Dark Pool Trading Strategies, Market Quality and Welfare

Online Appendix

Sabrina Buti* Barbara Rindi†

Ingrid M. Werner‡

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*University of Toronto, Rotman School of Management, sabrina.buti@rotman.utoronto.ca

†Bocconi University and IGIER, barbara.rindi@unibocconi.it

‡Fisher College of Business, Ohio State University, werner.47@osu.edu

Appendix B

B.1. Extension 1: Large and small traders

Under this framework we allow for two different types of traders: at each trading round t, a trader may arrive either with a two-share (large) order with probability α, or with a one-share (small) order with probability $1-\alpha$. Traders arriving with a large order can trade up to two shares, $j=\{0,1,2\}$, and we call them large traders (LT), whereas traders arriving with a small order can only trade one share and we call them small traders (ST).

B.1.1. The benchmark framework (B)

The strategies available to traders in this framework are presented in Table B1. As in the original model, both large and small traders can submit market orders to the first two levels of the price grid, $\varphi_M(j,p_i^z)$; they can post limit orders to the first level, $\varphi_L(j,p_1^z)$, and they can choose not to trade, $\varphi(0)$. In addition, a large trader can split his order and combine the two different strategies, $\varphi_{ML}(1,p_i^z;1,p_1^z)$, taking advantage of the better execution probability of a one-unit limit order.[1] In this case, only part of the large trader's profits depends on the probability of the order being executed in the following trading rounds:

$$\pi_t^e[\varphi_{ML}(1,p_i^z;1,p_1^z)] = \pi_t[\varphi_M(1,p_i^z)] + \pi_t^e[\varphi_L(1,p_1^z)] \ . \tag{44}$$

Moreover, the large trader can submit marketable orders that are large market orders walking up or down the book in search of execution, $\varphi_M(2,p^z)$.[2] For example, profits from a marketable sell order for the large trader are given by:

$$\pi_t[\varphi_M(2,p^B)] = (p_1^B + p_2^B) - 2\beta_t v \ . \tag{45}$$

As in the previous framework, at each trading round t the arriving risk-neutral trader selects the optimal order submission strategy which maximizes his expected profits, conditional on the state of the LOB, b_t, on his type represented by his personal valuation of the asset, β_t, and on whether he is a small or a large trader. The large trader chooses:

$$\max_{\varphi} \pi_t^e[\varphi_M(j,p_i^z), \varphi_M(2,p^z), \varphi_{ML}(1,p_i^z;1,p_1^z), \varphi_L(j,p_1^z), \varphi(0)\ |\beta_t, b_t] \ , \tag{46}$$

and the small trader still solves problem (7).

[**Insert Table B1 here**]

B.1.2. Limit order book and continuous dark pool ($L\&C$)

In this framework only large traders have access to the CDP and their action space includes, in addition to the dark pool orders presented for the original model, the possibility to split the order between the LOB and the CDP, by combining a one-unit dark pool order with either a market order, $\varphi_{MD}(1,p_i^z;\pm 1,\widetilde{p}_{Mid,t})$, or a limit order, $\varphi_{LD}(1,p_1^z;\pm 1,\widetilde{p}_{Mid,t})$:

$$\begin{aligned}
\varphi_{MD}(1,p_i^z;\pm 1,\widetilde{p}_{Mid,t}) &= \pi_t[\varphi_M(1,p_i^z)] + \pi_t^e[\varphi_D(\pm 1,\widetilde{p}_{Mid,t})], & (47)\\
\varphi_{LD}(1,p_1^z;\pm 1,\widetilde{p}_{Mid,t}) &= \pi_t^e[\varphi_L(1,p_1^z)] + \pi_t^e[\varphi_D(\pm 1,\widetilde{p}_{Mid,t})] \ . & (48)
\end{aligned}$$

[1] Because small traders trade only one unit, the execution probability of the first unit of a limit order is higher.

[2] We omit the subscript i for the level of the book since the order will be executed at different prices.

At each trading round the fully rational risk-neutral large trader chooses the optimal order submission strategy which maximizes his expected profits, conditional on his valuation of the asset, β_t, and his information set, Ω_t, respectively:

$$\max_{\varphi} \pi_t^e[\varphi_M(j, p_i^z), \varphi_M(2, p^z), \varphi_{ML}(1, p_i^z; 1, p_1^z), \varphi_{MD}(1, p_i^z; \pm 1, \widetilde{p}_{Mid,t}), \tag{49}$$
$$\varphi_D(\pm j, p_{Mid,t}, p_i^z), \varphi_D(\pm j, \widetilde{p}_{Mid,t}), \varphi_{LD}(1, p_1^z; \pm 1, \widetilde{p}_{Mid,t}), \varphi_L(j, p_1^z), \varphi(0) \, |\beta_t, \Omega_t] \ .$$

As before, small traders solve problem (7), and shape their strategies depending on the expected state of the CDP.

B.1.3. Results

In commenting the results from the model with both large and small traders and a CDP, we focus only on those findings which differ from our initial framework with only one-share trades. Overall the results from the original model concerning order flows, market quality and welfare continue to hold. Note that by comparing books with one and two shares on the best ask price, we can now differentiate between the effects of spread and depth. Moreover, we define a new order flow indicator, volume creation (VC), as the average LOB plus dark pool volume, $V_t^{L\&C}$, in excess of the total LOB volume in the B framework, V_t^B:

$$VC = \frac{1}{T} \sum_t (V_t^{L\&C} - V_t^B) \ , \tag{50}$$

where we measure volume in each period t by weighing the fill rate, FR_t, by the order size, q_t:

$$V_t^{L\&C,B} = \sum_{a=WA,NA} \Pr(a) E_{\Omega_t} \left[\int_0^2 q_t \cdot \varphi_a^n \cdot f(\beta_t) \, d\beta_t \right] \ . \tag{51}$$

Notice that with a unitary trade size, TC and VC coincide.

Order flows. When a CDP is introduced along side a LOB, we observe OM and TC (Fig. B1). Moreover, since orders may differ in size, we can also differentiate TC from volume creation (VC) (Fig. B2). The effects of the introduction of a dark pool on order flows are all weaker when traders can choose between large and small size than when all orders were one share. The reason is that starting from both an empty book and an empty dark pool, it takes longer to observe a book with two rather than one share at the best ask or bid price. For this reason traders have less incentive to opt for dark trading.[3]

Consider the new framework evaluated at t_2 with three periods remaining, and compare books with different amounts of liquidity, $b_{t_2} = [00]$, [10], and [20]. Fig. B1 shows that OM at t_2 is strongly increasing in the depth of the book. Dark orders become more attractive not only when depth increases, but also when spread becomes smaller. All else equal, a smaller spread increases competition for the provision of liquidity and makes the dark pool option more attractive for limit orders. A smaller spread also decreases the market order price improvement in the dark pool which reduces the incentives to send market orders to the dark, but this effect is compensated by the increased execution probability generated in the dark pool by the migration of the limit orders.

[Insert Fig. B1 and Fig. B2 here]

[3] Comparing Tables 2 and B2 it is evident that starting from $b_{t_1} = [00]$, the probability to observe $b_{t_2} = [10]$ in the framework with small (NA and WA) traders is equal to $a\varphi_L(1, p_1^A) + (1-a)\varphi_L(1, p_1^A) = \frac{1}{2}0.4066 + \frac{1}{2}0.4066 = 0.4066$, whereas the probability to observe $b_{t_2} = [20]$ in the framework with large and small traders is equal to $a\varphi_L(2, p_1^A) = \frac{1}{2}0.1791 = 0.0895$.

While OM increases as liquidity builds, TC and VC decrease when either spread narrows or depth builds up in the book with one or two shares on the best ask side. As we measure volume by weighing the fill rates by the size of the orders executed, VC has a pattern similar to TC, however, because the average size of the orders that migrate from the LOB to the dark pool is larger than the average size of the orders executed on the LOB, all the effects generated by the migration of orders to the dark pool are magnified when measured by volume rather than by fill rate, and so is the change in VC compared to the reduction in TC.

Market quality. As in the initial framework, our results show that the introduction of a dark pool has a negative effect on the liquidity of an empty LOB as both the inside spread and the depth at the best bid and offer worsen (Fig. B3, Panel A). The effects at work are very much similar to those discussed for the original model. At the beginning of the trading game when both the book and the dark pool open empty, the introduction of a dark pool makes traders switch from limit to market orders anticipating a future reduction in the execution probability of limit orders (Table B2).[4] As before, the resulting increased liquidity demand and reduced liquidity supply widen spread and reduce depth at the inside quotes. However, we note that the magnitude of the effect is smaller with large and small traders compared to the original model where all traders were small.

Consider again the new framework evaluated at t_2 with three periods remaining, and compare books with different amounts of liquidity, $b_{t_2} = [00]$, [10] and [20]. Fig. B3 shows that while the negative effect on spread decreases with the liquidity of the book, the effect on depth first improves and then, as traders more heavily move to the dark pool, further deteriorates.

[**Insert Fig. B3 and Table B2 here**]

When liquidity builds up in the LOB and the book opens with one or two shares at the best ask price, two effects occur. On the one hand, when the book becomes deeper traders generally tend to use more market than limit orders and so fewer traders switch from limit to market orders when a dark pool is introduced. This first effect attenuates the negative impact on liquidity of the switch from limit to market orders. On the other hand, however, as liquidity builds up and the queue at the top of the LOB becomes longer, more limit orders are attracted to the dark pool and this effect reduces liquidity of the LOB, especially depth.

When the book moves from being empty to having one share at the best ask price, the first effect dominates and both spread and depth improve (the impact of the dark pool introduction is less negative). When instead depth builds up substantially and the book opens with two shares at the best ask price, the second effect dominates and while spread improves, depth drops due to the migration of limit orders. The reason for why spread improves but depth deteriorates when the book opens with two shares at the best ask price is that a minimum of two shares at the best ask is required for the introduction of the dark pool to generate a migration of limit orders -not only a switch from market to limit orders. So the intense migration of the two-unit limit orders from the ask side does not impact spread heavily as the book already has two shares at p_1^A, whereas it does impact depth substantially.

Welfare. In this new framework small traders play the role of NA traders of the initial model, and large traders play the role of WA traders with the difference that they may submit orders of two shares. The conclusions are therefore very similar to the ones of the previous model and are illustrated in Fig. B3, Panel B. As before, the effect of the introduction of a dark pool on the welfare of small traders is mainly driven by the variation in the spread. Large traders also do not benefit from the introduction of a dark pool when both the book and the dark pool open empty because both spread and depth deteriorate.

[4] More precisely, traders switch from two-unit limit orders to two-unit market orders, and from two-unit limit orders to one-unit limit and one-unit market orders.

Finally, consider the new framework evaluated at t_2 with three periods remaining, and compare books with different amounts of liquidity, $b_{t_2} = [00]$, $[10]$ and $[20]$. Fig. B3, Panel B, shows that the negative effect on the welfare of small traders declines as book liquidity at t_2 increases. The reason is that spread deteriorates and then partially recovers due to the introduction of the dark pool. However, as the book becomes deeper and at the same time some liquidity builds up in the dark pool, large traders start using the dark pool intensively and they now benefit from the access to dark pools so that even aggregate welfare increases. Yet note that in this new model large traders' gains from trade outweigh small traders' losses only when liquidity builds up substantially and the book opens with two shares on the ask side.

B.2. Extension 2: LOB and periodic dark pool ($L\&P$)

We extend the framework with large and small traders by assuming that the dark venue has periodic execution. A PDP is organized like an opaque crossing network where time priority is enforced. In this trading venue, orders are crossed at the end of the trading game only if a matching number of orders on the opposite side has been submitted to the dark pool prior to the cross. The execution price is the spread mid-quote prevailing on the LOB at the end of period t_4 which we indicate with $\widetilde{p}_{Mid}$.

Similarly to the $L\&C$ framework, the large traders' action space includes, in addition to the orders presented for the benchmark model, the possibility to submit orders to buy or to sell the asset on the PDP, as shown in Table B1. In particular, large traders can split their orders between the LOB and the PDP, or they can submit their entire order to the PDP. Their optimal trading strategy is determined by solving at each round the following optimization problem:

$$\begin{aligned} \max_{\varphi}\ & \pi_t^e[\varphi_M(j,p_i^z), \varphi_M(2,p^z), \varphi_{ML}(1,p_i^z;1,p_1^z), \varphi_{MD}(1,p_i^z;\pm 1,\widetilde{p}_{Mid}), \\ & \varphi_D(\pm j,\widetilde{p}_{Mid}), \varphi_{LD}(1,p_1^z;\pm 1,\widetilde{p}_{Mid}), \varphi_L(j,p_1^z), \varphi(0)\ |\beta_t,\Omega_t]\ . \end{aligned} \tag{52}$$

Small traders still solve problem (7), however they now condition their strategies not only on their own β_t and on the state of the LOB but also on the inferred state of the PDP. The game is solved as before by backward induction starting from t_4.

B.3 Proof of Proposition 4

We first provide an example of how the extended version of the model with both small and large traders is solved, and then we discuss the results obtained.

B.3.1. B framework

An example of the extensive form of the game is presented in Fig. B4. Because the model is similar to the one with unitary trade size, for brevity we provide an example of how the model is solved only for the last two periods of the trading game. From now onwards, we assume that for large traders the optimal order size is $j^* = \max_j\ [\varphi\,|\,b_t]$, since $\partial\pi_t^e(\varphi)/\partial j \geq 0$ due to agents' risk neutrality.

Following Fig. B4, at t_4 we present as an example $b_{t_4} = [20]$, and focus on large trader's profits:

$$\pi_{t_4}[\varphi_M(2,p_2^B)] = 2(p_2^B - \beta_{t_4}v) = 2(1-\tfrac{3\tau}{2} - \beta_{t_4}) \tag{53}$$

$$\pi_{t_4}[\varphi_M(2,p_1^A)\,] = 2(\beta_{t_4}v - p_1^A) = 2(\beta_{t_4} - 1 - \tfrac{\tau}{2}) \tag{54}$$

$$\pi_{t_4}[\varphi(0)] = 0\ . \tag{55}$$

It is straightforward to show that all strategies are optimal in equilibrium ($N_{t_4} = 3$): $\varphi^1_{LT,b_{t_4}} = \varphi_M(2, p_2^B)$, $\varphi^2_{LT,b_{t_4}} = \varphi(0)$, and $\varphi^3_{LT,b_{t_4}} = \varphi_M(2, p_1^A)$. We refer to the proof of Proposition 1, Appendix A, for the optimal strategies of small traders.

Still as an example, and continuing to follow Fig. B4, we consider the opening LOB $b_{t_3} = [21]$. Small traders' profits are as follows:

$$\pi_{t_3}[\varphi_M(1, p_1^B)] = (p_1^B - \beta_{t_3} v) \tag{56}$$
$$\pi^e_{t_3}[\varphi_L(1, p_1^A)] = \pi_{t_3}[\varphi(0)] = 0 \tag{57}$$
$$\pi^e_{t_3}[\varphi_L(1, p_1^B)] = (\beta_{t_3} v - p_1^B)\alpha \Pr_{t_4}(\varphi_M(2, p_1^B) \mid b_{t_4} = [22]) \tag{58}$$
$$\pi_{t_3}[\varphi_M(1, p_1^A)] = (\beta_{t_3} v - p_1^A)\ . \tag{59}$$

Large traders' strategies are similar, the only difference being that $j = 2$ for a market sell order, and that conditioning on $b_{t_3} = [21]$ now traders willing to buy can combine market and limit orders, and traders willing to sell can walk down the book via a marketable order:

$$\pi^e_{t_3}[\varphi_{ML}(1, p_1^A; 1, p_1^B)] = (\beta_{t_3} v - p_1^A) + (\beta_{t_3} v - p_1^B)\alpha \Pr_{t_4}(\varphi_M(2, p_1^B) \mid b_{t_4} = [12]) \tag{60}$$
$$\pi_{t_3}[\varphi_M(2, p^B)] = (p_1^B - \beta_{t_3} v) + (p_2^B - \beta_{t_3} v)\ . \tag{61}$$

Profits for periods t_1 and t_2 are similar and are omitted, the main difference being that limit orders have additional periods to get executed.

B.3.2. L&P framework

An example of the extensive form of the game is presented in Fig. B5. The solution of the *L&P* framework follows the same methodology, but now the large trader solves Eq. (52). For brevity, we provide again examples only for the last two periods of the trading game. The remaining two periods are solved in a similar way.

Following Fig. B5, at t_4 we consider again the book $b_{t_4} = [20]$ with the following information set, $\Omega_{t_4} = [20, \varphi_L(2, p_1^A), \varphi_L(1, p_1^B), \varphi_M(1, p_1^B)]$. Notice that when at t_3 traders observe $\varphi_M(1, p_1^B)$, they don't know whether a small trader submitted a one-unit market order to sell, or whether a large trader submitted a market order combined with a dark pool order. Similarly, at t_2 when traders observe $\varphi_L(1, p_1^B)$ it could be a one-unit limit order to buy or a combination of a limit and a dark pool order. Traders coming at t_4 Bayesian update their expectations on the state of the dark pool as follows (we omit that all probabilities are conditional to Ω_{t_4}):

$$\widetilde{PDP}_{t_4} = \begin{cases} +1 \ \textit{with prob} = \dfrac{\alpha(1-\alpha) \Pr\limits_{t_2} \varphi_{LD}(1,p_1^B;+1,\widetilde{p}_{Mid}) \Pr\limits_{t_3} \varphi_M(1,p_1^B)}{[\alpha \Pr\limits_{t_2} \varphi_{LD}(1,p_1^B;+1,\widetilde{p}_{Mid}) + (1-\alpha) \Pr\limits_{t_2} \varphi_L(1,p_1^B)][(1-\alpha) \Pr\limits_{t_3} \varphi_M(1,p_1^B) + \alpha \Pr\limits_{t_3} \varphi_{MD}(1,p_1^B;-1,\widetilde{p}_{Mid})]} \\ 0 \ \textit{with prob} = \dfrac{\alpha^2 \Pr\limits_{t_2} \varphi_{LD}(1,p_1^B;+1,\widetilde{p}_{Mid}) \Pr\limits_{t_3} \varphi_{MD}(1,p_1^B;-1,\widetilde{p}_{Mid}) + (1-\alpha)^2 \Pr\limits_{t_2} \varphi_L(1,p_1^B) \Pr\limits_{t_3} \varphi_M(1,p_1^B)}{[\alpha \Pr\limits_{t_2} \varphi_{LD}(1,p_1^B;+1,\widetilde{p}_{Mid}) + (1-\alpha) \Pr\limits_{t_2} \varphi_L(1,p_1^B)][(1-\alpha) \Pr\limits_{t_3} \varphi_M(1,p_1^B) + \alpha \Pr\limits_{t_3} \varphi_{MD}(1,p_1^B;-1,\widetilde{p}_{Mid})]} \\ -1 \ \textit{with prob} = \dfrac{\alpha(1-\alpha) \Pr\limits_{t_2} \varphi_L(1,p_1^B) \Pr\limits_{t_3} \varphi_{MD}(1,p_1^B;-1,\widetilde{p}_{Mid})}{[\alpha \Pr\limits_{t_2} \varphi_{LD}(1,p_1^B;+1,\widetilde{p}_{Mid}) + (1-\alpha) \Pr\limits_{t_2} \varphi_L(1,p_1^B)][(1-\alpha) \Pr\limits_{t_3} \varphi_M(1,p_1^B) + \alpha \Pr\limits_{t_3} \varphi_{MD}(1,p_1^B;-1,\widetilde{p}_{Mid})]}. \end{cases} \tag{62}$$

We refer to the proof of Proposition 1 in Appendix A for the profits of the small trader, and discuss the large trader's profits. We omit regular market orders that we have already presented in the B framework, Eqs. (53)–(55), and focus only on orders that involve the use of the PDP:

$$\begin{aligned}
\pi^e_{t_4}[\varphi_{MD}(1,p^B_2;-1,\widetilde{p}_{Mid})] &= (p^B_2-\beta_{t_4}v)+(\tfrac{p^A_1+p^B_2}{2}-\beta_{t_4}v)\Pr(\widetilde{PDP}_{t_4}=+1) && (63)\\
\pi^e_{t_4}[\varphi_{D}(-1,\widetilde{p}_{Mid})] &= (\tfrac{p^A_1+p^B_2}{2}-\beta_{t_4}v)\Pr(\widetilde{PDP}_{t_4}=+1) && (64)\\
\pi^e_{t_4}[\varphi_{D}(+1,\widetilde{p}_{Mid})] &= (\beta_{t_4}v-\tfrac{p^A_1+p^B_2}{2})\Pr(\widetilde{PDP}_{t_4}=-1) && (65)\\
\pi^e_{t_4}[\varphi_{MD}(1,p^A_1;+1,\widetilde{p}_{Mid})] &= (\beta_{t_4}v-p^A_1)+(\beta_{t_4}v-\tfrac{p^A_1+p^B_2}{2})\Pr(\widetilde{PDP}_{t_4}=-1)\ . && (66)
\end{aligned}$$

To determine the equilibrium strategies $\varphi^n_{a,\Omega_{t_4}}$ at t_4 for $n\in N_{t_4}$, the model has to be solved up to period t_1. We anticipate that because in equilibrium $\Pr_{t_2}\varphi_{LD}(1,p^B_1;+1,\widetilde{p}_{Mid})=0$ and $\Pr_{t_3}\varphi_{MD}(1,p^B_1;-1,\widetilde{p}_{Mid})>0$, $N_{t_4}=5$ and the strategies of the large trader are as follows: $\varphi^1_{LT,\Omega_{t_4}}=\varphi_M(2,p^B_2)$, $\varphi^2_{LT,\Omega_{t_4}}=\varphi(0)$, $\varphi^3_{LT,\Omega_{t_4}}=\varphi_D(+1,\widetilde{p}_{Mid})$, $\varphi^4_{LT,\Omega_{t_4}}=\varphi_{MD}(1,p^A_1;+1,\widetilde{p}_{Mid})$, and $\varphi^5_{LT,\Omega_{t_4}}=\varphi_M(2,p^A_1)$.

As for the B framework, we now consider the case $b_{t_3}=[21]$ with the following information set, $\Omega_{t_3}=[21,\varphi_L(2,p^A_1),\varphi_L(1,p^B_1)]$. The expected state of the PDP is:

$$\widetilde{PDP}_{t_3}=\begin{cases} +1 \ \ \textit{with prob} = \dfrac{\alpha\Pr_{t_2}\varphi_{LD}(1,p^B_1;+1,\widetilde{p}_{Mid})}{\alpha\Pr_{t_2}\varphi_{LD}(1,p^B_1;+1,\widetilde{p}_{Mid})+(1-\alpha)\Pr_{t_2}\varphi_L(1,p^B_1)} \\ 0 \ \ \textit{with prob} = \dfrac{(1-\alpha)\Pr_{t_2}\varphi_L(1,p^B_1)}{\alpha\Pr_{t_2}\varphi_{LD}(1,p^B_1;+1,\widetilde{p}_{Mid})+(1-\alpha)\Pr_{t_2}\varphi_L(1,p^B_1)}\end{cases}. \qquad (67)$$

Compared to the B framework, the large trader has now additional strategies. He can combine market and dark orders:

$$\begin{aligned}
\pi^e_{t_3}[\varphi_{MD}(1,p^A_1;+1,\widetilde{p}_{Mid})]= \ & (\beta_{t_3}v-p^A_1)+\Pr(\widetilde{PDP}_{t_3}=+1)(\beta_{t_3}v-\tfrac{p^A_1+p^B_1}{2})\\
& \alpha\Pr_{t_4}(\varphi_D(-2,\widetilde{p}_{Mid})\,|\,\Omega_{t_4})+\Pr(\widetilde{PDP}_{t_3}{=}0)\alpha[(\beta_{t_3}v-\tfrac{p^A_1+p^B_1}{2})\\
& \Pr_{t_4}(\varphi_D(-2,\widetilde{p}_{Mid})|\,\Omega_{t_4})+(\beta_{t_3}v-\tfrac{p^A_1+p^B_2}{2})\Pr_{t_4}(\varphi_{MD}(1,p^B_1;-1,\widetilde{p}_{Mid})|\,\Omega_{t_4})]\ ,
\end{aligned} \qquad (68)$$

where $\Omega_{t_4}=[11,\varphi_L(2,p^A_1),\varphi_L(1,p^B_1),\varphi_M(1,p^A_1)]$;

$$\begin{aligned}
\pi^e_{t_3}[\varphi_{MD}(1,p^B_1;-1,\widetilde{p}_{Mid})]= \ & (p^B_1-\beta_{t_3}v)+\Pr(\widetilde{PDP}_{t_3}=+1)\{(\tfrac{p^A_1+p^B_2}{2}-\beta_{t_3}v)\\
& [(1-\alpha)+\alpha(1-\Pr_{t_4}(\varphi_M(2,p^A_1)|\,\Omega_{t_4}))]+(\tfrac{p^A_2+p^B_2}{2}-\beta_{t_3}v)\\
& \alpha\Pr_{t_4}(\varphi_M(2,p^A_1)|\,\Omega_{t_4})\}+\Pr(\widetilde{PDP}_{t_3}{=}0)\alpha(\tfrac{p^A_1+p^B_2}{2}-\beta_{t_3}v)\\
& [\Pr_{t_4}(\varphi_D(+1,\widetilde{p}_{Mid})\,|\,\Omega_{t_4})+\Pr_{t_4}(\varphi_{MD}(1,p^A_1;+1,\widetilde{p}_{Mid})\,|\,\Omega_{t_4})]\ ,
\end{aligned} \qquad (69)$$

where $\Omega_{t_4}=[20,\varphi_L(2,p^A_1),\varphi_L(1,p^B_1),\varphi_M(1,p^B_1)]$.

Alternatively, he can combine a limit and dark order on the bid side of the market:

$$\begin{aligned}\pi^e_{t_3}[\varphi_{LD}(1,p^B_1;+1,\widetilde{p}_{Mid})]= \quad & (\beta_{t_3}v-p^B_1)\alpha\Pr_{t_4}(\varphi_M(2,p^B_1)\,|\,\Omega_{t_4})+(\beta_{t_3}v-\tfrac{p^A_1+p^B_1}{2})\alpha\{\Pr(\widetilde{PDP}_{t_3}{=}0)\\ & [\Pr_{t_4}(\varphi_{MD}(1,p^B_1;-1,\widetilde{p}_{Mid})|\,\Omega_{t_4})+\Pr_{t_4}(\varphi_D(-2,\widetilde{p}_{Mid})|\,\Omega_{t_4})]\\ & +\Pr(\widetilde{PDP}_{t_3}=+1)\Pr_{t_4}(\varphi_D(-2,\widetilde{p}_{Mid})|\,\Omega_{t_4})\}\ ,\end{aligned} \tag{70}$$

where $\Omega_{t_4}=[22,\varphi_L(2,p^A_1),\varphi_L(1,p^B_1),\varphi_L(1,p^B_1)]$.

Finally, he can submit a pure dark pool order to sell or buy:

$$\begin{aligned}\pi^e_{t_3}[\varphi_D(-2,\widetilde{p}_{Mid})]= \quad & \Pr(\widetilde{PDP}_{t_3}{=}+1)\{(\tfrac{p^A_1+p^B_1}{2}-\beta_{t_3}v)[\alpha(2\Pr_{t_4}(\varphi_{MD}(1,p^A_1;+1,\widetilde{p}_{Mid})|\,\Omega_{t_4})\\ & +2\Pr_{t_4}(\varphi_D(+2,\widetilde{p}_{Mid})|\,\Omega_{t_4})+\Pr_{t_4}(\varphi_D(-2,\widetilde{p}_{Mid})|\,\Omega_{t_4})+\Pr_{t_4}(\varphi(0)|\,\Omega_{t_4}))+(1-\alpha)\\ & (\Pr_{t_4}(\varphi^s(0)|\,\Omega_{t_4})+\Pr_{t_4}(\varphi_M(1,p^A_1)|\,\Omega_{t_4}))]+(\tfrac{p^A_1+p^B_2}{2}-\beta_{t_3}v)[(1-\alpha)\\ & \Pr_{t_4}(\varphi_M(1,p^B_1)|\,\Omega_{t_4})+\alpha(\Pr_{t_4}(\varphi_M(1,p^B)|\,\Omega_{t_4})+\Pr_{t_4}(\varphi_{MD}(1,p^B_1;-1,\widetilde{p}_{Mid})|\,\Omega_{t_4}))]\\ & +(\tfrac{p^A_2+p^B_1}{2}-\beta_{t_3}v)\alpha\Pr_{t_4}(\varphi_M(2,p^A_1)|\,\Omega_{t_4})\}+\Pr(\widetilde{PDP}_{t_3}{=}0)\alpha(\tfrac{p^A_1+p^B_1}{2}-\beta_{t_3}v)\\ & [\Pr_{t_4}(\varphi_{MD}(1,p^A_1;+1,\widetilde{p}_{Mid})|\,\Omega_{t_4})+2\Pr_{t_4}(\varphi_D(+2,\widetilde{p}_{Mid})|\,\Omega_{t_4})]\end{aligned} \tag{71}$$

$$\begin{aligned}\pi^e_{t_3}[\varphi_D(+2,\widetilde{p}_{Mid})]= \quad & \Pr(\widetilde{PDP}_{t_3}{=}+1)\alpha(\beta_{t_3}v-\tfrac{p^A_1+p^B_1}{2})\Pr_{t_4}(\varphi_D(-2,\widetilde{p}_{Mid})|\,\Omega_{t_4})\\ & +\Pr(\widetilde{PDP}_{t_3}{=}0)\alpha[(\beta_{t_3}v-\tfrac{p^A_1+p^B_2}{2})\Pr_{t_4}(\varphi_{MD}(1,p^B_1;-1,\widetilde{p}_{Mid})|\,\Omega_{t_4})\\ & +(\beta_{t_3}v-\tfrac{p^A_1+p^B_1}{2})2\Pr_{t_4}(\varphi_D(-2,\widetilde{p}_{Mid})|\,\Omega_{t_4})]\ ,\end{aligned} \tag{72}$$

where in both cases $\Omega_{t_4}=[21,\varphi_L(2,p^A_1),\varphi_L(1,p^B_1),\varphi(0)]$.

B.3.3. L&C framework

An example of the extensive form of the game is presented in Fig. B6. For consistency, we consider again the example of the book $b_{t_4}=[20]$ with the following information set, $\Omega_{t_4}=[20,\varphi_L(2,p^A_1),$ $\varphi_L(1,p^B_1),\varphi_M(1,p^B_1)]$. In this case when at t_3 a trader observes $\varphi_M(1,p^B_1)$, he doesn't know whether a small trader submitted a one-unit market order, or whether a large trader submitted a market order combined with a dark pool order or a two-unit IOC dark pool order that was partially executed on the CDP. Therefore, traders coming at t_4 Bayesian update their expectations on the state of the dark pool by using an extended version of Eq. (62) which takes into account that $\widetilde{CDP}_{t_4}=0$ with the additional probability $\alpha^2\Pr_{t_2}\varphi_{LD}(1,p^B_1;+1,\widetilde{p}_{Mid,t})\Pr_{t_3}\varphi_D(-2,p_{Mid,t_3},p^B_1)$. Traders coming at t_3 instead face the same uncertainty as for the $L\&P$ framework.

Compared to the $L\&P$ framework, the large trader can now submit a IOC dark pool order with the following profits:

$$\pi^e_{t_4}[\varphi_D(+2, p_{Mid,t_4}, p_1^A)] = 2\beta_{t_4}v - p_1^A[1 + \Pr(\widetilde{CDP}_{t_4} = +1, 0)] - \tfrac{p_1^A + p_2^B}{2}\Pr(\widetilde{CDP}_{t_4} = -1) \quad (73)$$

$$\pi^e_{t_4}[\varphi_D(-2, p_{Mid,t_4}, p_2^B)] = p_2^B[1 + \Pr(\widetilde{CDP}_{t_4} = -1, 0)] + \tfrac{p_1^A + p_2^B}{2}\Pr(\widetilde{CDP}_{t_4} = +1) - 2\beta_{t_4}v \quad (74)$$

To determine the equilibrium strategies $\varphi^n_{a,\Omega_{t_4}}$ at t_4 for $n \in N_{t_4}$, the model has again to be solved up to period t_1. We anticipate that because in equilibrium $\Pr_{t_2}\varphi_{LD}(1, p_1^B; +1, \widetilde{p}_{Mid,t}) = 0$ and $\Pr_{t_3}\varphi_{MD}(1, p_1^B; -1, \widetilde{p}_{Mid,t}) > 0$, $N_{t_4} = 4$ and the strategies of the large trader are as follows: $\varphi^1_{LT,\Omega_{t_4}} = \varphi_M(2, p_2^B)$, $\varphi^2_{LT,\Omega_{t_4}} = \varphi(0)$, $\varphi^3_{LT,\Omega_{t_4}} = \varphi_D(+1, p_{Mid,t_4})$, $\varphi^4_{LT,\Omega_{t_4}} = \varphi_D(+2, p_{Mid,t_4}, p_1^A)$.

As for the other two frameworks, we now consider the case $b_{t_3} = [21]$ with the following information set, $\Omega_{t_3} = [20, \varphi_L(2, p_1^A), \varphi_L(1, p_1^B)]$. We refer to Eq. (67) for the probabilities of the expected state of the CDP, $\widetilde{CDP}_{t_3} = \{0, +1\}$. The large trader can now submit a IOC dark pool order to sell (the execution probability on the CDP of a IOC dark order to buy is zero, so traders never select this strategy):

$$\pi^e_{t_3}[\varphi_D(-2, p_{Mid,t_3}, p^B)] = p_1^B + p_2^B\Pr(\widetilde{CDP}_{t_3} = 0) + \tfrac{p_1^A + p_1^B}{2}\Pr(\widetilde{CDP}_{t_3} = +1) - 2\beta_{t_3}v \quad (75)$$

Notice that the profits of the combined market and dark orders, limit and dark orders and pure dark pool orders are similar to the ones presented for the $L\&P$ framework. The main two differences are that first in this framework the order is executed as soon as liquidity is available on the CDP and not at the end of the trading game; second that traders can use IOC dark pool orders. For example profits for a dark pool sell order are:

$$\begin{aligned}\pi^e_{t_3}[\varphi_D(-2, \widetilde{p}_{Mid,t})] = \;& \Pr(\widetilde{CDP}_{t_3} = +1)(\tfrac{p_1^A + p_1^B}{2} - \beta_{t_3}v)\{1 + \alpha[\Pr_{t_4}(\varphi_D(+2, p_{Mid,t_4}, p_1^A) |\, \Omega_{t_4}) \\ & + \Pr_{t_4}(\varphi_D(+2, p_{Mid,t_4}) |\, \Omega_{t_4})]\} + \Pr(\widetilde{CDP}_{t_3} = 0)\alpha(\tfrac{p_1^A + p_1^B}{2} - \beta_{t_3}v) \\ & 2[\Pr_{t_4}(\varphi_D(+2, p_{Mid,t_4}, p_1^A) |\, \Omega_{t_4}) + \Pr_{t_4}(\varphi_D(+2, p_{Mid,t_4}) |\, \Omega_{t_4})]\}\;,\end{aligned} \quad (76)$$

where, as for the $L\&P$, $\Omega_{t_4} = [21, \varphi_L(2, p_1^A), \varphi_L(1, p_1^B), \varphi(0)]$.

B.3.4. OM

Results for OM presented in Fig. B1 are derived by straightforward comparison of the equilibrium strategies for the three frameworks: B, $L\&P$ and $L\&C$. In Figs. B7–B12 we provide plots at t_2 for the large trader's profits as a function of β, for both the $L\&P$ and $L\&C$ frameworks. Following the exposition in the main text, we focus on selling strategies. Each figure provides a graphical representation of the traders' optimization problem. Fig. B7 shows how the introduction of a PDP changes the optimal order submission strategies of large traders by crowding out both market and limit orders, and generating OM. Consider first OM in the $L\&P$: compare Figs. B7 and B9 for the effect of market depth, B7 and B11 for the effect of spread. For the $L\&C$, compare instead Figs. B8 and B10, and B8 and B12, respectively.

B.3.5. TC and VC

Results for TC and VC presented in Figs. B1 and B2, respectively, are obtained by comparing fill rates and volumes for the B, $L\&P$ and $L\&C$ frameworks, as shown in Eqs. (14) and (50) respectively. As an example, we consider period t_1 of the B model and specify formulas for the estimated fill rate and volume in this period. Equilibrium strategies at t_1 for a large trader are as follows: $\varphi^1_{LT}= \varphi_M(2,p^B_2)$, $\varphi^2_{LT}= \varphi_{ML}(1,p^B_2;1,p^A_1)$, $\varphi^3_{LT}= \varphi_L(2,p^A_1)$, $\varphi^4_{LT}= \varphi_L(2,p^B_1)$, $\varphi^5_{LT}= \varphi_{ML}(1,p^A_2;1,p^B_1)$ and $\varphi^6_{LT}= \varphi_M(2,p^A_1)$. The ones for a small trader are: $\varphi^1_{ST}= \varphi_M(1,p^B_1)$, $\varphi^2_{ST}= \varphi_L(1,p^A_1)$, $\varphi^3_{ST}= \varphi_L(1,p^B_1)$ and $\varphi^4_{ST}= \varphi_M(1,p^A_1)$.

$$FR^B_{t_1,[22]} = \tfrac{1}{2}(\Pr_{t_1}\varphi^1_{ST} + \Pr_{t_1}\varphi^4_{ST})+\tfrac{1}{2}(\Pr_{t_1}\varphi^1_{LT} + \Pr_{t_1}\varphi^2_{LT} + \Pr_{t_1}\varphi^5_{LT} + \Pr_{t_1}\varphi^6_{LT}) \tag{77}$$

$$V^B_{t_1,[22]} = \tfrac{1}{2}(\Pr_{t_1}\varphi^1_{ST} + \Pr_{t_1}\varphi^4_{ST})+\tfrac{1}{2}(2\Pr_{t_1}\varphi^1_{LT} + \Pr_{t_1}\varphi^2_{LT} + \Pr_{t_1}\varphi^5_{LT} + 2\Pr_{t_1}\varphi^6_{LT})\ . \tag{78}$$

B.3.6. Spread, depth and welfare

Results for spread and depth (Fig. B3, Panel A), and for welfare (Fig. B3, Panel B) are obtained by comparing, respectively, the two market quality measures and welfare values for the B, $L\&P$ and $L\&C$ protocol. We refer to the proofs of Proposition 2 and Proposition 3 for an example of how to compute these measures.

Table B1
Order submission strategies.

This table reports the trading strategies, φ, available to large (LT) and small traders (ST) for the three different frameworks considered: a benchmark model (B) with a limit order book (LOB), and either a continuous ($L\&C$) or a periodic dark pool ($L\&P$) competing with a LOB. The LOB is characterized by a set of four prices, denoted by p_i^z, where $z = \{A, B\}$ indicates the ask or bid side of the market, and $i = \{1, 2\}$ the level on the price grid. In the $L\&C$, $\widetilde{p}_{Mid,t}$ indicates the spread mid-quote on the LOB prevailing in period t. In the $L\&P$, $\widetilde{p}_{Mid}$ indicates the spread mid-quote on the LOB prevailing at the end of period t_4, when the dark pool crosses orders. IOC indicates Immediate-or-Cancel orders. The LT trades up to two shares, $j = \{0, 1, 2\}$, while the ST trades up to one share.

Strategies LT (B)	Notation	Strategies ST (B – $L\&C$ – $L\&P$)	Notation
Market order of j shares	$\varphi_M(j, p_i^z)$	Market order of 1 share	$\varphi_M(1, p_i^z)$
Limit order of j shares	$\varphi_L(j, p_1^z)$	Limit order of 1 share	$\varphi_L(1, p_1^z)$
No trading	$\varphi(0)$	No trading	$\varphi(0)$
Market & limit order of 1 share each	$\varphi_{ML}(1, p_i^z; 1, p_1^z)$		
Marketable order of 2 shares	$\varphi_M(2, p^z)$		
Additional Strategies LT ($L\&C$)			
Market & dark pool order of 1 share each	$\varphi_{MD}(1, p_i^z; \pm 1, \widetilde{p}_{Mid,t})$		
Limit & dark pool order of 1 share each	$\varphi_{LD}(1, p_1^z; \pm 1, \widetilde{p}_{Mid,t})$		
Dark pool order of j shares	$\varphi_D(\pm j, \widetilde{p}_{Mid,t})$		
IOC on dark pool or market order of j shares	$\varphi_{DM}(\pm j, \widetilde{p}_{Mid,t}, p_i^z)$		
Additional Strategies LT ($L\&P$)			
Market & dark pool order of 1 share each	$\varphi_{MD}(1, p_i^z; \pm 1, \widetilde{p}_{Mid})$		
Limit & dark pool order of 1 share each	$\varphi_{LD}(1, p_1^z; \pm 1, \widetilde{p}_{Mid})$		
Dark pool order of j shares	$\varphi_D(\pm j, \widetilde{p}_{Mid})$		

Table B2

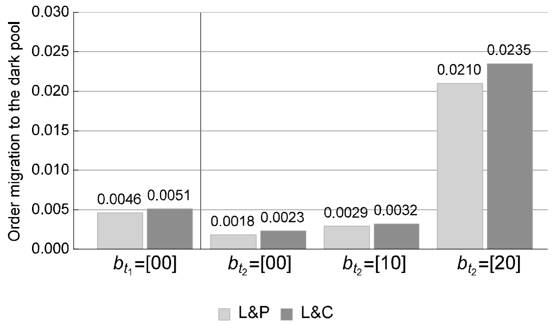
Order submission probabilities at t_1 and t_2.

This table reports the submission probabilities of large (LT) and small traders (ST) for the orders listed in column 1 for the benchmark framework (B), for the model with a limit order book and a periodic dark pool ($L\&P$) and for the model with a limit order book and a continuous dark pool ($L\&C$). We consider the t_1 equilibrium strategies from the four-period model that opens with an empty book at t_1, $b_{t_1} = [00]$, and the t_2 equilibrium strategies from the three-period models that open at t_2 according to the three equilibrium opening books (at t_2) from the four-period model, and that differ for the number of shares at the first level of the ask side of the book: $b_{t_2} = [00]$, $b_{t_2} = [10]$, and $b_{t_2} = [20]$. Results are computed assuming that the tick size is equal to $\tau = 0.08$ and the probability that a large trader arrives is $\alpha = 0.5$.

Panel A - ST	$b_{t_1} = [00]$			$b_{t_2} = [00]$			$b_{t_2} = [10]$			$b_{t_2} = [20]$		
Trading Strategy	B	$L\&C$	$L\&P$	B	$L\&C$	$L\&P$	B	$L\&C$	$L\&P$	B	$L\&C$	$L\&P$
$\varphi_M\left(1, p_2^B\right)$	0.0399	0.0429	0.0410	0.2376	0.2385	0.2384	0.3706	0.3724	0.3709	0.4245	0.4245	0.4245
$\varphi_L\left(1, p_1^A\right)$	0.4601	0.4571	0.4590	0.2624	0.2615	0.2616	0.1250	0.1231	0.1247	0.0628	0.0628	0.0628
$\varphi_L\left(1, p_1^B\right)$	0.4601	0.4571	0.4590	0.2624	0.2615	0.2616	0.1306	0.1305	0.1306	0.1406	0.1386	0.1377
$\varphi_M\left(1, p_1^A\right)$							0.3738	0.3741	0.3738	0.3721	0.3741	0.3750
$\varphi_M\left(1, p_2^A\right)$	0.0399	0.0429	0.0410	0.2376	0.2385	0.2384						

Panel B - LT	$b_{t_1} = [00]$			$b_{t_2} = [00]$			$b_{t_2} = [10]$			$b_{t_2} = [20]$		
Trading Strategy	B	$L\&C$	$L\&P$	B	$L\&C$	$L\&P$	B	$L\&C$	$L\&P$	B	$L\&C$	$L\&P$
$\varphi_M(2, p_2^B)$	0.0399	0.0429	0.0410	0.2376	0.2385	0.2384	0.3706	0.3724	0.3709	0.4245	0.4067	0.4061
$\varphi_{ML}(1, p_2^B; 1, p_1^A)$	0.2686	0.2780	0.2778	0.1330	0.1354	0.1324	0.0539	0.0521	0.0536	0.0110		
$\varphi_D(-2, \widetilde{p}_{Mid})$												0.0652
$\varphi_D(-2, \widetilde{p}_{Mid,t_2})$											0.0614	
$\varphi_L(2, p_1^A)$	0.1915	0.1791	0.1812	0.1294	0.1261	0.1291	0.0692	0.0692	0.0693	0.0505	0.0181	0.0147
$\varphi_L(2, p_1^B)$	0.1915	0.1791	0.1812	0.1294	0.1261	0.1291	0.0639	0.0624	0.0623	0.0714	0.0698	0.0713
$\varphi_{ML}(1, p_1^A; 1, p_1^B)$							0.2048	0.2054	0.2054	0.0687	0.0700	0.0689
$\varphi_{ML}(1, p_2^A; 1, p_1^B)$	0.2686	0.2780	0.2778	0.1330	0.1354	0.1324						
$\varphi_M(2, p_1^A)$										0.3738	0.3741	0.3738
$\varphi_M(2, p^A)$							0.2376	0.2385	0.2384			
$\varphi_M(2, p_2^A)$	0.0399	0.0429	0.0410	0.2376	0.2385	0.2384						

Panel A: Order migration



Panel B: Trade creation

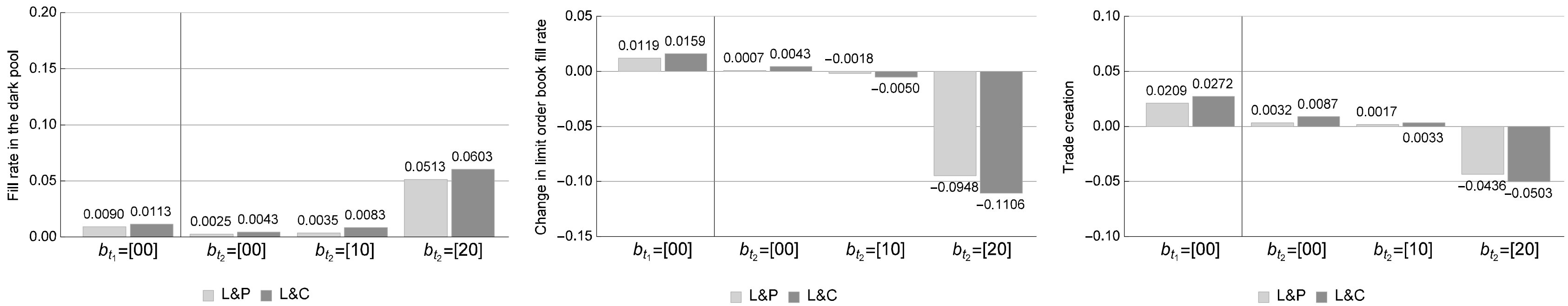


Fig. B1. Order migration and trade creation, small and large traders. This figure presents results for the two frameworks, $L\&P$ and $L\&C$. The first one combines a limit order book (LOB) and a periodic dark pool (PDP); the second combines a LOB and a continuous dark pool (CDP). For each framework we report in Panel A order migration which is the average probability that an order migrates to the dark pool, and in Panel B trade creation that is the sum of two components. The first one is the average fill rate in the dark pool. The second one is the difference between the average LOB fill rate in the $L\&P$ or $L\&C$, and the average LOB fill rate in the benchmark. We report results for both the four-period model that opens with an empty book at t_1, $b_{t_1} = [00]$, and for the three-period models that open at t_2 according to the three equilibrium opening books (at t_2) from the four-period model, and that differ for the number of shares at the first level of the ask side of the book: $b_{t_2} = [00]$, $b_{t_2} = [10]$, and $b_{t_2} = [20]$. Results are computed assuming a tick size equal to $\tau = 0.08$ and a probability that a large trader arrives $\alpha = 0.5$.

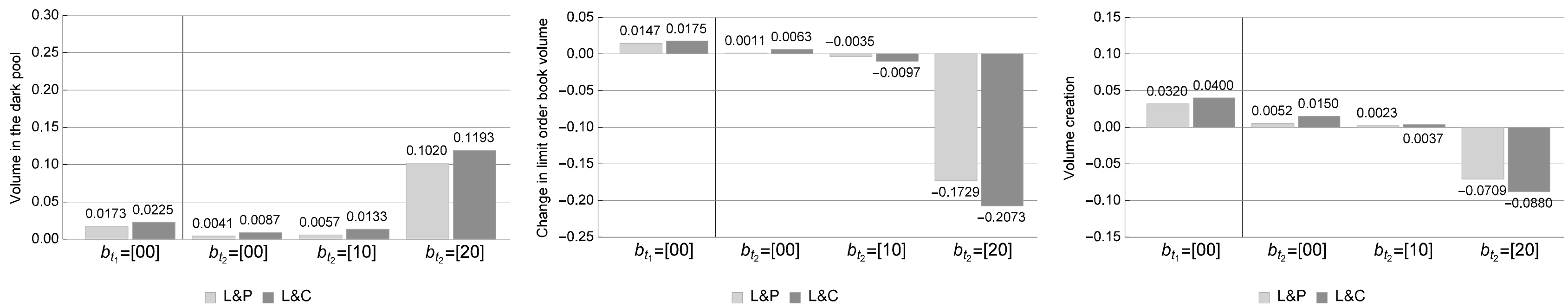
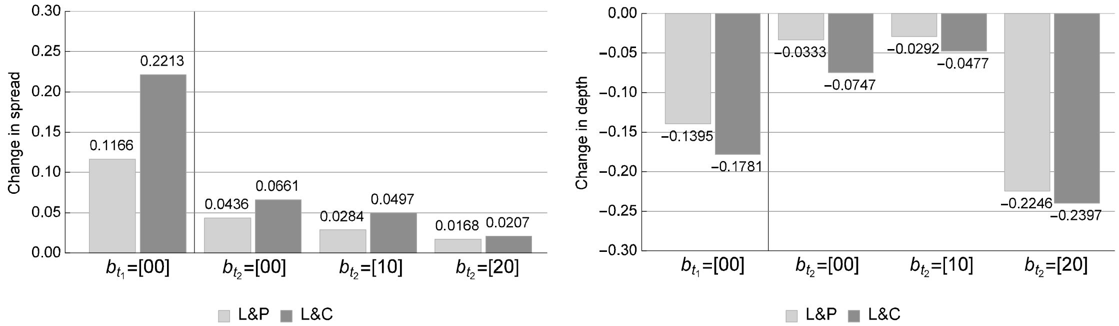


Fig. B2. Volume creation, small and large traders. This figure presents results for the two frameworks, $L\&P$ and $L\&C$. The first one combines a limit order book (LOB) and a periodic dark pool (PDP); the second combines a LOB and a continuous dark pool (CDP). For each framework we report volume creation (VC) that is the sum of two components. The first one is the average volume in the dark pool, $V(DARK\ POOL)$. The second one is the difference between the average LOB volume in the $L\&P$ or $L\&C$, $V(LOB)$, and the average LOB volume in the benchmark, $V(B)$. We report results for both the four-period model that opens with an empty book at t_1, $b_{t_1} = [00]$, and for the three-period models that open at t_2 according to the three equilibrium opening books (at t_2) from the four-period model, and that differ for the number of shares at the first level of the ask side of the book: $b_{t_2} = [00]$, $b_{t_2} = [10]$, and $b_{t_2} = [20]$. Results are computed assuming a tick size equal to $\tau = 0.08$ and a probability that a large trader arrives $\alpha = 0.5$.

Panel A: Market quality



Panel B: Welfare

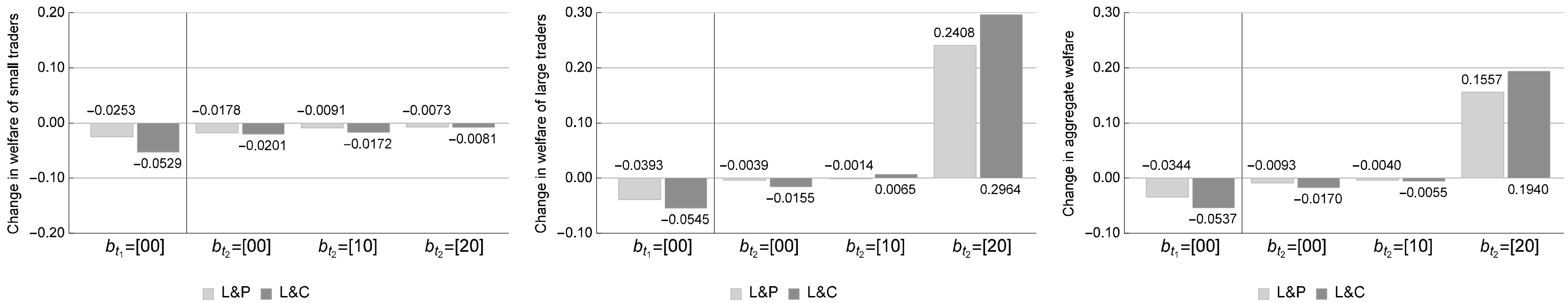


Fig. B3. Market quality and welfare, small and large traders. This figure presents results for spread, depth and welfare in the two frameworks, $L\&P$ and $L\&C$. $L\&P$ combines a limit order book (LOB) and a periodic dark pool (PDP); $L\&C$ combines a LOB and a continuous dark pool (CDP). All measures are computed as the average percentage difference between their value for the $L\&P$ or $L\&C$ framework and the benchmark framework. As spread and depth are exogenous in the initial period we do not include it in the average, while for welfare we consider three measures: the welfare of a small trader (ST), the welfare of a large trader (LT), and aggregate welfare.We report results for both the four-period model that opens with an empty book at t_1, $b_{t_1} = [00]$, and for the three-period models that open at t_2 according to the three equilibrium opening books (at t_2) from the four-period model, and that differ for the number of shares at the first level of the ask side of the book: $b_{t_2} = [00]$, $b_{t_2} = [10]$, and $b_{t_2} = [20]$. Results are computed assuming a tick size equal to $\tau = 0.08$ and a probability that a large trader arrives $\alpha = 0.5$.

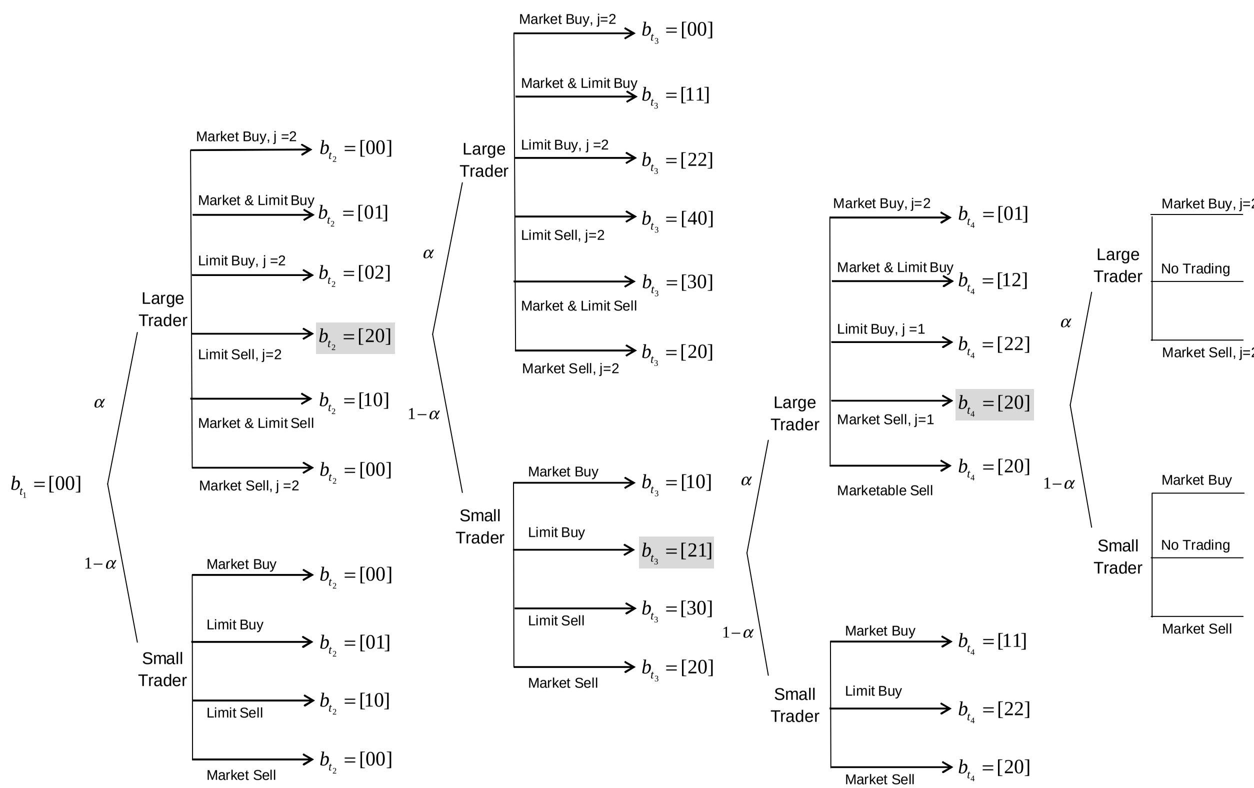


Fig. B4. Benchmark model of limit order book (B), small and large traders. Example of the extensive form of the game for the benchmark model when the opening book at t_1 is $b_{t_1} = [00]$, where j indicates the number of shares traded by the large trader. We only present the equilibrium strategies.

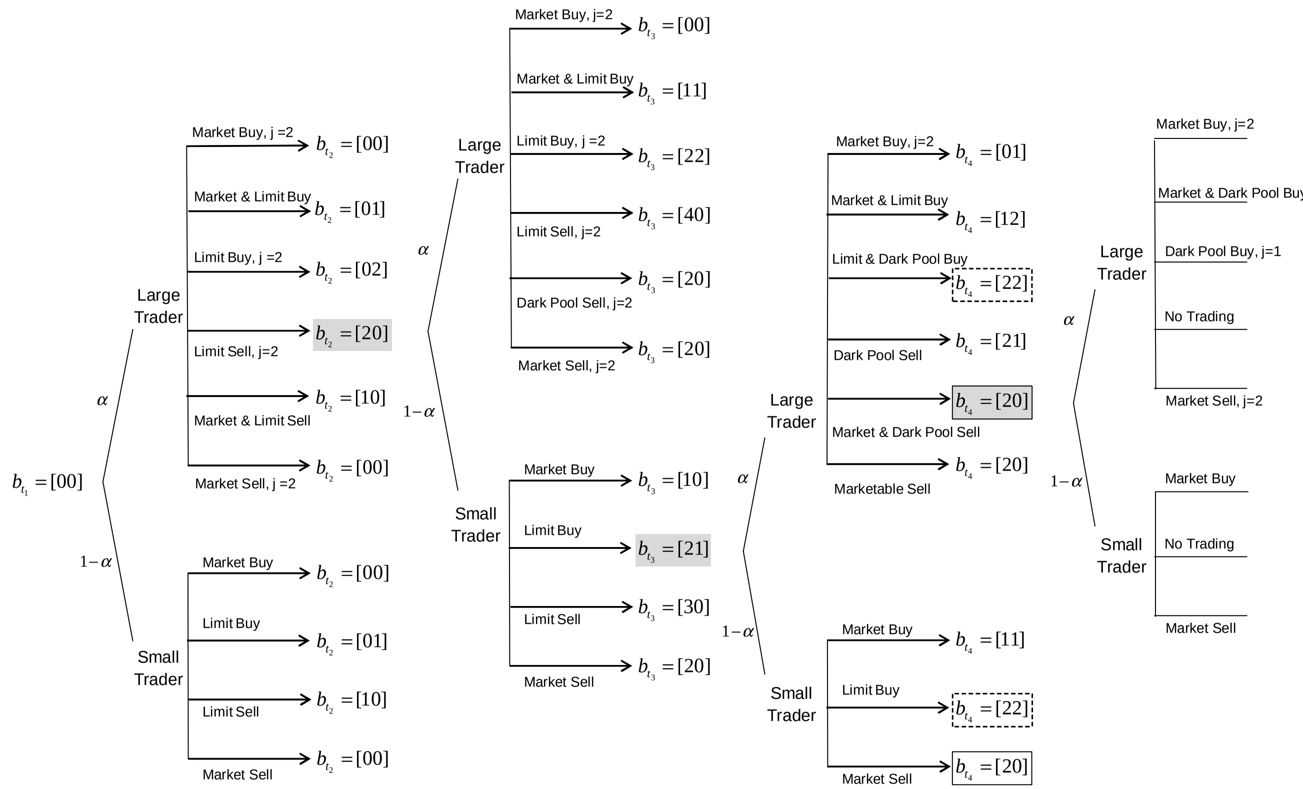


Fig. B5. Limit Order Book and Periodic Dark Pool (L&P), small and large traders. Example of the extensive form of the game for the model with a periodic dark pool when the opening book at t_1 is $b_{t_1} = [00]$, where j indicates the number of shares traded by the large trader. Books that belong to the same information set, and hence are undistinguishable, are in squared dashed boxes and have the same line format. For example, $b_{t_4} = [22]$ can be observed when at t_3 either a small trader arrives and submits a limit buy, or a large trader arrives and submits a combined limit and dark order to buy. We only present the equilibrium strategies.

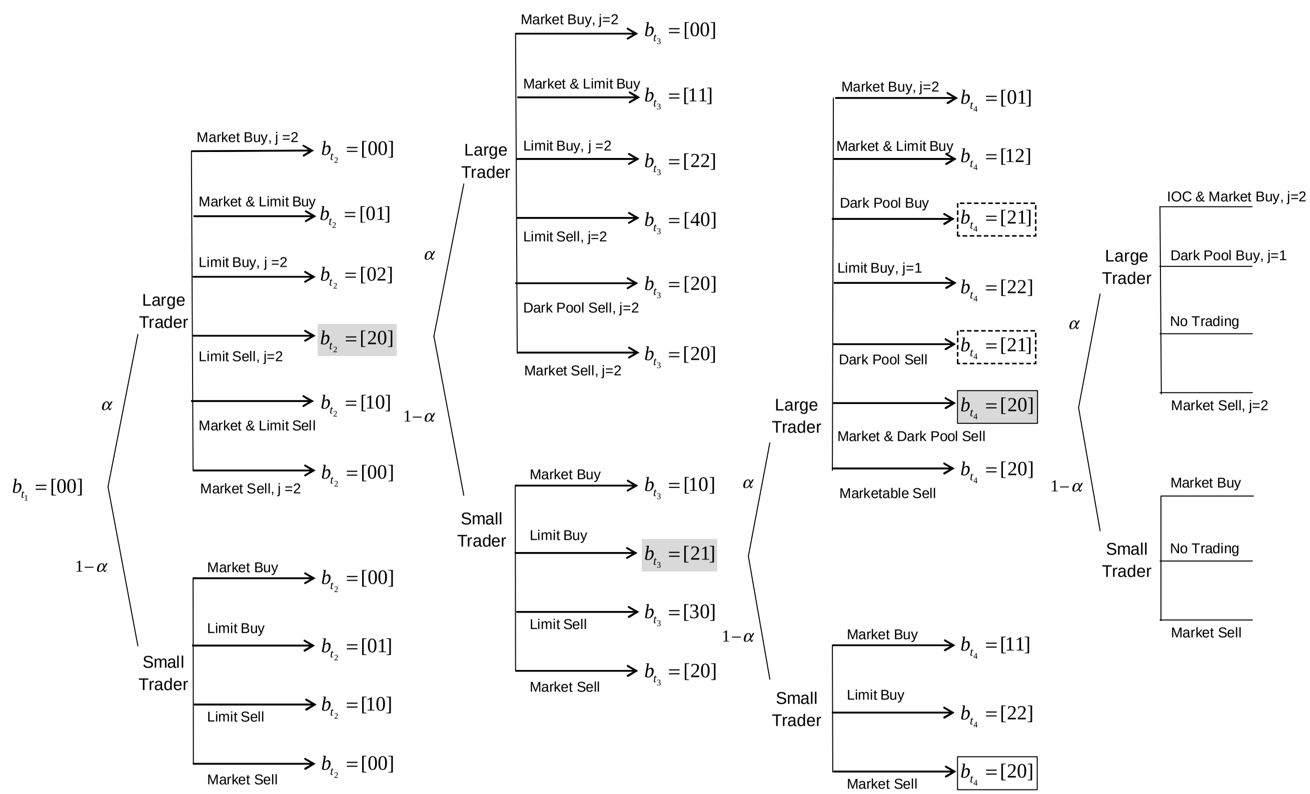


Fig. B6. Limit order book and continuous dark pool (L&C), small and large traders. Example of the extensive form of the game for the model with a continuous dark pool when the opening book at t_1 is $b_{t_1} = [00]$, where j indicates the number of shares traded by the large trader. Books that belong to the same information set, and hence are undistinguishable, are in squared dashed boxes and have the same line format. For example, $b_{t_4} = [21]$ can be observed when a large trader arrives at t_3 and submits either a dark pool order to sell or to buy. We only present the equilibrium strategies.

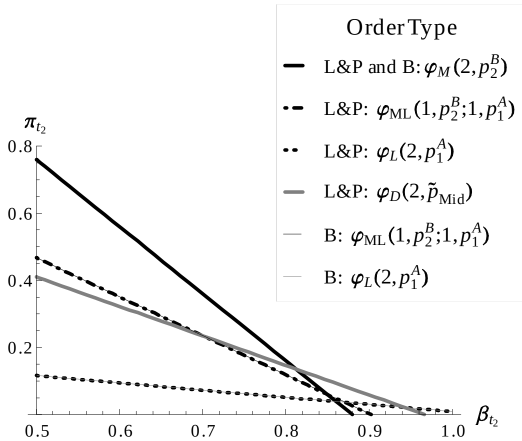


Fig. B7. Order migration on the L&P - b_{t_2}=[20]

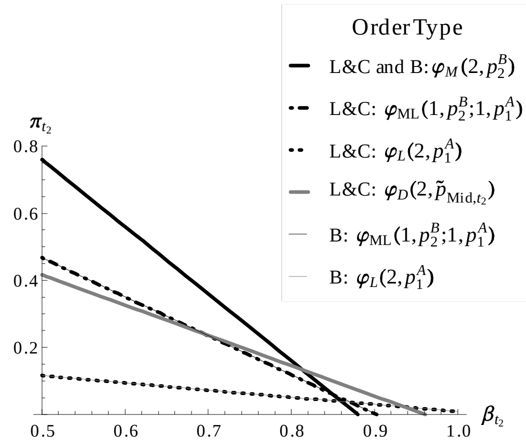


Fig. B8. Order migration on the L&C - b_{t_2}=[20]

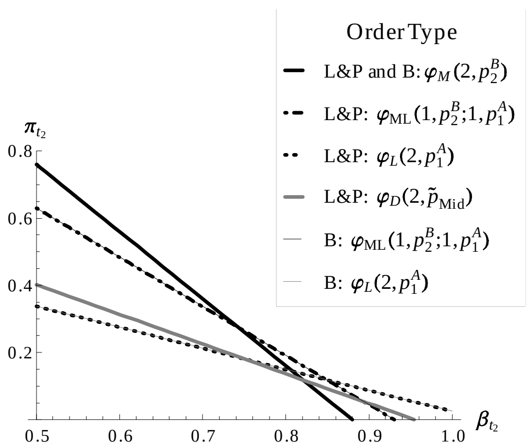


Fig. B9. Order migration on the L&P - b_{t_2}=[10]

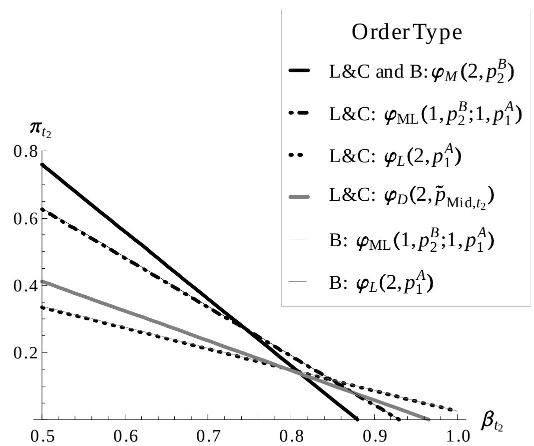


Fig. B10. Order migration on the L&C - b_{t_2}=[10]

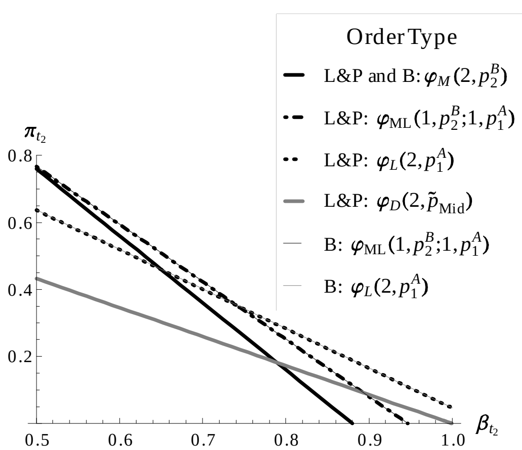


Fig. B11. Order migration on the L&P - b_{t_2}=[00]

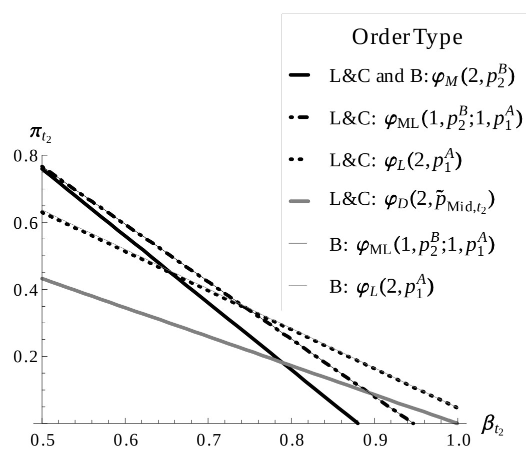


Fig. B12. Order migration on the L&C - b_{t_2}=[00]