Online Appendix to "Turning Alphas into Betas: Arbitrage and the Cross-Section of Risk"

Thummim Cho*

London School of Economics

September 2019

Abstract

The outline is as follows. Section 1 provides a simple model of multi-asset arbitrage in which predictions tested in the paper arise as analytical results, and Section 2 provides detailed derivation and proofs. Section 3 provides further robustness checks. Section 4 provides additional figures that may be useful in understanding my main results. Section 5 explains the construction of anomaly signals in detail. Finally, Section 6 explains how to construct the Adrian, Etula, and Muir (2014) funding-liquidity ("leverage") factor.

*Department of Finance, London School of Economics and Political Science, Houghton Street, London, UK. Email: t.cho@lse.ac.uk. I thank James Guo for replicating my construction of anomaly portfolios.

1 Theory

The economy has three periods ($t = 1, 2, 3$) and two types of security: a risk-free bond and a continuum of anomaly assets $i \in [0, 1]$. The risk-free bond is supplied elastically at a zero interest rate; hence excess return equals return. An anomaly asset ("asset") is a claim to a stream of dividends $\{d_{i,t}\}$ over $t \in \{2, 3\}$ and has a zero net supply.[1] The dividends are conditionally i.i.d. across assets with mean $v > 0$; hence there is zero aggregate cash-flow risk.

There are two types of investors: behavioral investors and a representative arbitrageur. Behavioral investors generate negative distortions in asset demands that push asset prices downward.[2] Importantly, these distortions are constant over time but *increasing* in magnitude in i. I model this as a distortion $-\phi i$ in the aggregate behavioral investor demand for asset i (in units of wealth) at time $t \in \{1, 2\}$:

$$B_{i,t} = E_t \left[r^e_{i,t+1} \right] - \phi i, \tag{1}$$

with $E_t \left[r^e_{i,t+1} \right]$ denoting the *objective* conditional expected (excess) return and $\phi > 0$. Besides the distortion, the demand function has three additional features:

- Demand falls as price rises (since price is inversely related to expected return).
- "Narrow framing" in that covariances do not matter (Barberis, Huang, and Thaler, 2006). This feature simplifies the model solution.
- Equal "size" or "liquidity" of all assets. That is, a marginal increase in the arbitrage position lowers the equilibrium expected return by an amount that is constant across all assets.[3]

A representative, risk-neutral arbitrageur with mass μ trades to maximize the expected wealth at time 3 but faces a capital constraint. Specifically, the arbitrageur is not short-sale constrained but faces a margin rate of one in all positions, which prevents the arbitrageur from raising cash by shorting an asset.[4] The arbitrageur can borrow up to an exogenous stochastic funding con-

[1] The zero net supply assumption is for simplicity; assuming a positive net supply does not affect the model's analytical results.

[2] The direction of the distortions is chosen for convenience and does not affect the model's predictions. Furthermore, I do not specify the reason for this distortion, which can be behavioral (e.g., sentiment) or rational (e.g., local risk to behavioral investors that arbitrageurs are willing to share).

[3] Suppose $\mu x_{i,t}$ is the aggregate arbitrageur demand for asset i at time t. Since market clearing implies $\mu x_{i,t} = -B_{i,t}$, $\partial E_t \left[r^e_{i,t+1} \right] / \partial \left(\mu x_{i,t} \right) = -\partial E_t \left[r^e_{i,t+1} \right] / \partial B_{i,t} = -1$ for all assets.

[4] This is analogous to how actual arbitrageurs, such as hedge funds, are not short-sale constrained but face a

straint $f_t \in [0, \infty)$ and additionally faces exogenous shocks to its wealth w_t, both of which are independent of dividends $\{d_{i,t}\}$ and generate shocks to the level of deployable capital of the unit arbitrageur ("arbitrage capital") k_t:[5]

$$k_t = w_t + f_t. \tag{2}$$

The presence of these shocks and the possibility of a binding capital constraint make the risk-neutral arbitrageur behave in a risk-averse manner through the intertemporal speculative motive (Merton, 1973). Finally, the arbitrageur faces a non-negativity constraint: a negative realized wealth forces the arbitrageur to exit the market immediately and pay an interest cost $c \geq \phi$ on the negative wealth in all future periods.[6]

To summarize, the arbitrageur's objective at time t is to choose asset positions $\boldsymbol{x}_t$ to maximize $E_t[w_3]$ s.t.

$$\begin{gathered} w_{t+1} = \begin{cases} w_t + \int_0^1 r_{i,t+1} x_{i,t} di + \tilde{w}_{t+1} & \text{if } w_t > 0 \\ (1+c)\, w_t & \text{if } w_t \leq 0 \end{cases} \\ \int_0^1 |x_{i,t}|\, di \leq \mathbf{1}(w_t > 0)\, k_t \\ k_t = w_t + \mathbf{1}(w_t > 0)\, f_t, \end{gathered} \tag{3}$$

where $\boldsymbol{x}_t$ is the unit arbitrageur's sequence of dollar positions on all assets over all remaining trading periods, $\tilde{w}_t$ is the wealth shock, $r_{i,t}$ is the asset return, and $\mathbf{1}(\cdot)$ is an indicator function.

I look for a competitive equilibrium in which (i) the aggregate behavioral investor demand $\{B_{i,1}\}$ and $\{B_{i,2}\}$ satisfy Eq. (1) given prices, (ii) the arbitrageur's chosen positions $\{x_{i,1}\}$ and $\{x_{i,2}\}$ solve problem (3) given prices, and (iii) all asset markets clear such that

$$\mu\, x_{i,t} + B_{i,t} = 0 \;\forall i, t. \tag{4}$$

I analyze the three-period equilibrium under two different assumptions about the arbitrageur's mass μ: the trivial "pre-arbitrage" equilibrium with $\mu = 0$ and the more interesting "post-arbitrage" equilibrium with $\mu = \frac{1}{2}\phi$. These two equilibria respectively capture sample periods before and

nonzero margin requirement. However, I hold the margin rate fixed rather than make it a function of asset volatility, as in Brunnermeier and Pedersen (2009) and Gromb and Vayanos (2018), to emphasize that arbitrage-driven betas can arise without differences in fundamental volatility.

[5]As the reader will see, the funding channel and the wealth channel play an identical role in the model, but I keep both channels for a tighter link to my empirical results.

[6]This allows me to obtain the arbitrageur's marginal value of wealth in the negative-wealth region.

after the growth of arbitrage on the assets. These two sets of three-period equilibria roughly correspond to the pre-1993 and post-1993 periods in the empirical analysis.

1.1 The pre-arbitrage equilibrium

It follows from the model setup that in the three-period "pre-arbitrage" economy with a negligible mass of arbitrageurs ($\mu = 0$), the assets feature different alphas but no systematic risk. (All proofs and derivations are in Section 2.)

Lemma 1. ***(Asset returns in the pre-arbitrage economy).*** *If* $\mu = 0$*, excess return on asset* i *is*

$$r^e_{i,t} = \phi i + \epsilon_{i,t} \tag{5}$$

$\forall t$ *where* $\epsilon_{i,t}$ *is a mean-zero idiosyncratic return and the pre-arbitrage abnormal return or "pre-arbitrage alpha,"*

$$\alpha_i^{pre} \equiv \phi i, \tag{6}$$

increases monotonically from asset $i = 0$ *to asset* $i = 1$*.*

This (trivial) result implies that the unobserved demand distortion ϕi is revealed in the abnormal return or "alpha" in the pre-arbitrage economy, α_i^{pre}, which continues to proxy for demand distortions latent in the post-arbitrage economy. This identification of the post-arbitrage-economy demand distortion using the pre-arbitrage alpha is valid up to the cross-sectional ordering if the relative ordering of the distortion is invariant over the two economies.[7]

1.2 The post-arbitrage equilibrium

Next, consider the three-period "post-arbitrage" economy in which the arbitrageur has a non-negligible mass of $\mu = \frac{1}{2}\phi$. Even in this economy, if the arbitrageur is always unconstrained with sufficient capital ($k_1, k_2 \geq 1$), all alphas are arbitraged away and no endogenous arbitrage-driven risk arises.

Lemma 2. ***(Asset returns with unconstrained arbitrageurs).*** *Suppose* $\mu = \frac{1}{2}\phi$ *and* $k_1, k_2 \geq 1$

[7] I maintain this assumption in my empirical tests using pre-arbitrage alphas. This is likely to be true despite the growth of institutional capital in the stock market if mutual fund managers exhibit behavioral patterns similar to those of retail investors (Frazzini, 2006; Frazzini and Lamont, 2008).

with certainty so that the arbitrageur is always unconstrained. Then, excess return on asset i is

$$r^e_{i,t} = \epsilon_{i,t} \tag{7}$$

$\forall i, t$ where $\epsilon_{i,t}$ is a mean-zero idiosyncratic return.

Hence, with the frictionless "textbook" arbitrage, assets that are subject to different degrees of demand distortion become effectively identical riskless assets. Comparing Eqs. (7) and (5), the pre-arbitrage alpha has disappeared completely with no emergence of endogenous risk.

However, the more realistic case is if, during the arbitrage, the level of arbitrage capital may fall below the value required to counteract all demand distortions, an assumption I maintain from hereon:[8]

Assumption 1. *$\mu = \frac{1}{2}\phi$ so that the arbitrageur is large and k_2 is in $[0, 1]$ with positive (conditional) probability so that the arbitrageur may be constrained during arbitrage.*

In this case, asset returns from time 1 to time 2 follow a factor structure, comoving endogenously with k_2. Since an asset's return beta with respect to k_2 arises as a result of arbitrage trades, I call it an "arbitrage-driven" beta.[9]

Lemma 3. ***(Asset returns with constrained arbitrageurs).*** *Under Assumption 1, the expected excess return on asset i from time 1 to time 2 approximately follows*

$$E_1 r^e_{i,2} = \alpha_{i,2} + \lambda_k \beta_{i,k}, \tag{8}$$

where $\beta_{i,k}$ is an "arbitrage-driven" beta with respect to k_2 defined as

$$\beta_{i,k} = \frac{Cov_1\left(r^e_{i,2}, k_2\right)}{Var_1\left(k_2\right)} \tag{9}$$

[8] The case in which k_1 may be below 1 but $k_2 \geq 1$ is not considered explicitly, since this case is analogous to the "perfect arbitrage" case except for a positive return from time 1 to time 2.

[9] Since the level of arbitrage capital k is the state variable in the model, the stochastic discount factor in this economy is a nonlinear function of k. I therefore state an approximate factor model with respect to k rather than the exact model with respect to the stochastic discount factor for better intuition. An analogous result for the exact factor model is available in Section 2.

with $\beta_{i,k} > 0\ \forall i \in (0,1]$, $\lambda_k > 0$ is the price of risk associated with k_2, and $\alpha_{i,2} \geq 0$ is the deviation of $E_1 r^e_{i,2}$ from risk premium $\lambda_k \beta_{i,k}$ that arises if the arbitrageur's shadow cost of capital is different from the risk-free rate. Since mispricing disappears at time 3 with certainty, the expected excess return on asset i from time 2 to time 3 is

$$E_2 r^e_{i,3} = \alpha_{i,3}, \tag{10}$$

where $\alpha_{i,3} \geq 0$ is the deviation of $E_2 r^e_{i,3}$ from zero risk premium and no arbitrage-driven beta arises.

Lemma 3 is intuitive. Equation (8) states the "limits of arbitrage" result of Shleifer and Vishny (1997) in a beta pricing framework. Arbitrage that requires capital is endogenously risky since the price of the arbitraged asset comoves with the level of arbitrage capital during arbitrage; i.e., since $\beta_{i,k}$ is positive for the arbitraged assets.[10] Although the arbitrageur is risk neutral, the arbitrageur perceives this beta as risk due to the intertemporal speculative motive; asset return tends to drop precisely when arbitrage capital drops and investment opportunity improves. However, this result does not depend on the risk preference, since a low-k state remains a high-marginal-value-of-wealth state under other risk preferences, as I explain further at the end of the section.

Comparing Lemma 3 with Lemma 2 shows that the entire cross-section of betas in Lemma 3 arises from frictions in the arbitrage, justifying the label "arbitrage-driven" beta. Furthermore, Eq. (10) shows that arbitrage-driven betas do not arise when demand distortion is about to disappear and asset prices are about to converge to the fundamental value, since the return on the asset would not comove with arbitrage capital in the next period. Hence, arbitrage-driven betas would not arise in assets or portfolios with a short mispricing horizon (Gromb and Vayanos, 2018).

In both Eqs. (8) and (10), the abnormal return $\alpha_{i,t}$ can differ across assets when the arbitrageur is constrained, violating the law of one price (e.g., Gârleanu and Pedersen, 2011; Geanakoplos and Zame, 2014). A positive margin rate means that when the capital constraint binds, the arbitrageur would not equalize all abnormal returns if doing so through a long-short trade makes less money than other trades the arbitrageur currently engages in. For example, if the arbitrageur's shadow cost of capital is 3% and the margin rate is 50%, the arbitrageur would not engage in a long-short trade on two portfolios with 1% and -1% abnormal returns and identical factor exposures to earn

[10] Also see Gromb and Vayanos (2018) and Kondor and Vayanos (2019).

a 2% return.

Since wealth w and funding conditions f determine the level of arbitrage capital, the beta pricing model in Eq. (8) can be restated in terms of wealth and funding betas:

Lemma 4. ***(Decomposing the arbitrage-capital beta).*** *Eq. (8) in Lemma 3 can be restated as*

$$E_1 r^e_{i,2} = \alpha_{i,2} + \lambda_w \beta_{i,w} + \lambda_f \beta_{i,f} \tag{11}$$

where $\beta_{i,w}$ and $\beta_{i,f}$ are betas with respect to $w_2 = w_1 + \int_0^1 r_{i,2}x_{i,2}di + \tilde{w}_2$ and f_2, respectively.

Hence, arbitrage-driven betas can arise with respect to two kinds of systematic factors. First, they arise with systematic shocks to the arbitrageur's wealth w_2 coming from the assets being arbitraged ($\int_0^1 r_{i,2}x_{i,2}di$) and from more exogenous shocks such as fund flows to institutional arbitrageurs ($\tilde{w}_2$). Hence an arbitraged asset with no prior factor exposure can attain betas with factors that other arbitraged assets are exposed to. Second, they arise with systematic funding shocks f_2. An arbitraged asset with no prior factor exposure can attain betas with factors that determine arbitrageur funding conditions. Restrictions on $\beta_{i,k}$ derived below apply analogously to both $\beta_{i,w}$ and $\beta_{i,f}$, but not necessarily to other systematic factors in the market.

1.3 Cross-sectional predictions

I present cross-sectional predictions of the model that allow us to detect endogenous betas with respect to arbitrage-capital shocks (dubbed "arbitrage-driven" betas) in the data. All comparative statistics are taken with respect to i, the degree of demand distortion.

Proposition 1. ***(Cross-section of arbitrage positions).*** *Expected arbitrage position increases in the demand distortion in the asset: $\frac{\partial E_1[\mu x_{i,2}]}{\partial i} > 0$.*

Proposition 1 clarifies the channel through which an asset with a larger demand distortion develops a larger arbitrage-driven beta. A larger demand distortion means that a larger fraction of the market capitalization of the asset is owned by the arbitrageur, so the price of the asset is more sensitive to the variation in the aggregate arbitrage capital. Hence, such an asset has a higher arbitrage-driven beta than other assets.

Although intuitive, Proposition 1 is not completely satisfactory since arbitrage position—the right-hand variable determining the level of arbitrage-driven beta—is itself an endogenous quantity determined in equilibrium. The next proposition shows that it is ultimately the asset's demand distortion that determines its arbitrage-driven beta. A larger demand distortion from the arbitrageur's perspective means that the arbitrageur plays a larger price-correcting role in the asset in equilibrium through a larger arbitrage position, which results in a higher arbitrage-driven beta. This demand distortion may be unobserved by the econometrician but is revealed by the pre-arbitrage alpha. The next proposition restates Proposition 1 using this "instrument" for the arbitrage position.

Proposition 2. ***(Cross-section of arbitrage-driven betas).*** *Arbitrage-driven beta increases in the magnitude of the demand distortion in the asset* $\frac{\partial \beta_{i,k}}{\partial i} > 0$. *Since pre-arbitrage alpha* $\alpha_i^{pre} = \phi i$ *is a scaled multiple of* i, *it follows that arbitrage-driven beta increases in pre-arbitrage alpha; that is, "alphas turn into betas."*

I illustrate Proposition 2 using an example. Consider assets A and B, which are claims to some deterministic payoff of \$10 in present value. Suppose also that, absent arbitrage capital, demand distortions in behavioral investors drive down the prices of the assets to $P_A = \$5$ and $P_B = \$8$, creating "pre-arbitrage" alphas of $\alpha_A = 100\%$ and $\alpha_B = 25\%$. Now suppose that arbitrageurs begin trading these assets but their capital loads positively on some factor k. Then in "normal" arbitrage times, the arbitrageurs would drive up P_A and P_B to nearly \$10. However, if during the arbitrage, a large negative-k shock depletes the arbitrage capital completely, P_A and P_B would drop 50% (\$10 to \$5) and 20% (\$10 to \$8) respectively, assuming that the behavioral investors' demand distortion stays. Hence, precisely because A has a larger pre-arbitrage alpha and arbitrageurs play a larger price-correcting role in the asset in normal times, A has a larger endogenous sensitivity to (i.e., higher beta with) factor k than B.[11]

Figure 1 illustrates Lemma 1 as well as propositions 1 and 2 to show that the model generates patterns observed in the data. In the pre-arbitrage economy, the assets have zero betas with respect to k_2 irrespective of their pre-arbitrage alphas, since arbitrageurs are too small to generate price pressure on the assets. However, in the post-arbitrage economy, the assets obtain a cross-section of different betas with k_2 that line up with their pre-arbitrage alpha or expected arbitrage position.

[11] And this endogenous risk means that P_A and P_B would actually be lower than \$10 even with sufficiently large arbitrage capital, except in the period immediately before the deterministic payoff.

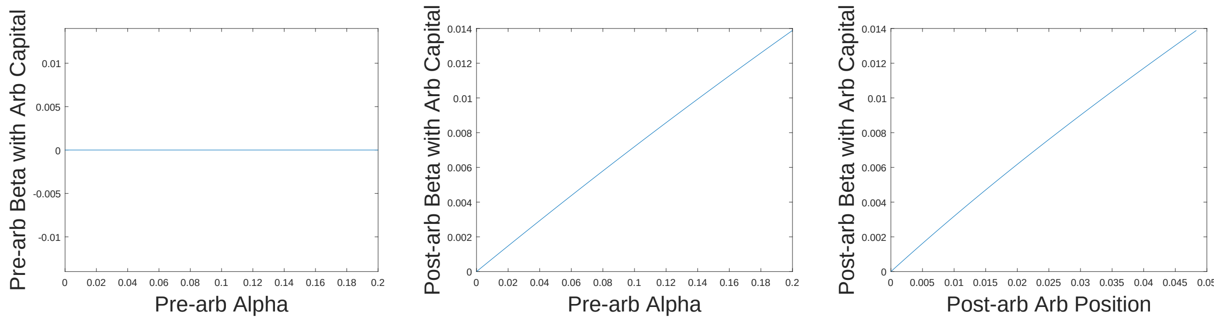

Figure 1: **"Turning Alphas into Betas" in the Model**

The first figure shows that assets' betas with respect to arbitrage capital in the pre-arbitrage economy cluster around zero. The next two figures show that the assets' arbitrage-capital betas in the post-arbitrage economy are explained by their pre-arbitrage alpha and expected arbitrage position. Parameter values used: $\phi = 0.2$, $\mu = \phi/2$, $k_2 \sim U[-10, 10]$, $k_1 \geq 1$, $c = 0.5$, and $\delta_{i,t}/v \sim N(0, 0.1)$.

Next, a useful restriction on arbitrage-driven betas is that the cross-section of different arbitrage-driven betas comes from the constrained states of time 2. Put differently, an arbitrageur does not generate endogenous βs in the assets when he has a "deep pocket," which was the case in Lemma 2:

Proposition 3. ***(Arbitrage-driven betas arise when the arbitrageur is constrained).*** *Arbitrage-driven betas arise only when the arbitrageur is constrained. That is,*

$$\begin{aligned} Cov_1\left(r^e_{i,2}, k_2 | k_2 \geq 1\right) &= 0 \\ Cov_1\left(r^e_{i,2}, k_2 | k_2 < 1\right) &> 0 \end{aligned} \tag{12}$$

for all $i \in (0, 1]$. For this reason, if k_t follows a process such that $k_1, k_2 \geq 1$ almost surely, then neither beta nor abnormal return arises:

$$\beta_{i,k} = 0 \textit{ and } E_1\left[r_{i,2}\right] = 0 \textit{ for all } i \in [0, 1]\,. \tag{13}$$

Although intuitive, the exact statement of Proposition 3 relies on the assumption that the arbitrageur is risk-neutral and dividends are i.i.d. However, a weaker version of the proposition would hold under risk aversion and undiversifiable dividends (in which case arbitrageurs would not correct asset prices completely despite high k_2): the arbitrage-driven beta is lower if k_2 is expected to be higher. Intuitively, the arbitrageur's optimization implies that $p_{i,2}$, a non-decreasing function of k_2, is capped at v. Hence $\partial p_{i,2}/\partial k_2$ should approach zero as k_2 increases, which implies a decreasing price sensitivity to k_2 for higher values of k_2.

Testing Proposition 3 requires empirically identifying constrained vs. unconstrained periods. In this model of time-varying arbitrage capacity, abnormal returns ($\alpha_{i,2}$ in Lemma 3) emerge only when the arbitrageur is constrained, providing one approach to identifying constrained periods for the arbitrageur.

Next, arbitrage-driven betas are "discount-rate" betas. They arise because a positive arbitrage-capital shock increases the *valuation*—as opposed to expected cash flows—of an underpriced asset while a negative arbitrage-capital shock lowers it. It follows that return predictability in the post-arbitrage economy increases in the demand distortion in the asset.

Proposition 4. ***(Cross-section of time-series return predictability).*** *Arbitrage-driven betas are discount-rate betas. Hence return predictability measured by the R^2 increases in the demand distortion in the asset i.*

$$\frac{\partial R_i^2}{\partial i} > 0,$$

where $R_i^2 = Var_1\left(E_2 r_{i,3}^e\right)/Var_1\left(r_{i,3}^e\right)$.

Intuitively, an asset with a larger demand distortion and therefore larger arbitrage-driven beta has a larger discount-rate variation generated by arbitrage capital. Hence, if return volatility unassociated with arbitrage activity is constant—as in this model—or similar across the assets, the explained part of the asset return increases in arbitrage-driven beta. It is important to note that the R^2 increases in the *absolute value* of the arbitrage-driven beta or its determinants. In a return predictive regression, the conditioning information can be either the level of arbitrage capital k_2 or past return $r_{i,2}$. I use the latter in my empirical tests.

Except for one part of Proposition 4 that relates predictability to the arbitrage position and the pre-arbitrage alpha, testing the previous propositions requires computing betas with respect to factors that arbitrage capital loads on. This means having to take a stance on which factor generates systematic shocks to arbitrage capital. However, one can circumvent this problem by observing asset returns during a severe crash of arbitrage capital, which reveals the assets' betas with respect to arbitrage capital shocks. Therefore, if arbitrage-capital beta increases in the asset's demand distortion (Propositions 1 and 2), the asset return response to the crash should also line up cross-sectionally with the arbitrage position and the pre-arbitrage alpha that proxy the distortion. The next proposition formalizes this idea, focusing on the case in which the arbitrageur is unconstrained at time 1 to state an analytical result.[12]

Proposition 5. ***(Cross-section of asset returns during a crash in arbitrage capital).*** *Asset return during a crash of arbitrage capital decreases in the asset's demand distortion. Specifically, if k_1 is sufficiently large, a crash in k_2 that leads to a complete unwinding of arbitrage positions on assets $[0, i_2^*]$ generates negative returns on these assets that increase in magnitude in i:*

$$r_{i,2}^e < 0 \textit{ and } \frac{\partial r_{i,2}^e}{\partial i} < 0 \quad \forall i \in [0, i_2^*] \, .$$

Furthermore, since these asset returns are discount-rate shocks, asset returns going forward display the opposite pattern:

$$\frac{\partial E_2 \left[r_{i,3}^e \right]}{\partial i} > 0 \quad \forall i \in [0, i_2^*] \, .$$

Proposition 5 implies that a large enough crash in arbitrage capital generates negative returns in almost all arbitraged assets and that the magnitude of the return response is greater in assets with larger demand distortions since the arbitrageur plays a larger price-correcting role in the assets. This allows me to test the predicted relationship between measures of demand distortion and arbitrage-capital beta without having to identify an arbitrage-capital factor.

1.4 Discussion

Although the model makes a few simplifying assumptions to deliver a simple framework, these assumptions are relatively innocuous in that the predictions I draw from the model are likely to survive various model extensions.

First, as in Shleifer and Vishny (1997), Brunneremeir and Pedersen (2009), and Brunnermeier and Sannikov (2014), the arbitrageur in my model is risk-neutral but perceives the arbitraged assets to be endogenously risky since shocks to arbitrage capital at time 2 makes the arbitrageur's marginal value of wealth (MVW) stochastic at time 2 and covary negatively with returns on the arbitraged assets. Adding risk aversion would not change the endogenous negative relationship

[12] See Lemma 7 in Section 2 for the exact condition.

between the MVW and returns on the arbitraged assets that generates the arbitrage-driven betas. In the presence of a margin constraint, a large negative arbitrage-capital shock during arbitrage would still force the arbitrageur to unwind its positions in the assets, generating negative asset returns. And low capital would still mean high marginal value of wealth.

Second, the cross-sectional relationship between demand distortion and arbitrage-driven beta would also remain with risk aversion, even if I reduce the number of assets to be finite so that the arbitrageur cannot disregard idiosyncratic risks. Despite idiosyncratic risk, the arbitrageur would take a larger position on the higher-distortion asset in equilibrium, since equal arbitrage position on two assets that have different demand distortions means that the arbitrageur should marginally increase his position on the higher-distortion asset. Then, the same example as above implies that the price of the higher-distortion asset with a larger arbitrage position would drop more in response to the arbitrage-capital shock.

Finally, arbitrageur wealth shocks in my model are "exogenous" in that they do not come from cash-flow shocks to the arbitraged assets, which are assumed i.i.d. to deliver analytical results. In reality, arbitrage-driven betas can arise with respect to "endogenous" wealth shocks coming from cash-flow shocks to arbitraged assets. Introducing cash-flow shocks (i.e., correlated dividends) to the model would not change the analytical result, since dividends are part of the arbitrageur portfolio return that determines arbitrageur wealth, and Lemma 4 already shows that arbitrage-driven betas arise with respect to arbitrageur wealth shocks. However, in the pre-arbitrage economy, correlated dividend shocks combined with arbitrageur risk aversion would mean that the pre-arbitrage alpha from the arbitrageur's perspective would need to be computed with respect to the common cash-flow factor.

2 Theory Appendix

2.1 Solving the pre-arbitrage equilibrium

Proof of Lemma 1 (Asset returns in the pre-arbitrage economy). Since the behavioral investors alone clear the market, Eq. (1) implies $B_{i,t} = 0 \implies \phi i = E_t\left[r^e_{i,t+1}\right]$. Hence, $r^e_{i,t} = E_{t-1}\left[r^e_{i,t}\right] + \epsilon_{i,t} = \phi i + \epsilon_{i,t}$ where $\epsilon_{i,t}$ is a mean-zero idiosyncratic return by the i.i.d.-dividend assumption. Finally, $\alpha_{i,t} = \phi i$ increases in i since $\phi > 0$.[13]

2.2 Solving the post-arbitrage equilibrium

Before proving the rest of the lemmas and propositions, I first solve the post-arbitrage equilibrium, highlighting important steps as new lemmas.

The equilibrium in the post-arbitrage economy with $\mu = \frac{\phi}{2}$ is solved backward from time 2, which represents the period immediately before mispricings disappear and asset prices converge to their fundamental value. Hence arbitrageurs at time 2 invest all available capital in the mispriced assets without worrying about asset returns covarying with the level of arbitrage capital in the future. Time 1 represents the earlier periods of arbitrage in which arbitrageurs do worry about asset returns covarying endogenously with their capital before the assets realize their fundamental value. The asset prices at time 1 therefore take this endogenous risk into account.

To find the equilibrium in each period, I express arbitrageur's optimization problem in (3) as a value function defined at $t \in \{1, 2\}$:

$$\begin{gathered} V_t\left(w_t, f_t\right) = \max_{\{x_{i,t}\}}\ E_t\left[V_{t+1}\left(w_{t+1}, f_{t+1}\right)\right] \quad \text{s.t.} \\ \int_0^1 \left|x_{i,t}\right| di \leq \left(w_t + f_t\right) \\ w_{t+1} = w_t + \int_0^1 \left(\frac{p_{i,t+1} + d_{i,t+1}}{p_{i,t}} - 1\right) x_{i,t} di + \tilde{w}_{t+1} \\ V_3 = w_3 \end{gathered} \tag{14}$$

in the non-default state ($w_t > 0$), and

$$V_t = (1 + c)^{3-t}\, w_t \tag{15}$$

in the default state ($w_t \leq 0$). Then, equilibrium prices at time 2 are given by the following lemma:

Lemma 5. ***(Time-2 equilibrium prices).*** *The equilibrium price of asset i at time 2 is*

$$p_{i,2} = m_{i,3} v \tag{16}$$

[13] To solve for prices, since the riskless rate is zero, $\phi i = E_t\left[r^e_{i,t+1}\right] = E_t\left[r_{i,t+1}\right] \implies p_t = E_t\left[\frac{1}{1+\phi i}\left(p_{i,t+1} + d_{i,t+1}\right)\right]$. That is, price at time t is the price and dividend at time $t+1$ discounted by the asset-specific constant discount factor $\frac{1}{1+\phi i}$ imposed by behavioral investors.

s.t. *(i)* $m_{i,3} = m_3^A = \frac{1}{1+\phi i_2^*}$ *for the "exploited" assets* $i \in (i_2^*, 1]$.
(ii) $m_{i,3} = m_{i,3}^B = \frac{1}{1+\phi i}$ *for the "unexploited" assets* $i \in [0, i_2^*]$.
(iii) i_2^* *is the marginal asset s.t.* $i_2^* = 1,\ 1 - \sqrt{k_2}$, *and* 0 *for* $k_2 \in (-\infty, 0]$, $(0, 1)$, *and* $[1, \infty)$, *respectively.*
(iv) For completeness, the equilibrium arbitrage position is $x_{i,2} = i - i_2^*$ *for* $i \geq i_2^*$ *and* $x_{i,2} = 0$ *for* $i < i_2^*$.

Proof. The arbitrageur's value function at time 2 in the non-default state ($w_2 > 0$) is

$$V_2 = w_2 + \max_{\{x_{i,2}\}} \left\{ \int_0^1 E_2 \left[r_{i,3} \right] x_{i,2} di + \psi_2 \left[w_2 + f_2 - \int_0^1 |x_{i,2}| \, di \right] \right\} \tag{17}$$

where ψ_2 is the shadow cost of capital at time 2 such that $\psi_2 = 0$ ($\psi_2 > 0$) if the arbitrageur is unconstrained (constrained). Since the arbitrageur takes nonnegative positions in the assets in equilibrium,[14] arbitrageur's optimal demand for asset i requires the following first order condition (FOC) with respect to $x_{i,2}$ inside the value function (17):

$$E_2 \left[r_{i,3} \right] \leq \psi_2 \quad \left(E_2 \left[r_{i,3} \right] = \psi_2 \text{ if and only if } x_{i,2} > 0 \right) \tag{18}$$

The arbitrageur faces a supply curve implied by (1) and (4):

$$E_2 \left[r_{i,3} \right] = \phi i - \frac{\phi}{2} x_{i,2} \tag{19}$$

Hence on assets $i \in (\frac{\psi_2}{\phi}, 1]$, the arbitrageur takes a strictly positive position $x_{i,2} = 2\left(i - \frac{\psi_2}{\phi}\right)$ implied by (18) and (19). On assets $i \in \left[0, \frac{\psi_2}{\phi}\right]$, the arbitrageur takes a zero position ($x_{i,2} = 0$) since $E_2 \left[r_{i,3} \right] |_{x_{i,2}=0} = \phi i \leq \psi_2$. Let $i_2^* \equiv \frac{\psi_2}{\phi}$ denote the marginal asset. If $k_2 \in (0, 1)$ such that the arbitrageur's capital constraint binds, the arbitrageur positions on different assets should integrate to k_2, allowing us to solve for i_2^* (and hence ψ_2) as a function of k_2:

$$k_2 = \int_0^1 x_{i,2} di = \int_{i_2^*}^1 x_{i,2} di = 2 \int_{i_2^*}^1 \left(i - i_2^* \right) di = \left(1 - i_2^* \right)^2 \implies i_2^* = 1 - \sqrt{k_2} \tag{20}$$

if $k_2 \in (0, 1)$. If $k_2 \in (-\infty, 0]$, no asset is exploited so that $i_2^* = 1$ and $E_2 \left[r_{i,3} \right] = \phi i \ \forall i$. If $k_2 \in [1, \infty)$, all assets are fully exploited so that $i_2^* = 0$ and $E_2 \left[r_{i,3} \right] = 0 \ \forall i$.

[14] If not, (1) ensures that the short position generates a negative expected return, which is dominated by the riskless rate of 0.

Intuitively, if $k_2 \geq 1$ and thus $i_2^* = 0$, the arbitrageur has enough capital to restore all asset prices to the correct level v. If $k_2 \leq 0$ and $i_2^* = 1$, all assets are priced by the behavioral investors. If $k_2 \in (0,1)$, the arbitrageur trades some assets but faces a capital constraint. In this case, the risk-neutral arbitrageur equalizes the expected return on all exploited assets $(i_2^*, 1]$ to ϕi_2^*, the arbitrageur's shadow cost of capital. The lower-i assets $[0, i_2^*]$ remain unexploited since their expected return is lower than ϕi_2^* even without arbitrage.

The equilibrium time-2 prices in Lemma 5 offer a glimpse into why high-i assets become endogenously riskier in this post-arbitrage equilibrium. It is because the prices of high-i assets respond more to the variation in k_2; as k_2 ranges from 0 to 1, the price of asset i rises from $\frac{v}{1+\phi i}$ to v, implying a ϕi-percent increase in its price. The intuition is that the an initially more-mispriced asset relies more heavily on the price-correcting role of arbitrage capital, which makes its price more sensitive to the variation in the level of arbitrage capital.

Next, to solve for equilibrium time-1 prices, I first show that the arbitrageur's marginal value of wealth at time 2 falls as k_2 rises:

Lemma 6. ***(Time-2 marginal value of wealth).*** *The arbitrageur's value function at time 2 is*

$$V_2 = \Lambda_2 w_2 \tag{21}$$

where the marginal value of wealth in the non-default state ($w_2 > 0$) is $\Lambda_2 = 1 + \phi i_2^*$ and that in the default state ($w_2 \leq 0$) is $\Lambda_2 = 1 + c$.

Proof. First, consider $w_2 > 0$. The derivative of the value function (17) with respect to w_2 gives $\Lambda_2 = 1 + \psi_2$. For ψ_2 , the derivative with respect to any exploited asset's $x_{i,2}$ within the bracket implies $\psi_2 = E_2[r_{i,3}] = \phi i_2^*$, where the second equality follows from equation (16). Next, Λ_2 for $w_2 \leq 0$ follows from equation (15). Finally, $V_2 = \Lambda_2 w_2$ since Lemma 5 implies that the marginal value of wealth $\Lambda_2 = 1 + \psi_2 = 1 + \phi i_2^*$ is also the average return on wealth in the non-default state and $w_3 = (1+c)\, w_2$ in the non-default case.

Lemma 6 implies that a low-k_2 state is a "bad" state in which the arbitrageur's marginal value of wealth is high: Λ_2 rises from 1 to $1 + \phi$ and to $1 + c$ as k_2 decreases from ∞ to 0^+ and to $-\infty$. This inverse relationship between Λ_2 and k_2 here is not driven by the preference for risk or intertemporal substitution, similarly to how the decreasing marginal utility of consumption does not rely on the curvature of the utility function. With risk-neutrality in particular, this happens because arbitrage capital k_2 falls precisely when the investment opportunity ϕi_2^* improves.

Given Lemma 6, the equilibrium price at time 1 depends on the extent to which the asset's return at time 2 covaries with the arbitrageur's marginal value of wealth Λ_2:

Lemma 7. ***(Time-1 equilibrium prices).*** *The equilibrium price of asset i at time 1 is*

$$p_{i,1} = E_1 \left[m_{i,2} \left(p_{i,2} + d_{i,2} \right) \right] \tag{22}$$

s.t. *(i) $m_{i,2} = m_2^A \equiv \frac{\Lambda_2}{\Lambda_1}$ for assets $i \in \boldsymbol{I}_1^*$, where $\boldsymbol{I}_1^*$ denotes the set of exploited assets.*
(ii) $m_{i,2} = m_{i,2}^B \equiv \frac{1}{1+\phi i}$ for the unexploited assets $i \in \boldsymbol{I}_1^*$.
(iii) Λ_1 is the time-1 marginal value of wealth s.t. $\Lambda_1 = E_1 [\Lambda_2] + \psi_1$ where $\psi_1 > 0$ if the arbitrageur is constrained and $\psi_1 = 0$ if the arbitrageur is unconstrained.
(iv) The arbitrageur is unconstrained if k_1 is above some threshold $k_1^* \leq 1$.

Proof. Eq. (14) and Lemma 6 imply that the arbitrageur's value function at time 1 is

$$V_1 = E_1 [\Lambda_2] w_1 + \max_{\{x_{i,1}\}} \left\{ \int_0^1 E_1 [\Lambda_2 r_{i,2}] x_{i,1} di + \psi_1 \left[w_1 + f_1 - \int_0^1 |x_{i,1}| di \right] \right\} \tag{23}$$

where ψ_1 is the shadow cost of capital at time 1 such that $\psi_1 = 0$ ($\psi_1 > 0$) if the arbitrageur is unconstrained. Since the arbitrageur takes nonnegative positions in the assets in equilibrium,[15] the arbitrageur's optimal demand for asset i requires the following FOC with respect to $x_{i,1}$ inside the value function (23):

$$p_{i,1} \geq E_1 \left[\frac{\Lambda_2}{E_1 [\Lambda_2] + \psi_1} \left(p_{i,2} + d_{i,2} \right) \right] \tag{24}$$

($p_{i,1} = \frac{\Lambda_2}{E_1[\Lambda_2]+\psi_1} \left(p_{i,2} + d_{i,2} \right)$ if and only if $x_{i,1} > 0$). Since the FOC of both sides of (23) with respect to w_1 implies $\Lambda_1 \equiv \frac{dV_1}{dw_1} = E_1 [\Lambda_2] + \psi_1$, we have $p_{i,1} = E_1 \left[\frac{\Lambda_2}{\Lambda_1} \left(p_{i,2} + d_{i,2} \right) \right]$ for exploited assets. All unexploited assets (assets such that $x_{i,1} = 0$) are priced by the behavioral investors so that $p_{i,1} = E_1 \left[\frac{1}{1+\phi i} \left(p_{i,2} + d_{i,2} \right) \right]$. To find k_1^*, assume that all assets are exploited and combine (1), (4), and (22) to get $E_1 \left[\frac{\Lambda_2}{E_1[\Lambda_2]} \left(p_{i,2} + d_{i,2} \right) \right] = \frac{E_1[p_{i,2}+d_{i,2}]}{1+\phi\left(i - \frac{1}{2} x_{i,1}\right)}$,

[15] The proof is by contradiction. Suppose $x_{i,1} < 0$. Then the FOC with respect to $x_{i,1}$ inside the value function (23) implies $E_2 [\Lambda_2 r_{i,2}] = -\psi_1 < 0$. However, when $x_{i,1} < 0$, (1) implies $E_1 [r_{i,2}] > \phi i \implies \frac{p_{i,2}}{p_{i,1}} > (1 + \phi i) \frac{p_{i,2}}{E_1[p_{i,2}]} \geq 1$ for any possible realization of $p_{i,2}$ since by Lemma 7, $p_{i,2} \in [v, (1 + \phi i) v]$. Since $p_{i,2}/p_{i,1} \geq 1$ and $\Lambda_2 > 0$, $E_1 [\Lambda_1 r_{i,2}] \geq 0$, which is a contradiction.

or $x_{i,1} = 2\left(i - \frac{1}{\phi}\left[\left(1 + Cov_1\left(\frac{\Lambda_2}{E_1[\Lambda_2]}, \frac{p_{i,2}+d_{i,2}}{E_1[p_{i,2}]}\right)\right)^{-1} - 1\right]\right)$. Rearranging and setting $k_1^* = \int_0^1 x_{i,1} di$ gives

$$k_1^* = 1 - \frac{2}{\phi}\int_0^1 \left\{\left(1 + Cov_1\left(\frac{\Lambda_2}{E_1[\Lambda_2]}, \frac{p_{i,2}+d_{i,2}}{E_1[p_{i,2}]}\right)\right)^{-1} - 1\right\}, \tag{25}$$

which is less than or equal to 1 since $Cov(\Lambda_2, p_{i,2} + d_{i,2}) = Cov(1 + \phi i_2^*, p_{i,2} + d_{i,2}) \leq 0$ $\forall i$ (because $p_{i,2} = v/(1 + \phi i_1^*)$ or $p_{i,2} = v/(1 + \phi i)$ and $i_2^* = 1 - \sqrt{k_2}$ where $k_2 = w_1 + \int_0^1 (p_{i,2} + d_{i,2}) x_{i,1} di$).

Lemma 8. ***(Asset pricing using the arbitrageur's SDF).*** *Under Assumption 1, the expected return on asset i at time 2 follows*

$$E_1 r_{i,2}^e = \alpha_{i,2} + \lambda_m \beta_{i,m} \tag{26}$$

s.t. *(i) $\beta_{i,m}$ is the negative of the beta with respect to the arbitrageur's time-2 stochastic discount factor (SDF), which depends negatively on k_2.*

(ii) $\alpha_{i,2}$ is the asset-specific zero-beta rate that is also the abnormal return by the zero-risk-free-rate assumption.

(iii) $\lambda_m > 0$ and $\beta_{i,m} > 0$ for $i > 0$.

Proof. The expected return formula follows from an algebraic manipulation of Lemma 7 where $\lambda_m > 0$ since k_2 is in $[0, 1]$ with positive probability. $\beta_{i,m} > 0$ is because

$$Cov_1\left(r_{i,2}, m_2^A\right) = Cov_1\left(p_{i,2} + d_{i,2}, m_2^A\right) = Cov_1\left(\frac{v}{m_{i,2}}, m_2^A\right),$$

where $m_{i,2} = m_2^A$ when $i > i_2^*$ and $m_{i,2} = (1 + \phi i)^{-1}$ when $i \leq i_2^*$.

2.3 Proof of lemmas 2-4 and propositions 1-5

Next, I prove the remaining lemmas and propositions in the main body of the paper.

Proof of Lemma 2. (Equilibrium with unconstrained arbitrageurs). Since $k_2 \geq 1$ with certainty, item (iii) of Lemma 5 implies that $\Lambda_2 = 1$ and $i_2^* = 1$ in all states. Hence item (i) of Lemma 5 shows that all assets are completely exploited such that $p_{i,2} = v$ and $r_{i,3}^e = r_{i,3} = \epsilon_{i,3} \equiv d_{i,3}/v$. Similarly, since $k_1 \geq 1$, item (iv) of Lemma 7 implies that k_1 is above the threshold value k_1^* that makes the arbitrageur unconstrained. Hence Lemma 6 and Lemma 7 imply that $m_{i,2} = 1$ for all assets such that $p_{i,1} = E_1[v + d_{i,2}] = v$ and $r^e_{i,2} = r_{i,2} = \epsilon_{i,2} = d_{i,2}/v$.

Proof of Lemma 3. (Asset returns with constrained arbitrageurs). Since $m_2^A = \left(1 + \phi\left(1 - \sqrt{k_2}\right)\right)/\Lambda_1$ at $k \in (0,1)$, a first-order approximation around $\overline{k}_2 \equiv \left(1 - \frac{E_1[\Lambda_2]-1}{\phi}\right)^2$ is $m_2^A \approx E_1\left[m_2^A\right] - \phi\left(2\Lambda_1\sqrt{\overline{k}_2}\right)^{-1}\left(k_2 - \overline{k}_2\right)$. Thus,

$$E[r_{i,2}] = \alpha_{i,2} + \lambda_m \beta_{i,m} \approx \alpha_{i,2} + \underbrace{\frac{\phi Var_1(k_2)}{2\Lambda_1 E_1\left[m_2^A\right]\sqrt{\overline{k}_2}}}_{\equiv\lambda_k} \underbrace{\frac{Cov_1(r_{i,2}, k_2)}{Var_1(k_2)}}_{\equiv\beta_{i,k}}. \tag{27}$$

To see that $\beta_{i,k} > 0$ for $i > 0$, note that $Cov_1(r_{i,2}, k_2) = p_{i,1}^{-1} Cov_1(p_{i,2} + d_{i,2}, k_2) = p_{i,1}^{-1} Cov_1\left(\frac{v}{m_{i,2}}, k_2\right)$, where we know $\frac{\partial m_{i,2}}{\partial k_2} \leq 0$ for $i > 0$. Also, for any random variable X, we know

$$\begin{gathered} Cov(X, f(X)) = E[(X - E[X])(f(X) - E[f(X)])] \\ = \underbrace{E[(X - E[X])(f(X) - f(E[X]))]}_{\geq 0} + \underbrace{E[(X - E[X])(f(E[X]) - E[f(X)])]}_{=0} \\ \geq 0 \end{gathered}$$

if $f'(X) \geq 0$, which is the case when X is k_2 and $f(X)$ is $m_{i,2}(k_2)$.

Proof of Lemma 4. (A factor model of asset returns). Substituting $k_2 = w_2 + f_2$ into eq. (27) gives

$$E[r_{i,2}] \approx \alpha_{i,2} + \underbrace{\frac{\phi Var_1(w_2)}{2\Lambda_1 E_1\left[m_2^A\right]\sqrt{\overline{k}_2}}}_{\equiv\lambda_w} \beta_{i,w} + \underbrace{\frac{\phi Var_1(f_2)}{2\Lambda_1 E_1\left[m_2^A\right]\sqrt{\overline{k}_2}}}_{\equiv\lambda_f} \beta_{i,f}.$$

Proof of Proposition 1. (Cross-section of arbitrage positions). Lemma 5 implies that $E_1[\mu x_{i,2}] = \int_{k_2(i)}^1 \left(\sqrt{k_2} - \sqrt{k_2(i)}\right) dF(k_2) + \int_1^\infty \left(1 - \sqrt{k_2(i)}\right) dF(k_2) = \int_{k_2(i)}^\infty \left[\min\left\{\sqrt{k_2}, 1\right\} - \sqrt{k_2(i)}\right] dF(k_2)$ where $k_2(i)$ is the level of k_2 that makes i the marginal asset. Since $\frac{\partial k_2(i)}{\partial i} < 0$ and $k_2(i) > 0$, $\frac{\partial E_1[\mu x_{i,2}]}{\partial i} = \left(\frac{\partial E_1[\mu x_{i,2}]}{\partial k_2(i)}\right)\left(\frac{\partial k_2(i)}{\partial i}\right) = \left(-\int_{k_2(i)}^\infty \frac{1}{2} k_2(i)^{-1/2} dF(k_2)\right)\frac{dk_2(i)}{di} > 0$.

Proof of Proposition 2. (Cross-section of arbitrage-driven betas). The proof has two steps: first prove that the prices of high-i assets respond more strongly to the variation in arbitrage capital and then prove that this implies that those assets have higher arbitrage capital betas.

(a) For the first step, since $Cov_1(p_{i,2}, k_2) = E_1[p_{i,2}k_2] - E_1[p_{i,2}] E_1[k_2]$,

$$\begin{aligned} Cov_1(p_{i,2}, k_2) = v \int_{-\infty}^{k_2(i)} \tfrac{k_2}{1+\phi i} dF(k_2) + v \int_{k_2(i)}^{\infty} \tfrac{k_2}{1+\phi i_2^*} dF(k_2) \\ -vE_1[k_2] \left(\int_{-\infty}^{k_2(i)} \tfrac{1}{1+\phi i} dF(k_2) + \int_{k_2(i)}^{\infty} \tfrac{1}{1+\phi i_2^*} dF(k_2) \right), \end{aligned}$$

where $k_2(i)$ denotes the value of k_2 that makes i the marginal asset and F is the conditional cumulative density function of k_2. The derivative of the covariance with respect to i gives

$$\frac{\partial Cov_1(p_{i,2}, k_2)}{\partial i} = -v \int_{-\infty}^{k_2(i)} \frac{k_2}{(1+\phi i)^2} dF(k_2) + vE_1[k_2] \int_{-\infty}^{k_2(i)} \frac{1}{(1+\phi i)^2} dF(k_2),$$

where the Leibniz terms cancel out by the fact that $i_2^*(k_2(i)) = i$. Rearranging the terms gives

$$\frac{\partial Cov_1(p_{i,2}, k_2)}{\partial i} = \frac{v}{(1+\phi i)^2} \left(E_1[k_2] - E_1[k_2 | k_2 \leq k_2(i)]\right) F(k_2(i)) > 0.$$

(b) Next, to show how this monotonicity of the price covariance implies $\partial Cov_1(r_{i,2}, k_2) / \partial i > 0$, it suffices to show that the equilibrium time-1 prices are non-increasing in i:

$$\frac{\partial p_{i,1}}{\partial i} \leq 0.$$

To see this, suppose for a contradiction that $A < B$ but $p_{A,1} < p_{B,1}$. Suppose also that B is priced by the arbitrageur so that $p_{B,1} = E_0 \left[\frac{\Lambda_2}{\Lambda_1} p_{B,2}\right]$. Since $p_{A,2} \geq p_{B,2}$ in all states of $t = 2$, it must be that

$$p_{A,1} \geq E_1 \left[\frac{\Lambda_1}{\Lambda_0} p_{A,2}\right] \geq E_1 \left[\frac{\Lambda_1}{\Lambda_0} p_{B,2}\right],$$

which is a contradiction. Now suppose that B is priced by the behavioral investors so that $p_{B,1} = \frac{1}{1+\phi B} E_1[p_{B,2}]$. Again, since $p_{A,2} \geq p_{B,2}$ in all states of $t = 2$, it must be that

$$p_{A,1} \geq \frac{1}{1+\phi A} E_1[p_{A,2}] \geq \frac{1}{1+\phi B} E_1[p_{B,2}],$$

which is also a contradiction. Hence, $p_{i,1}$ is non-increasing in i. Putting these together, we see that $Cov_1(r_{i,2}, k_2)$ is non-decreasing in i:

$$\frac{\partial Cov_1(r_{i,2}, k_2)}{\partial i} > 0.$$

It follows that

$$\frac{\partial \beta_{i,k}}{\partial i} = \frac{1}{Var_1(k_2)} \times \frac{\partial Cov_1(r_{i,2}, k_2)}{\partial i} > 0.$$

Finally, since $\alpha_i^{pre} = \phi i$, it also follows that $\frac{\partial \beta_{i,k}}{\partial i} > 0 \implies \frac{\partial \beta_{i,k}}{\partial(\phi i)} = \frac{\partial \beta_{i,k}}{\partial \alpha_i^{pre}} > 0$.

Proof of Proposition 3. (Arbitrage-driven betas arise when the arbitrageur is constrained). This follows trivially from the analysis in Lemma 2 and from Lemma 3.

Proof of Proposition 4. (Cross-section of time-series return predictability). First, I prove the statement that arbitrage-driven beta is a discount-rate beta:[16]

$$\beta_{i,k}^{DR} \equiv \frac{Cov_1(E_2[r_{i,3}], k_2)}{Var_1(k_2)} < 0 \quad \forall i \in (0,1].$$

Since the expected cash flow at time 3 is fixed, $E_2[r_{i,3}] = \frac{v}{p_{i,2}} - 1 = m_{i,3} - 1$ where $m_{i,3}$ is a non-increasing function of k_2 (and equals one if $i = 0$). Hence $\beta_{i,k}^{DR} < 0$ for $i > 0$. Next, to see the cross-sectional relationship between R^2 and i, note Lemma 5 implies

$$R_i^2 = \frac{Var_1\left(E_2 r_{i,3}^e\right)}{Var_1\left(r_{i,3}^e\right)} = \frac{Var_1\left(E_2 r_{i,3}^e\right)}{Var_1\left(\left(1 + \frac{d_{i,3}}{v}\right) E_2 r_{i,3}^e\right)} = \frac{Var_1\left(E_2 r_{i,3}^e\right)}{Var_1\left(1 + \frac{d_{i,3}}{v}\right)\left(Var_1\left(E_2 r_{i,3}^e\right) + \left(E_1\left[E_2 r_{i,3}^e\right]\right)^2\right)}$$

where the last equality follows from $d_{i,3}$ and $E_2 r_{i,3}^e$ being independent. Since $Var(X) + (E[X])^2 = E[X^2]$ for any random variable X,

$$R_i^2 = \frac{Var_1\left(E_2 r_{i,3}^e\right)}{Var_1\left(1 + \frac{d_{i,3}}{v}\right) E_1\left[\left(E_2 r_{i,3}^e\right)^2\right]}$$

Since $Var_1\left(1 + \frac{d_{i,3}}{v}\right)$ is the same for all assets by the i.i.d. assumption on $d_{i,3}$ and since $Var_1\left(E_2 r_{i,3}^e\right) = E_1\left(\left(E_2 r_{i,3}^e\right)^2\right) - \left(E_1\left(E_2 r_{i,3}^e\right)\right)^2$, it suffices to show that $\left(E_1\left(E_2 r_{i,3}^e\right)\right)^2 / E_1\left[\left(E_2 r_{i,3}^e\right)^2\right]$ is decreasing in i. Applying the formula for $r_{i,3}^e$ from Lemma 5, this is equivalent to proving that the function

$$S(i) \equiv \frac{(E[\min(Z,i)])^2}{E[\min(Z,i)^2]}$$

decreases in $i \in (0,1]$ where $Z = \max\left((1 - \sqrt{\max(k,0)}, 0)\right)$. Then, function $S(i)$ is

[16] I thank Georgii Riabov for help with this proof.

differentiable outside at most countable set of points $i \in (0,1]$ and its derivative is equal to

$$S'(i) = \frac{2P(Z>i)E\left[\min(Z,i)\right]}{(E\left[\min(Z,i)\right]^2)^2}\left(E\left[\min(Z,i)\right]^2 - iE\left[\min(Z,i)\right]\right). \tag{28}$$

As $\min(Z,i)^2 = \min(Z,i)\min(Z,i) \leq i\min(Z,i)$ with strict inequality at some values of i, it follows that $S'(i) < 0$ outside at most countable set of points $i \in (0,1]$. It remains to apply a well-known result from real analysis that if a continuous function on an interval has a negative derivative outside at most countable set of points, then it is decreasing (Dieudonné, 2006). The rest of the proof is to show (28). For a non-negative random variable ξ, the expectation can be written as $E[\xi] = \int_0^\infty P(\xi > \omega)d\omega$. It follows that

$$E\left[\min(Z,i)\right] = \int_0^i P(Z>\omega)d\omega \tag{29}$$

and it is a continuous function of i (a Lipschitz function). Similarly,

$$E\left[\min(Z,i)\right]^2 = E\left[\min(Z^2,i^2)\right] = \int_0^{i^2} P(Z^2>\omega)d\omega$$

It follows that $S(i)$ is continuous on $(0,1]$. From (29) it follows that $i \to E\left[\min(Z,i)\right]$ is differentiable at all points of continuity of the function $i \to P(Z>i)$, and the derivative is $P(Z>i)$. But the function $i \to P(Z>i)$ is monotone and hence possess at most countable set of discontinuities. The same applies to the function $i \to P(Z^2 > i)$. So, outside at most countable set of points on $(0,1]$ the derivative of the numerator $i \to (E\left[\min(Z,i)\right])^2$ is $2P(Z>i)E\left[\min(Z,i)\right]$ and the derivative of the denominator $i \to E(\min(Z,i))^2$ is $2iP(Z^2>i^2) = 2iP(Z>i)$. Hence (28).

Proof of Proposition 5. (Cross-section of asset returns during a crash of arbitrage capital). Suppose $k_2 = k_2(i_2^*)$ for some $i_2^* \in (0,1)$ where $k_2(i)$ denotes the level of k_2 that makes i the marginal asset. I proceed in three steps. (a) First, returns are negative for assets $[0, i_2^*]$. Since $p_{i,1} = E_1\left[\frac{\Lambda_2}{E_1[\Lambda_2]}(p_{i,2}+d_{i,2})\right]$ (Lemma 7) and $d_{i,2}$ and Λ_2 are independent (since $E[\Lambda_2|d_{i,2}] = E[\Lambda_2]$), $p_{i,1} = E_1\left[\frac{\Lambda_2}{E_1[\Lambda_2]}p_{i,2}\right] = \int_{-\infty}^0 \frac{1+c}{E_1[\Lambda_2]}\cdot\frac{v}{1+\phi i}dF(k_2) + \int_0^{k_2(i)}\frac{1+\phi\left(1-\sqrt{k_2}\right)}{E_1[\Lambda_2]}\cdot\frac{v}{1+\phi i}dF(k_2) + \int_{k_2(i)}^\infty \frac{1+\phi\left(1-\sqrt{k_2}\right)}{E_1[\Lambda_2]}\cdot\frac{v}{1+\phi\left(1-\sqrt{k_2}\right)}dF(k_2)$. Since $p_{i,2} = \frac{v}{1+\phi i}$ (Lemma 5), $\Delta p_{i,2} \equiv p_{i,2} - p_{i,1} = \frac{v}{1+\phi i}\left[1 - \frac{E_1\left[\tilde{\Lambda}_{i,2}\right]}{E_1[\Lambda_2]}\right]$ where $\tilde{\Lambda}_{i,2} \equiv 1+c$, $1+\phi\left(1-\sqrt{k_2}\right)$, and $1+\phi i$ for intervals $k_2 \leq 0$, $k_2 \in (0, k_2(i))$, and $k_2 \geq k_2(i)$, respectively. Under Assumption 1, $\tilde{\Lambda}_{i,2} > \Lambda_2 \implies \frac{E_1[\tilde{\Lambda}_{i,2}]}{E_1[\Lambda_2]} > 1 \implies \Delta p_{i,2} < 0 \implies r^e_{i,2} < 0\ \forall i \leq i^*_2$. (b) Next, $\Delta p_{i,2}$ is decreasing in $i \in [0, i^*_2]$. Let $g(i) \equiv \frac{E_1[\Lambda_2] - E_1[\tilde{\Lambda}_2]}{1+\phi i} = \frac{\int_{k_2(i)}^{\infty} \phi(i^*-i) dF(k_2)}{1+\phi i}$. Then, $g'(i) = -\frac{\phi}{(1+\phi i)^2}\left(E_1[\Lambda_2] - E_1\left[\tilde{\Lambda}_{i,2}\right]\right) - \frac{\phi \int_{k_2(i)}^{\infty} dF(k_2)}{1+\phi i} = -\frac{\phi}{(1+\phi i)^2}(E_1[\Lambda_2] - E_1\left[\tilde{\Lambda}_{i,2}\right] + (1+\phi i)$ $Pr(k_2 > k_2(i))) = -\frac{\phi}{(1+\phi i)^2}\left(\phi \int_{k_2(i)}^{\infty} i^* dF(k_2) + Pr(k_2 > k_2(i))\right) < 0$. (c) Finally, since $p_{i,1}$ is non-increasing in i (from the proof of Proposition 1) and $\Delta p_{i,2} < 0$ (first part of this proof), $r^e_{i,2} = \Delta p_{i,2}/p_{i,1}$ is decreasing in $i\ \forall i \leq i^*_2$.

3 Further Robustness Checks

3.1 Controlling for volatility

Next, I show that controlling for the portfolio's pre-arbitrage volatility proxied by the pre-1993 volatility does not strongly affect the ability of arbitrage-related variables (arbitrage position and pre-arbitrage alpha) to explain the cross-section of funding betas and arbitrageur wealth portfolio betas.

Table 1: **Explaining the Cross-Section of Funding-liquidity Betas (Adding Pre-1993 Volatility)**

$$\text{Baseline: } \beta^{post93}_{funding,i} = b_0 + b_1 \text{ Arbitrage position}^{post93}_i + b_2 \text{ Volatility}^{pre93}_i + u_i$$

The table repeats Table 2 of the paper but includes the cross-sectionally standardized pre-1993 volatility as an additional control variable. In the parentheses are t-statistics based on bootstrap standard errors that account for cross-portfolio covariances as well as generated regressors. Boldface denotes coefficient estimates greater than 1.96 times the standard error in absolute value.

	$\beta^{post93}_{funding}$							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Arbitrage Position	**1.88**	**2.05**	2.21					
	(2.38)	(2.54)	(1.88)					
α^{pre93}_{CAPM}				**0.15**	**0.15**	**0.17**		
				(2.58)	(2.51)	(2.09)		
Pre-1993 Volatility	0.26	0.36	0.33	-0.44	-0.44	-0.39	**-0.82**	-0.84
	(0.92)	(1.13)	(0.75)	(-1.29)	(-1.29)	(-1.06)	(-1.97)	(-1.73)
$\beta^{pre93}_{funding}$		0.28			-0.06		0.12	
		(1.15)			(-0.22)		(0.37)	
Size rank			0.06			0.01		-0.00
			(0.18)			(0.04)		(-0.01)
Value rank			0.07			-0.10		0.30
			(0.32)			(-0.33)		(1.32)
Profitability rank			-0.09			-0.08		0.20
			(-0.51)			(-0.40)		(1.06)
Investment rank			-0.09			-0.08		0.11
			(-0.48)			(-0.44)		(0.61)
Constant	-0.06	-0.04	0.54	-0.39	-0.38	0.92	-0.55	-3.47
	(-0.25)	(-0.16)	(0.22)	(-1.25)	(-1.22)	(0.31)	(-1.54)	(-1.40)
Observations	40	40	40	40	40	40	40	40
R^2_{adj}	0.73	0.76	0.75	0.76	0.76	0.75	0.46	0.51

Table 2: **Explaining the Cross-Section of Arbitrageur Wealth Portfolio Betas (Adding Pre-1993 Volatility)**

$$\text{Baseline: } \beta^{post93}_{wealth,i} = b_0 + b_1 \text{ Arbitrage position}^{post93}_i + b_2 \text{ Volatility}^{pre93}_i + u_i$$

The table repeats Table 6 of the paper but includes the cross-sectionally standardized pre-1993 volatility as an additional control variable. In the parentheses are t-statistics based on bootstrap standard errors that account for cross-portfolio covariances as well as generated regressors. Boldface denotes coefficient estimates greater than 1.96 times the standard error in absolute value.

	β^{post93}_{wealth}							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Arbitrage Position	**0.31**	**0.31**	0.39					
	(2.73)	(2.61)	(1.95)					
α^{pre93}_{CAPM}				**0.03**	**0.02**	**0.03**		
				(3.19)	(2.67)	(2.74)		
Share of wealth portfolio		0.39	0.40		0.31	0.25	1.16	0.64
		(0.61)	(0.57)		(0.56)	(0.40)	(1.78)	(0.88)
Pre-1993 Volatility	0.04	0.04	0.04	-0.07	-0.07	-0.09	**-0.12**	-0.17
	(0.68)	(0.64)	(0.51)	(-1.27)	(-1.21)	(-1.40)	(-1.99)	(-1.95)
β^{pre93}_{wealth}		0.11			-0.14		-0.16	
		(0.52)			(-0.58)		(-0.64)	
Size rank			0.04			0.03		0.03
			(0.51)			(0.70)		(0.39)
Value rank			-0.01			-0.05		0.03
			(-0.32)			(-1.00)		(0.69)
Profitability rank			-0.02			-0.02		0.03
			(-0.79)			(-0.73)		(0.90)
Investment rank			-0.03			-0.03		0.00
			(-1.07)			(-1.03)		(0.07)
Constant	0.01	0.03	0.25	-0.04	-0.02	0.37	-0.01	-0.41
	(0.30)	(0.58)	(0.66)	(-0.98)	(-0.48)	(0.83)	(-0.24)	(-0.85)
Observations	40	40	40	40	40	40	40	40
R^2_{adj}	0.65	0.65	0.70	0.70	0.70	0.75	0.47	0.45

3.2 Controlling for market liquidity

Similarly, controlling for the portfolio's pre-arbitrage market liquidity measured by pre-1993 Amihud (2002) does not strongly affect my results.

Table 3: **Explaining the Cross-Section of Funding-liquidity Betas (Adding Pre-1993 Market Liquidity)**

$$\text{Baseline: } \beta^{post93}_{funding,i} = b_0 + b_1 \text{ Arbitrage position}^{post93}_i + b_2 \text{ Amihud}^{pre93}_i + u_i$$

The table repeats Table 2 of the paper but includes the cross-sectionally standardized pre-1993 market liquidity as an additional control variable. In the parentheses are t-statistics based on bootstrap standard errors that account for cross-portfolio covariances as well as generated regressors. Boldface denotes coefficient estimates greater than 1.96 times the standard error in absolute value.

	$\beta^{post93}_{funding}$							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Arbitrage Position	**1.81**	**1.77**	**1.82**					
	(2.61)	(2.58)	(2.05)					
α^{pre93}_{CAPM}				**0.18**	**0.18**	**0.21**		
				(2.67)	(2.59)	(2.12)		
Pre-1993 Amihud rank	0.29	0.23	0.68	-0.27	-0.27	0.12	-0.53	0.52
	(1.33)	(1.04)	(1.20)	(-0.81)	(-0.85)	(0.17)	(-1.31)	(0.63)
$\beta^{pre93}_{funding}$		0.18			0.01		0.33	
		(0.79)			(0.03)		(0.88)	
Size rank			-0.29			-0.21		-0.72
			(-0.65)			(-0.38)		(-1.04)
Value rank			0.08			-0.16		0.36
			(0.36)			(-0.47)		(1.38)
Profitability rank			-0.06			-0.14		0.17
			(-0.39)			(-0.64)		(0.69)
Investment rank			-0.09			-0.11		0.14
			(-0.58)			(-0.56)		(0.80)
Constant	-0.07	-0.10	1.36	-0.35	-0.35	2.29	-0.57	-1.45
	(-0.38)	(-0.48)	(0.54)	(-1.15)	(-1.15)	(0.70)	(-1.55)	(-0.38)
Observations	40	40	40	40	40	40	40	40
R^2_{adj}	0.75	0.76	0.77	0.71	0.70	0.71	0.15	0.28

Table 4: **Explaining the Cross-Section of Arbitrageur Wealth Portfolio Betas (Adding Pre-1993 Market Liquidity)**

$$\text{Baseline: } \beta^{post93}_{wealth,i} = b_0 + b_1 \text{ Arbitrage position}^{post93}_i + b_2 \text{ Amihud}^{pre93}_i + u_i$$

The table repeats Table 6 of the paper but includes the cross-sectionally standardized pre-1993 market liquidity as an additional control variable. In the parentheses are t-statistics based on bootstrap standard errors that account for cross-portfolio covariances as well as generated regressors. Boldface denotes coefficient estimates greater than 1.96 times the standard error in absolute value.

	β^{post93}_{wealth}							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Arbitrage position	**0.33**	**0.32**	**0.36**					
	(3.57)	(2.93)	(2.14)					
α^{pre93}_{CAPM}				**0.03**	**0.03**	**0.04**		
				(3.29)	(2.84)	(2.79)		
Share of wealth portfolio		0.16	0.24		0.40	0.49	**1.79**	1.30
		(0.25)	(0.33)		(0.63)	(0.71)	(2.03)	(1.62)
Pre-1993 Amihud rank	**0.07**	0.07	0.10	-0.03	-0.02	-0.02	-0.06	0.04
	(2.23)	(1.56)	(1.10)	(-0.54)	(-0.27)	(-0.12)	(-0.82)	(0.27)
β^{pre93}_{wealth}		-0.02			-0.20		-0.23	
		(-0.12)			(-0.72)		(-0.71)	
Size rank			-0.01			0.01		-0.08
			(-0.13)			(0.08)		(-0.65)
Value rank			-0.01			-0.06		0.03
			(-0.28)			(-1.18)		(0.65)
Profitability rank			-0.02			-0.03		0.02
			(-0.74)			(-1.08)		(0.58)
Investment rank			-0.03			-0.04		0.00
			(-1.20)			(-1.26)		(0.08)
Constant	0.02	0.02	0.39	-0.04	-0.01	0.62	0.02	-0.02
	(0.53)	(0.56)	(0.90)	(-0.83)	(-0.27)	(1.17)	(0.37)	(-0.03)
Observations	40	40	40	40	40	40	40	40
R^2_{adj}	0.71	0.70	0.72	0.63	0.63	0.68	0.21	0.17

4 Additional Figures

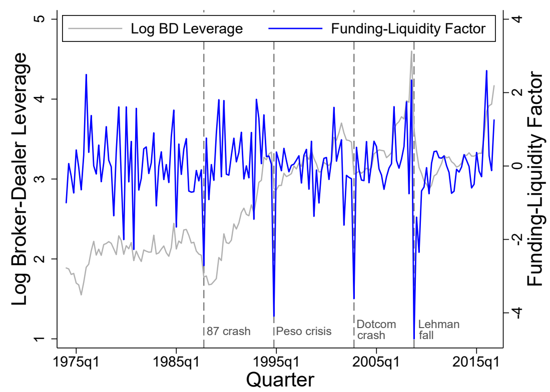

Figure 2: **Funding-liquidity Factor of Adrian, Etula, and Muir (2014)**

The figure plots the quarterly funding-liquidity factor used in the paper for 1974q1-2016q4.

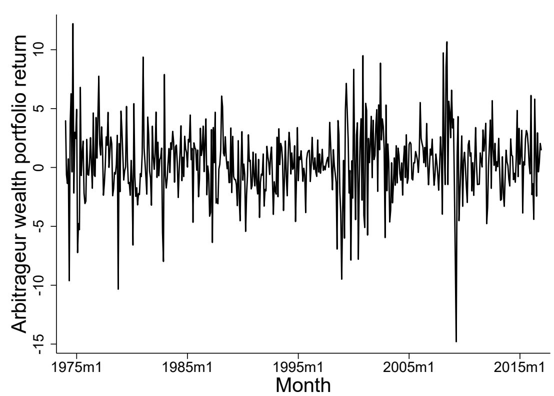

Figure 3: **Arbitrageur Wealth Portfolio Shocks Used in Cho (2019)**

The figure plots the monthly arbitrageur wealth portfolio shocks for 1974m1-2016m12.

5 Anomaly Signal Construction

This section borrows heavily from the supplementary appendix of Novy-Marx and Velikov (2016).

5.1 Cleaning and merging datasets

5.1.1 CRSP monthly

For CRSP monthly, I use domestically incorporated stocks listed on the three major exchanges ($shrcd$ 10 or 11 and $exchcd$ 1, 2, or 3). I use the bid-ask spread as prc (price) whenever prc is unavailable and remove observations with $prc = 0$. I also convert the CRSP code for missing ret (return) and $dlret$ (delisting return) to be a missing number.

I follow Green, Hand, and Zhang (2017) to add delisting returns back to returns in the spirit of Shumway and Warther (1999). That is, for the firms with $dlstcd = 500$ or $dlstcd \in [520, 584]$, I use $dlret = -0.35$ if $exchcd = 1$ or 2 and $dlret = -0.55$ if $exchcd = 3$.

5.1.2 Compustat annual

To clean this data, I remove duplicates where there are multiple $gvkey$ for the same $fyear$ and $permno$. I do this by using the observation with information about at or is newer (in that order) and remove any other observations with the same $fyear$ and $permno$.

5.1.3 Compustat quarterly

To clean this data, I remove duplicates where there are multiple $gvkey$ for the same rdq and $permno$. I do this by using the observation with information about atq or is newer (in that order) and remove any other observations with the same rdq and $permno$.

In the earlier data, the balance sheet information ($actq$, atq, $ceqq$, $dlcq$, $dlttq$, $lctq$, ltq) is often reported only annually. Hence, I replace the missing accounting data with the most recent accounting data up to 4 quarters ago. Likewise, if the identification information ($gvkey$ and sic) is missing, I replace the missing identification data with the most recent identification data.

5.1.4 Merging Compustat annual with CRSP monthly

I merge CRSP monthly with Compustat annual and use observations as of June of each year to form signals for the next 12 months. To do so, I match CRSP monthly data for June of year t with Compustat annual data with $t - 1$ as its fiscal year end. Similarly to Novy-Marx and Velikov (2016) and many other works, this avoids the look-ahead bias. The anomalies using Compustat annual data are rebalanced annually at the end of June of each year.

5.1.5 Merging Compustat quarterly with CRSP monthly

I merge CRSP monthly with the most recently reported (rdq) Compustat quarterly data, allowing for the maximum 12 months' difference between the CRSP month and the month-end of the rdq reported in Compustat quarterly. I do not use information about the accounting quarter end for the purpose of merging. Because rdq has been reported only since late 1971, this causes most anomaly signals that use quarterly accounting data to start in early 1970s.

5.1.6 For all anomalies

For all anomalies, I follow Stambaugh and Yuan (2016) to require that both extreme deciles of an anomaly includes at least 30 stocks each. This is to ensure that the extreme decile returns are not determined by only a few number of stocks.

5.2 Constructing anomalies based on Compustat annual and CRSP monthly datasets (Annually rebalanced)

1. Size (Banz, 1981; Fama and French, 1993):

$$size = -1 \times prc \times shrout \times 10^{-3}$$

   where prc is price per share and $shrout$ is the number of shares outstanding (in 10^3) from CRSP monthly.

2. Gross profitability (Balakrishnan, Bartov, and Faurel, 2010; Novy-Marx, 2013):

$$profit = \frac{gp}{at} \text{ if } sic \notin [6000, 6999]$$

   where gp is the gross profits and at is total assets from Compustat annual. Financial firms are excluded since the definition of total assets differs from that for non-financial firms. The gross profitability anomaly was first explored by Novy-Marx (2013), but its precursor—profitability—was first explored by Karthik, Bartov, and Faurel (2010).

3. Value (Rosenberg, Reid, and Lanstein, 1985; Fama and French, 1993):

$$value = \frac{be}{me}$$

where be is book value of equity and me is market value of equity. The book equity is defined as

$$be = seq + txditc - bpstk$$

where $bpstk$ equals $pstkrv$, $pstkl$, $upstk$, or zero depending on the availability. If $txditc$ is unavailable, it is assumed to be zero. If seq is unavailable, I use $seq = at - lt$. Finally, the market equity is

$$me = prc_{t-6mo} \times shrout_{t-6mo} \times 10^{-3}$$

where prc_{t-6mo} and $shrout_{t-6mo}$ are the price and shares outstanding as of December of year $t-1$ (as of six months ago).

4. Accruals (Sloan, 1996):

$$acc = -1 \times \frac{accruals}{(at + at_{t-1})/2}$$

where[17]

$$accruals = \Delta act - \Delta che - \Delta lct + \Delta dlc + \Delta txp - dp$$

and

$$\Delta x \equiv x - x_{t-1}$$

Here, txp (and txp_{t-1}) is assumed to be zero if missing.

5. Net issuance, rebalanced annually (Ikenberry, Lakonishok, and Vermaelen, 1995; Loughran and Ritter, 1995; Pontiff and Woodgate, 2008):

$$netissue = -1 \times (\ln(adjshrout) - \ln(adjshrout_{t-1}))$$

where

$$adjshrout = cfacshr \times shrout$$

6. Asset growth (Cooper, Gulen, and Schill, 2008):

$$atgrowth = -1 \times \frac{at}{at_{t-1}} \text{ if } sic \notin [6000, 6999]$$

[17] For the annually rebalanced anomalies, $t-1$ denotes lag one year.

7. Investment (Titman, Wei, and Xie, 2004; Lyandres, Sun, and Zhang, 2008; Chen, Novy-Marx, and Zhang, 2010):

$$invest = -1 \times \frac{\Delta ppegt + \Delta invt}{at_{t-1}} \text{ if } sic \notin [6000, 6999]$$

where $\Delta ppegt = ppegt - ppegt_{t-1}$ and $\Delta invt = invt - invt_{t-1}$.

8. Piotroski's F-score (Piotroski, 2000):

$$\begin{gathered} piotroski = \mathbf{1}\left(ib > 0\right) + \mathbf{1}\left(\Delta roa > 0\right) + \mathbf{1}\left(oancf > 0\right) + \mathbf{1}\left(oancf > ib\right) \\ +\mathbf{1}\left(\Delta dta < 0 \text{ or } dltt = 0 \text{ or } dltt_{t-1} = 0\right) + \mathbf{1}\left(\Delta atl > 0\right) \\ +\mathbf{1}\left(scstkc \leq prstkcc\right) + \mathbf{1}\left(\Delta ato\right) + \mathbf{1}\left(\Delta gm\right) \end{gathered}$$

where $\Delta x \equiv x - x_{t-1}$ and both ato and gm are defined below. Also, since $piotroski$ uses both ato and gm, I exclude non-financial firms from the $piotroski$ calculation. Novy-Marx and Velikov (2016) simply uses $piotroski + 1$ as the decile of a stock, but I find that this leaves me with too few stocks in the lowest decile. Hence, I assign piotroski deciles similarly to other anomalies.

9. Asset turnover (Soliman, 2008; Novy-Marx, 2013):

$$ato = \frac{sale}{at} \text{ if } sic \notin [6000, 6999]$$

10. Gross margins (Soliman, 2008; Novy-Marx, 2013):

$$gm = \frac{gp}{sale} \text{ if } sic \notin [6000, 6999]$$

where I use $gp = sale - cogs$ if gp is missing.

5.3 Constructing anomalies based on Compustat quarterly and CRSP monthly datasets (Monthly rebalanced)

11. Ohlson's O-score (Ohlson, 1980):

$$ohlson = -1 \times (-1.32 - 0.407 \ln (adjatq/cpi) + 6.03tlta - 1.43wcta + 0.076clca$$
$$-1.72oeneg - 2.37nita - 1.83futl + 0.285intwo - 0.521chin)$$

Here, adjusted asset is defined as

$$adjatq = atq + 0.1 \times (meq - beq)$$

where $meq = prc \times shrout \times 10^{-3}$ and $beq = seqq$, $ceqq + pstkq$, $atq - ltq$, $beq + txditcq$, or $beq - pstkq$ depending on the availability of data. The CPI index cpi is the annual Consumer Price Index (Not Seasonally Adjusted) downloaded from Federal Reserve Economic Data except that I normalize the series to ensure $cpi = 100$ for year 1968. Next, the other variables are defined as

$$tlta = \frac{dlcq + dlttq}{adjatq}$$
$$wcta = \frac{actq - lctq}{adjatq}$$
$$clca = \frac{lctq}{actq}$$
$$oeneg = \mathbf{1}(ltq > atq)$$
$$nita = \frac{niq}{adjatq}$$
$$futl = \frac{piq}{ltq}$$
$$intwo = \mathbf{1}(niq < 0) \times \mathbf{1}(niq_{t-3} < 0)$$
$$chin = \frac{niq - niq_{t-3}}{\mid niq \mid + \mid niq_{t-3} \mid}$$

where x_{t-3} denotes the value of x in the previous quarter.

12. Net issuance, rebalanced monthly (Ikenberry, Lakonishok, and Vermaelen, 1995; Loughran

and Ritter, 1995; Pontiff and Woodgate, 2008):

$$netissue = -1 \times \left(\ln\left(adjshrout\right) - \ln\left(adjshrout_{t-12}\right)\right)$$

where

$$adjshrout = cfacshr \times shrout$$

and x_{t-12} is the value of x as of 12 months ago.

13. Return on book equity (ROE) (Haugen and Baker, 1996; Chen, Novy-Marx, and Zhang, 2010):

$$roe = \frac{ibq}{beq_{t-3}}$$

where I use $beq = 10^{-6}$ if $beq \leq 0$ and x_{t-3} denotes the value of x in the previous quarter.

14. Failure probability (Dichev, 1998; Campbell, Hilscher, and Szilagyi, 2008):

$$\begin{aligned} failprob = -1 \times (&-9.164 - 20.264 nimtaavg + 1.416 tlmta - 7.129 exretavg + 1.411 sigma \\ &-0.045 rsize - 2.132 cashmta + 0.075 mb - 0.058 price) \end{aligned}$$

The individual items are defined as follows:

$$nimtaavg = \frac{1-\phi^3}{1-\phi^{12}}\left(nimta + \phi^3 nimta_{t-3} + \phi^6 nimta_{t-6} + \phi^9 nimta_{t-9}\right)$$

where $t-3q$ denotes data as of q quarters ago, $\phi = 2^{-1/3}$, and $nimta = niq/\left(meq + ltq\right)$;

$$tlmta = \frac{ltq}{meq + ltq};$$

$$exretavg = \frac{1-\phi^3}{1-\phi^{12}}\left(exret + \phi exret_{t-1} + ... + \phi^{11} exret_{t-11}\right)$$

where $exret_{t-s} = \ln\left(\left(1 + ret_{t-s}\right)/\left(1 + sprtrn_{t-s}\right)\right)$ is stock i's return relative to the S&P 500 return s months ago;

$$sigma = \sqrt{\frac{252}{N-1}\sum_{d\in\{t-2,t\}} r_d^2}$$

where r_d is daily return, $[t-2, t]$ denotes trading days in the last three months, and N is the number of trading days in the last three months;

$$rsize = \ln\left(\frac{meq}{totval \times 10^{-3}}\right)$$

where $totval$ is the total value of S&P 500 downloaded from the CRSP S&P 500 Indexes data;[18]

$$cashmta = \frac{cheq}{meq + ltq};$$

$$mb = \frac{meq}{beq2}$$

where $beq2 = \max\left\{beq + 0.1 \times (meq - beq), 10^{-6}\right\}$ following Cohen, Polk, and Vuolteenaho (2003) and Chen, Novy-Marx, and Zhang (2010);

$$price = \ln\left(\min\left\{prc, 15\right\}\right)$$

so that $price$ is log price per share, truncated above at \$15. Following Campbell, Hilscher, and Szilagyi (2008), I replace missing values of $nimta$, $exret$, $sigma$, $cashmta$, and mb with their cross-sectional means from the same month and winsorize all variables at 5% and 95%.

15. Idiosyncratic volatility (Ali, Hwang, and Trombly, 2003; Ang et al., 2006):

$$idiovol = -1 \times \sum_{d \in \{t-2,t\}} \hat{u}_{d,t}^2$$

where $\hat{u}_{d,t}$ is a residual on day d from regressing daily returns on daily Fama-French 3 factors using the last three months as of month t. To compure the residuals, I require at least 50 trading days in the last three months.

16. Momentum (Jegadeesh, 1990; Jegadeesh and Titman, 1993):

$$mom12m = \sum_{s=1}^{11} \ln\left(1 + r_{t-s}\right)$$

[18] It is unclear what would be a better measure of the total value of S&P 500 between $totval$ and $usdval$, but there is little difference between the two values.

where if the history of returns is shorter than 12 months is available since at least 6 months ago, I compute past returns using the available returns.

17. Long-run reversal (DeBondt and Tahler, 1985, 1987):

$$rev60m = -1 \times \sum_{s=12}^{59} \ln(1 + r_{t-s})$$

where if the data are shorter than 60 months, I require that at least 24 months be included in the calculation of the reversal.

18. Return on market equity (ROME) (Basu, 1977; Chen, Novy-Marx, and Zhang, 2010):

$$rome = \frac{ibq}{meq_{t-3}}$$

where meq_{t-3} is market equity as of three months ago. Note that earnings to price, first explored by Basu (1977) is the precusor of return on market equity of Chen, Novy-Marx, and Zhang (2010).

19. Return on assets (ROA) (Haugen and Baker, 1996; Chen, Novy-Marx, and Zhang, 2010):

$$roa = \frac{ibq}{atq_{t-3}} \; sic \notin [6000, 6999]$$

where atq_{t-3} is total assets as of three months ago.

20. Beta arbitrage (Black, 1972; Fama and MacBeth, 1973; Frazzini and Pedersen, 2014):

$$beta = -1 \times \left(0.6 \times \frac{\rho_{i,m}\sigma_i}{\sigma_m} + 0.4\right)$$

Here, $\rho_{i,m}$ is the correlation of overlapping three-day log returns $r_t^{3day} = \sum_{k=0}^{2} \ln(1 + r_{t-k})$ between stock i and the market (CRSP value-weighted index $vwretd$) over the last 5 years (or at least the last 750 trading days). The volatilities are estimated using daily returns over the last 1 year (or at least the last 6 months). The constant 0.6 multiplied to the time-series of betas is the shrinkage factor that reduces the influence of outliers. Finally, the beta arbitrage portfolio is constructed to neutralize the market exposure:

$$r_{beta,t+1} = \frac{1}{-beta_t^{low}} \left(r_{beta,t+1}^{low} - r_{f,t} \right) - \frac{1}{-beta_t^{high}} \left(r_{beta,t+1}^{high} - r_{f,t} \right)$$

Note that the construction of the beta arbitrage anomaly here is different from that of Novy-Marx and Velikov (2016), who do not compute betas by individually estimating correlations and standard deviations and do not make the shrinkage adjustment (which is way they do not call it the "betting against beta" anomaly).

6 Constructing the Funding-liquidity ("Leverage") Factor of Adrian et al.

This section explains constructing the funding-liquidity factor of Adrian, Etula, and Muir (2014).

6.1 Data location and information

As of August 2017, the underlying data for the 2016Q4 release are available to download from

https://www.federalreserve.gov/releases/z1/20170305/z1_csv_files.zip

Note that this file downloads automatically. Under the csv folder, select the file named L130.

6.2 Constructing the factor

To construct the leverage series, I compute

$$Leverage_t^{BD} = \frac{Total\ Financial\ Assets_t - Repo\ Assets_t}{Total\ Financial\ Assets_t - Total\ Liabilities_t - FDI\, in\, US_t}$$

where $Total\ Financial\ Assets$, $Repo\ Assets$, and $Total\ Liabilities$ are items FL664090005, FL662051003, and FL664190005 in the csv file for security brokers and dealers (currently L.130) and $FDI\, in\, US$ is the sum of items FL663192005 in the security brokers and dealers balance sheet (currently L.130) and L733192003 in the holding companies balance sheet (currently L.131). This formula to construct levearge appears different from the one used in AEM, but in fact, it ensures that the construction is identical to the original method used by AEM. I explain this briefly.

Previously, only the net repo amount (repo liabilities - repo assets) entered into total liabilities, whereas now repo assets (i.e., reverse repo) and repo liabilities are respectively included in total financial assets and total liabilities.[19] The interpretation is that only the relative increase in repo (i.e., increase in net repo) is taken as a good leverage shock; if the repo assets are not net out, then an increase in repo assets with no change in the net repo would still be taken as a good leverage shock.

Furthermore, since the 2016Q3 release, two changes occurred to the way security broker-dealer

[19] See https://www.federalreserve.gov/apps/fof/FOFHighlight.aspx (Highlights for the 2014Q1 release).

liabilities are treated in the flow of funds data. First, foreign direct investments (FDI) in the U.S. are excluded from liabilities by netting out the amount through miscellaneous liabilities. Second, U.S. subsidiaries of foreign banking organizations, previously included in the FDI in the U.S., are reported separately in the liabilities section of bank holding companies. Although this may be a more convenient way to represent flow of funds among different financial institutions, putting these liabilities back into security broker-dealer liabilities seems to be a better way to understand the actual leverage taken by US-operating security broker-dealers. Hence, I stick to the original method of AEM of adding these items to total liabilities of security broker-dealers. This amounts to adding the sum of items FL663192005 in the security brokers and dealers balance sheet (currently L.130) and FL733192003 in the holding companies balance sheet (currently L.131) back to total liabilities of security broker-dealers.

Then, I take the log difference and apply the seasonal adjustment based on a rolling regression of log leverage difference on quarterly seasonal dummies. I follow AEM to require a minimum 10 quarters to do the adjustment. The result is a seasonally-adjusted raw leverage shock series:

$$LevShock_t = \left[\Delta \ln \left(Leverage_t^{BD}\right)\right]^{SA}$$

AEM uses this series as the factor in their cross-sectional pricing. However, I find that this series experiences a 6-standard-deviation shock during the financial crises, causing any empirical results to depend too heavily on the financial crises period.

As a remedy, I winsorize the raw series above at the top 99% and bottom 1% levels, which is the final steps of generating the leverage factor:

$$LevFactor_t = \left[\Delta \ln \left(Leverage_t^{BD}\right)\right]^{SA,Winsorized}$$

The winsorization esssentially takes out the large negative and positive leverage shocks during the financial crisis.

It seems prudent to curb the effects of these shocks also because the liabilities of security broker-dealers around the crisis has been subject to substantial revisions. To illustrate, the total assets and total liabilities of security broker-dealers in 2008Q4 and 2009Q4 (in billions of dollars) reported in the flow of funds data of different release dates have been the following:

Release date	2009Q4	2010Q4	2011Q4	2012Q4
Total assets and liabilities in 2008Q4				
Total financial assets	2217.2	2217.2	2217.2	2217.2
Total liabilities	2165.9	2146.3	2158.1	2158.1
Leverage	43.22	31.27	37.52	37.52
Total assets and liabilities in 2009Q4				
Total financial assets	2080.0	2084.2	2084.2	2084.2
Total liabilities	1994.7	1998.5	1987.8	1977.7
Leverage	24.38	24.32	21.62	19.57
Implied growth in leverage from 2008Q4 to 2009Q4				
Log leverage change	−0.57	−0.25	−0.55	−0.65

Although the changes in the level of total liabilities do not seem large, these changes imply very large changes in the leverage. In this sense, winsorizing the series helps reduce the results' sensitivity to these restrospective changes in leverage values. The final series I use as a leverage factor uses the original AEM series for the period in which the series is available (–2009Q4) and use the newly constructed factor for the rest of the period (2010Q4–).

References

[1] Adrian, Tobias, Erkko Etula, and Tyler Muir, 2014, Financial intermediaries and the cross-section of asset returns, Journal of Finance 69(6), 2557-2596.

[2] Ali, Ashiq, Lee-Seok Hwang, and Mark A. Trombley, 2003, Arbitrage risk and the book-to-market anomaly, Journal of Financial Economics 69(2), 355-373.

[3] Amihud, Yakov, 2002, Illiquidity and stock returns: Cross-section and time-series effects, Journal of Financial Markets 5(1), 31-56.

[4] Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang, 2006, The cross-section of volatility and expected returns, Journal of Finance 61(1), 259-299.

[5] Asness, Clifford S., R. Burt Porter, and Ross L. Stevens, 1994, Predicting stock returns using industry-relative firm characteristics, working paper.

[6] Banz, Rolf W., 1981, The relationship between return and market value of common stocks, Journal of Financial Economics 9(1), 3-18.

[7] Balakrishnan, Karthik, Eli Bartov, and Lucile Faurel, 2010, Post loss/profit announcement drift, Journal of Accounting and Economics, 50(1), 20-41.

[8] Barberis, Nicholas, Ming Huang, and Richard H. Thaler, 2006, Individual preferences, monetary gambles, and stock market participation: A case for narrow framing, American Economic Review, 96(4), 1069-1090.

[9] Basu, Sanjoy, 1977, Investment performance of common stocks in relation to their price-earnings ratios: A test of the efficient market hypothesis, The Journal of Finance, 32(3), 663-682.

[10] Black, Fischer, 1972, Capital market equilibrium with restricted borrowing, Journal of Business 45(3), 444-455.

[11] Brandt, Michael W., Runeet Kishore, Pedro Santa-Clara, and Mohan Venkatachalam, 2008, Earnings announcements are full of surprises, working paper.

[12] Brunnermeier, Markus K., and Lasse Heje Pedersen, 2009, Market liquidity and funding liquidity, Review of Financial Studies 22(6), 2201-2238.

[13] Brunnermeier, Markus K., and Yuliy Sannikov, 2014, A macroeconomic model with a financial sector, American Economic Review 104(2), 379-421.

[14] Campbell, John Y., Jens Hilscher, and Jan Szilagyi, 2008, In search of distress risk, Journal of Finance 63(6), 2899-2939.

[15] Chan, Louis K. C., Yasushi Hamao, and Josef Lakonishok, 1991, Fundamentals and stock returns in Japan, Journal of Finance 46(5), 1739-1764.

[16] Chen, Long, Robert Novy-Marx, and Lu Zhang, 2010, An alternative three-factor model, working paper.

[17] Cho, Thummim, 2019, Turning alphas into betas: Arbitrage and the cross-section of risk, working paper.

[18] Cohen, Randolph B., Christopher Polk, and Tuomo Vuolteenaho, 2003, The value spread, Journal of Finance 58(2), 609-641.

[19] Cooper, Michael J., Huseyin Gulen, and Michael J. Schill, 2008, Asset growth and the cross-section of stock returns, Journal of Finance 63(4), 1609-1651.

[20] Corwin, Shane A., and Paul Schultz, 2012, A simple way to estimate bid-ask spreads from daily high and low prices, Journal of Finance 67(2), 719-760.

[21] Da, Zhi, Qianqiu Liu, and Ernst Schaumburg, 2013, A closer look at the short-term return reversal, Management Science 60(3), 658-674.

[22] De Bondt, Werner F.M., and Richard Thaler, 1985, Does the stock market overreact? Journal of finance 40(3), 793-805.

[23] De Bondt, Werner F.M., and Richard H. Thaler, 1987, Further evidence on investor overreaction and stock market seasonality, Journal of Finance 42(3), 557-581.

[24] Dichev, Ilia D., 1998, Is the risk of bankruptcy a systematic risk? Journal of Finance, 53(3), 1131-1147.

[25] Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, Journal of financial economics 33(1), 3-56.

[26] Fama, Eugene F., and James D. MacBeth, 1973, Risk, return, and equilibrium: Empirical tests, Journal of Political Economy, 81(3), 607-636.

[27] Foster, George, Chris Olsen, and Terry Shevlin, 1984, Earnings releases, anomalies, and the behavior of security returns, Accounting Review 59(4), 574-603.

[28] Frazzini, Andrea, 2006, The disposition effect and underreaction to news, Journal of Finance 61(4), 2017-2046

[29] Frazzini, Andrea, and Owen A. Lamont, 2008, Dumb money: Mutual fund flows and the cross-section of stock returns, Journal of financial economics, 88(2), 299-322.

[30] Frazzini, Andrea, and Lasse Heje Pedersen, 2014, Betting against beta, Journal of Financial Economics 111(1), 1-25.

[31] Gˆarleanu, Nicolae, and Lasse Heje Pedersen, 2011, Margin-based asset pricing and deviations from the law of one price, Review of Financial Studies 24(6), 1980-2022.

[32] Geanakoplos, John, and William R. Zame, 2014, Collateral equilibrium, I: A basic framework, Economic Theory, 56(3), 443-492.

[33] Green, Jeremiah, John R. M. Hand, and X. Frank Zhang, 2017, The characteristics that provide independent information about average us monthly stock returns, Review of Financial Studies, forthcoming.

[34] Gromb, Denis, and Dimitri Vayanos, 2018, The dynamics of financially constrained arbitrage, Journal of Finance, 73(4), 1713-1750.

[35] Haugen, Robert A., and Nardin L. Baker, 1996, Commonality in the determinants of expected stock returns, Journal of Financial Economics, 41(3), 401-439.

[36] Heston, Steven L., and Ronnie Sadka, 2008, Seasonality in the cross-section of stock returns, Journal of Financial Economics 87(2), 418-445.

[37] Ikenberry, David, Josef Lakonishok, and Theo Vermaelen, 1995, Market underreaction to open market share repurchases, Journal of Financial Economics 39(2), 181-208.

[38] Jegadeesh, Narasimhan, 1990, Evidence of predictable behavior of security returns, Journal of Finance 45(3), 881-898.

[39] Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, Journal of Finance 48(1), 65-91.

[40] Kondor, Péter, and Dimitri Vayanos, 2019, Liquidity risk and the dynamics of arbitrage capital, Journal of Finance, 74(3), 1139-1173.

[41] Lakonishok, Josef, Andrei Shleifer, and Robert W. Vishny, 1994, Contrarian investment, extrapolation, and risk, Journal of Finance 49(5), 1541-1578.

[42] Loughran, Tim, and Jay R. Ritter, 1995, The new issues puzzle, Journal of Finance 50(1), 23-51.

[43] Lyandres, Evgeny, Le Sun, and Lu Zhang, 2008, The new issues puzzle: Testing the investment-based explanation, Review of Financial Studies 21(6), 2825-855.

[44] Merton, Robert C, 1973, An intertemporal capital asset pricing model, Econometrica, 41(5), 867-887.

[45] Moskowitz, Tobias J., and Mark Grinblatt, 1999, Do industries explain momentum? Journal of Finance 54(4), 1249-1290.

[46] Novy-Marx, Robert, 2013, The other side of value: The gross profitability premium, Journal of Financial Economics 108(1), 1-28.

[47] Novy-Marx, Robert, and Mihail Velikov, 2016, A taxonomy of anomalies and their trading costs, Review of Financial Studies 29(1), 104-147.

[48] Ohlson, James A., 1980, Financial ratios and the probabilistic prediction of bankruptcy, Journal of Accounting Research 18(1), 109-131.

[49] Piotroski, Joseph D., 2000, Value investing: The use of historical financial statement information to separate winners from losers, Journal of Accounting Research 38(Suppl.), 1-41.

[50] Pontiff, Jeffrey, and Artemiza Woodgate, 2008, Share issuance and cross-sectional returns, Journal of Finance 63(2), 921-945.

[51] Rendleman, Richard J., Charles P. Jones, and Henry A. Latane, 1982, Empirical anomalies based on unexpected earnings and the importance of risk adjustments, Journal of Financial Economics 10(3), 269-287.

[52] Rosenberg, Barr, Kenneth Reid, and Ronald Lanstein, 1985, Persuasive evidence of market inefficiency, Journal of Portfolio Management 11(3), 9-16.

[53] Shleifer, Andrei, and Robert W. Vishny, 1997. The limits of arbitrage, Journal of Finance 52(1), 35-55.

[54] Shumway, Tyler, and Vincent A. Warther, 1999, The delisting bias in CRSP's Nasdaq data and its implications for the size effect, Journal of Finance 54(6), 2361-2379.

[55] Sloan, Richard, 1996, Do stock prices fully reflect information in accruals and cash flows about future earnings? Accounting Review 71(3), 289-315.

[56] Soliman, Mark T., 2008, The use of DuPont analysis by market participants, The Accounting Review, 83(3), 823-853.

[57] Stambaugh, Robert F., and Yu Yuan, 2016, Mispricing factors, Review of Financial Studies 30(4), 1270-1315.

[58] Titman, Sheridan, KC John Wei, and Feixue Xie, 2004, Capital investments and stock returns, Journal of Financial and Quantitative Analysis, 39(4), 677-700.