Pricing Structured Products with Economic Covariates
Online Appendix

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1 Overview

In this online appendix, we report on several empirical exercises that complement the analysis in the paper. Section 2 discusses model performance for alternative implementations of the model. Section 3 reports on the time variation in the value-at-risk implied by the model.

2 Alternative Model Implementations

The benchmark model is estimated using two intensities to describe the loss distribution. We now discuss model fit for a model with a single intensity and a model with three intensities. Figure A1 reports on model fit for the model with a single intensity. The model performs well in capturing the spreads of the junior tranches. This is not surprising since the estimated jump size with a single intensity is 0.0304. However, because the model does not have a large jump, it dramatically understates the spreads of the senior (15-30% and 15-100%) tranches. For those tranches that are adequately fit with the single intensity (the 0-3%, 3-7%, 7-10%, 7-15%, and 10-15% tranches), our conclusions regarding the economic impact of the covariates are consistent with the findings from our benchmark estimation. We conclude that a model with a single intensity is not able to adequately capture the cross-sectional and time-series variation in spreads. This demonstrates the importance of the second intensity in our benchmark specification.

We also investigated the performance of a model with three intensities. The model provided a better fit to the tranches that have the worst fit in the benchmark model, namely the 15-30% and 15-100% tranches. However, we found that in this model, some parameters are not well identified, and the resulting jump sizes are implausibly small.

In the benchmark model, we extract principal components using the levels of the covariates. Because the covariates are highly persistent, we now discuss the estimation of the benchmark model using a principal component analysis on the changes in the covariates rather than the levels. We obtain the principal component loadings ξ_j from the covariate changes. These loadings are reported in Panel A of Table A1, which also reports model parameters, deltas, and model fit for this implementation. The first principal component still loads heavily on volatility. The second principal component differs somewhat from the benchmark implementation; the correlation between the second principal components in both implementations is 67%. The parameter estimates are similar to the benchmark estimates reported in Table 3, with minor differences. The jump sizes are similar across both implementations. The sign of the deltas is also similar for all three covariates. The magnitude of the deltas is similar except for the leverage delta, which is lower relative to the benchmark model. Finally, Figure A2 and Panel F of Table A1 indicate that model fit is also similar across both implementations. We conclude that qualitatively, these results are similar to the benchmark results.

Finally, the benchmark model is estimated using the bootstrapped libor-swap rates as a proxy for the discount rate, but the discount rate is treated as deterministic. To alleviate concerns regarding the impact of nonzero correlation between the discount rate and the covariates, particularly the T-bill, we perform two additional robustness exercises. In the first, we re-estimate the model using a constant discount rate for all cash flows. This implementation uses the three-month libor rate to discount cash flows for all horizons on a given date, i.e. we use a flat term structure to discount the cash flows. The second exercise uses the average three-month libor rate during the sample to discount cash flows for all horizons, i.e. the discount rate is constant over time and across maturities. Both implementations ensure that the discount factor can be treated as deterministic in our model. We report the parameter estimates for these two robustness exercises in Table A2. For convenience, we also replicate the parameter estimates from the benchmark model. We obtain similar results for all three implementations. This is not surprising: the discount factor has a relatively minor impact on credit spreads, partly because it affects both the premium and the default leg of the pricing formula.

3 Value-at-Risk

Banks are required to undertake scenario analyses under Basel III, which are typically described in terms of changes in economic variables. Using our model, it is straightforward to comply with these regulatory requirements because we explicitly model the role of the economic factors in the valuation of the CDO. For example, what happens to the value of the bank's portfolio when there is a 20 percent increase in leverage and/or a 15 percent increase in volatility. Note that for the reporting of value-at-risk and other risk metrics such as expected shortfall, we require estimates of both the P and Q probability measures, which are readily provided by our model.

We recursively estimate value-at-risk (VaR) for all tranches. Each week, we estimate the model using the past one year of weekly data and compute the VaR for the one-month horizon. Using the estimated model parameters under the physical measure, we first simulate 10,000 paths of all three covariates for the relevant horizons. For each simulation, we then compute spreads for all tranches and generate the distribution of the change in the value of tranche at time $t + k$. We report the 1% and 5% VaR for each tranche.

Figure A3 presents the results on the one-month (4 weeks) VaR. For convenience, we assume that we are long credit risk and that the initial value of our position is zero. The fluctuations in the VaR over time are largely consistent with economic intuition. The VaR decreases in absolute value for more senior tranches, with the equity tranche having the largest absolute value.

VaR is relatively small in the beginning of the sample, which corresponds to a relatively quiet period in financial markets. The 1% VaR for the equity tranche is approximately 3 cents to the dollar in 2006, whereas it is near zero for the senior tranche. The collapse of two Bear Stearns hedge funds at the end of July of 2007 triggers the start of an increase in absolute VaR for all tranches. The VaR then remains elevated during the entire crisis period. For the equity tranche, the one month 1% VaR reaches a maximum of 7% during the financial crisis. The 1% VaR for the equity tranche fluctuates between 3% and 7% from 2011 onwards.

What is most interesting is the sharp increase in the VaR for the senior tranches. The VaR for the super senior tranche begins to increase in September 2007. By September 2008 the market perceives a one percent chance of an almost four cents loss on a dollar notional for the senior tranche over the next month. Assuming a 100 percent loss, there must be at least 15 defaults for this tranche to be impaired. Given that the names in the collateral pool are investment grade, this implies the possibility of a catastrophic event in the economy. The VaR remains elevated during the rest of the financial crisis period. The VaR for the super senior tranche reduces to around 1% and remains stable from 2011 onwards. The VaR is also relatively stable for the mezzanine tranches during this time period.

Table A1: Principal Components Based on Covariate Changes

Panel A: PC Loadings

	PC-1	PC-2
Volatility	0.989	-0.146
Leverage	0.146	0.989
T-bill	-0.006	-0.012

Panel B: Covariate Dynamics

	μ^Q	ρ^Q	μ^P	ρ^P	σ
Volatility	-0.0055	1.0095	0.0104	0.9646	0.0309
Leverage	-0.0067	1.0132	0.0083	0.9830	0.0059
T-bill	-0.0024	1.0398	0.00003	0.9979	0.0012

Panel C: Intensity Loadings

	Factor Loadings (κ)		α
	PC-1	PC-2	
λ_1	0.0201		0.0409
λ_2		-0.0037	0.0033

Panel D: Jump Properties

	Jump Size	Max Prob.	Collateral Loss
λ_1	0.0305	17.8%	0.0300
λ_2	0.3260	0.0180%	0.2782

Panel E: Average Deltas

	0-3%	3-7%	7-15%	15-100%
Volatility	26.57	12.07	3.27	0.04
Leverage	9.64	6.54	4.29	0.64
T-bill	-131.15	-86.35	-42.42	-4.81

Panel F: Model Fit

	0-3%	3-7%	7-10%	7-15%	10-15%	15-30%	15-100%
RMSE (%)	33.7	24.3	30.4	22.4	36.0	53.1	46.4
RMSE (bps)	702.8	113.2	70.7	39.7	45.1	26.1	14.7

Notes to Table: Panel A reports the loadings of the principal components on each of the covariates. The principal component analysis is performed using the changes in the covariates. Panel B reports the parameter estimates for the covariate dynamics under both the risk-neutral and physical measure. Panel C reports the intensity loadings of each of the principal components. For each of the intensities, Panel D reports the jump size, the maximum probability of observing at least one jump over one year, and the maximum collateral loss if one jump occurs. Panel E presents deltas from the the no-arbitrage model. The deltas represent the change in spreads (in basis points) for a one percentage point change in the covariates. Panel F reports the model fit (RMSE and relative RMSE).

Table A2: The Role of the Discount Factor

Panel A: Covariate Dynamics

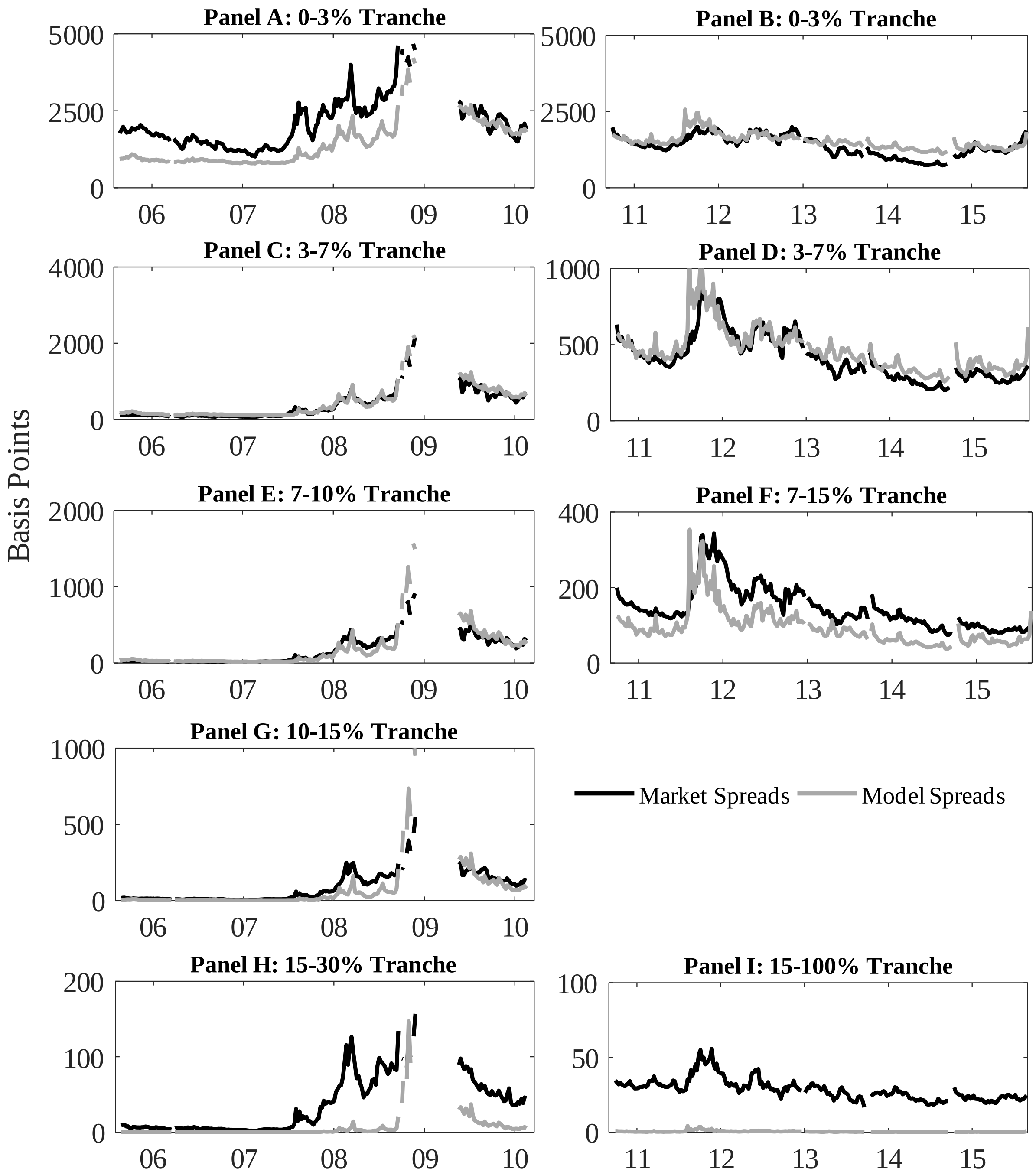
	Benchmark Model		Discount Factor: Flat Term Structure		Discount Factor: Flat Term Structure and Constant over Time	
	μ^Q	ρ^Q	μ^Q	ρ^Q	μ^Q	ρ^Q
Volatility	-0.0076	1.0075	-0.0077	1.0072	-0.0071	1.0076
Leverage	-0.0084	1.0195	-0.0084	1.0196	-0.0088	1.0196
T-Bill	-0.0020	1.0335	-0.0020	1.0333	-0.0020	1.0337

Panel B: Intensity Loadings

	Benchmark Model			Discount Factor: Flat Term Structure			Discount Factor: Flat Term Structure and Constant over Time		
	α	κ	Jump Size	α	κ	Jump Size	α	κ	Jump Size
PC-1	0.0471	0.0221	0.03	0.0468	0.0234	0.03	0.0462	0.0214	0.03
PC-2	0.0028	-0.0003	0.28	0.0028	-0.0003	0.30	0.0028	-0.0003	0.28

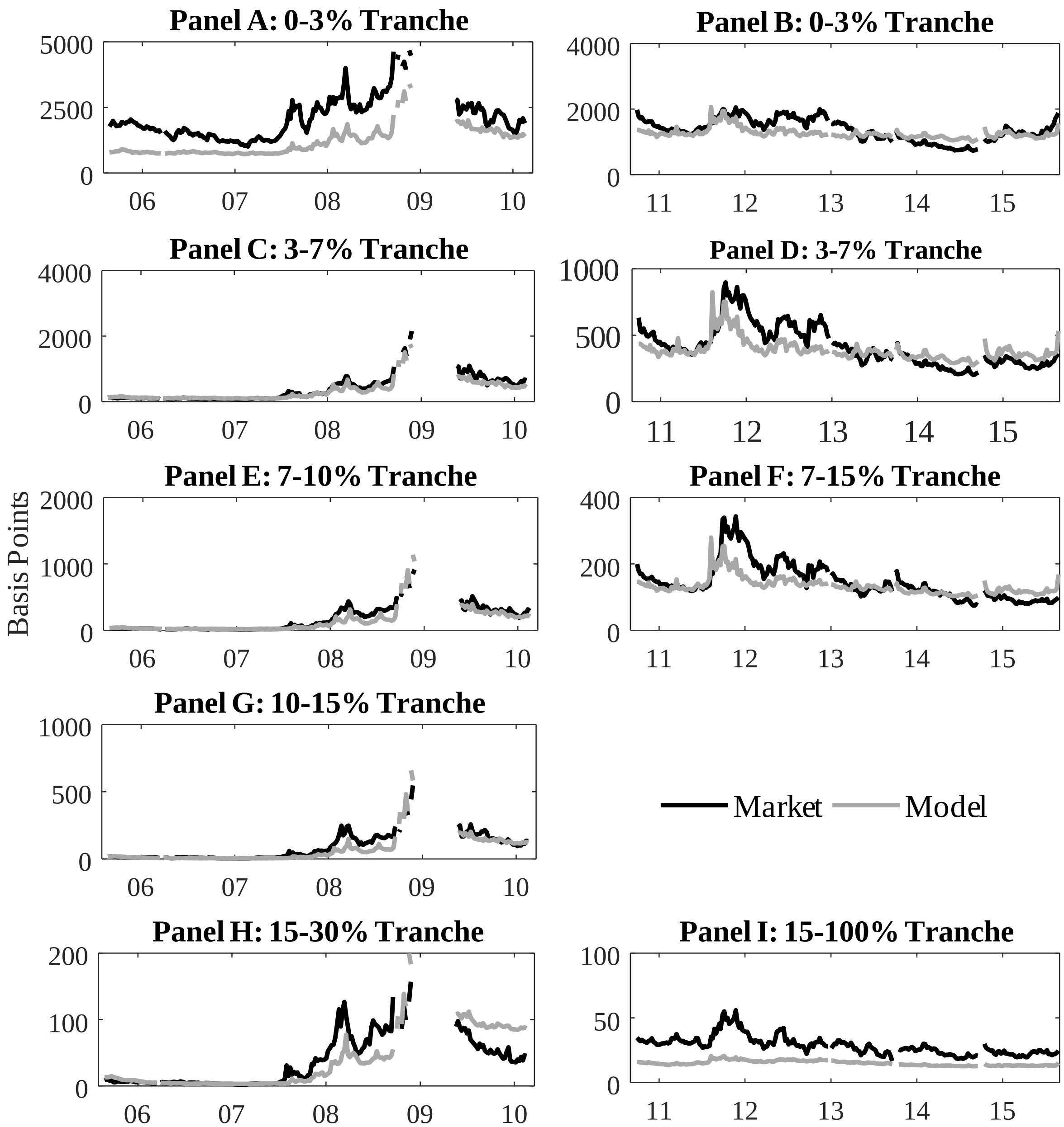
Notes to Table: Panel A reports the parameter estimates for the covariate dynamics under the risk-neutral measure for three different implementations: the benchmark model, estimation using a flat term structure for discount rates, and estimation using a flat term structure and a constant discount rate over time. Panel B reports the intensity loadings for each of the principal components and the jump size for each implementation. The benchmark implementation uses the term structure of bootstrapped libor-swap rates as a proxy for the discount factors. The implementation with a flat term structure uses the 3-month libor rate on each date to discount all cash flows. The constant over time specification uses the sample average of the 3-month libor rate to discount cash flows for all dates.

Figure A1: Model and Market Spreads: One Intensity



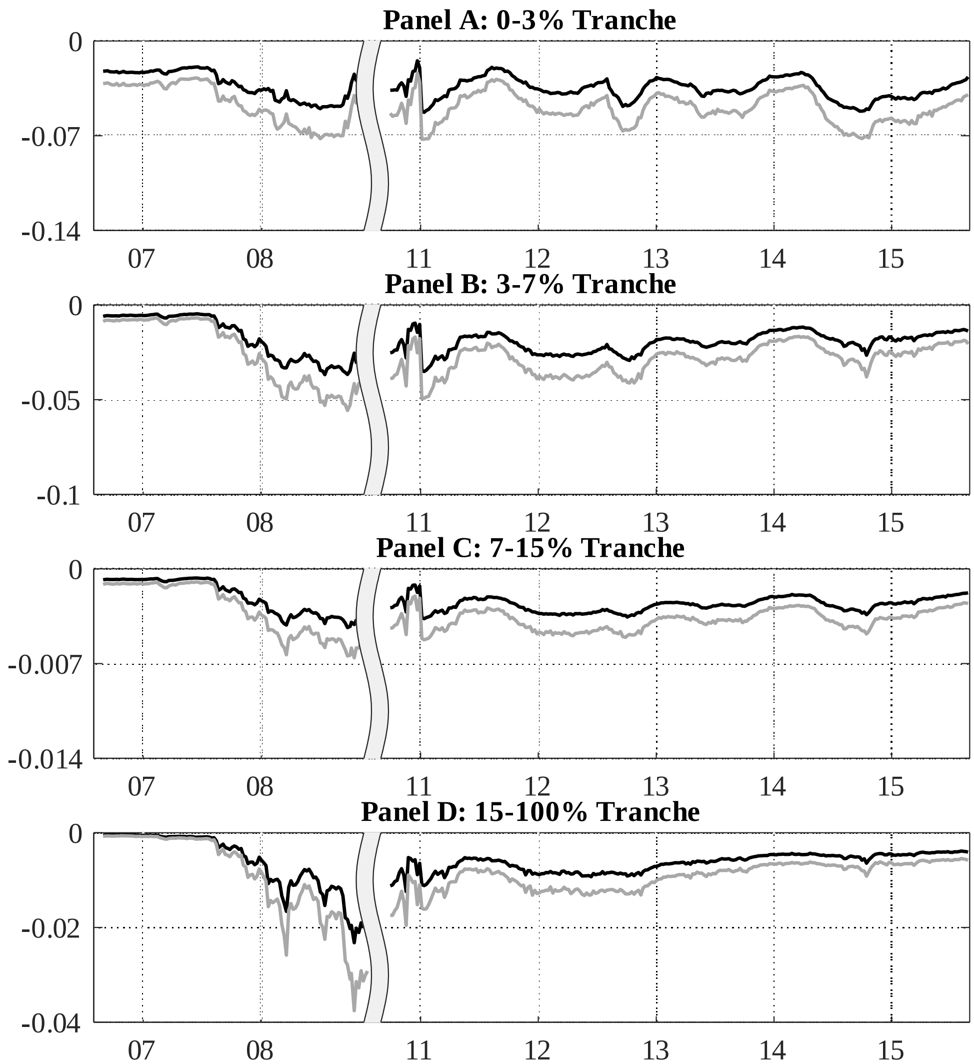
Notes to Figure: We plot the time series of market and model spreads for all tranches, using a model with one intensity. The sample period is from August 24, 2005 to August 26, 2015. The no-arbitrage model is estimated to fit weekly data on market CDX index tranche spreads. In each panel, the black line is for the market spreads and the grey line is for the model spreads.

Figure A2: Model and Market Spreads. Principal Components Extracted from Covariate Changes



Notes to Figure: We plot the time series of market and model spreads for all tranches. Principal components are extracted from covariate changes instead of the level of the covariates. The sample period is from August 24, 2005 to August 26, 2015. The no-arbitrage model is estimated to fit weekly data on market CDX index tranche spreads. In each panel, the black line is for the market spreads and the grey line is for the model spreads.

Figure A3: One-Month Value-at-Risk



Notes to Figure: We plot the one-month value-at-risk (VaR) for each tranche computed using rolling estimation. Each week, we estimate the model using the past one-year of weekly data and compute the VaR for the one-month horizon. We simulate 10000 iterations of all four covariates for 1 month horizon using the estimated VaR parameters under the physical measure. For each set of covariates, we obtain model spreads for all tranches, which are used to calculate the returns. The return is computed from the perspective of protection seller. For each CDX tranche, we plot the first and fifth percentile of returns. The gray line indicates the 1% VaR and the black line indicates the 5% VaR.