator				
	Normal	Low	High	**Steady state probabilities**
Normal	-0.0537051997	0.0537050000	0.0000001997	**0.7559**
	-	(0.013962)	(1.7331e-06)	
Low	0.2156900000	-0.2202637000	0.0045737000	**0.1843**
	(0.024922)	-	(0.0032995)	
High	0.0141050000	0.0000037906	-0.0141087906	**0.0598**
	(0.0073274)	(1.8853e-06)	-	

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