Internet Appendices for "Asset Pricing with Index Investing"

Internet Appendix A. Numerical methods

As follows from Proposition 1, all equilibrium processes in our model can be expressed in terms of the price-dividend ratio f and the wealth-consumption ratio h, which satisfy the system of quasilinear differential equations (A5) and (A6). Those equations do not admit an analytical solution, so we solve them numerically. We use the standard finite-difference approach, which prescribes to approximate an infinite-horizon economy by an economy with a large finite horizon T, discretize the time interval $[0, T]$ and domains of state variables, and solve the discretized equations backward as a sequence of systems of linear algebraic equations (e.g., Lapidus and Pinder, 1999).

Specifically, we introduce a vector of functions $F = [f \;\; h]'$, denote the first and second partial derivatives of F with respect to the state variables s and u as F_s, F_u, F_{ss}, F_{uu}, and F_{us}, and write the system of equations (A5) and (A6) adjusted for a finite horizon economy as

$$\begin{aligned} A_{ss}(F, F_s, F_u, s, u)F_{ss} + A_{uu}(F, F_s, F_u, s, u)F_{uu} + A_{us}(F, F_s, F_u, s, u)F_{us} + A_s(F, F_s, F_u, s, u)F_s \\ + A_u(F, F_s, F_u, s, u)F_u + A(F, F_s, F_u, s, u)F + 1 + \frac{\partial F}{\partial t} = 0, \quad \text{(IA1)} \end{aligned}$$

where A_{ss}, A_{uu}, A_{us}, A_s, A_u, and A are diagonal 2×2 matrices with the elements that correspond to the coefficients of differential equations (A5) and (A6). Note that Eq. (IA1) includes the time derivative $\partial F/\partial t$, which appears as an additional term in Itô's lemma applied to the time-dependent price-dividend ratio and indirect utility function in the derivation of Eqs. (A5) and (A6) presented in the Appendix.

Next, we set $T = 500$ and using a backward recursion solve Eq. (IA1) at the discrete moments $t = T,\ T - \Delta t, \ldots, \Delta t,\ 0$ and in the discrete states $s = 0,\ \Delta s,\ 2\Delta s, \ldots, 1$, and $u = 0,\ \Delta u,\ 2\Delta u, \ldots, 1$, where $\Delta t = 0.1$, $\Delta s = 0.01$, and $\Delta u = 0.01$. In particular, the time

t solution $F_{(t)}$ is found by solving discretized equation (IA1) in which all derivatives of $F_{(t)}$ are replaced with their finite-difference approximations, and the equation coefficients are computed using the solution $F_{(t+\Delta t)}$ at time $t+\Delta t$ obtained in the previous step. Thus, the coefficients of the discretized equation do not depend on the time t solution, and $F_{(t)}$ solves a system of linear algebraic equations. Because the time horizon T is large, the sequence $F_{(t)}$, $t = T,\ T - \Delta t, \dots, \Delta t,\ 0$, converges to a time-independent solution F, which describes an equilibrium in the infinite-horizon economy. We verify the convergence by observing that the discrete approximation of the derivative $\partial F/\partial t$ has the order of magnitude 10^{-7} at $t = 0$.

The iteration procedure starts from the terminal solution $F_{(T)} = [\Delta t \quad \Delta t]'$. Indeed, the index price and the type I investors' wealth at the terminal date are equal to $S_T = D_T \Delta t$ and $W_{IT} = C_{IT}\Delta t$, respectively, so the price-dividend ratio and wealth-consumption ratio at time T are $f_{(T)} = \Delta t$ and $h_{(T)} = \Delta t$. The spacial boundary conditions for the discretized version of Eq. (IA1) are obtained by taking the limits $s \to 0$, $u \to 0$, $s \to 1$, and $u \to 1$ in Eq. (IA1). The computation of the boundary conditions is incorporated directly into the numerical algorithm. Appendix B in Chabakauri (2013) provides further details.

Having solved Eq. (IA1) and obtained f and h, we find r, μ_s, Σ_s, and Σ_I as functions of the state variables using Eqs. (A1) – (A4). Also, we compute η and μ_I from Eq. (A7). To find the price-dividend ratios f_i, we solve differential equations (A8). Note that those equations are linear because their coefficients are known functions of the state variables. Therefore, they are solved using the finite-difference approximation that no longer requires a backward recursion. The remaining equilibrium variables are obtained from Eq. (A9).

To find the equilibrium in the unconstrained benchmark economy, we also use the finite-difference approximation. However, in this case the differential equations for the price-dividend ratios and wealth-consumption ratio are linear and decoupled, so each of them is solved individually without a backward recursion. Those computations closely follow Chabakauri (2013).

Internet Appendix B. Benchmark economies

In this Internet Appendix, we present the equilibrium characteristics of the unconstrained economy and pre-indexing economy, which are the benchmarks in the analysis conducted in the main part of the paper.

Unconstrained economy

Consider first an unconstrained economy in which all investors can trade all assets individually. The equilibrium in this economy is characterized by Proposition 2 in Chabakauri (2013), and the corresponding Σ_s, μ_s, and r are given by Eqs. (15), (16), and (17). For the parameters from Section 3.1, the equilibrium characteristics are presented in Fig. IB.1. Because the parameters of the stock dividend processes are identical, we plot the characteristics of the first stock only.

Fig. IB.1 demonstrates that the volatilities of returns on the market and individual stocks tend to be higher than the volatilities of the corresponding dividends, and this is a consequence of dynamic risk sharing among investors with different risk preferences (e.g., Bhamra and Uppal, 2009; Longstaff and Wang, 2012). Fig. IB.1 also shows that the volatility is more amplified for the larger stock (the first stock when $u > 1/2$). The stock returns are positively correlated even though the correlation between dividends is zero because the prices of both stocks are affected by time varying aggregate risk aversion (e.g., Cochrane et al., 2008; Ehling and Heyerdahl-Larsen, 2016). Also, dividend shocks of the larger stock have a higher price of risk. This is not surprising because the larger stock is a better proxy for the whole market, and the risk associated with it has a stronger effect on the investors' consumption.

Fig. IB.1 also demonstrates that the risk-free rate r and the market price of risk η_1 are increasing functions of the type I investors' consumption share s. As discussed in Wang (1996), the risk-free rate is determined by both the investors' equilibrium expected consumption growth and consumption volatility. On the one hand, the investors who are more risk averse have a lower elasticity of intertemporal substitution (EIS) and prefer to borrow more to smooth their consumption over time. This drives the equilibrium risk-free rate up. On the other hand, the higher risk aversion of the investors makes them less tolerant to consumption volatility and increases their

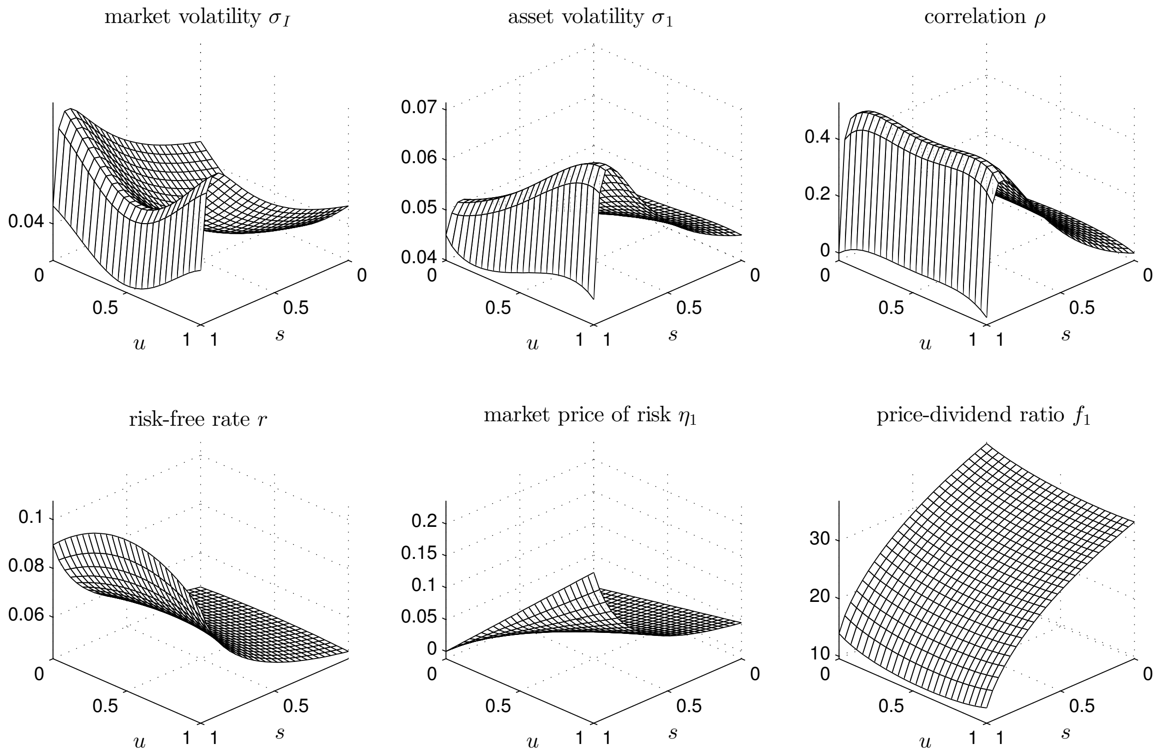

Figure IB.1: This figure presents the equilibrium characteristics of the unconstrained economy as functions of the consumption share s of the type I investors and the share u of the first dividend D_1 in the aggregate dividend D. The model parameters are as follows: $\mu_{D1} = \mu_{D2} = 0.018$, $\Sigma_{D1} = [0.045 \;\; 0]$, $\Sigma_{D2} = [0 \;\; 0.045]$, $\beta = 0.03$, $\gamma_I = 5$, and $\gamma_P = 1$.

precautionary savings. This drives the equilibrium risk-free rate down. In our case, the first effect is stronger than the second one.[1] As a result, the risk-free rate is higher when more risk averse type I investors dominate the market (when s is close to 1) and lower when the type P investors dominate (when s is close to 0).

The increasing relation between η_1 and s is also intuitive: the more risk averse type I investors, whose impact increases with s, require a higher compensation for holding risk than the type P investors. Also, η_1 is an increasing function of the size of the first stock u because investors require a higher compensation for holding the risk associated with a shock to a larger stock. Finally, because both the risk-free rate and market prices of risk are increasing functions of s, future dividends are more heavily discounted when s is high, and the price-dividend ratio f_1 decreases with s.

[1] As demonstrated by Wang (1996), the relation between the risk aversion and risk-free rate can be nonmonotonic.

Pre-indexing economy

As another benchmark, we consider the pre-indexing economy, in which the type I investors cannot trade at all, and the prices are determined by the optimal behavior of the type P investors. More details on this economy are provided in Section 3.3. Because the type P investors have logarithmic preferences, this benchmark economy has exactly the same properties as the economy in Cochrane et al. (2008). We find the equilibrium variables using the analytical formulas from Martin (2013).

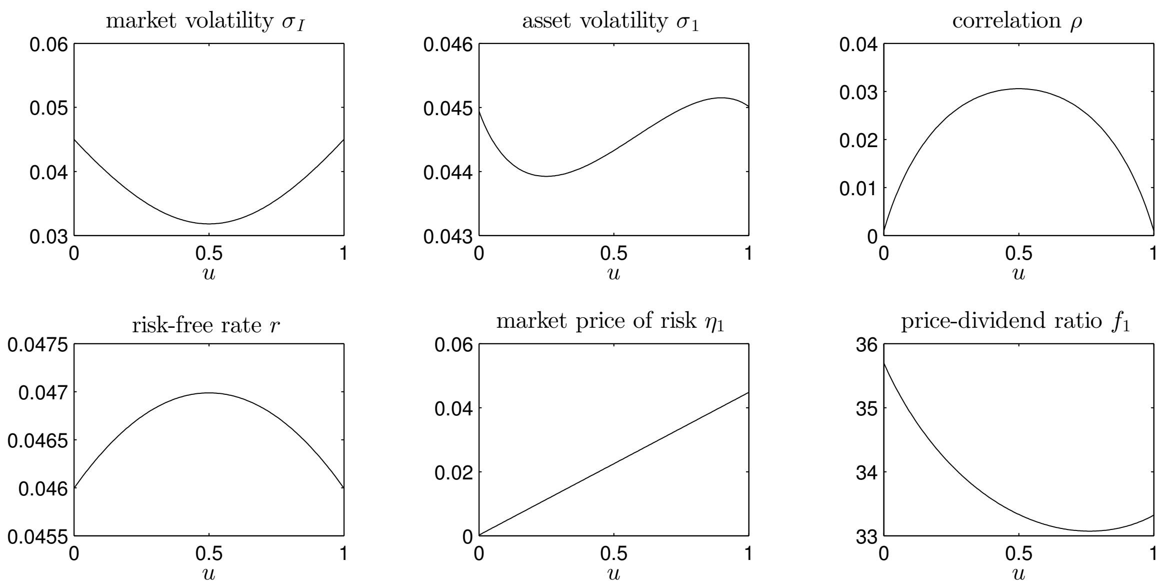

Figure IB.2: This figure presents the equilibrium characteristics of the pre-indexing economy as functions of the share u of the first dividend D_1 in the aggregate dividend D. The model parameters are as follows: $\mu_{D1} = \mu_{D2} = 0.018$, $\Sigma_{D1} = [0.045 \;\; 0]$, and $\Sigma_{D2} = [0 \;\; 0.045]$, $\beta = 0.03$, and $\gamma_P = 1$.

The main equilibrium characteristics for the parameters from Section 3.1 are presented in Fig. IB.2. As before, we plot only the characteristics of the first stock because those of the second stock can be obtained by flipping the graphs around $u = 1/2$. For a better comparison with the economy with indexing, the price-dividend ratio in Fig. IB.2 uses the tradable fraction of the dividend as the denominator.

The graphs highlight several properties of the equilibrium. As in Cochrane et al. (2008), the volatilities of returns differ from the dividend volatilities, the returns are correlated even though the dividend shocks of the stocks are independent, and the market price of risk rises with the size of the stock to compensate investors for a higher risk associated with a larger stock.

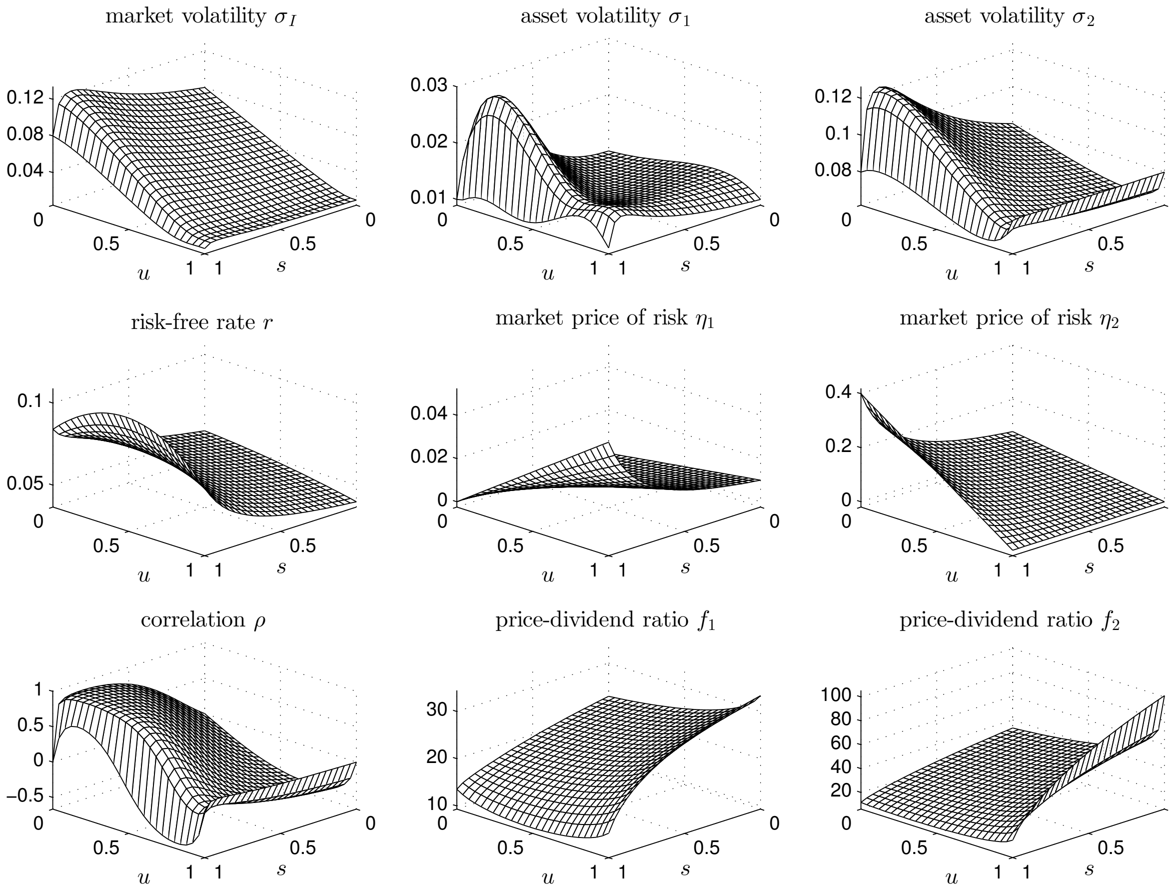

Figure IC.1: This figure presents the equilibrium variables in the unconstrained economy as functions of the consumption share s of the type I investors and the share u of the first dividend D_1 in the aggregate dividend D. The model parameters are as follows: $\mu_{D1} = 0.01$, $\mu_{D2} = 0.03$, $\Sigma_{D1} = [0.01\ \ 0]$, $\Sigma_{D2} = [0\ \ 0.08]$, $\beta = 0.03$, $\gamma_I = 5$, and $\gamma_P = 1$.

Internet Appendix C. Alternative parameters of the model

In this Internet Appendix, we explore the robustness of our results to various changes in the model parameters. In particular, we consider stocks that have different dividend processes and investors with alternative coefficients of risk aversion.

Heterogeneous trees

In our main analysis, we assume that the dividend growth rates and diffusions of the dividend processes are the same for both stocks, so all heterogeneity across the stocks is produced by the endogenous difference in the stock sizes. As a result, the portfolio distortions caused by indexing are relatively small, and all effects are relatively weak. However, in reality there is a substantial heterogeneity in the stock dividend processes, which may amplify the impact of indexing. To entertain this possibility, we consider a specification with heterogeneous dividend processes in which the expected dividend growth rates and diffusions of dividends are different and calibrated as $\mu_{D1} = 0.01$, $\mu_{D2} = 0.03$, $\Sigma_{D1} = [0.01 \quad 0]$, and $\Sigma_{D2} = [0 \quad 0.08]$. Thus, the first tree can be interpreted as a mature firm with a relatively low expected dividend growth rate but stable cash flows, and the second tree can be viewed as a young firm with a high dividend growth rate but relatively volatile cash flows. All other parameters are the same as in our main specification described in Section 3.1.

Fig. IC.1 presents the equilibrium in the unconstrained economy with heterogeneous trees. Because the stocks' dividend processes no longer have identical parameters, we plot the characteristics for both stocks. The graphs confirm many observations from our main analysis and reveal new patterns. In particular, the volatilities of both individual stocks and the market tend to be higher when the economy is dominated by the more volatile second stock. This happens because both the fundamental volatility and the volatility produced by risk sharing are higher.

Fig. IC.1 also shows that the correlation between stock returns is high and positive in some states but negative in the others. To explain the sign of the correlation, we follow Cochrane et al. (2008) and decompose the covariance between stock returns as

$$\operatorname{cov}(dQ_1, dQ_2) = \operatorname{cov}\left(\frac{dD_1}{D_1}, \frac{dD_2}{D_2}\right) + \operatorname{cov}\left(\frac{df_1}{f_1}, \frac{df_2}{f_2}\right) + \operatorname{cov}\left(\frac{dD_1}{D_1}, \frac{df_2}{f_2}\right) + \operatorname{cov}\left(\frac{dD_2}{D_2}, \frac{df_1}{f_1}\right). \quad \text{(IC1)}$$

The first term in Eq. (IC1) is equal to zero because the dividends are uncorrelated. The second term is small, so, as in Cochrane et al. (2008), the covariance between stock returns is mainly determined by the last two terms. The third term is positive because f_2 is an increasing function of u, and the shock dD_1 is positively correlated with innovations in $u = D_1/(D_1 + D_2)$. The fourth term is negative when $u \in (0.5, 1)$ because in that interval f_1 is an increasing function of

u, and innovations in u are negatively correlated with the shock dD_2. Moreover, the fourth term is large in absolute terms because the dividend D_2 is more volatile than the dividend D_1. As a result, the correlation of stock returns is positive when $u \in (0, 0.5)$, but can become negative when $u \in (0.5, 1)$.

It remains to explain why the price-dividend ratio f_1 increases with u when $u \in (0.5, 1)$. Fig. IC.1 shows that in this region the risk-free rate r is a decreasing function of u. As a result, the dividends are discounted at a lower rate in the vicinity of $u = 1$ (where the first stock dominates the economy) than in the vicinity of $u = 1/2$, and the price-dividend ratios of both stocks are higher. Thus, the ratios f_i are increasing functions of u, and this fact gives rise to the negative correlation between stock returns. Note that the described effect crucially relies on the heterogeneity of both drifts and diffusions of the dividend processes: when either drifts or diffusions are the same for both stocks, the correlation is positive in all states of the economy.

Fig. IC.2 compares the equilibrium characteristics in the economy with indexing and in the unconstrained economy in the case of heterogeneous trees. It demonstrates that many effects of indexing are qualitatively similar to those in the economy from the main part of the paper and illustrated in Figs. 1 and 2. In particular, we find that indexing reduces the volatility of market returns σ_I and the risk-free rate r. It also again has an ambiguous effect on the correlation between stock returns. As before, the presence of index investors tends to increase (decrease) the volatility of the larger (smaller) stock. Finally, all effects tend to be strong when the risk-averse type I investors consume a substantial fraction of the total dividend. Thus, our main conclusions are robust to the heterogeneity in the stock dividend processes.

Nevertheless, the heterogeneity in the dividend processes quantitatively changes the impact of indexing on the equilibrium characteristics. The graphs in Fig. IC.2 show that it makes many effects much stronger than they are in our main model. For example, the correlation between stock returns in the economy with indexing can be lower by almost 0.15 than in the unconstrained economy, whereas this difference does not exceed 0.01 when the parameters of the dividend processes are identical. Similarly, the differences in the volatilities and risk-free rates reported in Fig. IC.2 differ by an order of magnitude from their counterparts in Figs. 1 and 2.

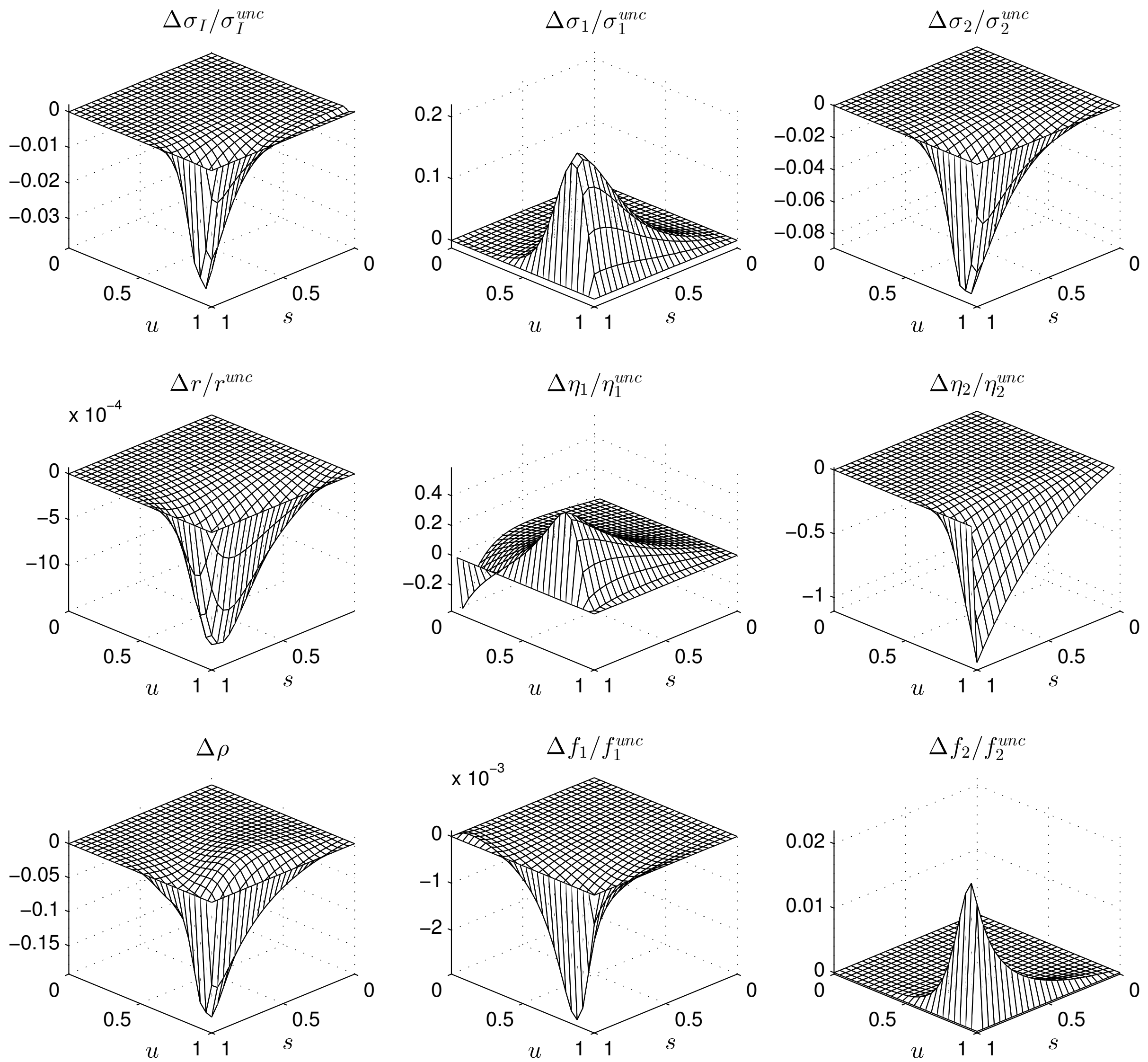

Figure IC.2: This figure shows how the equilibrium in the unconstrained economy with heterogeneous trees changes due to indexing. All variables are functions of the consumption share s of the type I investors and the share u of the first dividend D_1 in the aggregate dividend D. The model parameters are as follows: $\mu_{D1} = 0.01$, $\mu_{D2} = 0.03$, $\Sigma_{D1} = [0.01 \;\; 0]$, $\Sigma_{D2} = [0 \;\; 0.08]$, $\beta = 0.03$, $\gamma_I = 5$, and $\gamma_P = 1$.

Alternative coefficients of risk aversion

Next, we investigate the sensitivity of our conclusions to the assumption $\gamma_P = 1$ and $\gamma_I = 5$, which implies that the index investors are more risk averse than the unconstrained investors. We consider the equilibrium in exactly the same model as in the main part of the paper but set $\gamma_P = 5$ and $\gamma_I = 1$. Because now the type I investors have logarithmic preferences, their wealth-consumption ratio is constant: $h = 1/\beta$. As a result, the system of equations that describes the equilibrium simplifies, and instead of two differential equations (for the wealth-consumption ratio h and index price-dividend ratio f) it contains only one of them (for the index price-dividend ratio f). Nevertheless, the equation does not have an analytical solution, and, as in Section 3, we solve it numerically.

Because in the unconstrained case the reassignment of the coefficients of risk aversion is equivalent to relabeling the agents, the graphs of all equilibrium variables in the unconstrained economy can be obtained from the graphs in Fig. IB.1 by flipping them around the plane $s = 1/2$. Therefore, we present only the graphs for the changes in the equilibrium variables produced by indexing.

The comparison of Fig. IC.3 with Figs. 1, 2, and 3 shows that our conclusions about the impact of indexing on the volatilities of returns, risk-free rate, and correlation between returns are insensitive to whether the more risk-averse or less risk-averse investors are assumed to be constrained. We again observe that indexing decreases the volatility of market returns and the risk-free rate, but has an ambiguous effect on the individual volatilities and correlation. In particular, indexing increases (decreases) the volatility of the larger (smaller) stock. However, now all effects are more pronounced when s is close to 0, not to 1. To understand this pattern, recall that one of the channels through which indexing affects the equilibrium characteristics is the reduction in risk sharing. Because the general equilibrium effects of risk sharing among investors are stronger when more risk averse investors dominate the economy (Fig. IB.1 clearly illustrates this fact), the impact of indexing is also likely to be stronger when a representative investor is more risk averse. This is exactly what happens around $s = 0$, where the more risk averse type P investors dominate the economy.

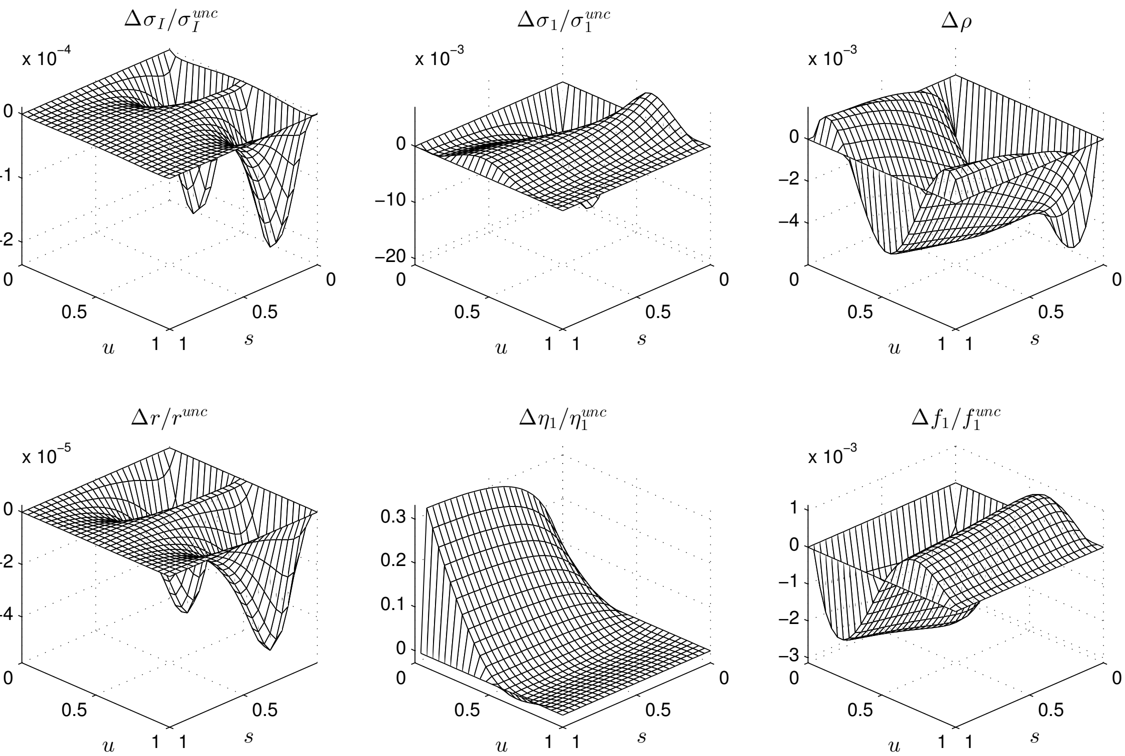

Figure IC.3: This figure shows how the unconstrained equilibrium changes due to indexing when the type I investors are less risk averse than the type P investors. All variables are functions of the consumption share s of the type I investors and the share u of the first dividend D_1 in the aggregate dividend D. The model parameters are as follows: $\mu_{D1} = \mu_{D2} = 0.018$, $\Sigma_{D1} = [0.045\ \ 0]$, $\Sigma_{D2} = [0\ \ 0.045]$, $\beta = 0.03$, $\gamma_I = 1$, and $\gamma_P = 5$.

Internet Appendix D. Alternative preferences

In this Internet Appendix, we consider the same model as in the main part of the paper but assume that all investors have recursive preferences in the form of Duffie and Epstein (1992). In particular, each investor solves the following optimization problem:

$$J_t = \max_{\{C_t, \omega_t \in \Omega\}} \mathbb{E}_t \left[\int_t^\infty f(C_\tau, J_\tau) d\tau \right], \tag{ID1}$$

where

$$f(C,V) = \begin{cases} \frac{\beta(1-\gamma)V}{1-1/\psi} \left[\left(\frac{C}{((1-\gamma)V)^{\frac{1}{1-\gamma}}} \right)^{1-1/\psi} - 1 \right], & \psi \neq 1, \\ \beta(1-\gamma)V \left[\log(C) - \frac{\log((1-\gamma)V)}{1-\gamma} \right], & \psi = 1, \end{cases} \tag{ID2}$$

subject to the budget constraint

$$dW_t = (r_t W_t - C_t)dt + W_t \omega_t' (\mu_{Qt} dt + \Sigma_{Qt} dB_t). \tag{ID3}$$

The risk aversion parameters γ and elasticities of intertemporal substitution ψ are $\gamma = \gamma_P$, $\psi = \psi_P$ for the type P investors and $\gamma = \gamma_I$, $\psi = \psi_I$ for the type I investors. As in the main part of the paper, the type P investors are unconstrained ($\Omega_P = \mathbb{R}^2$), but the type I investors can invest only in the risk-free bond and index (Ω_I consists of index portfolios). The following proposition, which is an analog of Proposition 1, describes the equilibrium in the model.

Proposition ID1. *The equilibrium in the model is characterized by the functions r, η, μ_s, Σ_s, Σ_I, h, h_P, and f that solve a system of algebraic and differential equations. The functions h and h_P satisfy the following partial differential equations:*

$$\begin{aligned}
&\frac{1}{2} h_{Pss} \Sigma_s \Sigma_s' + \frac{1}{2} h_{Puu} \Sigma_u \Sigma_u' + h_{Psu} \Sigma_s \Sigma_u' \\
&+ h_{Ps} \left(\mu_s + \frac{1}{2} \Sigma_s \left(\frac{1 - 2\gamma_P}{\gamma_P} \eta + \Sigma_D - \frac{\Sigma_s}{1-s} \right)' \right) + h_{Pu} \left(\mu_u + \frac{1}{2} \Sigma_u \left(\frac{1 - 2\gamma_P}{\gamma_P} \eta + \Sigma_D - \frac{\Sigma_s}{1-s} \right)' \right) \\
&+ h_P \left((\psi_P - 1) r + \frac{\psi_P - 1}{2\gamma_P} \eta' \eta - \beta \psi_P \right) + 1 = 0,
\end{aligned} \tag{ID4}$$

$$\begin{aligned}
&\frac{1}{2} h_{ss} \Sigma_s \Sigma_s' + \frac{1}{2} h_{uu} \Sigma_u \Sigma_u' + h_{su} \Sigma_s \Sigma_u' \\
&+ h_s \left(\mu_s + \frac{1}{2} \Sigma_s \Pi_I \left(\frac{1 - 2\gamma_I}{\gamma_I} \eta + \Sigma_D + \frac{\Sigma_s}{s} \right)' \right) + h_u \left(\mu_u + \frac{1}{2} \Sigma_u \Pi_I \left(\frac{1 - 2\gamma_I}{\gamma_I} \eta + \Sigma_D + \frac{\Sigma_s}{s} \right)' \right) \\
&+ h \left((\psi_I - 1) r + \frac{\psi_I - 1}{2\gamma_I} \eta' \Pi_I \eta - \beta \psi_I + \frac{2 - \gamma_I - \psi_I}{2(\psi_I - 1)} \left(\Sigma_D + \frac{\Sigma_s}{s} \right) (I - \Pi_I) \left(\Sigma_D + \frac{\Sigma_s}{s} \right)' \right) + 1 = 0,
\end{aligned} \tag{ID5}$$

where the market price of risk η is given by

$$\eta = \gamma_P \left(\Sigma_D - \frac{\Sigma_s}{1-s} \right) + \frac{\gamma_P \psi_P - 1}{\psi_P - 1} \left(\frac{h_{Ps}}{h_P} \Sigma_s + \frac{h_{Pu}}{h_P} \Sigma_u \right) \tag{ID6}$$

and $\Pi_I = (\Sigma_I' \Sigma_I)/(\Sigma_I \Sigma_I')$. *The rest of the equations are algebraic:*

$$\begin{aligned} r = \beta + \frac{1}{s\psi_I + (1-s)\psi_P} \Bigg[\mu_D + \frac{(\psi_I - \gamma_I)}{2(1-\psi_I)} s \left(\Sigma_D + \frac{\Sigma_s}{s} \right) \left(\Sigma_D + \frac{\Sigma_s}{s} \right)' \\ - \frac{\psi_I(\gamma_I \psi_I - 1)}{2(\psi_I - 1)} s \left(\Sigma_D + \frac{\Sigma_s}{s} + \frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u \right) \left(\Sigma_D + \frac{\Sigma_s}{s} + \frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u \right)' \\ + \frac{(\gamma_P - \psi_P)}{2(\psi_P - 1)} (1-s) \left(\Sigma_D - \frac{\Sigma_s}{1-s} \right) \left(\Sigma_D - \frac{\Sigma_s}{1-s} \right)' - \frac{\psi_P(\gamma_P \psi_P - 1)}{2(\psi_P - 1)} (1-s) \times \\ \times \left(\Sigma_D - \frac{\Sigma_s}{1-s} + \frac{h_{Ps}}{h_P}\Sigma_s + \frac{h_{Pu}}{h_P}\Sigma_u \right) \left(\Sigma_D - \frac{\Sigma_s}{1-s} + \frac{h_{Ps}}{h_P}\Sigma_s + \frac{h_{Pu}}{h_P}\Sigma_u \right)' \Bigg], \end{aligned} \tag{ID7}$$

$$\begin{aligned} \mu_s = -\Sigma_s \Sigma_D' + \frac{s(1-s)}{s\psi_I + (1-s)\psi_P} \Bigg[(\psi_I - \psi_P)\mu_D + \frac{(\psi_I - \gamma_I)\psi_P}{2(\psi_I - 1)} \left(\Sigma_D + \frac{\Sigma_s}{s} \right) \left(\Sigma_D + \frac{\Sigma_s}{s} \right)' \\ + \frac{\psi_I \psi_P(\gamma_I \psi_I - 1)}{2(\psi_I - 1)} \left(\Sigma_D + \frac{\Sigma_s}{s} + \frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u \right) \left(\Sigma_D + \frac{\Sigma_s}{s} + \frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u \right)' \\ + \frac{(\gamma_P - \psi_P)\psi_I}{2(\psi_P - 1)} \left(\Sigma_D - \frac{\Sigma_s}{1-s} \right) \left(\Sigma_D - \frac{\Sigma_s}{1-s} \right)' + \frac{\psi_I \psi_P(1 - \gamma_P \psi_P)}{2(\psi_P - 1)} \times \\ \times \left(\Sigma_D - \frac{\Sigma_s}{1-s} + \frac{h_{Ps}}{h_P}\Sigma_s + \frac{h_{Pu}}{h_P}\Sigma_u \right) \left(\Sigma_D - \frac{\Sigma_s}{1-s} + \frac{h_{Ps}}{h_P}\Sigma_s + \frac{h_{Pu}}{h_P}\Sigma_u \right)' \Bigg], \end{aligned} \tag{ID8}$$

$$f = (1-s)h_P + sh, \tag{ID9}$$

$$\Sigma_I = \frac{(f\Sigma_D + f_u\Sigma_u + f_s\Psi_1)(f\Sigma_D + f_u\Sigma_u + f_s\Psi_2)'}{(f\Sigma_D + f_u\Sigma_u + f_s\Psi_2)(f\Sigma_D + f_u\Sigma_u + f_s\Psi_2)'} \left(\Sigma_D + \frac{f_u}{f}\Sigma_u + \frac{f_s}{f}\Psi_2 \right), \tag{ID10}$$

$$\Sigma_s = \Psi_2 + (\Psi_1 - \Psi_2)\Pi_I, \tag{ID11}$$

where Ψ_1 *and* Ψ_2 *are given by*

$$\Psi_1 = \frac{(\gamma_P - \gamma_I)\Sigma_D + \left(\frac{\psi_P\gamma_P - 1}{\psi_P - 1}\frac{h_{Pu}}{h_P} - \frac{\psi_I\gamma_I - 1}{\psi_I - 1}\frac{h_u}{h} \right)\Sigma_u}{\frac{\gamma_P}{1-s} + \frac{\gamma_I}{s} - \left(\frac{\psi_P\gamma_P - 1}{\psi_P - 1}\frac{h_{Ps}}{h_P} - \frac{\psi_I\gamma_I - 1}{\psi_I - 1}\frac{h_s}{h} \right)}, \tag{ID12}$$

$$\Psi_2 = -\frac{s}{h + sh_s}\left(h\Sigma_D + h_u\Sigma_u \right). \tag{ID13}$$

Proof. Because many steps of the proof are identical or very similar to those of the proof of Proposition 1, we present them with less details and focus on the modifications produced by recursive preferences. In the derivations, we assume that $\psi \neq 1$.

A. Utility maximization problem

The type P investors solve the optimization problem (ID1) – (ID3) with $\gamma = \gamma_P$ and $\psi = \psi_P$, whereas the type I investors solve the same problem but with $\gamma = \gamma_I$ and $\psi = \psi_I$ and an additional portfolio constraint $\omega_t \in \Omega_I$. Below we solve the optimization problem (ID1) – (ID3) with arbitrary γ, ψ, and Ω and omit the subscripts P and I. To simplify notations, we also omit the subscript t.

The value function J solves the following HJB equation:

$$\max_{\{C,\omega\in\Omega\}} \Big\{ f(C,J) + J_W(W(r+\omega'\mu_Q) - C) + \frac{1}{2}J_{WW}W^2\omega'\Sigma_Q\Sigma_Q'\omega + J_{Ws}W\omega'\Sigma_Q\Sigma_s' + J_{Wu}W\omega'\Sigma_Q\Sigma_u' + J_s\mu_s + J_u\mu_u + \frac{1}{2}J_{ss}\Sigma_s\Sigma_s' + \frac{1}{2}J_{uu}\Sigma_u\Sigma_u' + J_{us}\Sigma_s\Sigma_u' \Big\} = 0. \quad \text{(ID14)}$$

The first-order condition with respect to C gives

$$\frac{\beta(1-\gamma)JC^{-1/\psi}}{((1-\gamma)J)^{\frac{1-1/\psi}{1-\gamma}}} = J_W. \quad \text{(ID15)}$$

Assuming that the value function is

$$J = (\beta^\psi h)^{\frac{1-\gamma}{\psi-1}} \frac{W^{1-\gamma}}{1-\gamma}, \quad \text{(ID16)}$$

where the function h depends on the state variables s and u, and substituting (ID16) into (ID15), we find that $C = Wh^{-1}$. Thus, the function h coincides with the investor's wealth-consumption ratio. Note that

$$f(C,J) - CJ_W = \frac{\beta(1-\gamma)J}{\psi-1}\left(\frac{C}{((1-\gamma)J)^{\frac{1}{1-\gamma}}}\right)^{1-1/\psi} - \frac{\beta(1-\gamma)J}{1-1/\psi} = \left(\frac{1}{h} - \beta\psi\right)\frac{(1-\gamma)J}{\psi-1}.$$

Using this fact and substituting the value function from (ID16) into Eq. (ID14), we obtain the following equation for h:

$$\frac{1}{2}h_{ss}\Sigma_s\Sigma_s' + \frac{1}{2}h_{uu}\Sigma_u\Sigma_u' + h_{su}\Sigma_s\Sigma_u' + h_s\mu_s + h_u\mu_u + h((\psi-1)r - \beta\psi) + 1 + \frac{(2-\gamma-\psi)h}{2(\psi-1)}\left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)\left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)' + (\psi-1)h\max_{\{\omega\in\Omega\}}\left\{\omega'\mu_Q + \frac{1-\gamma}{\psi-1}\omega'\Sigma_Q\left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)' - \frac{\gamma}{2}\omega'\Sigma_Q\Sigma_Q'\omega\right\} = 0. \quad \text{(ID17)}$$

In the case of the type P investors, $\Omega_P = \mathbb{R}^2$ and the optimization with respect to ω yields

$$\omega = \frac{1}{\gamma_P}\left(\Sigma_Q'\right)^{-1}\left(\eta + \frac{1-\gamma_P}{\psi_P-1}\left(\frac{h_{Ps}}{h_P}\Sigma_s + \frac{h_{Pu}}{h_P}\Sigma_u\right)\right)', \quad \text{(ID18)}$$

where $\eta' = \Sigma_Q^{-1}\mu_Q$. Putting this solution back into Eq. (ID17) and rearranging the terms, we get

$$\tfrac{1}{2}h_{Pss}\Sigma_s\Sigma_s' + \tfrac{1}{2}h_{Puu}\Sigma_u\Sigma_u' + h_{Psu}\Sigma_s\Sigma_u' + h_{Ps}\left(\mu_s + \tfrac{1-\gamma_P}{\gamma_P}\Sigma_s\eta'\right) + h_{Pu}\left(\mu_u + \tfrac{1-\gamma_P}{\gamma_P}\Sigma_u\eta'\right) + h_P\left((\psi_P-1)r + \tfrac{\psi_P-1}{2\gamma_P}\eta\eta' - \beta\psi_P + \tfrac{1-\gamma_P\psi_P}{2\gamma_P(\psi_P-1)}\left(\tfrac{h_{Ps}}{h_P}\Sigma_s + \tfrac{h_{Pu}}{h_P}\Sigma_u\right)\left(\tfrac{h_{Ps}}{h_P}\Sigma_s + \tfrac{h_{Pu}}{h_P}\Sigma_u\right)'\right) + 1 = 0. \quad \text{(ID19)}$$

In the case of the type I investors, $\Omega = \{\omega : \omega = \hat{\omega}[S_1/S \;\; S_2/S]'\}$, where $\hat{\omega}$ is a scalar weight of the index in the investor's portfolio. Therefore, Eq. (ID17) becomes

$$\frac{1}{2}h_{ss}\Sigma_s\Sigma_s' + \frac{1}{2}h_{uu}\Sigma_u\Sigma_u' + h_{su}\Sigma_s\Sigma_u' + h_s\mu_s + h_u\mu_u + h((\psi_I-1)r - \beta\psi_I) + 1 + \frac{(2-\gamma_I-\psi_I)h}{2(\psi_I-1)}\left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)\left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)' + (\psi_I-1)h\max_{\hat{\omega}}\left\{\hat{\omega}\mu_I + \frac{1-\gamma_I}{\psi_I-1}\hat{\omega}\Sigma_I\left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)' - \frac{\gamma_I}{2}\hat{\omega}^2\Sigma_I\Sigma_I'\right\} = 0. \quad \text{(ID20)}$$

The optimal portfolio of the type I investors is given by

$$\hat{\omega}_I = \frac{1}{\gamma_I\Sigma_I\Sigma_I'}\left(\mu_I + \frac{1-\gamma_I}{\psi_I-1}\Sigma_I\left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)'\right), \quad \text{(ID21)}$$

and Eq. (ID20) reduces to

$$\frac{1}{2}h_{ss}\Sigma_s\Sigma_s' + \frac{1}{2}h_{uu}\Sigma_u\Sigma_u' + h_{su}\Sigma_s\Sigma_u' + h_s\mu_s + h_u\mu_u + h((\psi_I - 1)r - \beta\psi_I) + 1 \\ + \frac{(2-\gamma_I-\psi_I)h}{2(\psi_I - 1)}\left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)\left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)' \\ + \frac{(\psi_I - 1)h}{2\gamma_I\Sigma_I\Sigma_I'}\left(\mu_I + \frac{1-\gamma_I}{\psi_I - 1}\Sigma_I\left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)'\right)^2 = 0. \quad \text{(ID22)}$$

B. Dynamics of the state variable s and returns on the index

The definition of the consumption share s implies that $C_P = (1-s)D$ and $C_I = sD$. Using Itô's lemma, we obtain the consumption processes of the type P and type I investors:

$$\frac{dC_P}{C_P} = \mu_{CP}dt + \Sigma_{CP}dB, \qquad \frac{dC_I}{C_I} = \mu_{CI}dt + \Sigma_{CI}dB, \quad \text{(ID23)}$$

where

$$\mu_{CP} = \mu_D - \frac{\mu_s + \Sigma_D\Sigma_s'}{1-s}, \qquad \Sigma_{CP} = \Sigma_D - \frac{\Sigma_s}{1-s}, \quad \text{(ID24)}$$

and

$$\mu_{CI} = \mu_D + \frac{\mu_s + \Sigma_D\Sigma_s'}{s}, \qquad \Sigma_{CI} = \Sigma_D + \frac{\Sigma_s}{s}. \quad \text{(ID25)}$$

Next, we represent dJ/J of both types of investors in two different ways and match the drifts and diffusions of the obtained processes. Because the equations below are valid for both types of investors, we omit the subscripts P and I. On the one hand, using $h = W/C$, we rewrite Eq. (ID16) as

$$J = \frac{1}{1-\gamma}(\beta W)^{\frac{\psi(1-\gamma)}{\psi-1}}C^{\frac{1-\gamma}{1-\psi}}.$$

Applying Itô's lemma to this equation and taking into account Eqs. (ID3) and (ID23), we get

$$\frac{dJ}{J} = \frac{\psi(1-\gamma)}{\psi-1}\Big(r - h^{-1} - \frac{1}{\psi}\mu_C + \omega'\left(\mu_Q - \frac{1-\gamma}{\psi-1}\Sigma_Q\Sigma_C'\right) - \frac{\psi-\gamma}{2\psi(1-\psi)}\Sigma_C\Sigma_C' \\ + \frac{1-\psi\gamma}{2(\psi-1)}\omega'\Sigma_Q\Sigma_Q'\omega\Big)dt + \frac{\psi(1-\gamma)}{\psi-1}\Big(\omega'\Sigma_Q - \frac{1}{\psi}\Sigma_C\Big)dB. \quad \text{(ID26)}$$

On the other hand, Itô's lemma applied to Eq. (ID16) yields

$$\frac{dJ}{J} = \frac{\mathcal{D}J}{J}dt + (1-\gamma)\left(\omega'\Sigma_Q + \frac{1}{\psi-1}\left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)\right)dB. \tag{ID27}$$

The HJB equation (ID14) implies that $f(C,J) + \mathcal{D}J = 0$, so

$$\frac{\mathcal{D}J}{J} = -\frac{f(C,J)}{J} = \frac{\psi(1-\gamma)}{\psi-1}\left(\beta - \frac{1}{h}\right),$$

where we use $f(C,J)$ from (ID2) and J from (ID16). Matching the drifts and diffusions in Eqs. (ID26) and (ID27), we get

$$r - \beta - \frac{1}{\psi}\mu_C + \omega'\left(\mu_Q - \frac{1-\gamma}{\psi-1}\Sigma_Q\Sigma_C'\right) - \frac{\psi-\gamma}{2\psi(1-\psi)}\Sigma_C\Sigma_C' + \frac{1-\psi\gamma}{2(\psi-1)}\omega'\Sigma_Q\Sigma_Q'\omega = 0, \tag{ID28}$$

$$\omega'\Sigma_Q = \Sigma_C + \frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u. \tag{ID29}$$

First, we consider this system of equations for the type P investors. Plugging the optimal portfolio weight ω from (ID18) and Σ_{CP} from (ID24) into (ID29), solving for η, and restoring the subscript P, we get Eq. (ID6). Using that $\mu_Q = \Sigma_Q\eta'$, $\omega'\Sigma_Q$ from (ID29), Σ_{CP} from (ID24), and η from (ID6), we transform Eq. (ID28) into

$$\begin{aligned} & r - \beta - \frac{1}{\psi_P}\left(\mu_D - \frac{\mu_s + \Sigma_D\Sigma_s'}{1-s}\right) - \frac{\gamma_P - \psi_P}{2\psi_P(\psi_P-1)}\left(\Sigma_D - \frac{\Sigma_s}{1-s}\right)\left(\Sigma_D - \frac{\Sigma_s}{1-s}\right)' \\ & - \frac{1-\gamma_P\psi_P}{2(\psi_P-1)}\left(\Sigma_D - \frac{\Sigma_s}{1-s} + \frac{h_{Ps}}{h_P}\Sigma_s + \frac{h_{Pu}}{h_P}\Sigma_u\right)\left(\Sigma_D - \frac{\Sigma_s}{1-s} + \frac{h_{Ps}}{h_P}\Sigma_s + \frac{h_{Pu}}{h_P}\Sigma_u\right)' = 0. \end{aligned} \tag{ID30}$$

Eqs. (ID28) and (ID29) for the type I investors yield two additional relations for the equilibrium functions. Using that for the type I investors $\omega'\Sigma_Q = \hat{\omega}\Sigma_I$, where $\hat{\omega}$ is given by Eq. (ID21), noting that Σ_C is given by Eq. (ID25), and multiplying both sides of Eq. (ID29) by Σ_I', we get

$$\mu_I = \gamma_I\left(\Sigma_D + \frac{\Sigma_s}{s}\right)\Sigma_I' + \frac{\psi_I\gamma_I - 1}{\psi_I - 1}\left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)\Sigma_I'. \tag{ID31}$$

Plugging this representation of μ_I back into Eq. (ID21), we find that

$$\hat{\omega} = \frac{\Sigma_I}{\Sigma_I \Sigma_I'} \left(\Sigma_D + \frac{\Sigma_s}{s} + \frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u \right)'. \tag{ID32}$$

Using Eq. (ID29) again, we get

$$\left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u + \Sigma_D + \frac{1}{s}\Sigma_s \right)(I_2 - \Pi_I) = 0. \tag{ID33}$$

Note that Eq. (ID33) coincides with Eq. (A50) from the proof of Proposition 1. To simplify Eq. (ID28) for the type I investors, note that $\omega'\mu_Q = \hat{\omega}\mu_I$ and $\omega'\Sigma_Q\Sigma_Q'\omega = \hat{\omega}^2\Sigma_I\Sigma_I'$, where μ_I and $\hat{\omega}$ are given by (ID31) and (ID32), respectively. Using μ_C and Σ_C from (ID25), Eq. (ID28) for the type I investors becomes

$$\begin{aligned} r - \beta - \frac{1}{\psi_I}\left(\mu_D + \frac{\mu_s + \Sigma_D\Sigma_s'}{s}\right) - \frac{\gamma_I - \psi_I}{2\psi_I(\psi_I - 1)}\left(\Sigma_D + \frac{\Sigma_s}{s}\right)\left(\Sigma_D + \frac{\Sigma_s}{s}\right)' \\ - \frac{1 - \gamma_I\psi_I}{2(\psi_I - 1)}\left(\Sigma_D + \frac{\Sigma_s}{s} + \frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)\Pi_I\left(\Sigma_D + \frac{\Sigma_s}{s} + \frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)' = 0, \end{aligned}$$

which together with (ID33) yields

$$\begin{aligned} r - \beta - \frac{1}{\psi_I}\left(\mu_D + \frac{\mu_s + \Sigma_D\Sigma_s'}{s}\right) - \frac{\gamma_I - \psi_I}{2\psi_I(\psi_I - 1)}\left(\Sigma_D + \frac{\Sigma_s}{s}\right)\left(\Sigma_D + \frac{\Sigma_s}{s}\right)' \\ - \frac{1 - \gamma_I\psi_I}{2(\psi_I - 1)}\left(\Sigma_D + \frac{\Sigma_s}{s} + \frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)\left(\Sigma_D + \frac{\Sigma_s}{s} + \frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)' = 0. \end{aligned} \tag{ID34}$$

Eqs. (ID30) and (ID34) can be viewed as a system of linear equations for r and μ_s. Its solution is given by Eqs. (ID7) and (ID8).

To find equations for Σ_s and Σ_I, we use that $\mu_I = \eta\Sigma_I'$, where μ_I is from Eq. (ID31) and η is from Eq. (ID6). After rearranging the terms, this equation yields $\Sigma_s\Sigma_I' = \Psi_1\Sigma_I'$ or $\Sigma_s\Pi_I = \Psi_1\Pi_I$, where Ψ_1 is defined by (ID12). Using this fact, Eq. (ID33) can be resolved for Σ_s as

$$\Sigma_s = \Psi_1\Pi_I + \Psi_2(I_2 - \Pi_I), \tag{ID35}$$

where Ψ_2 is defined by (ID13). This is Eq. (ID11). As in the proof of Proposition 1, the second equation in the system for Σ_s and Σ_I is given by the diffusion of index returns represented in terms of the index price-dividend ratio f:

$$\Sigma_I = \Sigma_D + \frac{f_s}{f}\Sigma_s + \frac{f_u}{f}\Sigma_u. \tag{ID36}$$

This is Eq. (A17). Substituting Σ_s from (ID35) into (ID36) and rearranging the terms, we get

$$\Sigma_I = \frac{f_s}{f}(\Psi_1 - \Psi_2)\Pi_I + \left(\Sigma_D + \frac{f_u}{f}\Sigma_u + \frac{f_s}{f}\Psi_2\right). \tag{ID37}$$

This is an analog of Eq. (A45) from the proof of Proposition 1. As there, we solve it by applying Lemma 1, which yields (ID10).

C. Equations for h_P, h, and f

Finally, we obtain quasilinear differential equations for h_P and h. Note that Eq. (ID6) implies that

$$\frac{1-\gamma_P\psi_P}{\psi_P - 1}\left(\frac{h_{Ps}}{h_P}\Sigma_s + \frac{h_{Pu}}{h_P}\Sigma_u\right) = \gamma_P\left(\Sigma_D - \frac{\Sigma_s}{1-s}\right) - \eta. \tag{ID38}$$

Therefore, Eq. (ID19) can be rewritten as (ID4). To transform Eq. (ID22) into quasilinear differential equation, note first that Eq. (ID33) can be rewritten as

$$\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u = -\left(\Sigma_D + \frac{1}{s}\Sigma_s\right)(I_2 - \Pi_I) + \left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)\Pi_I$$

and, therefore,

$$\left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)\left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)' = \left(\Sigma_D + \frac{1}{s}\Sigma_s\right)(I_2 - \Pi_I)\left(\Sigma_D + \frac{1}{s}\Sigma_s\right)' + \left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)\Pi_I\left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)'. \tag{ID39}$$

Using (ID39), the last two terms of Eq. (ID22) become

$$\begin{aligned}
&\frac{(2-\gamma_I-\psi_I)h}{2(\psi_I-1)}\left(\left(\Sigma_D+\frac{1}{s}\Sigma_s\right)(I_2-\Pi_I)\left(\Sigma_D+\frac{1}{s}\Sigma_s\right)'+\left(\frac{h_s}{h}\Sigma_s+\frac{h_u}{h}\Sigma_u\right)\Pi_I\left(\frac{h_s}{h}\Sigma_s+\frac{h_u}{h}\Sigma_u\right)'\right)\\
&\qquad+\frac{(\psi_I-1)h}{2\gamma_I}\left(\frac{\mu_I^2}{\Sigma_I\Sigma_I'}+\frac{2(1-\gamma_I)\mu_I}{(\psi_I-1)\Sigma_I\Sigma_I'}\Sigma_I\left(\frac{h_s}{h}\Sigma_s+\frac{h_u}{h}\Sigma_u\right)'+\right.\\
&\qquad\qquad\left.\frac{(1-\gamma_I)^2}{(\psi_I-1)^2}\left(\frac{h_s}{h}\Sigma_s+\frac{h_u}{h}\Sigma_u\right)\Pi_I\left(\frac{h_s}{h}\Sigma_s+\frac{h_u}{h}\Sigma_u\right)'\right)\\
&=\frac{(2-\gamma_I-\psi_I)h}{2(\psi_I-1)}\left(\Sigma_D+\frac{1}{s}\Sigma_s\right)(I_2-\Pi_I)\left(\Sigma_D+\frac{1}{s}\Sigma_s\right)'+\frac{(\psi_I-1)h}{2\gamma_I}\eta\Pi_I\eta'\\
&+\frac{(1-\gamma_I)h}{\gamma_I}\eta\Pi_I\left(\frac{h_s}{h}\Sigma_s+\frac{h_u}{h}\Sigma_u\right)'+\frac{(1-\psi_I\gamma_I)h}{2\gamma_I(\psi_I-1)}\left(\frac{h_s}{h}\Sigma_s+\frac{h_u}{h}\Sigma_u\right)\Pi_I\left(\frac{h_s}{h}\Sigma_s+\frac{h_u}{h}\Sigma_u\right)',
\end{aligned}\tag{ID40}$$

where it has been used that $\mu_I=\Sigma_I\eta$. Multiplying Eq. (ID31) by $\Sigma_I\left(\frac{h_s}{h}\Sigma_s+\frac{h_u}{h}\Sigma_u\right)'/(\Sigma_I\Sigma_I')$ and using again that $\mu_I=\Sigma_I\eta$, we get

$$\left(\gamma_I\left(\Sigma_D+\frac{1}{s}\Sigma_s\right)-\eta\right)\Pi_I\left(\frac{h_s}{h}\Sigma_s+\frac{h_u}{h}\Sigma_u\right)'=\frac{1-\psi_I\gamma_I}{\psi_I-1}\left(\frac{h_s}{h}\Sigma_s+\frac{h_u}{h}\Sigma_u\right)\Pi_I\left(\frac{h_s}{h}\Sigma_s+\frac{h_u}{h}\Sigma_u\right)'.$$

Combining this equation with (ID40) and collecting like terms, we arrive at Eq. (ID5).

Finally, we derive the equation for f. The market clearing condition implies that $W_P+W_I=S$. Noting that $W_I=hC_I$, $W_P=h_PC_P$, and $S=fD$, we get that $f=(1-s)h_P+sh$, and this is Eq. (ID9). Q.E.D.

As in the main part of the paper, we demonstrate the impact of indexing by comparing equilibrium variables in the economy with indexing and in the unconstrained economy. The equilibrium in the unconstrained economy with recursive preferences is described by Proposition ID2.

Proposition ID2. *The equilibrium in the economy without indexing is characterized by the functions r, η, μ_s, Σ_s, h, and h_P, that solve a system of algebraic and differential equations. The* *functions h and h_P satisfy the following partial differential equations:*

$$\frac{1}{2}h_{Pss}\Sigma_s\Sigma_s' + \frac{1}{2}h_{Puu}\Sigma_u\Sigma_u' + h_{Psu}\Sigma_s\Sigma_u' + h_{Ps}\left(\mu_s + \frac{1}{2}\Sigma_s\left(\frac{1-2\gamma_P}{\gamma_P}\eta + \Sigma_D - \frac{\Sigma_s}{1-s}\right)'\right) + h_{Pu}\left(\mu_u + \frac{1}{2}\Sigma_u\left(\frac{1-2\gamma_P}{\gamma_P}\eta + \Sigma_D - \frac{\Sigma_s}{1-s}\right)'\right) + h_P\left((\psi_P - 1)r + \frac{\psi_P - 1}{2\gamma_P}\eta'\eta - \beta\psi_P\right) + 1 = 0, \quad \text{(ID41)}$$

$$\frac{1}{2}h_{ss}\Sigma_s\Sigma_s' + \frac{1}{2}h_{uu}\Sigma_u\Sigma_u' + h_{su}\Sigma_s\Sigma_u' + h_s\left(\mu_s + \frac{1}{2}\Sigma_s\left(\frac{1-2\gamma_I}{\gamma_I}\eta + \Sigma_D + \frac{\Sigma_s}{s}\right)'\right) + h_u\left(\mu_u + \frac{1}{2}\Sigma_u\left(\frac{1-2\gamma_I}{\gamma_I}\eta + \Sigma_D + \frac{\Sigma_s}{s}\right)'\right) + h\left((\psi_I - 1)r + \frac{\psi_I - 1}{2\gamma_I}\eta'\eta - \beta\psi_I\right) + 1 = 0, \quad \text{(ID42)}$$

where the market price of risk η is given by (ID6) and

$$\Sigma_s = \frac{(\gamma_P - \gamma_I)\Sigma_D + \left(\frac{\psi_P\gamma_P - 1}{\psi_P - 1}\frac{h_{Pu}}{h_P} - \frac{\psi_I\gamma_I - 1}{\psi_I - 1}\frac{h_u}{h}\right)\Sigma_u}{\frac{\gamma_P}{1-s} + \frac{\gamma_I}{s} - \left(\frac{\psi_P\gamma_P - 1}{\psi_P - 1}\frac{h_{Ps}}{h_P} - \frac{\psi_I\gamma_I - 1}{\psi_I - 1}\frac{h_s}{h}\right)}. \quad \text{(ID43)}$$

The other equations coincide with Eqs. (ID7) and (ID8).

Proof. The proof of Proposition ID2 closely follows the proof of Proposition ID1. In particular, the utility maximization problem of the type P investors yields the same equation (ID19) for h_P. Because the type I investors are also unconstrained, the equation for h is identical to (ID19) in which ψ_P is replaced with ψ_I and γ_P is replaced with γ_I:

$$\begin{aligned} &\tfrac{1}{2}h_{ss}\Sigma_s\Sigma_s' + \tfrac{1}{2}h_{uu}\Sigma_u\Sigma_u' + h_{su}\Sigma_s\Sigma_u' + h_s\left(\mu_s + \tfrac{1-\gamma_I}{\gamma_I}\Sigma_s\eta'\right) + h_u\left(\mu_u + \tfrac{1-\gamma_I}{\gamma_I}\Sigma_u\eta'\right) \\ &+ h\left((\psi_I - 1)r + \tfrac{\psi_I - 1}{2\gamma_I}\eta\eta' - \beta\psi_I + \tfrac{1-\gamma_I\psi_I}{2\gamma_I(\psi_I - 1)}\left(\tfrac{h_s}{h}\Sigma_s + \tfrac{h_u}{h}\Sigma_u\right)\left(\tfrac{h_s}{h}\Sigma_s + \tfrac{h_u}{h}\Sigma_u\right)'\right) + 1 = 0. \end{aligned} \quad \text{(ID44)}$$

Also, removing of the indexing constraint does not change Eqs. (ID23) – (ID30), and the equation for the market price of risk η is again given by (ID6). This equation can be rewritten as (ID38), and a simple algebraic manipulation yields Eq. (ID41).

Because the type I investors are unconstrained, Eqs. (ID28) and (ID29) that follow from their optimization problem produce equations that are similar to those in the case of type P investors. Plugging the optimal portfolio weight

$$\omega = \frac{1}{\gamma_I} \left(\Sigma_Q'\right)^{-1} \left(\eta + \frac{1-\gamma_I}{\psi_I - 1} \left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)\right)' \tag{ID45}$$

and Σ_{CI} from (ID25) into (ID29), solving for η, and restoring the subscript I, we get

$$\eta = \gamma_I \left(\Sigma_D + \frac{\Sigma_s}{s}\right) + \frac{\gamma_I \psi_I - 1}{\psi_I - 1} \left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right). \tag{ID46}$$

Combining Eqs. (ID46) and (ID6) and solving for Σ_s, we get Eq. (ID43). Also, rewriting (ID46) as

$$\frac{1-\gamma_I \psi_I}{\psi_I - 1} \left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right) = \gamma_I \left(\Sigma_D + \frac{\Sigma_s}{s}\right) - \eta,$$

we transform Eq. (ID44) into (ID42).

Finally, using that $\mu_Q = \Sigma_Q \eta'$, $\omega' \Sigma_Q$ from (ID29), Σ_{CI} from (ID25), and η from (ID46), we transform Eq. (ID28) into (ID34). Solving Eqs. (ID30) and (ID34) for r and μ_s yields Eqs. (ID7) and (ID8). Q.E.D.

To compare the equilibria in the constrained and unconstrained economies, we solve the systems of equations from Propositions ID1 and ID2 numerically using the same techniques as in the main part of the paper. We calibrate the preference parameters following Gârleanu and Panageas (2015) and set $\gamma_I = 10$, $\gamma_P = 1.5$, $\psi_I = 0.05$, $\psi_P = 0.7$, and $\beta = 0.02$. The dividends follow the same processes as in the main part of the paper: $\mu_{D1} = \mu_{D2} = 0.018$, $\Sigma_{D1} = [0.045 \;\; 0]$, and $\Sigma_{D2} = [0 \;\; 0.045]$. Fig. ID.1 reports the changes in various equilibrium characteristics produced by indexing.

The comparison of Fig. ID.1 with Figs. 1 and 2 from the main part of the paper reveals that the graphs for the economies with the CRRA and recursive utility functions are qualitatively similar. We again observe that indexing typically decreases the market volatility and risk-free rate, but its effect on the correlation between the stocks and on the stock volatilities is ambiguous. In particular, indexing tends to increase the correlation when the stocks have comparable sizes but

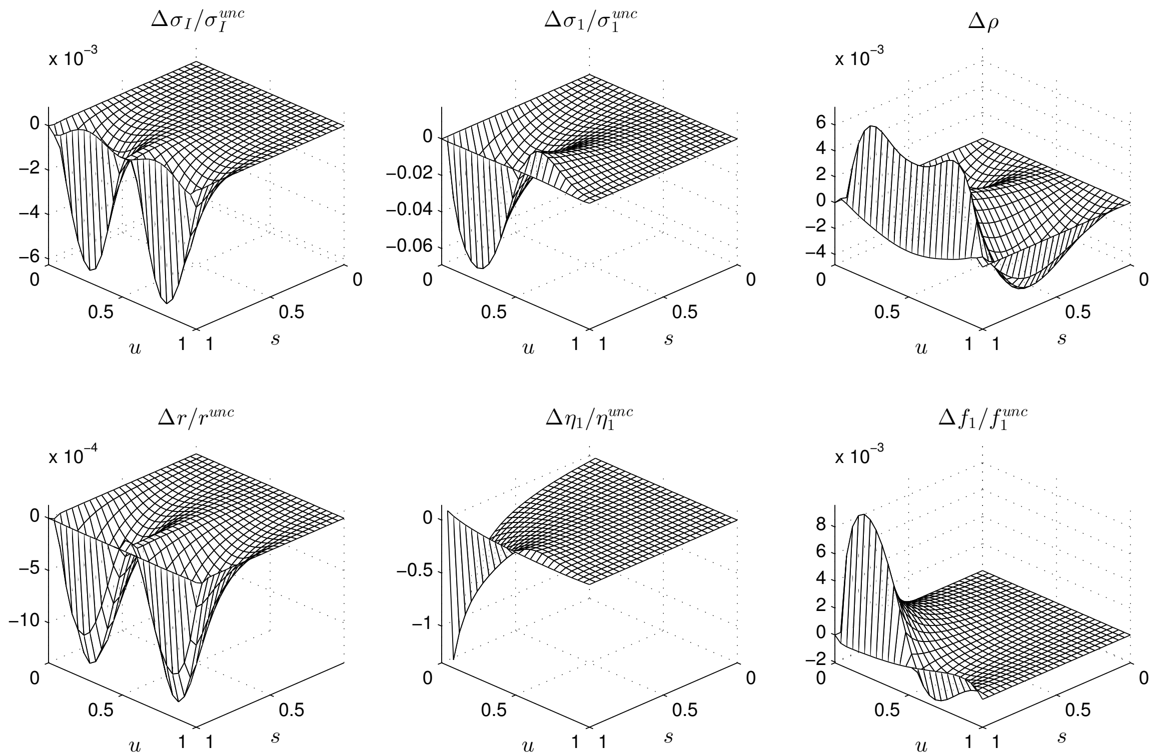

Figure ID.1: This figure shows how the unconstrained equilibrium changes due to indexing in the model with recursive preferences. All variables are functions of the consumption share s of the type I investors and the share u of the first dividend D_1 in the aggregate dividend D. The model parameters are as follows: $\mu_{D1} = \mu_{D2} = 0.018$, $\Sigma_{D1} = [0.045 \quad 0]$, $\Sigma_{D2} = [0 \quad 0.045]$, $\gamma_I = 10$, $\gamma_P = 1.5$, $\psi_I = 0.05$, $\psi_P = 0.7$, and $\beta = 0.02$.

decrease when one of the stocks is substantially larger than the other. Thus, our main conclusions are robust to the choice of the preferences. Finally, note that quantitatively many effects of indexing are substantially stronger in the economy with recursive preferences than their analogs in the economy with the CRRA preferences. Hence, a more realistic specification of preferences can make the impact of indexing more pronounced and practically relevant.

Internet Appendix E. Index investing as an outcome of complexity aversion

In our main model, we assume that the type I investors trade only the index because of unspecified exogenous reasons. In this Internet Appendix, we present a modification of the model that demonstrates how the equilibrium with index investing endogenously arises when the type I investors are unconstrained in their portfolio choice but have complexity aversion, that is, derive disutility from managing a complex portfolio of individual risky assets.[2]

The main components of the modified model are the same as in the main model of the paper. We again consider a pure exchange economy with two Lucas trees that follow geometric Brownian motions and with two types of investors. The type P investors have the standard CRRA preferences and can trade all assets. The difference between the models is in the preferences of the type I investors. Specifically, we assume that the type I investors have the following utility function:

$$U_t = \mathbb{E}_t \left[\int_t^{\infty} e^{-\beta\tilde{t}} \frac{C_{I\tilde{t}}^{1-\gamma_I}}{1-\gamma_I} g(A_{I\tilde{t}}) d\tilde{t} \right], \tag{IE1}$$

where $\gamma_I > 1$. Thus, the type I investors' utility is determined not only by the consumption C_{It} but also by A_{It}, which is a set of risky assets in their portfolio at time t and a subset of $A = \{\text{stock 1, stock 2, index}\}$. Eq. (IE1) represents the CRRA preferences with the new factor $g(A_{It})$ that describes complexity aversion: it decreases utility when the investor's portfolio includes assets that are hard to analyze and understand. Without losing generality, we set $g(\emptyset) = 1$. Also, we assume that $g(\{\text{stock 1}\}) = g(\{\text{stock 2}\}) = g(\{\text{stock 1, stock 2}\}) = g(\{\text{stock 1, index}\}) = g(\{\text{stock 2, index}\}) = \bar{g} > 1$. The assumption implies that (i) the individual stocks are "complex," so holding them reduces the investor's utility and (ii) the disutility of a portfolio with the individual stocks is the same irrespectively of how many individual assets the portfolio contains. To capture the idea that indexes simplify the portfolio choice and reduce the disutility from complexity of financial markets, we set $g(\{\text{index}\}) = 1$. Because the index can be replicated by a portfolio of individual stocks and the investors cannot benefit from combining individual stocks with the index, the portfolios $\{\text{stock 1, index}\}$ and $\{\text{stock 2, index}\}$ are redundant and can be excluded from consideration.

[2] The idea that economic agents have bounded rationality and prefer to reduce the complexity of decision making goes back to Simon (1955, 1956). Rubinstein (1998) provides a textbook exposition of various approaches to modeling bounded rationality. Recent theoretical studies of preferences that involve complexity include Ortoleva (2013), who provides an axiomatic treatment of "thinking aversion," and Fudenberg and Strzalecki (2015), who introduce "choice aversion" into a dynamic choice problem. Experimental evidence of complexity aversion is presented by Huck and Weizsäcker (1999), Sonsino et al. (2002), and Moffatt et al. (2015), among others.

The utility function (IE1) is maximized subject to the standard budget constraint

$$dW_{It} = (r_t W_{It} - C_{It})dt + W_{It}\omega'_{It}(\mu_{Qt}dt + \Sigma_{Qt}dB_t), \tag{IE2}$$

where $\omega'_{It} = [\omega_{I(1)t} \;\; \omega_{I(2)t}]$ is a vector of portfolio weights consistent with the composition of the portfolio A_{It}.[3] This condition implies that

$$\omega'_{It} = \begin{cases} [0 \;\; 0], & \text{if } A_{It} = \{\emptyset\}, \\ [\omega_{I(1)t} \;\; 0], & \text{if } A_{It} = \{\text{stock 1}\}, \\ [0 \;\; \omega_{I(2)t}], & \text{if } A_{It} = \{\text{stock 2}\}, \\ [\omega_{I(1)t} \;\; \omega_{I(2)t}], & \text{if } A_{It} = \{\text{stock 1, stock 2}\}, \\ \omega_{I(ind)t}\left[\frac{uf_1}{uf_1+(1-u)f_2} \;\; \frac{(1-u)f_2}{uf_1+(1-u)f_2}\right], & \text{if } A_{It} = \{\text{index}\}. \end{cases} \tag{IE3}$$

Note that in contrast to the main part of the paper, the type I investors optimally choose both the composition of the portfolio A_{It} and the portfolio weights ω_{It}.

The main result of Internet Appendix E is the following proposition.

Proposition IE1. *There exists $\bar{g}_{min} > 1$ such that for any $\bar{g} > \bar{g}_{min}$ the equilibrium in the modified economy coincides with the equilibrium described in Proposition 1. In particular, in any state of the economy, the type I investors hold only the risk-free bond and the index, and the functions r, μ_s, Σ_s, Σ_I, f, and h solve the system of algebraic and differential equations (A1) – (A6). The market price of risk η and the expected excess returns on the index μ_I are given by Eq. (A7). The price-dividend ratio f_i of stock $i = 1, 2$ solves Eq. (A8). The expected excess returns on individual stocks μ_{Qi}, $i = 1, 2$, and return diffusions Σ_{Qi}, $i = 1, 2$, are given by Eq. (A9).*

Proof. The proof closely follows the proof of Proposition 1 from the main part of the paper, which is presented in the Appendix. We do not reproduce the equations that are the same in both proofs.

[3] For expositional convenience, we have slightly changed the notations compared to the main part of the paper. In particular, the portfolio weights of the type I investors are now denoted as $\omega_{I(1)t}$ and $\omega_{I(2)t}$ instead of ω_{I1t} and ω_{I2t}, and the scalar weight of the index is denoted as $\omega_{I(ind)t}$ instead of $\hat{\omega}_{It}$.

A. Price-dividend ratios

This part of the proof is the same as in the proof of Proposition 1, and the equations for the price-dividend ratios f_i, expected returns μ_{Qi}, and diffusions Σ_{Qi} coincide with Eqs. (A15), (A13), and (A14), respectively.

B. Utility maximization problem of the type P investors

This part is also the same as in the proof of Proposition 1 and yields equations for the risk-free rate and expected returns that are identical to Eqs. (A23) and (A24).

C. Utility maximization problem of the type I investors

Because of the modified utility function and the lack of exogenous constraints on the portfolio weights, the utility maximization problem of the type I investors is different from its analog in the main part of the paper. In particular, the indirect utility function of the type I investors $J(s,u)$ solves a different HJB equation:

$$\max_{\{C_I,A_I,\omega_I\}} \left[e^{-\beta t} \frac{C_I^{1-\gamma_I}}{1-\gamma_I} g(A_I) + \mathcal{D}J \right] = 0, \tag{IE4}$$

where $\mathcal{D}J = \mathbb{E}[dJ]/dt$ is given by

$$\mathcal{D}J = J_W(rW_I - C_I) + J_s\mu_s + J_u\mu_u + \frac{1}{2}J_{ss}\Sigma_s\Sigma_s' + \frac{1}{2}J_{uu}\Sigma_u\Sigma_u' + J_{us}\Sigma_s\Sigma_u' + J_t + W_I \times$$

$$\times \begin{cases} 0, & A_I = \{\emptyset\}, \\ J_W\omega_{I(1)}\mu_{Q1} + \frac{1}{2}J_{WW}W_I\omega_{I(1)}^2\Sigma_{Q1}\Sigma_{Q1}' + J_{Ws}\omega_{I(1)}\Sigma_{Q1}\Sigma_s' + J_{Wu}\omega_{I(1)}\Sigma_{Q1}\Sigma_u', & A_I = \{\text{stock 1}\}, \\ J_W\omega_{I(2)}\mu_{Q2} + \frac{1}{2}J_{WW}W_I\omega_{I(2)}^2\Sigma_{Q2}\Sigma_{Q2}' + J_{Ws}\omega_{I(2)}\Sigma_{Q2}\Sigma_s' + J_{Wu}\omega_{I(2)}\Sigma_{Q2}\Sigma_u', & A_I = \{\text{stock 2}\}, \\ J_W\omega_I'\mu_Q + \frac{1}{2}J_{WW}W_I\omega_I'\Sigma_Q\Sigma_Q'\omega_I + J_{Ws}\omega_I'\Sigma_Q\Sigma_s' + J_{Wu}\omega_I'\Sigma_Q\Sigma_u', & A_I = \{\text{stocks 1, 2}\}, \\ J_W\omega_{I(ind)}\mu_I + \frac{1}{2}J_{WW}W_I\omega_{I(ind)}^2\Sigma_I\Sigma_I' + J_{Ws}\omega_{I(ind)}\Sigma_I\Sigma_s' + J_{Wu}\omega_{I(ind)}\Sigma_I\Sigma_u', & A_I = \{\text{index}\}. \end{cases}$$

The subscripts of J denote derivatives, and μ_I and Σ_I are the drift and diffusion of the index returns. As in the proof of Proposition 1, we look for the indirect utility function in the following form:

$$J = \frac{1}{1-\gamma_I} W_I^{1-\gamma_I} h(s,u)^{\gamma_I} \exp(-\beta t). \quad \text{(IE5)}$$

The maximization in Eq. (IE4) with respect to C_I together with Eq. (IE5) yields the optimal consumption:

$$C_I = W_I h^{-1} g(A_I)^{\frac{1}{\gamma_I}}. \quad \text{(IE6)}$$

The optimal portfolio weights depend on the choice of A_I:

$$\omega'_{It} = \begin{cases} [0 \;\; 0], & \text{if } A_{It} = \{\emptyset\}, \\ \left[\frac{1}{\Sigma_{Q1}\Sigma'_{Q1}} \left(\frac{\mu_{Q1}}{\gamma_I} + \frac{h_s}{h}\Sigma_{Q1}\Sigma'_s + \frac{h_u}{h}\Sigma_{Q1}\Sigma'_u\right) \;\; 0\right], & \text{if } A_{It} = \{\text{stock 1}\}, \\ \left[0 \;\; \frac{1}{\Sigma_{Q2}\Sigma'_{Q2}} \left(\frac{\mu_{Q2}}{\gamma_I} + \frac{h_s}{h}\Sigma_{Q2}\Sigma'_s + \frac{h_u}{h}\Sigma_{Q2}\Sigma'_u\right)\right], & \text{if } A_{It} = \{\text{stock 2}\}, \\ \left(\frac{\mu_Q}{\gamma_I} + \frac{h_s}{h}\Sigma_Q\Sigma'_s + \frac{h_u}{h}\Sigma_Q\Sigma'_u\right)' (\Sigma_Q\Sigma'_Q)^{-1}, & \text{if } A_{It} = \{\text{stocks 1, 2}\}, \\ \frac{1}{\Sigma_I\Sigma'_I} \left(\frac{\mu_I}{\gamma_I} + \frac{h_s}{h}\Sigma_I\Sigma'_s + \frac{h_u}{h}\Sigma_I\Sigma'_u\right) \left[\frac{uf_1}{uf_1+(1-u)f_2} \;\; \frac{(1-u)f_2}{uf_1+(1-u)f_2}\right], & \text{if } A_{It} = \{\text{index}\}. \end{cases} \quad \text{(IE7)}$$

Plugging Eqs. (IE6) and (IE7) into Eq. (IE4), we get a differential equation for h:

$$\begin{aligned} &\frac{1}{2}h_{ss}\Sigma_s\Sigma'_s + \frac{1}{2}h_{uu}\Sigma_u\Sigma'_u + h_{us}\Sigma_u\Sigma'_s + h_s\mu_s + h_u\mu_u \\ &\quad + \frac{\gamma_I - 1}{2}\left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)\left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u\right)' h + \frac{1}{\gamma_I}((1-\gamma_I)r - \beta)h + \min_{A_I}\left[\frac{(1-\gamma_I)h}{2} \times \right. \\ &\times \begin{cases} 0, & A_I = \{\emptyset\}, \\ \frac{1}{\Sigma_{Q1}\Sigma'_{Q1}} \left(\frac{\mu_{Q1}}{\gamma_I} + \frac{h_s}{h}\Sigma_{Q1}\Sigma'_s + \frac{h_u}{h}\Sigma_{Q1}\Sigma'_u\right)^2, & A_I = \{\text{stock 1}\}, \\ \frac{1}{\Sigma_{Q2}\Sigma'_{Q2}} \left(\frac{\mu_{Q2}}{\gamma_I} + \frac{h_s}{h}\Sigma_{Q2}\Sigma'_s + \frac{h_u}{h}\Sigma_{Q2}\Sigma'_u\right)^2, & A_I = \{\text{stock 2}\}, \\ \left(\frac{\mu_Q}{\gamma_I} + \frac{h_s}{h}\Sigma_Q\Sigma'_s + \frac{h_u}{h}\Sigma_Q\Sigma'_u\right)' \left(\Sigma_Q\Sigma'_Q\right)^{-1} \left(\frac{\mu_Q}{\gamma_I} + \frac{h_s}{h}\Sigma_Q\Sigma'_s + \frac{h_u}{h}\Sigma_Q\Sigma'_u\right), & A_I = \{\text{stocks 1, 2}\}, \\ \frac{1}{\Sigma_I\Sigma'_I} \left(\frac{\mu_I}{\gamma_I} + \frac{h_s}{h}\Sigma_I\Sigma'_s + \frac{h_u}{h}\Sigma_I\Sigma'_u\right)^2, & A_I = \{\text{index}\}, \end{cases} \\ &\left. \qquad + g(A_I)^{\frac{1}{\gamma_I}}\right] = 0. \quad \text{(IE8)} \end{aligned}$$

Note that when $A_I = \{\text{index}\}$, Eq. (IE8) coincides with Eq. (A30).

D. Dynamics of the state variable s

The definition of the consumption share s implies that $C_I = sD$, so using Itô's lemma, we get

$$\frac{dC_I}{C_I} = \mu_C dt + \Sigma_C dB, \qquad \frac{dC_I^{-\gamma_I}}{C_I^{-\gamma_I}} = \left(-\gamma_I \mu_C + \frac{1}{2}\gamma_I(\gamma_I + 1)\Sigma_C \Sigma_C'\right) dt - \gamma_I \Sigma_C dB, \tag{IE9}$$

where

$$\mu_C = \mu_D + \frac{\mu_s + \Sigma_s \Sigma_D'}{s}, \qquad \Sigma_C = \Sigma_D + \frac{1}{s}\Sigma_s. \tag{IE10}$$

Taking into account Eq. (IE6), the indirect utility function from Eq. (IE5) can be rewritten as

$$J = \frac{1}{1-\gamma_I} C_I^{-\gamma_I} W_I g(A_I) \exp(-\beta t).$$

Applying Itô's lemma to this equation and using Eqs. (IE2) and (IE9), we get

$$\frac{dJ}{J} = \Big(-\beta - \gamma_I \mu_C + \frac{1}{2}\gamma_I(\gamma_I + 1)\Sigma_C \Sigma_C' + r - h^{-1} g(A_I)^{\frac{1}{\gamma_I}} + \omega_I'(\mu_Q - \gamma_I \Sigma_Q \Sigma_C')\Big) dt + (\omega_I' \Sigma_Q - \gamma_I \Sigma_C) dB. \tag{IE11}$$

Alternatively, Itô's lemma applied to Eq. (IE5) yields

$$\frac{dJ}{J} = \frac{\mathcal{D}J}{J} dt + \left((1-\gamma_I)\omega_I' \Sigma_Q + \gamma_I \frac{h_s}{h}\Sigma_s + \gamma_I \frac{h_u}{h}\Sigma_u\right) dB. \tag{IE12}$$

Noting that Eqs. (IE5) and (IE6) imply that

$$e^{-\beta t} \frac{C_I^{1-\gamma_I}}{1-\gamma_I} = J h^{-1} g(A_I)^{\frac{1-\gamma_I}{\gamma_I}}$$

and using the HJB equation (IE4), we get $\mathcal{D}J = -Jh^{-1}g(A_I)^{\frac{1}{\gamma_I}}$ and rewrite Eq. (IE12) as

$$\frac{dJ}{J} = -h^{-1} g(A_I)^{\frac{1}{\gamma_I}} dt + \left((1-\gamma_I)\omega_I' \Sigma_Q + \gamma_I \frac{h_s}{h}\Sigma_s + \gamma_I \frac{h_u}{h}\Sigma_u\right) dB. \tag{IE13}$$

Matching the drifts and diffusions in Eqs. (IE11) and (IE13) and using μ_C and Σ_C from Eq.

(IE10), we get

$$\frac{1+\gamma_I}{2}\left(\Sigma_D+\frac{1}{s}\Sigma_s\right)\left(\Sigma_D+\frac{1}{s}\Sigma_s\right)'+\frac{r-\beta}{\gamma_I}+\omega_I'\left(\frac{\mu_Q}{\gamma_I}-\Sigma_Q\left(\Sigma_D+\frac{1}{s}\Sigma_s\right)'\right)$$
$$=\mu_D+\frac{1}{s}(\mu_s+\Sigma_s\Sigma_D'), \quad \text{(IE14)}$$

$$\omega_I\Sigma_Q-\frac{h_s}{h}\Sigma_s-\frac{h_u}{h}\Sigma_u=\Sigma_D+\frac{1}{s}\Sigma_s. \quad \text{(IE15)}$$

Note that Eqs. (IE14) and (IE15) hold for any set of assets A_I. Moreover, when $A_I = \{\text{index}\}$, they coincide with Eqs. (A38) and (A39), so in this case Eqs. (A40) – (A49) also apply. As a result, the dynamics of the state variable s are identical to its dynamics in the economy with indexing constraints and described by Eqs. (A2) – (A4).

E. Optimal choice of A_I

The arguments presented above demonstrate that if the type I investors choose $A_I = \{\text{index}\}$ in all states of the economy, the equilibrium coincides with the equilibrium from Proposition 1. Hence, to prove Proposition IE1, it is sufficient to show that there exists $\bar{g}_{min} > 1$ such that for any $\bar{g} > \bar{g}_{min}$ the type I investors do not deviate from $A_I = \{\text{index}\}$ taking the investment opportunities as given.

Note that the value of the objective function at the solution of an unconstrained maximization problem cannot be lower than that produced by maximization with constraints. Applying this observation to the maximization in the HJB equation with respect to the portfolio weights, we get that in all states

$$\frac{1}{\Sigma_{Q1}\Sigma_{Q1}'}\left(\frac{\mu_{Q1}}{\gamma_I}+\frac{h_s}{h}\Sigma_{Q1}\Sigma_s'+\frac{h_u}{h}\Sigma_{Q1}\Sigma_u'\right)^2$$
$$\leq\left(\frac{\mu_Q}{\gamma_I}+\frac{h_s}{h}\Sigma_Q\Sigma_s'+\frac{h_u}{h}\Sigma_Q\Sigma_u'\right)'\left(\Sigma_Q\Sigma_Q'\right)^{-1}\left(\frac{\mu_Q}{\gamma_I}+\frac{h_s}{h}\Sigma_Q\Sigma_s'+\frac{h_u}{h}\Sigma_Q\Sigma_u'\right),$$

$$\frac{1}{\Sigma_{Q2}\Sigma'_{Q2}}\left(\frac{\mu_{Q2}}{\gamma_I}+\frac{h_s}{h}\Sigma_{Q2}\Sigma'_s+\frac{h_u}{h}\Sigma_{Q2}\Sigma'_u\right)^2$$
$$\leq\left(\frac{\mu_Q}{\gamma_I}+\frac{h_s}{h}\Sigma_Q\Sigma'_s+\frac{h_u}{h}\Sigma_Q\Sigma'_u\right)'\left(\Sigma_Q\Sigma'_Q\right)^{-1}\left(\frac{\mu_Q}{\gamma_I}+\frac{h_s}{h}\Sigma_Q\Sigma'_s+\frac{h_u}{h}\Sigma_Q\Sigma'_u\right),$$

and

$$\frac{1}{\Sigma_I\Sigma'_I}\left(\frac{\mu_I}{\gamma_I}+\frac{h_s}{h}\Sigma_I\Sigma'_s+\frac{h_u}{h}\Sigma_I\Sigma'_u\right)^2$$
$$\leq\left(\frac{\mu_Q}{\gamma_I}+\frac{h_s}{h}\Sigma_Q\Sigma'_s+\frac{h_u}{h}\Sigma_Q\Sigma'_u\right)'\left(\Sigma_Q\Sigma'_Q\right)^{-1}\left(\frac{\mu_Q}{\gamma_I}+\frac{h_s}{h}\Sigma_Q\Sigma'_s+\frac{h_u}{h}\Sigma_Q\Sigma'_u\right).$$

In particular, those inequalities hold for the equilibrium with indexing. Also note that by definition, $h > 0$, so $(1-\gamma_I)h < 0$. Hence, the optimization with respect to A_I in Eq. (IE8) yields $A_I = \{\text{index}\}$ when in all states of the economy

$$\frac{(1-\gamma_I)h}{2}\frac{1}{\Sigma_I\Sigma'_I}\left(\frac{\mu_I}{\gamma_I}+\frac{h_s}{h}\Sigma_I\Sigma'_s+\frac{h_u}{h}\Sigma_I\Sigma'_u\right)^2+1$$
$$<\frac{(1-\gamma_I)h}{2}\left(\frac{\mu_Q}{\gamma_I}+\frac{h_s}{h}\Sigma_Q\Sigma'_s+\frac{h_u}{h}\Sigma_Q\Sigma'_u\right)'\left(\Sigma_Q\Sigma'_Q\right)^{-1}\left(\frac{\mu_Q}{\gamma_I}+\frac{h_s}{h}\Sigma_Q\Sigma'_s+\frac{h_u}{h}\Sigma_Q\Sigma'_u\right)+\bar{g}^{\frac{1}{\gamma_I}},$$

where all functions are the same as in the equilibrium with indexing. This condition holds if

$$\bar{g} > \bar{g}_{min} = \max_{s\in[0,1],\, u\in[0,1]} \bar{g}(s,u)^{\gamma_I},$$

where

$$\bar{g}(s,u) = \frac{(\gamma_I-1)h}{2}\left[\left(\frac{\mu_Q}{\gamma_I}+\frac{h_s}{h}\Sigma_Q\Sigma'_s+\frac{h_u}{h}\Sigma_Q\Sigma'_u\right)'\left(\Sigma_Q\Sigma'_Q\right)^{-1}\left(\frac{\mu_Q}{\gamma_I}+\frac{h_s}{h}\Sigma_Q\Sigma'_s+\frac{h_u}{h}\Sigma_Q\Sigma'_u\right)\right.$$
$$\left.-\frac{1}{\Sigma_I\Sigma'_I}\left(\frac{\mu_I}{\gamma_I}+\frac{h_s}{h}\Sigma_I\Sigma'_s+\frac{h_u}{h}\Sigma_I\Sigma'_u\right)^2\right]+1. \quad \text{(IE16)}$$

This completes the proof. Q.E.D.

To demonstrate how the condition on $\bar{g}$ from Proposition IE1 applies in the numerical example presented in the paper, we compute $\bar{g}(s,u)$ and plot $\bar{g}(s,u)-1$ in Fig. IE.1.

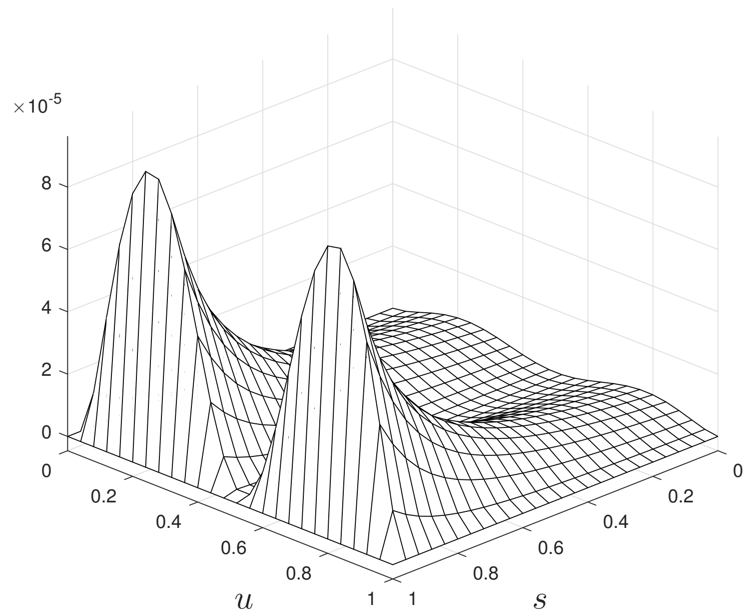

Figure IE.1: This figure plots the function $\bar{g}(s,u) - 1$ with $\bar{g}(s,u)$ from equation (IE16). The model parameters are as follows: $\mu_{D1} = \mu_{D2} = 0.018$, $\Sigma_{D1} = [0.045 \;\; 0]$, $\Sigma_{D2} = [0 \;\; 0.045]$, $\beta = 0.03$, $\gamma_I = 5$, and $\gamma_I = 5$.

The figure demonstrates that, as expected, $\bar{g}(s,u) \geq 1$ in all states, so investors hold the index instead of the individual stocks only when they receive disutility from the latter choice. However, the magnitude of disutility that justifies indexing is small: investors shun the individual stocks in all states if $\bar{g} > 1.0001$. This observation is consistent with a relatively small welfare loss produced by switching from trading two stocks to trading the index that we find in Section 3.4. It also demonstrates that the equilibrium with indexing constructed in the main part of the paper endogenously arises for a quantitatively realistic cost of complexity.

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