Online Appendix

Moral Hazard in Active Asset Management

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OA.1 Moral Hazard Under Symmetric Information

In our base model, managers cannot credibly reveal their types to investors. In this online appendix, we consider a setting in which each manager's type is known by investors. One interpretation is that managers credibly communicate their types to investors through a perfectly revealing signal. We maintain all other assumptions from the base model, and highlight that a manager's effort allocation decision is not observable despite the observability of manager types.

Consider a manager type $\theta' \in [0, 1]$. A manager with type θ' uses a mixing probability $\rho \in [0, 1]$ in determining whether or not to incur the effort cost c. Specifically, the manager exerts effort with probability ρ and shirks otherwise. If $\rho \in \{0, 1\}$ the the manager's decision is deterministic, otherwise he mixes between his options.

Investors are rational and have accurate beliefs about the manager's strategy. Consequently, investors at $t = 1$ believe that a manager will earn excess returns with probability,

$$\Pr(S_1|\theta', K_1, \eta, A) = \frac{\rho\theta'}{1 + \eta K_1}, \tag{OA1}$$

and investors allocate capital to the manager until the point that they are indifferent between investing in the manager's fund or their outside option of a passive product. Investors' indifference condition yields,

$$F_E = \alpha \frac{\rho\theta'}{1 + \eta K_1}. \tag{OA2}$$

At the conclusion of $t = 1$ investors observe realized performance and update their beliefs. If a manager succeeds at earning excess returns, investors' expect the manager to subsequently earn excess returns with probability,

$$\Pr(S_2|\theta', \mathbb{1}_{S_1} = 1) = \frac{\theta'}{1 + \eta K_2}. \tag{OA3}$$

The mixing probability ρ does not enter into the preceding expression because the manager's $t = 1$ performance perfectly reveals that he incurred the research cost c. That is, $\Pr(A|\mathbb{1}_{S_1} = 1) = 1$. If the manager does not succeed at $t = 1$, investors cannot tell whether the manager shirked or if the manager was unlucky. As such, investors expect the manager to earn $t = 2$ excess returns with probability

$$\Pr(S_2|\theta', \mathbb{1}_{S_1} = 0) = \frac{\theta' \Pr(A|\mathbb{1}_{S_1} = 0)}{1 + \eta K_2}, \tag{OA4}$$

where $\Pr(A|\mathbb{1}_{S_1} = 0)$ is the posterior belief of the manager's effort choice conditional on failure. Based on the manager's performance, investors' indifference condition at $t = 2$ is given by,

$$F_E = \alpha \frac{\theta' \Pr(A|\mathbb{1}_{S_1})}{1 + \eta K_2(\mathbb{1}_{S_1})}. \tag{OA5}$$

Using investors' unconditional beliefs at $t = 1$ and their conditional beliefs at $t = 2$, the following lemma provides the manager's asset allocations in each period,

Lemma OA1. *With symmetric information about a manager's type, a fund's capital allocations in periods 1 and 2 are given by,*

$$K_1 = \max\left\{\frac{\alpha\rho\theta' - F_E}{\eta F_E}, 0\right\}. \tag{OA6}$$

$$K_2(\mathbb{1}_{S_1}) = \begin{cases} \max\left\{\frac{\alpha\theta' - F_E}{\eta F_E}, 0\right\} & \text{if } \mathbb{1}_{S_1} = 1 \\ \max\left\{\frac{F_E^2 + \alpha^2\rho\theta' - \alpha F_E(1+\theta')}{\eta F_E(\alpha - F_E)}, 0\right\} & \text{if } \mathbb{1}_{S_1} = 0. \end{cases} \tag{OA7}$$

At $t = 1$, the manager chooses to exert effort based on two payoffs: his expected payoff from allocating effort and his expected payoff from shirking. In an equilibrium with mixing between both actions, i.e., ρ is an interior value, the manager's best response and investors' best response require that the manager is indifferent between shirking and being truly-active,

$$0 = (F_E + F_P)\left(\frac{\theta'\left(K_2(1) - K_2(0)\right)}{1 + \eta K_1}\right) - c. \tag{OA8}$$

In this setup, the equilibrium mixing probability is akin to the threshold θ^* explored in the base model section. It is important to mention that larger values of ρ imply a less severe moral hazard problem (in contrast, larger values of θ^* imply a more severe moral hazard conflict).

Lemma OA2. *The equilibrium mixing probabilities as a function of θ' are given by,*

$$\rho(\theta') = \begin{cases} 0 & \text{if } \theta' < \underline{\theta} \\ \frac{(F_E+F_P)(\alpha\theta' - F_E)}{\alpha c\eta} & \text{if } \theta' \in [\underline{\theta}, \overline{\theta}) \\ \frac{\alpha\theta'(F_E+F_P)}{c\eta(\alpha - F_E) + \alpha\theta'(F_E+F_P)} & \text{if } \theta' \geq \overline{\theta}, \end{cases} \tag{OA9}$$

where $\underline{\theta}$ and $\overline{\theta}$ are implicitly defined by the conditions,

$$0 = (F_E + F_P)(\alpha\underline{\theta} - F_E)\underline{\theta} - F_E c\eta, \tag{OA10}$$

$$0 = (F_E + F_P)(\alpha\overline{\theta} - F_E)\overline{\theta} - F_E c\eta\left(1 + \overline{\theta} - \left(\frac{F_E}{\alpha}\right)\right). \tag{OA11}$$

The piecewise mixing probability outlined in Lemma OA2 critically depends on the manager's type. If the manager's type is less than the threshold $\underline{\theta}$, investors do not contribute capital to him because their expected excess returns are smaller than the incremental fee F_E. The manager's best response is to choose $\rho = 0$ since he will not be rewarded for incurring the effort cost c. Only if the manager's type is sufficiently large is there positive probability on him being truly-active. In fact, it is never the case that the manager chooses a mixing probability equal to or near zero. If the manager did choose a ρ near zero, investors would not provide him with any capital as their expected excess returns would fall below the incremental fee F_E. To formalize this point, the following lemma provides a lower bound on the mixing probability employed by the manager.

Lemma OA3. *The lowest-skilled active manager who plays a non-zero mixing strategy uses a mixing probability equal to,*

$$\rho(\underline{\theta}) = \frac{2\sqrt{F_E(F_E + F_P)}}{\sqrt{F_E(F_E + F_P)} + \sqrt{4\alpha c\eta + F_E(F_E + F_P)}}. \tag{OA12}$$

As demonstrated in Lemma OA3, the manager will never choose $\rho = 0$. The lower-bound on ρ is increasing with F_P and F_E and decreasing in α, η, and c. While the mixing probability is fully characterized shortly, the result of Lemma OA3 suggests that severity of the moral hazard conflict is inversely related to F_P and positively related to η and c.

Similar to never choosing $\rho = 0$, even the most skilled manager type, i.e., $\theta' = 1$, will never choose $\rho = 1$. Using the piecewise function outlined in Lemma OA2, it is straightforward that the most skilled manager chooses a mixing probability equal to,

$$\rho(1) = \frac{\alpha(F_E + F_P)}{c\eta(\alpha - F_E) + \alpha(F_E + F_P)}, \tag{OA13}$$

which is strictly less than one if c and η are positive valued and $\alpha \neq F_E$. To understand the intuition behind the result, consider a hypothetical equilibrium in which a manager with type $\theta' = 1$ chooses $\rho = 1$. So long as $\eta > 0$, the manager faces positive probability of not earning excess returns. Specifically, the probability of failing is equal to,

$$1 - \frac{1}{1 + \eta K_t}. \tag{OA14}$$

In the hypothetical equilibrium, investors believe the manager incurs the research cost c with probability one. Knowing the investors' beliefs, the manager has the incentive to deviate from the equilibrium and forgo paying c. After the realization of $t = 1$ returns, the manager can then blame the poor performance on bad luck. Thus, the hypothetical equilibrium unravels implying that a moral hazard conflict exists across manager types, even with symmetric information. The following proposition characterizes the equilibrium mixing probability ρ.

Proposition OA1. *The equilibrium mixing probability ρ^* is* *(i) increasing in F_P,*

(ii) decreasing in c,

(iii) decreasing in η.

The comparative statics of ρ^* with respect to F_E and α are equivocal.

A comparison of Proposition OA1 to Proposition 1 in Section 2 of the manuscript shows that the drivers of the moral hazard conflict with asymmetric information are exactly the same as the drivers with symmetric information. In other words, the parameters that increase θ^* in the base model also decrease ρ^* in this extension. While the analysis in this section is focused on a single manager type θ' it applies to all manager types, i.e., the analysis holds pointwise. Therefore, even if different manager types use different management fees F_E or if different manager types have different effort costs c, the economic intuition is qualitatively unchanged. So long as F_P is a global parameter, that is, investors' opportunity cost for delegated portfolio management is equal across manager types, any decrease in F_P will result in a more severe moral hazard conflict across all active managers. Similarly, if the parameter η — which proxies for the scalability of an investment strategy or the sensitivity of performance to dollars employed in the manager's strategy — increases across manager types, the moral hazard conflict will become more severe.

Online Appendix Analytic Proofs

Proof of Lemma OA1:

The formulations of K_1 and $K_2(1)$ are straightforward and omitted for the sake of brevity. The formulation of $K_2(0)$ relies on the explicit form of $\Pr(A|\mathbb{1}_{S_1}=0)$ which is given by,

$$\Pr(A|\mathbb{1}_{S_1}=0) = \frac{\rho\left(1-\frac{\theta'}{1+\eta K_1}\right)}{\rho\left(1-\frac{\theta'}{1+\eta K_1}\right)+(1-\rho)}. \tag{OA15}$$

Using the preceding expression, $\Pr(S|\theta', \mathbb{1}_{S_1}=0)$ is given by,

$$\Pr(S|\theta', \mathbb{1}_{S_1}=0) = \frac{\frac{\theta'\left(1-\frac{\theta'}{1+\eta K_1}\right)\rho}{\rho\left(1-\frac{\theta'}{1+\eta K_1}\right)+(1-\rho)}}{1+\eta K_2}, \tag{OA16}$$

which simplifies to,

$$= \frac{(\alpha\rho - F_E)\theta'}{(\alpha - F_E)(1+\eta K_2)}. \tag{OA17}$$

The explicit form of $K_2(0)$ is achieved via investors' indifference condition.

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Proof of Lemma OA2:

Noting that $\Pr(S|\mathbb{1}_{S_1} = 0) \leq \Pr(S)$, there exists a region of manager types for which $K_2(0) = 0$ if α is finite and F_E is non-zero. In this setting where $K_2(0) = 0$, the condition that uniquely pins down a type-θ manager's mixing probability is implied by,

$$0 = (F_E + F_P)\left(\frac{\theta\left(\frac{\alpha\theta - F_E}{\eta F_E}\right)}{1 + \eta\left(\frac{\alpha\underline{\rho}\theta - F_E}{\eta F_E}\right)}\right) - c, \tag{OA18}$$

and is explicitly equal to,

$$\underline{\rho}(\theta) = \frac{(F_E + F_P)(\alpha\theta - F_E)}{\alpha c\eta}. \tag{OA19}$$

The mixing probability outlined in (OA19) implies that any manager with type $\theta < \frac{F_E}{\alpha}$ will choose $\rho = 0$ and will effectively exit the game since he receives no assets from investors. Moreover, other managers will also choose to not participate based whether or not they receive any assets from investors at $t = 1$. Using the $t = 1$ asset allocation from investors outlined in (OA6) in combination with (OA19), the manager type $\underline{\theta}$ that is just indifferent between allocating effort and exiting the game all together is implicitly defined by,

$$0 = \frac{\alpha\left(\frac{(F_E+F_P)(\alpha\underline{\theta}-F_E)}{\alpha c\eta}\right)\underline{\theta} - F_E}{\eta F_E}, \tag{OA20}$$

which simplifies to,

$$0 = (F_E + F_P)(\alpha\underline{\theta} - F_E)\underline{\theta} - F_E c\eta. \tag{OA21}$$

An explicit form of $\underline{\theta}$ is given by,

$$\underline{\theta} \equiv \frac{F_E}{2\alpha} + \frac{\sqrt{F_E\left(4\alpha c\eta + F_E(F_E + F_P)\right)}}{2\alpha\sqrt{F_E + F_P}}. \tag{OA22}$$

Managers with types $\theta \geq \underline{\theta}$ will choose the mixing probability in (OA18) unless their type exceeds $\overline{\theta}$ where $\overline{\theta}$ is implicitly defined by,

$$0 = \frac{\alpha\left(\frac{\overline{\theta}\left(1-\frac{\overline{\theta}}{1+\eta K_1}\right)\left(\frac{(F_E+F_P)(\alpha\overline{\theta}-F_E)}{\alpha c\eta}\right)}{\left(\frac{(F_E+F_P)(\alpha\overline{\theta}-F_E)}{\alpha c\eta}\right)\left(1-\frac{\overline{\theta}}{1+\eta K_1}\right)+\left(1-\left(\frac{(F_E+F_P)(\alpha\overline{\theta}-F_E)}{\alpha c\eta}\right)\right)}\right) - F_E}{\eta F_E}, \tag{OA23}$$

which can be simplified as,

$$0 = \alpha\overline{\theta}\left(1 - \frac{F_E c\eta}{(F_E+F_P)(\alpha\overline{\theta}-F_E)}\right)\left(\frac{(F_E+F_P)(\alpha\overline{\theta}-F_E)}{\alpha c\eta}\right) - F_E\left(1 - \left(\frac{(F_E+F_P)(\alpha\overline{\theta}-F_E)}{\alpha c\eta}\right)\left(\frac{F_E c\eta}{(F_E+F_P)(\alpha\overline{\theta}-F_E)}\right)\right), \tag{OA24}$$

and further simplified to,

$$0 = (F_E+F_P)(\alpha\overline{\theta}-F_E)\overline{\theta} - F_E c\eta\left(1+\overline{\theta}-\left(\frac{F_E}{\alpha}\right)\right). \tag{OA25}$$

The explicit form of $\overline{\theta}$ is given by,

$$\overline{\theta} \equiv \frac{\alpha F_E(c\eta+F_E+F_P) - \sqrt{\alpha^2 F_E(4\alpha c\eta(F_E+F_P)+F_E(F_E+F_P-c\eta)^2)}}{2\alpha^2(F_E+F_P)}. \tag{OA26}$$

Any manager with type $\theta > \overline{\theta}$ expects a strictly positive capital allocation even after failing to earn excess returns at $t = 1$. Thus, the mixing probability in (OA19) cannot be the equilibrium mixing probability for managers with type $\theta > \overline{\theta}$. Instead, the mixing probability is implicitly defined by,

$$0 = (F_E+F_P)\theta\left(\frac{\left(\frac{\alpha\theta-F_E}{\eta F_E}\right) - \left(\frac{F_E^2+\alpha^2\rho\theta-\alpha F_E(1+\theta)}{\eta F_E(\alpha-F_E)}\right)}{1+\eta\left(\frac{\alpha\rho\theta-F_E}{\eta F_E}\right)}\right) - c, \tag{OA27}$$

and is explicitly equal to,

$$\overline{\rho}(\theta) = \frac{\alpha\theta(F_E+F_P)}{c\eta(\alpha-F_E)+\alpha\theta(F_E+F_P)}. \tag{OA28}$$

It is also straightforward to show that $\overline{\theta} \geq \underline{\theta}$ by comparing (OA21) and (OA25), nothing that any fund playing a positive mixing probability must have a type $\theta \geq \frac{F_E}{\alpha}$.

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Proof of Lemma OA3:

The mixing probability is achieved by substituting the explicit form of $\underline{\theta}$ found in (OA22) into the mixing probability function outlined in (OA19).

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Proof of Proposition OA1:

We now consider the comparative statics of $\rho(\theta')$ with respect to $\{F_P, c, \eta, \alpha, F_E\}$. First, consider the comparative static with respect to F_P,

$$\frac{\partial\rho(\theta')}{\partial F_P} = \begin{cases} 0 & \text{if } \theta' < \underline{\theta} \\ \frac{\alpha\theta'-F_E}{\alpha c\eta} & \text{if } \theta' \in [\underline{\theta}, \overline{\theta}) \\ \frac{\alpha c\eta\theta'(\alpha-F_E)}{(c\eta(\alpha-F_E)+\alpha(F_E+F_P)\theta)^2} & \text{if } \theta' \geq \overline{\theta}, \end{cases} \tag{OA29}$$

$$\geq 0. \tag{OA30}$$

Now, consider the comparative static with respect to η,

$$\frac{\partial \rho(\theta')}{\partial \eta} = \begin{cases} 0 & \text{if } \theta' < \underline{\theta} \\ -\frac{(F_E+F_P)(\alpha\theta'-F_E)}{\alpha c \eta^2} & \text{if } \theta' \in [\underline{\theta}, \overline{\theta}) \\ -\frac{\alpha c(\alpha-F_E)(F_E+F_P)\theta'}{(c\eta(\alpha-F_E)+\alpha(F_E+F_P)\theta')^2} & \text{if } \theta' \geq \overline{\theta}, \end{cases} \tag{OA31}$$

$$\leq 0. \tag{OA32}$$

Now, consider the comparative static with respect to c,

$$\frac{\partial \rho(\theta')}{\partial c} = \begin{cases} 0 & \text{if } \theta' < \underline{\theta} \\ -\frac{(F_E+F_P)(\alpha\theta'-F_E)}{\alpha c^2 \eta} & \text{if } \theta' \in [\underline{\theta}, \overline{\theta}) \\ -\frac{\alpha \eta(\alpha-F_E)(F_E+F_P)\theta'}{(c\eta(\alpha-F_E)+\alpha(F_E+F_P)\theta')^2} & \text{if } \theta' \geq \overline{\theta}, \end{cases} \tag{OA33}$$

$$\leq 0. \tag{OA34}$$

Now, consider the comparative static with respect to α,

$$\frac{\partial \rho(\theta')}{\partial \alpha} = \begin{cases} 0 & \text{if } \theta' < \underline{\theta} \\ \frac{F_E(F_E+F_P)}{\alpha^2 c \eta} & \text{if } \theta' \in [\underline{\theta}, \overline{\theta}) \\ -\frac{c\eta F_E(F_E+F_P)\theta'}{(c\eta(\alpha-F_E)+\alpha(F_E+F_P)\theta')^2} & \text{if } \theta' \geq \overline{\theta}, \end{cases} \tag{OA35}$$

$$\gtreqless 0. \tag{OA36}$$

Finally, consider the comparative static with respect to F_E,

$$\frac{\partial \rho(\theta')}{\partial F_E} = \begin{cases} 0 & \text{if } \theta' < \underline{\theta} \\ -\frac{2F_E+F_P-\alpha\theta'}{\alpha c \eta} & \text{if } \theta' \in [\underline{\theta}, \overline{\theta}) \\ \frac{\alpha c \eta(\alpha+F_P)\theta'}{(c\eta(\alpha-F_E)+\alpha(F_E+F_P)\theta')^2} & \text{if } \theta' \geq \overline{\theta}, \end{cases} \tag{OA37}$$

$$\gtreqless 0. \tag{OA38}$$

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