D Internet Appendix

Jin Hyuk Choi, Ulsan National Institute of Science and Technology (UNIST)
Kasper Larsen, Rutgers University
Duane J. Seppi, Carnegie Mellon University

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This Internet Appendix provides supporting proofs and additional numerical results for our paper "Information and Trading Targets in a Dynamic Market Equilibrium" in the *Journal of Financial Economics*.

D.1 Sufficient conditions for the modified FV model

Our derivation of sufficient conditions for a linear Bayesian Nash equilibrium for the modified FV model follows the same logic as in our dynamic-rebalancing model. Given a set $\{\lambda_n, \phi_n, \beta_n^I, \beta_n^L\}_{n=1}^N$ of model parameters, we define the following set of conjectured "hat" processes:

$$\Delta\hat{\theta}_n^I := \beta_n^I(\tilde{v} - \hat{p}_{n-1}), \quad \hat{\theta}_0^I := 0, \tag{D.1}$$

$$\Delta\hat{\theta}_n^L := \beta_n^L(\hat{s}_{n-1} - \hat{p}_{n-1}), \quad \hat{\theta}_0^L := 0, \tag{D.2}$$

$$\hat{y}_n := \Delta\hat{\theta}_n^I + \Delta\hat{\theta}_n^L + \Delta w_n, \quad \hat{y}_0 := 0, \tag{D.3}$$

$$\Delta\hat{p}_n := \lambda_n \hat{y}_n, \quad \hat{p}_0 := 0, \tag{D.4}$$

$$\Delta\hat{s}_n := \phi_n\Big(\hat{y}_n - (\beta_n^L + \beta_n^I)(\hat{s}_{n-1} - \hat{p}_{n-1})\Big), \quad \hat{s}_0 := \rho\frac{\sigma_{\tilde{v}}}{\sigma_{\tilde{a}}}\tilde{a}. \tag{D.5}$$

These processes must satisfy a variety of restrictions to be a linear Bayesian equilibrium. We derive these restrictions in two steps.

Step 1: The conjectured price and less-informed investor expectation processes must satisfy:

$$\hat{p}_n = \mathbb{E}[\tilde{v} \,|\, \hat{y}_1, ..., \hat{y}_n], \tag{D.6}$$

$$\hat{s}_n = \mathbb{E}[\tilde{v} \,|\, \tilde{a}, \hat{y}_1, ..., \hat{y}_n]. \tag{D.7}$$

We define the conditional moments for $n = 1, \dots N$:

$$\Sigma_n^{(1)} := \mathbb{V}\big[\tilde{v} - \hat{p}_n\big], \tag{D.8}$$

$$\Sigma_n^{(2)} := \mathbb{V}\big[\hat{s}_n - \hat{p}_n\big], \tag{D.9}$$

$$\Sigma_n^{(3)} := \mathbb{E}\big[\big(\hat{s}_n - \hat{p}_n\big)(\tilde{v} - \hat{p}_n)\big] = \Sigma_n^{(2)}, \tag{D.10}$$

where the last equality follows from iterated expectations. The starting values are:

$$\Sigma_0^{(1)} = \sigma_{\tilde{v}}^2, \quad \Sigma_0^{(2)} = \mathbb{V}\Big[\rho\frac{\sigma_{\tilde{v}}}{\sigma_{\tilde{a}}}\tilde{a}\Big] = \rho^2\sigma_{\tilde{v}}^2. \tag{D.11}$$

Furthermore, $\Sigma_n^{(1)} \geq \Sigma_n^{(2)}$ because we have

$$0 \leq \mathbb{V}\big[\tilde{v} - \hat{s}_n\big] = \mathbb{V}\big[\tilde{v} - \hat{p}_n + \hat{p}_n - \hat{s}_n\big] = \Sigma_n^{(1)} + \Sigma_n^{(2)} - 2\Sigma_n^{(3)} = \Sigma_n^{(1)} - \Sigma_n^{(2)}. \tag{D.12}$$

The filter dynamics are given by:

$$\begin{aligned}
\Sigma_n^{(1)} &= \mathbb{V}\big[\tilde{v} - \hat{p}_{n-1} - \Delta\hat{p}_n\big] \\
&= \mathbb{V}\big[\tilde{v} - \hat{p}_{n-1} - \lambda_n(\beta_n^I(\tilde{v} - \hat{p}_{n-1}) + \beta_n^L(\hat{s}_{n-1} - \hat{p}_{n-1}) + \Delta w_n)\big] \qquad \text{(D.13)} \\
&= (1 - \lambda_n\beta_n^I)^2\Sigma_{n-1}^{(1)} + (\lambda_n\beta_n^L)^2\Sigma_{n-1}^{(2)} - 2\lambda_n\beta_n^L(1 - \lambda_n\beta_n^I)\Sigma_{n-1}^{(3)} + \lambda_n^2\Delta\sigma_w^2, \\
\Sigma_n^{(2)} &= \mathbb{V}\big[\hat{s}_{n-1} + \Delta\hat{s}_n - (\hat{p}_{n-1} + \Delta\hat{p}_n)\big] \\
&= \mathbb{V}\big[\hat{s}_{n-1} + \phi_n(\beta_n^I(\tilde{v} - \hat{p}_{n-1} + \hat{p}_{n-1} - \hat{s}_{n-1}) + \Delta w_n) \\
&\quad - \hat{p}_{n-1} - \lambda_n(\beta_n^I(\tilde{v} - \hat{p}_{n-1}) + \beta_n^L(\hat{s}_{n-1} - \hat{p}_{n-1}) + \Delta w_n)\big] \qquad \text{(D.14)} \\
&= (\beta_n^I)^2(\phi_n - \lambda_n)^2\Sigma_{n-1}^{(1)} + (1 - \beta_n^I\phi_n - \beta_n^L\lambda_n)^2\Sigma_{n-1}^{(2)} \\
&\quad + 2\beta_n^I(\phi_n - \lambda_n)(1 - \beta_n^I\phi_n - \beta_n^L\lambda_n)\Sigma_{n-1}^{(3)} + (\phi_n - \lambda_n)^2\Delta\sigma_w^2.
\end{aligned}$$

To find the equations for the constants λ_n and ϕ_n appearing in (D.4) and (D.5) we need the investors' innovation processes. The informed investor (who knows $\tilde{v}$) has innovations defined by

$$\begin{aligned}
z_n^I :&= \hat{y}_n - \Big(\beta_n^I + \beta_n^L\frac{\Sigma_{n-1}^{(3)}}{\Sigma_{n-1}^{(1)}}\Big)(\tilde{v} - \hat{p}_{n-1}) \\
&= \Delta w_n + \beta_n^L(\hat{s}_{n-1} - \hat{p}_{n-1}) - \beta_n^L\frac{\Sigma_{n-1}^{(3)}}{\Sigma_{n-1}^{(1)}}(\tilde{v} - \hat{p}_{n-1}).
\end{aligned} \tag{D.15}$$

The less-informed investor (who knows $\tilde{a}$) learns about $\tilde{v}$ over time by filtering the aggregate order-flow process to construct the estimate process s_n given by (D.7). His innovations are defined by

$$\begin{aligned} z_n^L &:= \hat{y}_n - (\beta_n^I + \beta_n^L)(\hat{s}_{n-1} - \hat{p}_{n-1}) \\ &= \Delta w_n + \beta_n^I(\tilde{v} - \hat{s}_{n-1}). \end{aligned} \tag{D.16}$$

Finally, the market makers' innovations are defined by

$$z_n^M := \hat{y}_n, \tag{D.17}$$

because all trades of the forms (D.1) and (D.2) are unpredictable for the market makers. Based on the requirement (D.6), we can use (D.17) to obtain the representation

$$\Delta \hat{p}_n = \frac{\mathbb{E}[(\tilde{v} - \hat{p}_{n-1})z_n^M]}{\mathbb{V}[z_n^M]} z_n^M. \tag{D.18}$$

We then use the projection theorem for multivariate normals to see that the price coefficient in (D.4) is given by

$$\lambda_n = \frac{\beta_n^I \Sigma_{n-1}^{(1)} + \beta_n^L \Sigma_{n-1}^{(2)}}{(\beta_n^I)^2 \Sigma_{n-1}^{(1)} + (\beta_n^L)^2 \Sigma_{n-1}^{(2)} + 2\beta_n^I \beta_n^L \Sigma_{n-1}^{(2)} + \Delta\sigma_w^2}. \tag{D.19}$$

Similarly, we can use the less-informed investor's innovation process (D.16) to re-write (D.7) as

$$\Delta \hat{s}_n = \frac{\mathbb{E}[(\tilde{v} - \hat{s}_{n-1})z_n^L]}{\mathbb{V}[z_n^L]} z_n^L. \tag{D.20}$$

Consequently, we find the coefficient requirement

$$\phi_n = \frac{\beta_n^I (\Sigma_{n-1}^{(1)} - \Sigma_{n-1}^{(2)})}{(\beta_n^I)^2 (\Sigma_{n-1}^{(1)} - \Sigma_{n-1}^{(2)}) + \Delta\sigma_w^2}. \tag{D.21}$$

Step 2: The price and updating processes as well as the order-flow coefficients also need to be consistent with the two informed investors' optimization problems. First, we consider the better-informed investor where the less-informed investor's strategy is fixed to be the conjectured strategy (B.5). Then, for $\Delta\theta_n^I \in \sigma(\tilde{v}, y_1, ..., y_{n-1})$, we have

$$
\begin{aligned}
&\mathbb{E}[(\tilde{v}-p_n)\Delta\theta_n^I|\tilde{v}-\hat{p}_{n-1}]\\
&=\Delta\theta_n^I\mathbb{E}[\tilde{v}-p_{n-1}-\lambda_n\Delta\theta_n^I-\lambda_n\beta_n^L(s_{n-1}-p_{n-1})|\tilde{v}-\hat{p}_{n-1}]\\
&=\Delta\theta_n^I(\tilde{v}-p_{n-1})-\lambda_n(\Delta\theta_n^I)^2\\
&-\lambda_n\beta_n^L\Delta\theta_n^I\mathbb{E}[s_{n-1}+\hat{s}_{n-1}-\hat{s}_{n-1}+\hat{p}_{n-1}-\hat{p}_{n-1}-p_{n-1}|\tilde{v}-\hat{p}_{n-1}]\\
&=\Delta\theta_n^I(\tilde{v}-p_{n-1})-\lambda_n(\Delta\theta_n^I)^2-\lambda_n\beta_n^L\Delta\theta_n^I\Big(s_{n-1}-\hat{s}_{n-1}+\hat{p}_{n-1}-p_{n-1}+\frac{\Sigma_{n-1}^{(2)}}{\Sigma_{n-1}^{(1)}}(\tilde{v}-\hat{p}_{n-1})\Big)\\
&=\Delta\theta_n^IX_{n-1}^{(1)}-\lambda_n(\Delta\theta_n^I)^2-\lambda_n\beta_n^L\Delta\theta_n^IX_{n-1}^{(2)}, \qquad \text{(D.22)}
\end{aligned}
$$

where we have defined the two state-variables:

$$
X_n^{(1)}:=\tilde{v}-p_n,\quad X_n^{(2)}:=s_n-\hat{s}_n+\hat{p}_n-p_n+\frac{\Sigma_n^{(2)}}{\Sigma_n^{(1)}}(\tilde{v}-\hat{p}_n). \qquad \text{(D.23)}
$$

The dynamics of the first state-variable are given by

$$
\begin{aligned}
\Delta X_n^{(1)}&=-\Delta p_n\\
&=-\lambda_n\Big(\Delta\theta_n^I+\beta_n^L(s_{n-1}-p_{n-1})+\Delta w_n\Big)\\
&=-\lambda_n\Big(\Delta\theta_n^I+\beta_n^L(s_{n-1}-p_{n-1})+z_n^I-\beta_n^L(\hat{s}_{n-1}-\hat{p}_{n-1})+\beta_n^L\frac{\Sigma_{n-1}^{(2)}}{\Sigma_{n-1}^{(1)}}(\tilde{v}-\hat{p}_{n-1})\Big)\\
&=-\lambda_n\Big(\Delta\theta_n^I+\beta_n^LX_{n-1}^{(2)}+z_n^I\Big). \qquad \text{(D.24)}
\end{aligned}
$$

Similarly, by using (D.13)-(D.14) and (D.19)-(D.21) we find the dynamics of the second state-variable to be:

$$
\Delta X_n^{(2)}=(\phi_n-\lambda_n)\Delta\theta_n^I-(\phi_n\beta_n^I+\lambda_n\beta_n^L)X_{n-1}^{(2)}-\lambda_n\frac{\Sigma_n^{(2)}}{\Sigma_n^{(1)}}z_n^I. \qquad \text{(D.25)}
$$

Second, we consider the less-informed investor and here the better-informed investor's strategy is fixed as in (B.4). Then, for $\Delta\theta_n^L \in \sigma(\tilde{a}, y_1, ..., y_{n-1})$, we have

$$\begin{aligned}
\mathbb{E}[(\tilde{v} - p_n)\Delta\theta_n^L|\tilde{a} - \hat{p}_{n-1}] &= \Delta\theta_n^L \mathbb{E}[(\tilde{v} - p_{n-1} - \lambda_n\Delta\theta_n^L - \lambda_n\beta_n^I(\tilde{v} - p_{n-1})|\tilde{a} - \hat{p}_{n-1}] \\
&= -\lambda_n(\Delta\theta_n^L)^2 + (1 - \lambda_n\beta_n^I)\Delta\theta_n^L \mathbb{E}[\tilde{v} - p_{n-1}|\tilde{a} - \hat{p}_{n-1}] \\
&= -\lambda_n(\Delta\theta_n^L)^2 + (1 - \lambda_n\beta_n^I)\Delta\theta_n^L\Big(\hat{s}_{n-1} - p_{n-1}\Big) \\
&= -\lambda_n(\Delta\theta_n^L)^2 + (1 - \lambda_n\beta_n^I)\Delta\theta_n^L(Y_{n-1}^{(2)} + Y_{n-1}^{(1)}), \qquad \text{(D.26)}
\end{aligned}$$

where we have defined the two state-variables:

$$Y_n^{(1)} := s_n - p_n, \quad Y_n^{(2)} := \hat{s}_n - s_n. \qquad \text{(D.27)}$$

Similarly to the better-informed investor considered before, we find the dynamics

$$\Delta Y_n^{(1)} = \phi_n\Big(z_n^L + \beta_n^I Y_{n-1}^{(2)} + \Delta\theta_n^L - \beta_n^L Y_{n-1}^{(1)}\Big) - \lambda_n\Big(\beta_n^I(Y_{n-1}^{(1)} + Y_{n-1}^{(2)}) + \Delta\theta_n^L + z_n^L\Big), \qquad \text{(D.28)}$$

$$\Delta Y_n^{(2)} = -\phi_n\Big(\beta_n^I Y_{n-1}^{(2)} + \Delta\theta_n^L - \beta_n^L Y_{n-1}^{(1)}\Big). \qquad \text{(D.29)}$$

The above dynamics of the state-variables (D.23) and expression for the conditional expectation (D.22) ensure that the better-informed investor's problem (not stated for brevity) is a quadratic maximization problem. Therefore, subject to second-order conditions, the optimal orders $\Delta\hat{\theta}_n^I$ are linear in the state-variables (D.23). Similarly, given the above dynamics of the state-variables (D.27) and the expression for the conditional expectation (D.26) ensure that the less-informed investor's problem is also quadratic with linear optimal orders $\Delta\hat{\theta}_n^L$. By inserting the respective optimal linear orders into their respective quadratic optimization problems, we find recursions for the coefficients describing the two quadratic value functions.

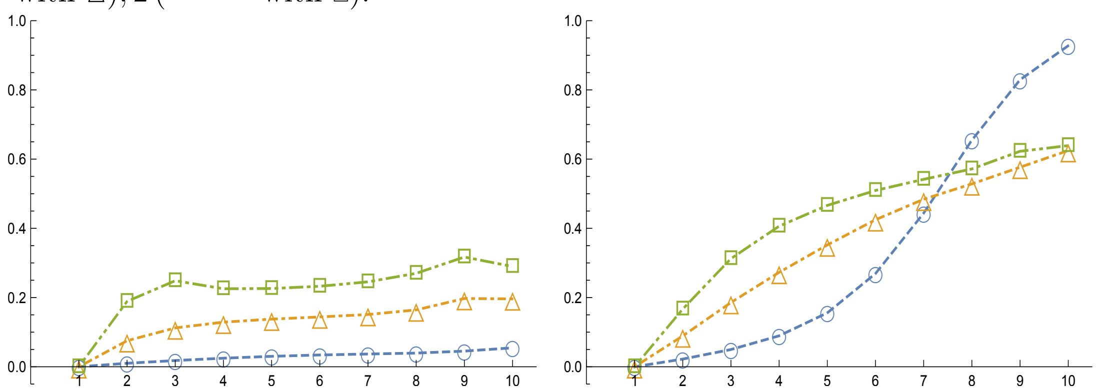
D.2 Intraday patterns in sunshine trading

The rebalancer's expected orders given his target $\tilde{a}$ can be further decomposed to isolate predictable sunshine trading relative to the other expected drivers of the rebalancer's strategy. We do this by computing the ratio of the rebalancer's expectation of his sunshine trading relative to his total expected orders given $\tilde{a} \neq 0$

$$\frac{\mathbb{E}[\mathbb{E}[\Delta\theta_n^R \,|\, y_1, \ldots, y_{n-1}] \,|\, \tilde{a}]}{\mathbb{E}[\Delta\theta_n^R \,|\, \tilde{a}]} = \frac{(\alpha_n^R + \beta_n^R)\mathbb{E}[q_{n-1} \,|\, \tilde{a}]}{\mathbb{E}[\Delta\theta_n^R \,|\, \tilde{a}]}. \tag{D.30}$$

Figures 12A and B show that expected sunshine trading is increasing in the target variance $\sigma_{\tilde{a}}^2$ and correlation ρ.[34] When the target $\tilde{a}$ is ex ante uninformative (i.e., $\rho = 0$), sunshine trading accounts for between 5% and 30% of the rebalancer's total expected orders. However, when $\tilde{a}$ is informative, then sunshine trading can account for an even larger portion of trading later in the day. Interestingly, Figure 12B shows that the impact of greater target variance $\sigma_{\tilde{a}}^2$ can be non-monotone at later dates (e.g., at time $n = 9$, the intermediate target variance $(- \cdot -)$ line is below both the lower variance $(--)$ line and the higher variance $(- \cdot \cdot -)$ line). However, the main point here is that other deterministic trading components can also be large.

Figure 12: Intraday patterns for the ratio of expected sunshine trading relative to the rebalancer's total expected orders given a target $\tilde{a} \neq 0$ for times $n = 1, \ldots, 10$. The parameters are $N = 10$, $\sigma_{\tilde{v}}^2 = 1$, $\sigma_w^2 = 1$, and $\sigma_{\tilde{a}}^2 = 0.2\,(- - \text{ with } \circ), 1\,(- \cdot - \text{ with } \triangle), 2\,(- \cdot \cdot - \text{ with } \square)$.



A: $\rho = 0$ B: $\rho = 0.45$

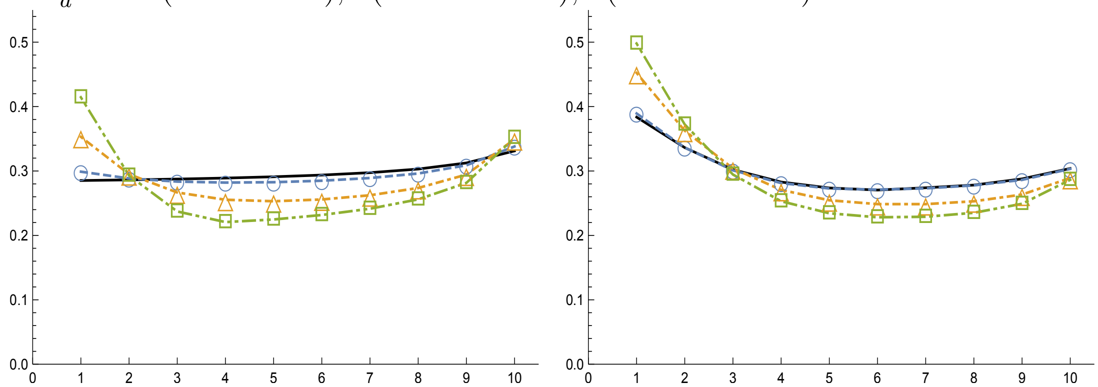
D.3 Intraday patterns in price volatility

Figure 13 shows the unconditional standard deviation for the price changes $p_n - p_{n-1}$ over time. Price volatility is monotonely increasing over time in the Kyle model (solid black line) in Figure 13A, whereas our rebalancing model produces U-shaped

[34] In the modified FV model, with no trading constraint, there is no sunshine trading.

price volatility (dotted lines). In addition, the U-shape becomes larger when rebalancing volatility is higher. When $\rho > 0$, Figure 13B shows that price volatility has a downward-sloping U-shape in both the modified FV model and in our dynamic rebalancing model.

Figure 13: Intraday patterns for the unconditional price-change standard deviations $\mathrm{SD}[p_n - p_{n-1}]$ for times $n = 1, 2, ..., 10$. The parameters are $N = 10$, $\sigma_{\tilde{v}}^2 = 1$, $\sigma_w^2 = 1$, and $\sigma_{\tilde{a}}^2 = 0.2\,(- - \text{ with } \circ), 1\,(- \cdot - \text{ with } \triangle), 2\,(- \cdot \cdot - \text{ with } \square)$.



A: $\rho = 0$, [Kyle]: (——) B: $\rho = 0.45$, [FV]: (——)

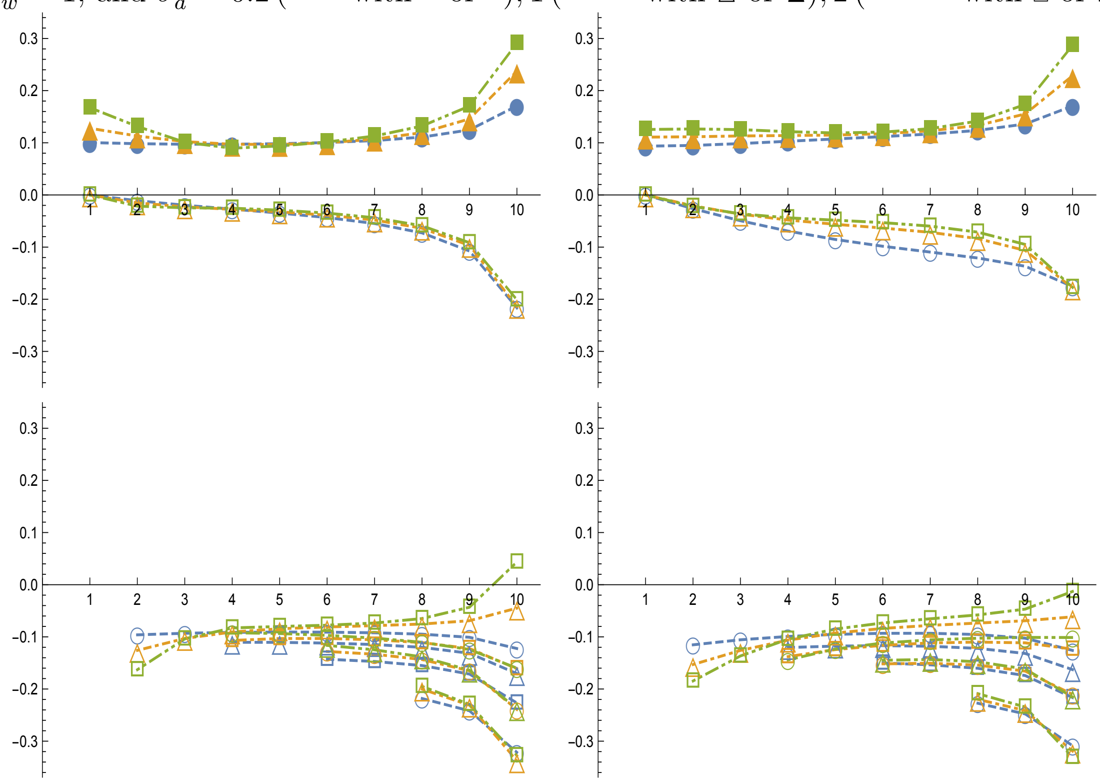
D.4 Intraday patterns in informed-investor trading

Equation (56) gives the linear decomposition for the informed-investor orders in terms of the asset value $\tilde{v}$ (which the informed investor knows), the trading target $\tilde{a}$ (which she does not know), and the noise-trader orders. Figure 14 shows the informed investor's decomposition coefficients for our six reference parameterizations. As expected, the informed-investor orders load positive on the stock value $\tilde{v}$ and load negatively on the target $\tilde{a}$ and the noise-trader orders Δw_j since both inject price-pressure noise in prices.

D.5 Negative cross-correlation of orders

The rebalancer and hedge-fund order decompositions (52) and (56) can be used to understand our correlation results by dividing the order correlations into components

Figure 14: Intraday patterns for the linear-decomposition coefficients for the informed-trader orders for times $n = 1, \ldots, 10$. The top figures show the coefficients A^I_n on the rebalancer's target $\tilde{a}$ (lines with $\circ$, $\triangle$, $\square$) and B^I_n on the asset value $\tilde{v}$ (lines with $\bullet$, $\blacktriangle$, $\blacksquare$), and the lower figures show the coefficients $c^I_{j,n}$ on Δw_j for for noise-trader order arrival times $j = 1, 3, 5$, and 7. The parameters are $N = 10$, $\sigma^2_{\tilde{v}} = 1$, $\sigma^2_w = 1$, and $\sigma^2_{\tilde{a}} = 0.2$ ($- -$ with $\circ$ or $\bullet$), 1 ($- \cdot -$ with $\triangle$ or $\blacktriangle$), 2 ($- \cdot \cdot -$ with $\square$ or $\blacksquare$).



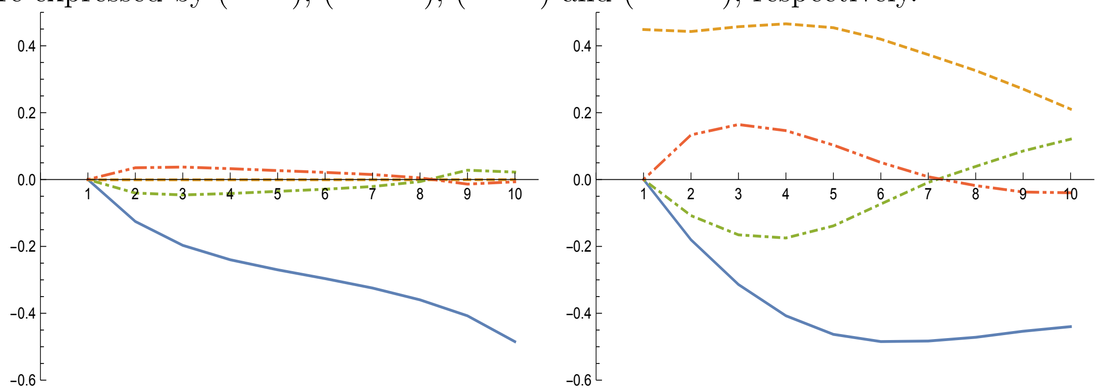
A: $\rho = 0$ B: $\rho = 0.45$

due to the two investors' loadings on $\tilde{a}$, $\tilde{v}$, and the noise-trading orders

$$\begin{aligned} \text{corr}(\Delta\theta^R_n, \Delta\theta^I_n) = & \frac{A^R_n A^I_n \, \sigma^2_{\tilde{a}}}{\text{SD}[\Delta\theta^R_n]\,\text{SD}[\Delta\theta^I_n]} + \frac{(A^R_n B^I_n + B^R_n A^I_n)\,\sigma_{\tilde{a}}\sigma_{\tilde{v}}\rho}{\text{SD}[\Delta\theta^R_n]\,\text{SD}[\Delta\theta^I_n]} \\ & + \frac{B^R_n B^I_n \, \sigma^2_{\tilde{v}}}{\text{SD}[\Delta\theta^R_n]\,\text{SD}[\Delta\theta^I_n]} + \sum_{j=1,\ldots n-1} \frac{c^R_{j,n} c^I_{j,n} \sigma^2_w \Delta}{\text{SD}[\Delta\theta^R_n]\,\text{SD}[\Delta\theta^I_n]}. \end{aligned} \tag{D.31}$$

Figure 15 shows for the case of $\sigma^2_{\tilde{a}} = 1$ and $\rho = 0$ that the negative correlation component due to the target $\tilde{a}$ can account for a large part of the negative correlation between the rebalancer and informed hedge-fund orders. The rebalancer trades in the direction of his target $\tilde{a}$ while the hedge fund trades opposite the noise that rebalancing induces in prices (i.e., see the negative loading A_n^I in Figure 14).

Figure 15: Intraday patterns for the correlation components for times $n = 1, 2, ..., 10$. The parameters are $N = 10$, $\sigma_{\tilde{v}}^2 = 1$, $\sigma_w^2 = 1$, and $\sigma_{\tilde{a}}^2 = 1$. The four terms in (D.31) are expressed by (——), (– – –), (– · –) and (– · · –), respectively.



A: $\rho = 0$ B: $\rho = 0.45$

D.6 Analysis using ad hoc strategies

The relative importance of different economic considerations driving the rebalancer orders can be quantified using a second decomposition. This approach takes the equilibrium pricing rule $\{p_n\}$ and informed-investor strategy $\{\theta_n^I\}$ as given, and then computes a number of ad hoc rebalancer trading strategies that ignore various combinations of the different economic considerations for the rebalancer's optimal equilibrium strategy. Specifically, we consider five ad hoc strategies in which the rebalancer

1. Trades just once to reach his full target $\tilde{a}$ but optimizes the time he trades so as to minimize his expected trading cost. This is the Admati and Pfleiderer (1988) strategy.

2. Trades deterministically to reach his target $\tilde{a}$ by splitting his orders equally over time (i.e., trading $\tilde{a}/N$ at each time n). This is the *time-weighted average price (TWAP)* strategy.

3. Trades deterministically to reach his target $\tilde{a}$ and minimizes his expected cost taking into account the time pattern of the equilibrium price-impacts λ_n, but

ignoring the effects of trading by the informed investor and sunshine trading.[35]

4. Trades deterministically to reach his target $\tilde{a}$ and minimizes his expected cost taking into account the time pattern of the equilibrium λ_ns and predictable effects of the informed investor's trading but ignoring sunshine-trading.[36]

5. Trades to reach his target $\tilde{a}$ using the optimal deterministic strategy in Proposition 2.

The orders for these strategies $j = 1, \ldots, 5$ at time n are denoted here by x_n^j. These ad hoc strategies are off-equilibrium deviations from the rebalancer's equilibrium orders. Comparing different ad hoc strategies and the equilibrium strategy disentangles the impacts of various omitted and included economic considerations in the rebalancer's trading behavior. Conveniently, each of the ad hoc strategies is linear in the target $\tilde{a}$.

Table 1 measures the distance between the ad hoc strategies and the equilibrium rebalancer strategy using mean squared errors (MSEs). We average the squared errors $(\Delta\theta_n^R - x_n^j)^2$ at each time n given a target level $\tilde{a}$ across different simulated values of $\tilde{v}$ and noise trader orders and then sum them across all N dates. The table also includes the expected total sum-of-squares (TSS) for the equilibrium strategy $\sum_{n=1}^{N} \mathbb{E}[(\Delta\hat{\theta}_n^R)^2|\tilde{a}]$ for some context about size. Note, first, that the TSS intercepts — which are the contribution of the adaptive component to the total sum-of-squares of the equilibrium strategy — are small.[37] This indicates that the adaptive component has a quantitatively small impact on rebalancer orders. Second, the MSE coefficients multiplying $\tilde{a}^2$ for the ad hoc strategies measure suboptimality due to omitted deterministic trading considerations. Not surprisingly, these coefficients are large for the single-date Admati-Pfleiderer strategy #1 but can be quite small for the multiperiod strategies. However, strategy #3 is an outlier and deviates from strategy #4 by a large amount. This shows that predictable interactions with the informed-investor orders can have a quantitatively important effect on the rebalancer's trading strategy.

[35] These orders are optimized for a perceived pricing rule $\Delta p_n = \lambda_n(\Delta\theta_n^R + \Delta w_n)$ that excludes the informed investor's orders $\Delta\theta_n^I$ and the sunshine-trading adjustment $-\lambda_n\,(\alpha_n^R + \beta_n^R)\,q_{n-1}$.

[36] These orders are optimized for a perceived price rule $\Delta p_n = \lambda_n(\Delta\theta_n^R + \Delta\theta_n^I + \Delta w_n)$ with the correct aggregate order flow but missing the sunshine-trading adjustment $-\lambda_n\,(\alpha_n^R + \beta_n^R)q_{n-1}$.

[37] In particular, the TSS intercepts are small relative to the variability induced by a one standard deviation rebalancing shock $a = \sigma_{\tilde{a}}$. The intercepts are also the same in all the MSEs since all of the ad hoc strategies are deterministic and thus omit the adaptive order variability.

Table 1: Mean squared errors for ad hoc strategies $x_n^1, \dots, x_n^5$ relative to the equilibrium rebalancer strategy $\Delta\theta_n^R$ and the equilibrium total sum of squares.

	$\rho = 0$, $\sigma_{\tilde{a}}^2 = 0.2$	$\rho = 0$, $\sigma_{\tilde{a}}^2 = 1$	$\rho = 0$, $\sigma_{\tilde{a}}^2 = 2$
MSE for strategy #1	$0.0001 + 0.6213\,\tilde{a}^2$	$0.0007 + 0.6097\,\tilde{a}^2$	$0.0008 + 1.0615\,\tilde{a}^2$
MSE for strategy #2	$0.0001 + 0.0324\,\tilde{a}^2$	$0.0007 + 0.0375\,\tilde{a}^2$	$0.0008 + 0.0410\,\tilde{a}^2$
MSE for strategy #3	$0.0001 + 0.0259\,\tilde{a}^2$	$0.0007 + 0.1205\,\tilde{a}^2$	$0.0008 + 0.3540\,\tilde{a}^2$
MSE for strategy #4	$0.0001 + 0.0003\,\tilde{a}^2$	$0.0007 + 0.0068\,\tilde{a}^2$	$0.0008 + 0.0256\,\tilde{a}^2$
MSE for strategy #5	0.0001	0.0007	0.0008
Equilibrium TSS	$0.0001 + 0.1324\,\tilde{a}^2$	$0.0007 + 0.1375\,\tilde{a}^2$	$0.0008 + 0.1410\,\tilde{a}^2$
	$\rho = 0.45$, $\sigma_{\tilde{a}}^2 = 0.2$	$\rho = 0.45$, $\sigma_{\tilde{a}}^2 = 1$	$\rho = 0.45$, $\sigma_{\tilde{a}}^2 = 2$
MSE for strategy #1	$0.0062 + 0.8508\,\tilde{a}^2$	$0.0093 + 1.0005\,\tilde{a}^2$	$0.0093 + 1.0211\,\tilde{a}^2$
MSE for strategy #2	$0.0062 + 0.0411\,\tilde{a}^2$	$0.0093 + 0.0242\,\tilde{a}^2$	$0.0093 + 0.0300\,\tilde{a}^2$
MSE for strategy #3	$0.0062 + 0.3991\,\tilde{a}^2$	$0.0093 + 0.8027\,\tilde{a}^2$	$0.0093 + 1.4935\,\tilde{a}^2$
MSE for strategy #4	$0.0062 + 0.0015\,\tilde{a}^2$	$0.0093 + 0.0045\,\tilde{a}^2$	$0.0093 + 0.0084\,\tilde{a}^2$
MSE for strategy #5	0.0062	0.0093	0.0093
Equilibrium TSS	$0.0062 + 0.1411\,\tilde{a}^2$	$0.0093 + 0.1242\,\tilde{a}^2$	$0.0093 + 0.1300\,\tilde{a}^2$

Table 2 shows the rebalancer's expected trading profits conditional on the target $\tilde{a}$ for each of the ad hoc strategies $x_n^1, \dots, x_n^5$ and for the rebalancer's equilibrium strategy $\Delta\theta_n^R$ ("Equilibrium"). Given risk neutrality and the linearity of the prices and informed orders, the rebalancer's value function is quadratic in $\tilde{a}$. We average over $\tilde{v}$ and the noise trader orders for the various parameterizations.

There are several things to note in Table 2: First, the rebalancer's value function based on his equilibrium strategy includes a positive constant that reflects the contribution of adaptive sunshine trading and speculation based on endogenous learning. However, the incremental impact of adaptive trading is often numerically small relative to the contribution from deterministic trading. This reinforces the earlier point in Table 1 about adaptive trading being economically small. Second, when ρ is zero (or sufficiently small), there is a negative coefficient on the $\tilde{a}^2$ term indicating that the contribution to the rebalancer's expected profits from trading towards the target $\tilde{a}$ is negative. This is because the uninformed deterministic part of the rebalancer's orders on average push the price away from $\tilde{v}$. However, when the target has significant information content (i.e., when ρ is large), then the coefficient on $\tilde{a}$ can be positive or negative. In particular, reducing the size of the target relative to its information content — i.e., when $\sigma_{\tilde{a}}$ is small — can make the rebalancer's expected profit be positive by not constraining him to trade larger quantities than he would optimally choose to trade given the information in $\tilde{a}$. This can be seen by comparing the rebalancer expected profit when $\rho = 0.45$ in the case $\sigma_{\tilde{a}}^2 = 0.2$ (when the rebalancer is constrained to trade less than he would like given his information and his equilibrium expected profit from trading on $\tilde{a}$ is actually positive) and the cases of $\sigma_{\tilde{a}}^2 = 1$ and 2 (when the rebalancer has a negative expected profit from trading on $\tilde{a}$ because his orders are constrained to be too large relative to the informativeness of his target). Third, the rebalancer's expected profit increases significantly when the rebalancer splits his orders over time relative to just trading once at an optimally chosen single time. Taking the intraday pattern of price impact and predictable interactions with the informed-investor orders into account also has significant positive effects. However, the incremental impact of sunshine trading (comparing the expected profits for strategies #4 and #5) seems small.

Table 2: Expected profit for ad hoc rebalancer strategies $x_n^1, \ldots, x_n^5$ and using the equilibrium strategy $\Delta\theta_n^R$ conditional on a parent target $\tilde{a}$.

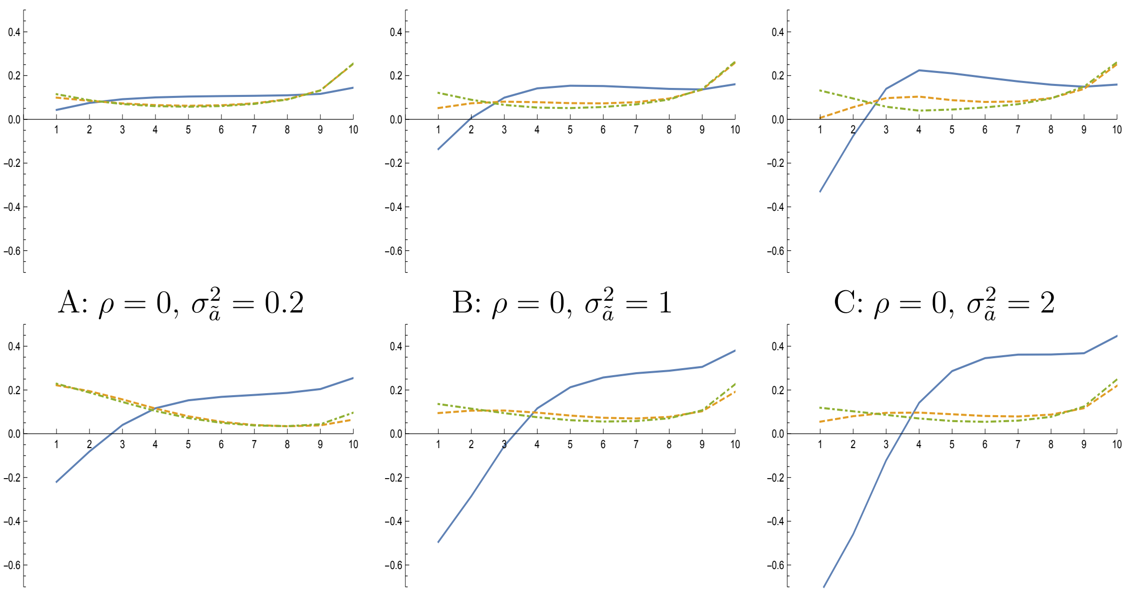
Strategy	$\rho = 0,\ \sigma_{\tilde{a}}^2 = 0.2$	$\rho = 0,\ \sigma_{\tilde{a}}^2 = 1$	$\rho = 0,\ \sigma_{\tilde{a}}^2 = 2$
1	-0.7343 $\tilde{a}^2$	-0.7159 $\tilde{a}^2$	-0.6657 $\tilde{a}^2$
2	-0.2844 $\tilde{a}^2$	-0.2575 $\tilde{a}^2$	-0.2336 $\tilde{a}^2$
3	-0.2754 $\tilde{a}^2$	-0.3131 $\tilde{a}^2$	-0.4592 $\tilde{a}^2$
4	-0.2560 $\tilde{a}^2$	-0.2304 $\tilde{a}^2$	-0.2182 $\tilde{a}^2$
5	-0.2558 $\tilde{a}^2$	-0.2255 $\tilde{a}^2$	-0.2013 $\tilde{a}^2$
Equilibrium	0.0001 - 0.2558 $\tilde{a}^2$	0.0007 - 0.2255 $\tilde{a}^2$	0.0007 - 0.2013 $\tilde{a}^2$
	$\rho = 0.45,\ \sigma_{\tilde{a}}^2 = 0.2$	$\rho = 0.45,\ \sigma_{\tilde{a}}^2 = 1$	$\rho = 0.45,\ \sigma_{\tilde{a}}^2 = 2$
1	-0.1323 $\tilde{a}^2$	-0.4990 $\tilde{a}^2$	-0.5128 $\tilde{a}^2$
2	0.2469 $\tilde{a}^2$	-0.0233 $\tilde{a}^2$	-0.0659 $\tilde{a}^2$
3	-0.1713 $\tilde{a}^2$	-1.0306 $\tilde{a}^2$	-2.0148 $\tilde{a}^2$
4	0.2913 $\tilde{a}^2$	-0.0104 $\tilde{a}^2$	-0.0516 $\tilde{a}^2$
5	0.2925 $\tilde{a}^2$	-0.0072 $\tilde{a}^2$	-0.0448 $\tilde{a}^2$
Equilibrium	0.0062 + 0.2925 $\tilde{a}^2$	0.0095 - 0.0072 $\tilde{a}^2$	0.0095 - 0.0448 $\tilde{a}^2$

Our ad hoc trading strategy analysis identifies two sources of the U-shape in the deterministic component of the equilibrium rebalancer orders in Figure 3. This can be seen in Figure 16. First, ad hoc strategy #5 (which, from Proposition 1, gives the expected equilibrium orders) is slightly more U-shaped than strategy #4. Thus, sunshine trading (which is omitted in strategy #4) is one source of the U-shape in the equilibrium orders. In particular, the rebalancer trades more early in the day so as to be able to trade more later in the day with no price impact.

Second, predictable interactions between rebalancer and informed-investor orders are a quantitatively significant reason for the U-shape in expected rebalancer orders. This can be seen by comparing ad hoc strategies #3 and #4. The expected orders given the target $\tilde{a}$ for strategy #3 (which excludes consideration of both sunshine trading and interactions with the informed-investor orders) are not U-shaped at all, but rather increasing over the trading day. In contrast, the expected orders in strategy #4 (which also excludes consideration of sunshine trading but does include consideration of predictable responses in informed-investor orders to the rebalancer's orders) are U-shaped. Thus, another reason for the U-shape in the rebalancer orders is that the rebalancer optimally trades a large amount early in the day and then gives the informed investor time to correct the price pressure from these early trades before trading more later in the day.[38]

[38]The negative portion of the strategy #3 orders indicates that — given the equilibrium price impacts and given an assumption that the price-correction mechanism from informed investor trading is missing — the rebalancer tries to manipulate prices in the opposite direction before then trading to meet his target. This explains the large negative coefficients on $\tilde{a}^2$ for strategy #3 in the expected profits in Table 2 where informed trading, contrary to the ad hoc assumption, is actually present.

Figure 16: Intraday patterns of the ratio $\mathbb{E}[\Delta\theta_n^R \,|\, \tilde{a}]/\tilde{a}$ for $\tilde{a} \neq 0$ for three ad hoc deterministic strategies for times $n = 1, \ldots, 10$. The strategies are x_n^3 (——), which ignores sunshine trading and order interactions with the informed investor, x_n^4 (−−), which just ignores sunshine trading, and the optimal deterministic strategy x_n^5 (−·−). The parameters are $N = 10$, $\sigma_{\tilde{v}}^2 = 1$, and $\sigma_w^2 = 1$.



A: $\rho = 0$, $\sigma_{\tilde{a}}^2 = 0.2$ B: $\rho = 0$, $\sigma_{\tilde{a}}^2 = 1$ C: $\rho = 0$, $\sigma_{\tilde{a}}^2 = 2$

D: $\rho = 0.45$, $\sigma_{\tilde{a}}^2 = 0.2$ E: $\rho = 0.45$, $\sigma_{\tilde{a}}^2 = 1$ F: $\rho = 0.45$, $\sigma_{\tilde{a}}^2 = 2$