Internet Appendix for "On the performance of volatility-managed portfolios"

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Abstract

This internet appendix provides material that is supplemental to the paper "On the performance of volatility-managed portfolios." Section 1 presents additional empirical results. Section 2 provides analytical results on spanning regression alphas for volatility-managed portfolios.

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1. Supplementary empirical results

Section 1.1 presents results on the performance of each of the 103 volatility-managed trading strategies considered in the paper. Section 1.2 details the bootstrap test used in the direct comparisons of volatility-managed and original portfolios in Section 3 of the paper. Section 1.3 considers reverse spanning regressions. Section 1.4 examines whether the poor performance for real-time combination strategies is attributable to structural instability in the risk-return relation for the various factors and anomaly portfolios.

1.1. Results for individual trading strategies

Table IA1 shows the mean, standard deviation, and Sharpe ratio for the original and volatility-managed versions of 103 trading strategies. We also present results from tests of whether the Sharpe ratio differences are statistically significant. The results in Table IA1 are summarized in Table 2 of the paper.

Table IA2 presents results from spanning regressions of volatility-managed portfolio returns on returns for the corresponding original portfolio. The table also details the portfolio properties of the ex post optimal combinations of volatility-managed and original portfolios. The spanning regression results are summarized in Table 4 of the paper.

1.2. Bootstrap analysis

Section 3 of the paper focuses on direct performance comparisons of volatility-managed and original strategies. In particular, we examine the Sharpe ratios of volatility-managed and original versions of the factors and anomaly portfolios. Overall, the volatility-managed portfolio outperforms the original portfolio in 53 of 103 comparisons as shown in Table 2. In this section, we describe a bootstrap analysis to examine whether observing 53 positive Sharpe ratio differences across the 103 strategies is statistically significant relative to a null hypothesis. The null hypothesis has a data generating process in which volatility is persistent (as in the data) but there is no risk-return relation. Our bootstrap design generates 100,000 bootstrap samples under the null and tabulates the number of positive Sharpe ratio differences across the 103 strategies for each bootstrap sample. We then examine whether the number of positive performance differences that we observe in the data (53) is statistically significant relative to the bootstrap distribution of the number of positive differences.

To better understand the performance of volatility-managed portfolios under the null hypothesis, we introduce a decomposition of the difference between the average returns of the volatility-scaled and original portfolios. Given that the variance of the volatility-scaled portfolio is equal to that of the original portfolio by construction, a positive Sharpe ratio difference is equivalent to the condition that the volatility-scaled portfolio earns a higher average return compared with the original portfolio. The difference in average returns can be decomposed as

$$\bar{f}_{\sigma,t} - \bar{f}_t = \text{cov}(w_t, f_t) + \bar{f}_t(\bar{w}_t - 1), \qquad \text{(IA1)}$$

where $w_t = c^*/\hat{\sigma}^2_{t-1}$ is the volatility-managed portfolio's investment position in the original portfolio. The first component, $\text{cov}(w_t, f_t)$, measures the relation between the volatility-managed strategy's investment weight and the unscaled strategy return. This component is negative if lagged volatility is a positive predictor of strategy return. There is no risk-return relation under the null hypothesis such that $\text{cov}(w_t, f_t) = 0$ under the null. The second component, $\bar{f}_t(\bar{w}_t - 1)$, captures the effect of the average investment weight for the volatility-managed portfolio. To the extent that the volatility-managed portfolio has an average investment of more (less) than 100% in the original portfolio, the volatility-managed portfolio will tend to have a higher (lower) average return compared with the original portfolio because of this term (assuming $\bar{f}_t > 0$). Given that the volatility-managed portfolio is constructed to have equal variance to the original portfolio, the behavior of the volatility-managed portfolio time series prior to calculating the scaling parameter c^* (i.e., $\frac{1}{\hat{\sigma}^2_{t-1}} f_t$) is important for determining the sign of the average weight term. If volatility is persistent, as in the null hypothesis, the $\frac{1}{\hat{\sigma}^2_{t-1}} f_t$ time series has fairly low volatility because the extreme outcomes of f_t tend to be downweighted by a low value of $\frac{1}{\hat{\sigma}^2_{t-1}}$. In this case, c^* is relatively large and the average weight is greater than one. We thus expect that Sharpe ratio differences tend to be positive under the null.

The following steps produce a distribution of the number of positive Sharpe ratio differences under the null hypothesis that volatility is persistent and there is no risk-return relation. The risk-return relation in the data can affect Sharpe ratios to the extent that the volatility scaling parameter, $c^*/\hat{\sigma}^2_{t-1}$, is able to forecast the factor return realization, f_t. Thus, the bootstrap design is centered around orthogonalizing f_t with respect to $c^*/\hat{\sigma}^2_{t-1}$ while maintaining the time-series relation between $c^*/\hat{\sigma}^2_{t-1}$ and the volatility of f_t.

1. For a given factor or anomaly portfolio, we first run a predictive regression to estimate the relation between $c^*/\hat{\sigma}^2_{t-1}$ and f_t,
$$f_t = a + b\frac{c^*}{\hat{\sigma}^2_{t-1}} + \varepsilon_t. \tag{IA2}$$
The ε_t error terms from this regression are orthogonal to $c^*/\hat{\sigma}^2_{t-1}$, but they inherit time variation and persistence in volatility from f_t such that the ε_t from high-volatility periods will tend to be matched with low values of $c^*/\hat{\sigma}^2_{t-1}$. We use these error terms as the basis for a new time series of hypothetical portfolio returns under the null hypothesis. We adjust the mean to equal the original portfolio mean by adding $\hat{a} + \hat{b}\overline{c^*/\hat{\sigma}^2_{t-1}}$. The unconditional variance of the new time series is also slightly lower than the original portfolio variance (it equals $(1-R^2)\sigma^2_f$), so we add a small white noise component to cause the time series generated under the null to have variance equal to that of the original portfolio. In sum, this process creates a new time series of portfolio returns denoted by $\tilde{f}_t$ with mean and variance that are equal to those of the original portfolio returns, the same degree of persistence in volatility as is present in the data, and no predictive relation between the scaling factor and subsequent realizations of portfolio returns.
2. Using the newly constructed portfolio returns from the previous step, we generate a single bootstrap sample by drawing with replacement T matched observations of $\hat{\sigma}^2_{t-1}$ and $\tilde{f}_t$. That is, we maintain the

relative timing of the $\hat{\sigma}^2_{t-1}$ and $\tilde{f}_t$ observations while drawing observations to retain the property that volatility is persistent. Denote these draws as $\hat{\sigma}^2_{b,t-1}$ and $f_{b,t}$.

3. Construct the volatility-managed portfolio return as $f_{b,\sigma,t} = \frac{c_b^*}{\hat{\sigma}^2_{b,t-1}} f_{b,t}$, where c_b^* is a constant that is set to equalize the variances of $f_{b,t}$ and $f_{b,\sigma,t}$ in the bootstrap sample.
4. Calculate the Sharpe ratios of $f_{b,t}$ and $f_{b,\sigma,t}$ and record whether the Sharpe ratio difference between the volatility-managed and original portfolio is positive.
5. Repeat steps 1 to 4 to produce 100,000 direct performance comparisons across bootstrap samples for each factor or anomaly portfolio.
6. Count the number of positive performance differences across the 103 strategies in each of the 100,000 draws and produce the distribution of the number of positive performance differences under the null hypothesis.

Fig. 1 in the paper shows the bootstrap distribution of the number of positive Sharpe ratio differences with the null hypothesis of persistent volatility with no risk-return relation. Across bootstrap draws, about 66 of 103 performance differences are positive on average. The two-tailed bootstrap p-value for the observation from the data that 53 of 103 performance differences are positive is 0.01. Given that the persistence of volatility matches the data in the null hypothesis, the finding that fewer performance differences are positive in the data compared with the outcomes in the bootstrap distribution indicates that the $\text{cov}(w_t, f_t)$ term is sufficiently negative in the data to produce a statistically significant number of positive performance differences. Negative values of $\text{cov}(w_t, f_t)$ correspond to a positive risk-return relation given persistent volatility such that the bootstrap results indicate the presence of systematically positive risk-return relations across the 103 strategies.

1.3. Reverse regressions

Panel A.1 of Table 3 in the paper shows results from univariate spanning regressions of volatility-managed factor returns on the corresponding original factor returns. Across the nine equity factors considered in the paper, eight of the volatility-managed portfolios exhibit positive regression intercepts, and five of the estimates are statistically significant at the 5% level. The spanning regressions in Panel B of Table 3 add controls for the Fama-French (1993) three factors. Seven of the nine regressions produce positive intercepts, and six alphas are statistically significant. Table 4 of the paper extends these results to the broader sample of 103 trading strategies. We find that 77 of the 103 volatility-managed portfolios earn positive alphas in univariate spanning tests and 70 of the 103 portfolios have positive alphas in spanning tests that control for exposure to the Fama-French (1993) factors.

Table IA3 presents results from reverse specifications of the spanning regressions for the nine equity factors. These regressions are given by

$$f_t = \alpha^r + \beta^r f_{\sigma,t} + \varepsilon_t, \tag{IA3}$$

where f_t ($f_{\sigma,t}$) is the monthly return for the original (volatility-managed) factor. The univariate reverse regressions in Panel A produce positive intercepts for the original factors in all nine cases, and four of the estimates are significant at the 5% level. The reverse regressions in Panel B incorporate controls for the Fama-French (1993) factors. Eight of the nine intercepts are positive under this specification, and five are statistically significant. The results highlight that spanning tests are unsuitable for identifying the superior version of a given strategy.

Table IA4 confirms this conclusion in the broader sample. Relative to the performance measures for the volatility-managed portfolios summarized in Table 4 of the paper, the reverse spanning tests generate even stronger support for the original portfolios. The reverse regressions in Table IA4 without and with controls for the Fama-French (1993) factors generate 82 and 81 positive intercepts, respectively. Both totals exceed the corresponding numbers of positive spanning test alphas reported in Table 4.

1.4. Structural breaks analysis

Table 9 of the paper summarizes results from multiple structural break tests for spanning regressions and anomaly regressions in the broad sample of 103 trading strategies. In this section, we describe the design of these tests and discuss detailed results for the nine equity factors.

To test for structural breaks in the regression models, we use Bai and Perron's (1998, 2003) method, which allows for an unknown number of breaks. In particular, we use the sequential procedure at the 10% significance level. The maximum number of breaks allowed is five, but this constraint is not binding for any model in the paper. The minimum subperiod length is set to 15% of the sample period length.

Table IA5 shows results from these tests for the nine equity factors. Panel A focuses on univariate spanning regressions. For each factor, we report the $UDmax$ and $WDmax(10\%)$ statistics for tests of the null hypothesis of zero breaks.[1] We also show the estimated break dates and the spanning regression alpha estimates for each subperiod.

Panel A of Table IA5 provides strong evidence of structural instability in the spanning regressions. The null hypothesis of zero breaks is rejected for each factor, and the structural break tests identify either two or three break dates in each case. Further, the spanning regression alphas tend to vary substantially across subperiods. Seven of the nine factors have at least one subperiod with a negative alpha estimate such that just two factors (MOM and ROE) consistently produce positive spanning regression intercepts.

These results are helpful in understanding why real-time investors attempting to adopt combination strategies often experience poor performance. That is, investors who use historical information to make investment decisions are adversely affected by structural instability. For example, the strong results for volatility management early in the sample for the MKT and RMW factors lead real-time investors to trade aggressively based on volatility, but volatility management produces reduced or negative performance

[1] In a few cases, the $UDmax$ statistic indicates a strong statistical rejection of the null hypothesis of zero breaks, but the sequential procedure breaks down by failing to reject the null hypothesis of zero versus one break. In these instances, we follow the suggestions of Bai and Perron (2003) to select the number of breakpoints.

over much of the investment period. Conversely, it would have been difficult for investors to anticipate that volatility-managed factors with poor early performance (e.g., CMA and IA) would eventually realize improved results. The findings in Panel A of Table IA5 underscore the difficulty of real-time investing with volatility-managed strategies.

For completeness, Panel B of Table IA5 presents structural break tests for spanning regressions that control for the Fama-French (1993) factors. The estimated break dates and patterns in subperiod alpha estimates closely resemble those in Panel A. Panels C and D report results from structural break tests for the CAPM and Fama-French (1993) regressions, respectively, for the unscaled factor portfolios.

2. Supplementary analytical results

Section 2.1 presents a decomposition of the spanning regression alpha for the market portfolio. This relation is useful in linking the empirical evidence in Moreira and Muir (2017) on the performance of the volatility-managed market portfolio to models in the macrofinance literature. Section 2.2 characterizes the conditions under which spanning tests produce positive alphas for volatility-managed portfolios.

2.1. A decomposition of spanning regression alpha

An important contribution of Moreira and Muir (2017) is the demonstration that leading asset pricing models from the macrofinance literature are unable to generate the large, positive spanning regression alpha for the market portfolio. In particular, the habit formation (Campbell and Cochrane, 1999), long-run risk (Bansal and Yaron, 2004; Bansal et al., 2012), rare disasters (Barro, 2009; Wachter, 2013), and intermediary-based asset pricing (He and Krishnamurthy, 2013) models each produce spanning regression alphas that are relatively close to zero, and the alpha estimated from the data falls outside of the reasonable range of alphas from model simulations. Thus, the spanning regression alpha diagnostic provides a challenge for the macrofinance literature by introducing an additional hurdle that models should produce similar information about time-varying compensation for risk as is observed from the data.

In this section, we complement Moreira and Muir's (2017) diagnostic by introducing a two-term decomposition of the spanning regression alpha. We build on decompositions of alpha from the cross-sectional asset pricing literature (e.g., Lewellen and Nagel, 2006; Boguth et al., 2011). To derive our decomposition, start with the definition of the scaled portfolio return, $f_{\sigma,t} = w_t f_t$, where $w_t = c^*/\hat{\sigma}^2_{t-1}$. Using a covariance decomposition, the sample mean of $f_{\sigma,t}$ is

$$\bar{f}_{\sigma,t} = \text{cov}(w_t, f_t) + \bar{w}_t \bar{f}_t. \tag{IA4}$$

The definition of the spanning regression alpha is

$$\hat{\alpha} = \bar{f}_{\sigma,t} - \hat{\beta}\bar{f}_t \tag{IA5}$$

$$= \text{cov}(w_t, f_t) + \bar{f}_t(\bar{w}_t - \hat{\beta}). \tag{IA6}$$

Further, let $w_t = \bar{w}_t + e_t$, where e_t is the time-varying component of the investment position in the underlying factor. The unconditional beta is

$$\hat{\beta} = \text{cov}(f_{\sigma,t}, f_t)/\hat{\sigma}_f^2 \tag{IA7}$$

$$= \text{cov}[(\bar{w}_t + e_t)f_t, f_t]/\hat{\sigma}_f^2 \tag{IA8}$$

$$= [\bar{w}_t\hat{\sigma}_f^2 + \text{cov}(e_t, f_t^2) - \text{cov}(e_t, f_t)\bar{f}_t]/\hat{\sigma}_f^2 \tag{IA9}$$

$$= \bar{w}_t - (\bar{f}_t/\hat{\sigma}_f^2)\text{cov}(w_t, f_t) + \text{cov}(w_t, f_t^2)/\hat{\sigma}_f^2. \tag{IA10}$$

Finally, substituting Eq. (IA10) into Eq. (IA6) yields an expression for the spanning regression alpha,

$$\hat{\alpha} = \left(1 + \frac{\bar{f}_t^2}{\hat{\sigma}_f^2}\right)\text{cov}(w_t, f_t) - \left(\frac{\bar{f}_t}{\hat{\sigma}_f^2}\right)\text{cov}(w_t, f_t^2). \tag{IA11}$$

The spanning regression alpha decomposition in Eq. (IA11) has two terms. The first term, $(1 + \bar{f}_t^2/\hat{\sigma}_f^2)\text{cov}(w_t, f_t)$, measures the effect on alpha of the investment position's relation with the factor return. The second term, $-(\bar{f}_t/\hat{\sigma}_f^2)\text{cov}(w_t, f_t^2)$, is indicative of the effect on alpha from the relation between investment weight and factor volatility. Given the construction of the weight as $w_t = c^*/\hat{\sigma}_{t-1}^2$, this term is likely to be positive when volatility is persistent.

We apply the decomposition in Eq. (IA11) to the spanning regression alpha for the market factor. To generate the spanning regression alpha estimate of 4.63% from Table 3 of the paper, the first component contributes −0.24% and the second component contributes 4.87%. This finding is consistent with Moreira and Muir's (2017) discussion of the volatility-managed strategy for the market portfolio, as lagged volatility is only weakly related to average return but is strongly related to return volatility. The spanning regression alpha decomposition formalizes this intuition, and it provides additional diagnostics and moments for macrofinance models to match.

A formal analysis of macrofinance model shortcomings is beyond the scope of our paper, but we note that generating the large, positive value for the second component, $-(\bar{f}_t/\hat{\sigma}_f^2)\text{cov}(w_t, f_t^2)$, that is estimated from the data requires market factor volatility to be relatively persistent and highly volatile. When variance is persistent and volatile, the unconditional covariance of the inverse of lagged variance with the current variance will be negative and can be large enough to produce a large spanning regression alpha.

Traditional macrofinance models, such as the habit formation and long-run risk models, typically do not closely fit the stylized fact from the data that market return volatility is highly volatile. Taking the long-run risk model as a specific example, return volatility is highly persistent and less volatile in the model compared with actual data. Return volatility in the model only varies over time as a function of the expected growth and economic uncertainty state variables, which are calibrated to vary at low frequencies. Volatility behavior in the long-run risk model can be observed in the tests by Bansal et al. (2012) that regress the one-year conditional variance of excess market return on the lagged price-dividend ratio in simulated model data. These predictive regressions produce a population R^2 of 0.44 in the model compared with an estimate of 0.11 in the data. The strong predictability of variance using price-dividend ratio in the model may be expected given that the two processes are functions of the same two state variables. Further, a back-of-the-envelope calculation based on the slope and R^2 from the predictive regression along with the variance of the price-dividend ratio predictor from Bansal et al. (2012) suggests that the unconditional variance of conditional one-year return variance is about three times higher in the data compared with the model.[2] Spanning regression alpha tests in the long-run risk model are, therefore, unlikely to produce the large, positive value of the second component that we estimate from the data.

To successfully match the spanning regression alpha estimate and its components, new macrofinance models will likely need to generate additional volatility in volatility compared with traditional models. In addition, a negative relation between the conditional reward-to-risk ratio and the conditional variance is necessary, as discussed by Moreira and Muir (2017). Models that have multiple time-varying components of return volatility that carry different prices of risk and that have different frequencies may have the potential to reconcile with the spanning regression alpha and its components that we derive in Eq. (IA11).

2.2. Properties of spanning regression alpha

Consider a volatility-managed portfolio constructed according to Eq. (3) in the paper. The following proposition characterizes the conditions under which the intercept from a regression of the volatility-managed portfolio's returns on returns for the corresponding unscaled portfolio [i.e., Eq. (5)] is positive.

Proposition 1 *Assume that $\bar{f}_{\sigma,t} > 0$ and $\bar{f}_t > 0$. The alpha estimate in the spanning regression is positive if*

$$\bar{f}_{\sigma,t} > \hat{\rho}\bar{f}_t, \tag{IA12}$$

where $\hat{\rho}$ is the unconditional correlation between the scaled and unscaled portfolios.

Corollary 1.1 *Consider a reverse spanning regression given by*

$$f_t = \alpha^r + \beta^r f_{\sigma,t} + \varepsilon_t. \tag{IA13}$$

[2]Given the predictive regression $\sigma_t^2 = a + b(p_{t-1} - d_{t-1}) + \varepsilon_t$, $R^2 = (\hat{b}\text{var}(p_{t-1} - d_{t-1}))/\text{var}(\sigma_t^2)$. The calculations using estimates from Bansal et al. (2012) give $\text{var}(\sigma_t^2) = ((-0.08)^2 \times 0.26^2)/0.44 = 0.00098$ for the long-run risk model versus $\text{var}(\sigma_t^2) = (-0.04^2 \times 0.45^2)/0.11 = 0.00295$ for the data.

Alphas in the spanning regression and the reverse regression are simultaneously positive as long as

$$\hat{\rho}^{-1}\bar{f}_t > \bar{f}_{\sigma,t} > \hat{\rho}\bar{f}_t. \tag{IA14}$$

Proof 1 *Start with the definition of alpha in the spanning regression given in Eq. (5) in the paper,*

$$\hat{\alpha} = \bar{f}_{\sigma,t} - \hat{\beta}\bar{f}_t. \tag{IA15}$$

The unconditional beta is

$$\hat{\beta} = cov(f_{\sigma,t}, f_t)/\hat{\sigma}_f^2, \tag{IA16}$$

where $\hat{\sigma}_f^2$ is the unconditional variance of the unscaled factor return. Because $f_{\sigma,t}$ is constructed to have the same unconditional standard deviation as f_t,

$$\hat{\beta} = \hat{\rho}. \tag{IA17}$$

Substituting Eq. (IA17) *into Eq.* (IA15) *leads to the result in Eq.* (IA12). □

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Table IA1
Volatility-managed and original portfolios: broad sample

The table compares the performance of volatility-managed and original versions of 103 trading strategies. For a given factor or anomaly portfolio, the volatility-managed strategy return in month t is $f_{\sigma,t} = (c^*/\hat{\sigma}^2_{t-1})f_t$, where f_t is the monthly return for the original portfolio, $\hat{\sigma}^2_{t-1}$ is the realized variance of daily portfolio returns in month $t-1$, and c^* is a constant chosen so that f_t and $f_{\sigma,t}$ have the same unconditional standard deviation over the full sample period. We present the mean return, standard deviation, and annualized Sharpe ratio for each original and volatility-managed portfolio. The means and standard deviations are reported in percentage per year. The table also shows the difference between the Sharpe ratio of the volatility-managed portfolio and that of the original portfolio (ΔSR) and the corresponding Jobson and Korkie (1981) p-value. Panels of the table are organized by trading strategy type (i.e., accruals, intangibles, investment, market, momentum, profitability, trading, and value).

	Original portfolio			Volatility-managed portfolio			Difference	
Strategy (1)	Mean (2)	Std. dev. (3)	Sharpe ratio (4)	Mean (5)	Std. dev. (6)	Sharpe ratio (7)	ΔSR (8)	p-value (9)
Panel A: Accruals								
IvC	4.74	11.63	0.41	5.69	11.63	0.49	0.08	0.39
IvG	4.37	12.44	0.35	3.32	12.44	0.27	−0.08	0.38
NOA	4.26	12.65	0.34	2.51	12.65	0.20	−0.14	0.17
OA	4.71	12.38	0.38	3.79	12.38	0.31	−0.07	0.39
POA	4.84	11.48	0.42	5.15	11.48	0.45	0.03	0.77
PTA	4.84	10.55	0.46	6.01	10.55	0.57	0.11	0.26
TA	3.67	11.54	0.32	3.13	11.54	0.27	−0.05	0.67
ΔNCO	5.60	10.33	0.54	4.49	10.33	0.43	−0.11	0.28
ΔNWC	5.24	12.31	0.43	6.21	12.31	0.50	0.08	0.43
NoaG	4.87	11.13	0.44	2.87	11.13	0.26	−0.18	0.13
Panel B: Intangibles								
AccQ	−1.28	17.97	−0.07	−2.05	17.97	−0.11	−0.04	0.69
AD/M	7.82	21.79	0.36	10.62	21.79	0.49	0.13	0.35
BC	0.12	14.69	0.01	0.91	14.69	0.06	0.05	0.68
H/N	2.56	13.88	0.18	1.24	13.88	0.09	−0.10	0.34
OC/A	5.56	16.37	0.34	4.84	16.37	0.30	−0.04	0.68
OL	5.04	13.35	0.38	3.96	13.35	0.30	−0.08	0.40
RC/A	4.47	25.57	0.17	3.12	25.57	0.12	−0.05	0.69
RD/M	5.34	17.91	0.30	3.70	17.91	0.21	−0.09	0.39
RD/S	−0.25	28.46	−0.01	1.51	28.46	0.05	0.06	0.64
Age	−1.20	17.86	−0.07	−2.26	17.86	−0.13	−0.06	0.63
Panel C: Investment								
CMA	3.72	6.97	0.53	2.79	6.97	0.40	−0.13	0.23
IA	4.99	6.48	0.77	4.69	6.48	0.72	−0.05	0.68
$\Delta PI/A$	4.92	10.93	0.45	3.84	10.93	0.35	−0.10	0.28
ACI	3.25	10.00	0.33	4.42	10.00	0.44	0.12	0.26
CEI	1.75	10.37	0.17	2.73	10.37	0.26	0.09	0.29
I/A	4.79	12.90	0.37	2.97	12.90	0.23	−0.14	0.19
IG	3.78	11.28	0.34	3.51	11.28	0.31	−0.02	0.83
NSI	3.78	11.06	0.34	4.30	11.06	0.39	0.05	0.62
NXF	5.18	15.94	0.32	3.28	15.94	0.21	−0.12	0.33
BeG	3.86	13.07	0.30	0.53	13.07	0.04	−0.25	0.01
I-ADJ	3.02	10.58	0.29	2.06	10.58	0.19	−0.09	0.35

(*continued on next page*)

Table IA1 (*continued*)

	Original portfolio			Volatility-managed portfolio			Difference	
Strategy (1)	Mean (2)	Std. dev. (3)	Sharpe ratio (4)	Mean (5)	Std. dev. (6)	Sharpe ratio (7)	ΔSR (8)	p-value (9)
				Panel D: Market				
MKT	7.80	18.61	0.42	9.55	18.61	0.51	0.09	0.30
				Panel E: Momentum				
MOM	7.94	16.39	0.48	16.17	16.39	0.99	0.50	0.00
Abr-1	11.84	13.28	0.89	14.33	13.28	1.08	0.19	0.12
R11-1	14.03	25.79	0.54	23.45	25.79	0.91	0.37	0.00
R6-1	10.51	25.27	0.42	22.81	25.27	0.90	0.49	0.00
RE-1	7.66	16.62	0.46	11.86	16.62	0.71	0.25	0.04
SUE-1	4.94	10.36	0.48	6.36	10.36	0.61	0.14	0.24
R6-Lag	14.19	22.53	0.63	17.52	22.53	0.78	0.15	0.15
Season	9.54	13.13	0.73	11.98	13.13	0.91	0.19	0.05
W52	1.10	26.71	0.04	11.92	26.71	0.45	0.40	0.00
				Panel F: Profitability				
RMW	2.92	7.71	0.38	3.94	7.71	0.51	0.13	0.29
ROE	6.52	8.83	0.74	9.39	8.83	1.06	0.32	0.01
ATO	2.52	14.45	0.17	2.49	14.45	0.17	−0.00	0.98
CTO	3.11	13.67	0.23	4.40	13.67	0.32	0.09	0.30
F	3.87	11.50	0.34	4.89	11.50	0.42	0.09	0.44
FP	6.73	23.31	0.29	8.85	23.31	0.38	0.09	0.52
GP/A	4.61	12.82	0.36	5.01	12.82	0.39	0.03	0.74
O	2.16	14.73	0.15	3.00	14.73	0.20	0.06	0.56
PM	−0.00	18.57	−0.00	−1.33	18.57	−0.07	−0.07	0.51
RNA	0.31	16.45	0.02	1.68	16.45	0.10	0.08	0.43
ROA	7.80	18.80	0.41	8.25	18.80	0.44	0.02	0.86
ROE-HB	7.65	17.58	0.44	7.04	17.58	0.40	−0.03	0.79
RS	4.02	11.38	0.35	4.44	11.38	0.39	0.04	0.74
TES	3.49	12.96	0.27	4.56	12.96	0.35	0.08	0.51
TI/BI	1.63	10.88	0.15	1.42	10.88	0.13	−0.02	0.83
ΔATO	−1.46	11.78	−0.12	−0.22	11.78	−0.02	0.11	0.33
ΔPM	1.83	12.33	0.15	2.71	12.33	0.22	0.07	0.47
E-con	−0.50	9.82	−0.05	−0.21	9.82	−0.02	0.03	0.76
S/IV	3.55	10.39	0.34	1.37	10.39	0.13	−0.21	0.03
S/P	6.34	17.83	0.36	5.81	17.83	0.33	−0.03	0.79
S/SG&A	−0.86	11.52	−0.07	−0.49	11.52	−0.04	0.03	0.73
Z	0.10	16.78	0.01	−0.74	16.78	−0.04	−0.05	0.64

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Table IA1 (*continued*)

	Original portfolio			Volatility-managed portfolio			Difference	
Strategy (1)	Mean (2)	Std. dev. (3)	Sharpe ratio (4)	Mean (5)	Std. dev. (6)	Sharpe ratio (7)	ΔSR (8)	p-value (9)
Panel G: Trading								
SMB	2.57	11.12	0.23	0.86	11.12	0.08	−0.15	0.09
BAB	8.23	10.71	0.77	10.81	10.71	1.01	0.24	0.01
β-D	−0.51	23.68	−0.02	0.89	23.68	0.04	0.06	0.56
β-FP	−0.39	29.37	−0.01	2.65	29.37	0.09	0.10	0.30
1/P	0.60	21.84	0.03	−5.63	21.84	−0.26	−0.29	0.00
Disp	2.98	19.12	0.16	5.93	19.12	0.31	0.15	0.23
Dvol	1.33	13.50	0.10	1.91	13.50	0.14	0.04	0.65
Illiq	2.09	15.09	0.14	1.78	15.09	0.12	−0.02	0.83
Ivol	6.93	22.49	0.31	11.13	22.49	0.50	0.19	0.05
MDR	5.50	22.54	0.24	8.42	22.54	0.37	0.13	0.19
ME	2.98	16.37	0.18	1.34	16.37	0.08	−0.10	0.25
S-Rev	6.93	19.48	0.36	5.92	19.48	0.30	−0.05	0.59
Svol	6.29	17.32	0.36	3.12	17.32	0.18	−0.18	0.24
Turn	0.05	22.13	0.00	−0.27	22.13	−0.01	−0.01	0.87
Tvol	6.01	25.91	0.23	9.93	25.91	0.38	0.15	0.13
β-M	−2.11	28.88	−0.07	0.36	28.88	0.01	0.09	0.39
σ(Dvol)	3.55	13.55	0.26	3.79	13.55	0.28	0.02	0.86
B-A	−4.38	24.12	−0.18	−5.45	24.12	−0.23	−0.04	0.72
Short	2.08	15.22	0.14	2.50	15.22	0.16	0.03	0.79
Skew	2.01	13.78	0.15	0.79	13.78	0.06	−0.09	0.32
Vol-T	5.33	14.92	0.36	4.54	14.92	0.30	−0.05	0.58
Panel H: Value								
HML	4.84	12.14	0.40	4.64	12.14	0.38	−0.02	0.86
A/ME	4.88	16.72	0.29	4.72	16.72	0.28	−0.01	0.93
B/M	4.43	16.09	0.28	4.03	16.09	0.25	−0.02	0.81
CF/P	3.73	16.71	0.22	3.57	16.71	0.21	−0.01	0.93
D/P	2.09	18.41	0.11	0.11	18.41	0.01	−0.11	0.24
Dur	6.14	17.78	0.35	4.35	17.78	0.24	−0.10	0.35
E/P	5.10	16.75	0.30	2.87	16.75	0.17	−0.13	0.22
EF/P	9.15	22.11	0.41	10.48	22.11	0.47	0.06	0.66
LTG	0.78	30.58	0.03	−4.65	30.58	−0.15	−0.18	0.25
NO/P	4.44	13.65	0.33	3.26	13.65	0.24	−0.09	0.48
O/P	4.16	16.09	0.26	2.58	16.09	0.16	−0.10	0.41
Rev	3.77	18.31	0.21	−0.71	18.31	−0.04	−0.24	0.01
SG	0.96	15.78	0.06	2.41	15.78	0.15	0.09	0.43
An-V	8.47	21.97	0.39	11.32	21.97	0.52	0.13	0.35
σ(CF)	4.47	19.19	0.23	4.57	19.19	0.24	0.01	0.97
B/P-E	2.65	9.93	0.27	2.95	9.93	0.30	0.03	0.80
B/P-Lev	1.05	10.30	0.10	2.21	10.30	0.21	0.11	0.33
Enter	1.82	12.34	0.15	4.57	12.34	0.37	0.22	0.02
Pension	0.22	19.13	0.01	−0.22	19.13	−0.01	−0.02	0.86

Table IA2
Spanning regressions: broad sample

The table reports results from univariate spanning regressions of volatility-managed portfolio returns on the corresponding original portfolio returns and details properties of the ex post optimal combinations of volatility-managed and original portfolios. The spanning regressions are given by $f_{\sigma,t} = \alpha + \beta f_t + \varepsilon_t$, where $f_{\sigma,t}$ (f_t) is the monthly return for the volatility-managed (original) portfolio. The estimates of α are reported in percentage per year, and the t-statistics are based on White (1980) standard errors. For the ex post optimal combinations, the table presents the scaling parameter (c^*) for the volatility-managed portfolio, the ex post optimal total weight in risky assets ($x_\sigma^* + x^*$), and the ex post optimal relative weight in the volatility-managed portfolio (w_σ^*). The vector of portfolio weights is $[x_\sigma^* \quad x^*]^\top = (1/\gamma)\hat{\Sigma}^{-1}\hat{\mu}$, where γ is the risk aversion parameter, $\hat{\Sigma}$ is the 2×2 variance-covariance matrix of $f_{\sigma,t}$ and f_t, and $\hat{\mu}$ is the 2×1 vector of mean excess returns for $f_{\sigma,t}$ and f_t. The vector of relative weights is computed as $[w_\sigma^* \quad w^*]^\top = [x_\sigma^* \quad x^*]^\top / |x_\sigma^* + x^*|$. The table also shows the difference in annualized Sharpe ratio (ΔSR) and the difference in certainty equivalent return (ΔCER) for the following two strategies: (i) the ex post optimal combination of original portfolio, volatility-managed portfolio, and risk-free asset and (ii) the ex post optimal combination of original portfolio and risk-free asset. The results correspond to a risk aversion parameter of $\gamma = 5$. Panels of the table are organized by trading strategy type (i.e., accruals, intangibles, investment, market, momentum, profitability, trading, and value).

	Regression results				Ex post optimization parameters			Portfolio performance	
Strategy (1)	α (%) (2)	$t(\alpha)$ (3)	β (4)	$t(\beta)$ (5)	c^* (6)	$x_\sigma^* + x^*$ (7)	w_σ^* (8)	ΔSR (9)	ΔCER (%) (10)
Panel A: Accruals									
NOA	2.08	(2.09)	0.76	(20.23)	6.26	0.88	0.84	0.08	0.76
OA	0.03	(0.03)	0.75	(14.85)	7.62	0.57	0.02	0.00	0.00
POA	−0.58	(−0.49)	0.72	(13.00)	7.33	0.49	−0.31	0.01	0.04
TA	0.02	(0.02)	0.80	(17.17)	6.85	0.62	0.01	0.00	0.00
ΔNCO	1.35	(1.39)	0.78	(18.12)	6.15	0.85	0.63	0.04	0.36
ΔNWC	2.39	(2.13)	0.75	(9.54)	4.99	1.12	0.88	0.11	1.17
NoaG	0.60	(0.52)	0.69	(11.32)	6.03	0.60	0.28	0.01	0.05
IvC	0.36	(0.37)	0.74	(13.24)	5.66	1.09	0.14	0.00	0.03
IvG	2.34	(1.97)	0.74	(13.38)	6.43	0.87	0.78	0.08	0.80
PTA	−0.22	(−0.19)	0.64	(10.60)	5.66	0.77	−0.08	0.00	0.01
Panel B: Intangibles									
AccQ	−1.16	(−0.64)	0.70	(11.93)	10.14	−0.12	−1.15	0.04	0.08
AD/M	5.96	(2.22)	0.60	(8.22)	16.33	0.49	0.80	0.14	1.16
BC	0.82	(0.47)	0.69	(10.09)	11.73	0.06	2.58	0.07	0.06
H/N	−0.64	(−0.49)	0.73	(12.47)	8.58	0.23	−0.63	0.01	0.05
OC/A	0.96	(0.62)	0.70	(14.02)	8.55	0.46	0.31	0.01	0.07
OL	0.12	(0.10)	0.76	(14.49)	5.50	0.57	0.06	0.00	0.00
RC/A	0.09	(0.03)	0.68	(6.50)	25.66	0.14	0.04	0.00	0.00
RD/M	−0.41	(−0.23)	0.77	(12.47)	15.83	0.32	−0.20	0.00	0.01
RD/S	1.67	(0.49)	0.64	(7.14)	25.93	0.02	3.70	0.07	0.06
Age	−1.55	(−0.78)	0.59	(6.86)	8.70	−0.14	−1.10	0.06	0.12

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Table IA2 (*continued*)

	Regression results				Ex post optimization parameters			Portfolio performance	
Strategy (1)	α (%) (2)	$t(\alpha)$ (3)	β (4)	$t(\beta)$ (5)	c^* (6)	$x^*_\sigma + x^*$ (7)	w^*_σ (8)	ΔSR (9)	ΔCER (%) (10)
Panel C: Investment									
CMA	0.26	(0.39)	0.68	(13.82)	1.53	1.60	0.12	0.00	0.03
IA	1.18	(1.83)	0.70	(13.59)	1.64	2.70	0.41	0.04	0.65
$\Delta PI/A$	0.02	(0.02)	0.78	(20.50)	6.21	0.83	0.01	0.00	0.00
ACI	2.03	(2.24)	0.73	(13.60)	5.43	0.88	1.00	0.12	0.90
CEI	1.57	(1.89)	0.66	(10.40)	4.54	0.50	1.04	0.09	0.41
I/A	−0.35	(−0.28)	0.69	(13.18)	7.40	0.55	−0.15	0.00	0.01
IG	0.94	(0.88)	0.68	(11.58)	6.21	0.68	0.40	0.02	0.13
NSI	1.41	(1.48)	0.76	(17.52)	6.42	0.75	0.74	0.05	0.39
NXF	−0.23	(−0.13)	0.68	(11.23)	10.36	0.40	−0.09	0.00	0.00
BeG	−2.41	(−2.01)	0.76	(15.27)	7.85	0.29	−2.30	0.11	0.81
I-ADJ	−0.22	(−0.22)	0.75	(13.74)	5.85	0.52	−0.17	0.00	0.01
Panel D: Market									
MKT	4.63	(3.08)	0.63	(11.32)	10.33	0.61	0.72	0.11	1.03
Panel E: Momentum									
MOM	12.39	(7.31)	0.48	(7.13)	4.60	1.22	0.98	0.50	7.39
Abr-1	5.82	(4.73)	0.72	(9.82)	8.26	1.73	0.79	0.20	3.97
R11-1	15.63	(6.13)	0.56	(8.53)	16.05	0.72	0.94	0.37	5.33
R6-1	16.98	(7.34)	0.56	(10.12)	16.53	0.67	1.15	0.49	6.52
RE-1	6.47	(3.55)	0.70	(11.61)	12.18	0.83	1.12	0.26	3.01
SUE-1	2.74	(2.49)	0.73	(13.36)	5.15	1.22	0.91	0.14	1.51
R6-Lag	9.74	(3.87)	0.55	(6.82)	12.65	0.81	0.68	0.19	2.67
Season	5.43	(4.02)	0.69	(9.93)	5.99	1.48	0.81	0.20	3.24
W52	11.37	(4.62)	0.50	(7.51)	14.83	0.24	1.73	0.45	2.40

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Table IA2 (*continued*)

	Regression results				Ex post optimization parameters			Portfolio performance	
Strategy (1)	α (%) (2)	$t(\alpha)$ (3)	β (4)	$t(\beta)$ (5)	c^* (6)	$x^*_\sigma + x^*$ (7)	w^*_σ (8)	ΔSR (9)	ΔCER (%) (10)
Panel F: Profitability									
RMW	2.23	(2.57)	0.59	(7.10)	1.48	1.45	0.79	0.14	1.27
ROE	4.97	(5.10)	0.68	(11.12)	2.06	2.44	0.97	0.32	5.86
ATO	0.59	(0.45)	0.75	(15.38)	7.23	0.27	0.48	0.01	0.04
CTO	1.95	(1.72)	0.79	(18.51)	6.37	0.45	1.21	0.10	0.53
F	2.10	(1.79)	0.72	(12.28)	6.20	0.77	0.86	0.09	0.69
FP	4.88	(1.62)	0.59	(7.82)	13.67	0.36	0.77	0.10	0.67
GP/A	1.54	(1.38)	0.75	(15.82)	6.49	0.67	0.65	0.04	0.33
O	1.40	(1.03)	0.74	(14.87)	7.99	0.27	1.05	0.06	0.20
PM	−1.33	(−0.72)	0.69	(12.36)	9.25	−0.05	−3.20	0.10	0.10
RNA	1.46	(0.92)	0.71	(14.46)	9.17	0.09	2.50	0.11	0.16
ROA	3.54	(1.61)	0.60	(8.10)	11.05	0.57	0.56	0.06	0.56
ROE-HB	2.16	(1.10)	0.64	(8.86)	9.97	0.58	0.41	0.03	0.26
RS	1.41	(1.21)	0.75	(14.16)	5.60	0.74	0.67	0.05	0.35
TES	2.19	(1.45)	0.68	(10.28)	7.64	0.57	0.85	0.09	0.53
TI/BI	0.16	(0.17)	0.77	(16.04)	5.93	0.29	0.23	0.00	0.01
ΔATO	0.79	(0.68)	0.69	(11.62)	6.05	−0.14	1.51	0.03	0.09
ΔPM	1.35	(1.20)	0.74	(15.46)	6.94	0.34	1.16	0.07	0.27
E-con	0.19	(0.21)	0.78	(15.49)	6.21	−0.08	1.21	0.01	0.01
S/IV	−1.26	(−1.34)	0.74	(17.99)	5.53	0.52	−0.99	0.04	0.33
S/P	1.42	(0.84)	0.69	(11.90)	9.87	0.45	0.38	0.02	0.12
S/SG&A	0.17	(0.16)	0.76	(17.35)	7.46	−0.11	0.52	0.00	0.00
Z	−0.81	(−0.50)	0.70	(13.19)	9.41	−0.03	−4.23	0.06	0.05
Panel G: Trading									
SMB	−0.76	(−0.87)	0.63	(7.75)	2.63	0.34	−0.60	0.02	0.08
BAB	5.74	(5.97)	0.62	(12.97)	3.20	2.05	0.78	0.26	4.63
β*-D*	1.16	(0.55)	0.53	(10.08)	24.06	0.01	6.56	0.04	0.03
β*-FP*	2.88	(1.11)	0.58	(11.18)	31.93	0.03	3.03	0.11	0.14
1/P	−5.99	(−3.27)	0.60	(11.13)	9.50	−0.13	−2.98	0.32	1.18
Disp	3.93	(1.77)	0.67	(10.03)	10.60	0.29	1.35	0.16	0.77
Dvol	1.12	(0.97)	0.60	(9.88)	9.74	0.22	0.85	0.04	0.11
Illiq	0.59	(0.46)	0.57	(9.10)	8.64	0.22	0.36	0.01	0.02
Ivol	7.10	(3.79)	0.58	(11.20)	15.41	0.45	0.94	0.19	1.51
MDR	5.34	(2.68)	0.56	(10.35)	16.11	0.35	0.87	0.13	0.82
ME	−0.64	(−0.51)	0.67	(13.27)	6.59	0.19	−0.45	0.01	0.03
S-Rev	1.89	(1.13)	0.58	(12.30)	12.33	0.43	0.35	0.02	0.14
Svol	−0.82	(−0.33)	0.63	(7.81)	10.13	0.39	−0.23	0.01	0.04
Turn	−0.30	(−0.17)	0.63	(16.54)	21.02	−0.01	−3.72	0.02	0.00
Tvol	6.56	(2.90)	0.56	(12.13)	20.83	0.30	0.94	0.15	0.93
β*-M*	1.59	(0.63)	0.58	(10.44)	23.06	−0.03	2.19	0.03	0.05
σ*(Dvol)*	1.75	(1.42)	0.57	(9.36)	10.00	0.51	0.56	0.04	0.25
B-A	−2.93	(−1.10)	0.58	(9.26)	15.39	−0.21	−0.70	0.05	0.22
Short	0.90	(0.60)	0.76	(15.43)	11.91	0.22	0.84	0.03	0.08
Skew	−0.54	(−0.49)	0.66	(16.03)	7.05	0.18	−0.57	0.01	0.03
Vol-T	1.28	(1.02)	0.61	(13.24)	8.64	0.55	0.33	0.02	0.12

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Table IA2 (*continued*)

	Regression results				Ex post optimization parameters			Portfolio performance	
Strategy	α (%)	$t(\alpha)$	β	$t(\beta)$	c^*	$x^*_\sigma + x^*$	w^*_σ	ΔSR	ΔCER (%)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Panel H: Value									
HML	1.87	(1.88)	0.57	(7.65)	2.95	0.82	0.46	0.04	0.35
A/ME	1.40	(0.87)	0.68	(11.72)	7.58	0.41	0.46	0.02	0.13
B/M	0.84	(0.57)	0.72	(13.22)	8.22	0.38	0.36	0.01	0.06
CF/P	0.97	(0.60)	0.70	(13.37)	8.44	0.31	0.44	0.01	0.07
D/P	−1.21	(−0.81)	0.63	(14.20)	9.48	0.08	−1.50	0.03	0.07
Dur	0.13	(0.08)	0.69	(10.98)	9.64	0.39	0.04	0.00	0.00
E/P	−0.65	(−0.40)	0.69	(10.85)	9.48	0.34	−0.26	0.00	0.03
EF/P	4.81	(1.83)	0.62	(9.09)	14.05	0.50	0.64	0.08	0.77
LTG	−5.11	(−1.21)	0.58	(7.88)	26.97	−0.05	−3.16	0.18	0.42
NO/P	0.24	(0.16)	0.68	(11.77)	6.65	0.49	0.10	0.00	0.01
O/P	−0.33	(−0.19)	0.70	(11.04)	9.66	0.31	−0.16	0.00	0.01
Rev	−3.10	(−2.06)	0.64	(12.30)	9.56	0.11	−2.79	0.10	0.48
SG	1.78	(1.05)	0.66	(10.96)	9.37	0.16	1.55	0.10	0.22
An-V	6.11	(2.18)	0.62	(7.60)	14.56	0.51	0.80	0.14	1.24
σ(CF)	1.87	(0.77)	0.61	(7.86)	13.04	0.31	0.52	0.03	0.15
B/P-E	0.92	(0.80)	0.76	(13.15)	5.81	0.64	0.70	0.04	0.21
B/P-Lev	1.38	(1.23)	0.78	(14.54)	5.63	0.34	1.97	0.14	0.47
Enter	3.18	(2.91)	0.76	(13.37)	6.41	0.48	2.09	0.28	1.58
Pension	−0.38	(−0.17)	0.71	(13.33)	15.07	0.00	−475.76	0.02	0.01

Table IA3
Reverse spanning regressions

Panel A reports results from univariate spanning regressions of original factor returns on the corresponding volatility-managed factor returns. The spanning regressions are given by $f_t = \alpha^r + \beta^r f_{\sigma,t} + \varepsilon_t$, where f_t ($f_{\sigma,t}$) is the monthly return for the original (volatility-managed) factor. The estimates of α^r are reported in percentage per year, and the numbers in parentheses are t-statistics based on White (1980) standard errors. For each regression, R^2 is the adjusted R^2 value, and the appraisal ratio is computed as the ratio of alpha to root mean square error. Panel B adds the Fama-French (1993) three factors as controls in the spanning regressions.

	Factor								
	MKT (1)	SMB (2)	HML (3)	MOM (4)	RMW (5)	CMA (6)	ROE (7)	IA (8)	BAB (9)
Panel A: Univariate regressions									
Alpha, α^r (%)	1.79	2.03	2.19	0.24	0.61	1.82	0.17	1.68	1.58
	(1.15)	(2.25)	(2.12)	(0.13)	(0.70)	(2.67)	(0.16)	(2.54)	(1.58)
Beta, β^r	0.63	0.63	0.57	0.48	0.59	0.68	0.68	0.70	0.62
	(19.48)	(15.81)	(16.08)	(11.69)	(15.52)	(15.17)	(16.26)	(16.85)	(19.13)
R^2	0.40	0.40	0.33	0.23	0.34	0.46	0.46	0.50	0.38
Appraisal ratio, AR	0.12	0.24	0.22	0.02	0.10	0.36	0.03	0.37	0.19
Panel B: Additional controls for Fama-French (1993) three factors									
Alpha, α^r (%)	−0.00	0.90	0.85	4.57	0.99	1.67	1.93	2.02	2.02
	(−0.00)	(1.14)	(0.94)	(3.18)	(1.26)	(3.03)	(2.03)	(3.86)	(2.17)
R^2	0.48	0.44	0.39	0.42	0.46	0.67	0.53	0.69	0.39
Appraisal ratio, AR	−0.00	0.11	0.09	0.37	0.17	0.42	0.32	0.56	0.24

Table IA4
Summary of reverse spanning regressions: broad sample

The table summarizes results from reverse spanning regressions for 103 trading strategies. The reverse spanning regressions are given by $f_t = \alpha^r + \beta^r f_{\sigma,t} + \varepsilon_t$, where f_t ($f_{\sigma,t}$) is the monthly return for the original (volatility-managed) anomaly portfolio. The results in columns (3) and (4) correspond to univariate spanning regressions, and those in columns (5) and (6) are for regressions that add the Fama-French (1993) three factors as controls. Panel A reports results for the full set of 103 trading strategies. Panel B presents separate results for the 9 factors and the 94 anomaly portfolios. Panel C breaks the results down by trading strategy type. For each set of regressions, the table reports the number of alphas that are positive, positive and significant at the 5% level, negative, and negative and significant at the 5% level. We assess statistical significance of the alpha estimates using White (1980) standard errors.

		Univariate regressions		Additional controls for Fama-French (1993) factors	
Sample (1)	Total (2)	$\alpha^r > 0$ [Signif.] (3)	$\alpha^r < 0$ [Signif.] (4)	$\alpha^r > 0$ [Signif.] (5)	$\alpha^r < 0$ [Signif.] (6)
Panel A: Combined sample					
All trading strategies	103	82 [16]	21 [0]	81 [35]	22 [4]
Panel B: By category					
Factors	9	9 [4]	0 [0]	8 [5]	1 [0]
Anomaly portfolios	94	73 [12]	21 [0]	73 [30]	21 [4]
Panel C: By trading strategy type					
Accruals	10	10 [3]	0 [0]	10 [4]	0 [0]
Intangibles	10	8 [0]	2 [0]	5 [0]	5 [0]
Investment	11	10 [5]	1 [0]	10 [6]	1 [0]
Market	1	1 [0]	0 [0]	0 [0]	1 [0]
Momentum	9	6 [1]	3 [0]	9 [4]	0 [0]
Profitability	22	15 [1]	7 [0]	18 [6]	4 [0]
Trading	21	16 [4]	5 [0]	17 [13]	4 [2]
Value	19	16 [2]	3 [0]	12 [2]	7 [2]

Table IA5
Multiple structural break tests for spanning regressions and anomaly regressions

The table reports results from structural break tests in univariate spanning regressions of volatility-managed factor returns on the corresponding original factor returns (Panel A), spanning regressions with additional controls for the Fama-French (1993) three factors (Panel B), CAPM regressions (Panel C), and Fama-French (1993) three-factor model regressions (Panel D). The tests follow Bai and Perron (1998, 2003) and allow for an unknown number of structural breaks. Column (2) reports the $UDmax$ statistic and *** indicates significance at the 1% level in a test of the null hypothesis of zero breaks. Column (3) reports the $WDmax(10\%)$ statistic, which specifies the 10% significance level, and * indicates significance at the 10% level. Columns (5)–(7) show the estimated structural break dates. The estimates of α for each subperiod in columns (8)–(11) are reported in percentage per year, and the numbers in parentheses are t-statistics based on Andrews (1991) standard errors.

	Bai and Perron (1998) statistics			Bai and Perron (1998) break dates			Subperiod alphas (%)			
Factor (1)	$UDmax$ (2)	$WDmax(10\%)$ (3)	Sample start (4)	1st break (5)	2nd break (6)	3rd break (7)	$\alpha(1)$ (8)	$\alpha(2)$ (9)	$\alpha(3)$ (10)	$\alpha(4)$ (11)
Panel A: Spanning regressions										
MKT	82.73***	154.00*	1926:08	1940:02	1969:06	1997:03	6.51	−1.29	2.03	2.44
							(2.23)	(−0.46)	(1.09)	(1.74)
SMB	62.94***	117.16*	1926:08	1940:07	1973:03	1997:03	−0.89	−3.09	0.72	−0.17
							(−0.66)	(−2.00)	(0.79)	(−0.21)
HML	42.79***	79.65*	1926:08	1940:12	1997:02	—	−1.15	−0.46	1.35	—
							(−0.74)	(−0.39)	(0.70)	—
MOM	42.03***	78.23*	1927:01	1940:12	1998:07	—	5.08	10.07	4.26	—
							(2.34)	(5.14)	(2.74)	—
RMW	148.90***	277.17*	1963:08	1998:08	2006:08	—	2.05	−0.64	−1.12	—
							(2.55)	(−0.66)	(−1.18)	—
CMA	34.47***	64.17*	1963:08	1971:07	1996:07	2004:07	−1.78	0.57	0.59	−0.61
							(−1.17)	(0.57)	(0.70)	(−0.70)
ROE	49.80***	92.70*	1967:02	1983:03	1998:09	—	2.03	1.66	3.51	—
							(1.77)	(0.96)	(2.83)	—
IA	30.27***	56.34*	1967:02	1975:06	1996:07	2004:05	−0.21	1.12	1.54	−0.22
							(−0.20)	(1.21)	(1.41)	(−0.21)
BAB	48.98***	91.18*	1931:02	1943:12	1997:09	—	−1.38	2.44	8.41	—
							(−0.64)	(2.49)	(4.87)	—

(*continued on next page*)

Table IA5 (*continued*)

	Bai and Perron (1998) statistics			Bai and Perron (1998) break dates			Subperiod alphas (%)			
Factor (1)	$UDmax$ (2)	$WDmax(10\%)$ (3)	Sample start (4)	1st break (5)	2nd break (6)	3rd break (7)	$\alpha(1)$ (8)	$\alpha(2)$ (9)	$\alpha(3)$ (10)	$\alpha(4)$ (11)
			Panel B: Spanning regressions with Fama-French factors							
MKT	93.58***	159.43*	1926:08	1940:02	1969:06	1997:03	6.64	−1.67	1.33	2.25
							(2.23)	(−0.62)	(0.69)	(1.60)
SMB	75.03***	127.83*	1926:08	1940:07	1973:04	1997:03	−0.88	−2.65	1.28	−0.54
							(−0.65)	(−1.68)	(1.36)	(−0.69)
HML	49.93***	85.07*	1926:08	1944:05	1997:02	—	0.60	−0.48	1.02	—
							(0.42)	(−0.41)	(0.48)	—
MOM	45.33***	74.74*	1927:01	1940:12	1998:10	—	4.32	10.72	4.07	—
							(1.85)	(5.39)	(2.71)	—
RMW	191.91***	316.39*	1963:08	1998:08	2006:08	—	1.97	−0.28	−1.73	—
							(2.33)	(−0.27)	(−1.54)	—
CMA	36.96***	60.93*	1963:08	1971:07	1996:07	2004:07	−2.14	−0.06	0.04	−0.90
							(−1.39)	(−0.06)	(0.06)	(−0.99)
ROE	107.18***	176.70*	1967:02	1999:03	—	—	2.25	3.94	—	—
							(2.38)	(2.79)	—	—
IA	37.25***	61.42*	1967:02	1975:06	1995:07	2003:01	−0.55	0.83	0.80	−0.02
							(−0.55)	(0.81)	(0.76)	(−0.02)
BAB	52.82***	87.08*	1931:02	1943:12	1997:09	—	−0.50	3.05	7.95	—
							(−0.26)	(2.99)	(4.74)	—

(*continued on next page*)

Table IA5 (*continued*)

	Bai and Perron (1998) statistics			Bai and Perron (1998) break dates			Subperiod alphas (%)			
Factor (1)	$UDmax$ (2)	$WDmax(10\%)$ (3)	Sample start (4)	1st break (5)	2nd break (6)	3rd break (7)	$\alpha(1)$ (8)	$\alpha(2)$ (9)	$\alpha(3)$ (10)	$\alpha(4)$ (11)
Panel C: CAPM regressions										
SMB	12.38**	23.05*	—	—	—	—	—	—	—	—
							—	—	—	—
HML	34.49***	64.21*	1926:08	1955:09	1980:05	2002:12	1.80	5.26	7.83	−0.12
							(0.77)	(3.52)	(3.79)	(−0.05)
MOM	14.83**	27.61*	1927:01	1942:11	2000:12	—	7.54	10.29	4.08	—
							(1.52)	(7.00)	(1.05)	—
RMW	40.92***	76.17*	1963:08	2000:03	2008:03	—	1.38	8.52	4.80	—
							(1.26)	(3.11)	(2.64)	—
CMA	36.53***	67.99*	1963:08	1994:03	2002:06	—	4.88	10.45	0.97	—
							(4.46)	(4.21)	(0.73)	—
ROE	40.99***	76.29*	1967:02	2000:03	2007:12	—	7.50	7.13	3.56	—
							(5.96)	(2.32)	(1.50)	—
IA	44.76***	83.33*	1967:02	1994:11	2002:07	—	7.01	10.32	0.81	—
							(7.10)	(3.88)	(0.61)	—
BAB	13.94**	25.96*	1931:02	1970:10	1990:10	2003:09	6.17	10.55	16.86	8.05
							(4.33)	(5.41)	(4.29)	(2.79)
Panel D: Fama-French regressions										
MOM	22.61**	38.52*	1927:01	2000:11	—	—	13.65	4.52	—	—
							(8.91)	(1.21)	—	—
RMW	153.14***	260.90*	1963:08	1982:09	1996:05	2005:02	3.04	5.85	6.33	4.05
							(2.95)	(4.69)	(2.13)	(2.92)
CMA	36.12***	61.54*	1963:08	1972:01	1990:05	2002:07	0.82	2.63	4.06	0.90
							(0.58)	(2.77)	(2.83)	(0.72)
ROE	85.58***	145.64*	1967:02	1975:06	1998:12	—	9.43	11.48	5.81	—
							(4.97)	(9.38)	(3.18)	—
IA	29.42***	50.13*	1967:02	1974:09	1994:11	2002:07	3.83	4.61	6.42	0.72
							(2.31)	(5.24)	(3.35)	(0.59)
BAB	94.61**	161.19*	1931:02	1955:11	1979:07	1998:08	5.03	7.50	11.31	7.49
							(3.24)	(5.01)	(5.67)	(2.47)