

**INTERNET APPENDIX**  
Business Cycles and Currency Returns

NOT FOR PUBLICATION

# Estimating the Output Gap In-Sample

## Hodrick-Prescott Filter

The Hodrick-Prescott filter minimizes the following expression to produce a new time series of trend output,  $y_{i,t}^{tr}$

$$\sum_{t=1}^T (y_{i,t} - y_{i,t}^{tr})^2 + \lambda \sum_{t=2}^{T-1} [(y_{i,t+1}^{gr} - y_{i,t}^{gr}) - (y_{i,t}^{tr} - y_{i,t-1}^{tr})]^2 \quad (\text{IA.1})$$

where  $y_{i,t}$  is the logarithm of industrial production at time  $t$  for country  $i$ ,  $\lambda$  is a weighing factor to control the smoothness of the trend line. The lower the value of  $\lambda$ , the more the resulting trend will resemble the raw data series. We follow the suggestion of Hodrick and Prescott (1980) and Kydland and Prescott (1990) and set  $\lambda = 1600$  to smooth quarterly data and  $\lambda = 14400$  to smooth monthly data. The output gap (cyclical component) is then constructed as the difference between  $y_{i,t}$  and the trend series extracted from the filter  $c_{i,t} = y_{i,t} - y_{i,t}^{tr}$ .

## Baxter-King Filter

The Baxter-King filter removes both low and high frequency components from a time series. Specifically it involves the estimation of the moving average model

$$\hat{y}_{i,t} = \sum_{n=-K}^K B_n x_{t-n} \quad (\text{IA.2})$$

The values  $B_n$  can be estimated using an inverse Fourier transform such that they minimize the mean squared error between  $y_t$  and  $\hat{y}_t$  (see Priestly, 1981). We follow the suggestion of Baxter and King (1999) and set  $K=12$  for quarterly data and  $K=36$  for monthly data. We also set standard upper and lower limits for the cutoff frequency of 6 and 32 quarters for quarterly data and 8 and 96 months for monthly data.

## Linear Projection

We follow the linear projection method proposed by Hamilton (2018) and project (log) industrial production at time  $t$  on 12-lags of industrial production beginning 24-months earlier<sup>1</sup>

$$y_{i,t} = \alpha_i + \sum_{s=0}^{11} \beta_{i,s} y_{i,t-24-s} + \varepsilon_{i,t} \quad (\text{IA.3})$$

The output gap (cyclical component) is then constructed as the difference between  $y_{i,t}$  and the fitted value from the above regression  $c_{i,t} = y_{i,t} - \hat{y}_{i,t}$ .

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<sup>1</sup>For quarterly industrial production time series we project onto four lags beginning eight-quarters earlier.

## Quadratic Time Trend

The quadratic time trend projects the logarithm of industrial production on a time trend  $t$  and quadratic time trend  $t^2$

$$y_{i,t} = \alpha_i + \beta_{i,1}t + \beta_{i,2}t^2 + \varepsilon_{i,t} \quad (\text{IA.4})$$

The output gap (cyclical component) is then constructed as the difference between  $y_{i,t}$  and the fitted value from the above regression  $c_{i,t} = y_{i,t} - \hat{y}_{i,t}$ .

## Alternative Currency Portfolios, Factors, and Strategies

**Carry Trade Portfolios.** At the end of each month  $t$ , we allocate currencies to five portfolios on the basis of their forward discounts (or interest rate differential relative to the US). This exercise implies that currencies with the lowest forward discounts (or lowest interest rate differential relative to the US) are assigned to Portfolio 1, whereas currencies with the highest forward discounts (or highest interest rate differential relative to the US) are assigned to Portfolio 5. We compute the excess return for each portfolio as an equally weighted average of the currency excess returns within that portfolio. The strategy that is long Portfolio 5 and short Portfolio 1 is referred to as *HML<sub>FX</sub>*.

**Currency Momentum Portfolios.** At the end of each month  $t$ , we form five portfolios based on exchange rate returns over the previous month. We assign the 20% of all currencies with the lowest lagged exchange rate returns to Portfolio 1 and the 20% of all currencies with the highest lagged exchange rate returns to Portfolio 5. We then compute the excess return for each portfolio as an equally weighted average of the currency excess returns within that portfolio. A strategy long in Portfolio 5 (*winner currencies*) and short in Portfolio 1 (*loser currencies*) is denoted as *MOM*.

**Value Portfolios.** At the end of each month  $t$ , we form five portfolios based on the lagged five-year real exchange rate return as in Asness et al. (2013). This measure of currency value is based on calculating the deviation from relative purchasing power parity. Specifically, relative inflation over a 5-year window vis-à-vis the US is compared with the foreign exchange (FX) rate appreciation over the same period versus the US dollar. To provide a more stable measure of the FX rate appreciation, Asness et al. (2013) calculate the appreciation as today's FX rate minus the average FX rate observed 4.5 to 5.5 years earlier. If inflation growth in the foreign economy outpaced that in the US but the US dollar did not appreciate against the foreign currency by an offsetting amount, then the foreign currency is considered 'overvalued'.

To construct currency value portfolios, we collect monthly data on consumer price indices from the IMF's *International Financial Statistics* database beginning in October 1978 and also collect additional FX spot rate data from *Global Financial Data* beginning in April 1978, such that the first currency value signals are obtained in October 1983. We assign the 20% of all currencies with the highest lagged real exchange rate return to Portfolio 1 and the 20% of all currencies with the lowest lagged real exchange rate return to Portfolio 5. We compute the excess

return for each portfolio as an equally weighted average of the currency excess returns within that portfolio. A strategy long in Portfolio 5 (*undervalued currencies*) and short in Portfolio 1 (*overvalued currencies*) is denoted as *VAL*.

**Other Factors and Portfolios.** In addition to the portfolios described above, we also compare the properties of the output-gap portfolios against other popular strategies and factors in the literature. These include: (i) the **Dollar** factor, proposed by Lustig et al. (2011), which is essentially a market factor in currency space, equal to the average return of a large basket of foreign currencies against the US dollar; (ii) the **Dollar Carry** strategy as proposed by Lustig et al. (2014), which conditions the Dollar factor on the average forward premia of currencies against the US and thus goes long (short) the US dollar whenever interest rates are relatively high (low) in the US; (iii) the **Global Imbalance** factor of Della Corte et al. (2016b), which is a factor that compensates investors for financing risky economies with large stocks of liabilities that issue the majority of those in foreign currency; (iv) the **Trend-Following** risk factors proposed by Fung and Hsieh (2001), which reflect the option-like returns typically generated by hedge funds (we use the FX and interest-rate trend-following returns); (v) the Pástor and Stambaugh (2003) measure of **Aggregate Market Liquidity**, and (vi) the **Market Risk Premium** collected from Kenneth French’s website.<sup>2</sup>

## Asset Pricing Tests

We provide further details on the asset pricing methods used in Sections 5.1 and 5.2 of the paper to evaluate whether the relationship between currency returns and business cycles can be understood from a risk-return perspective.

We denote the discrete excess returns on portfolio  $j$  in period  $t$  as  $RX_t^j$ , for  $j = 1, \dots, N$  and  $t = 1, \dots, T$ ; and let  $RX_t$  be a  $N$ -dimensional vector of test asset excess returns. In the absence of arbitrage opportunities, risk-adjusted excess returns have a price of zero and satisfy the following Euler equation:

$$E_t[M_{t+1}RX_{t+1}^j] = 0 \tag{IA.5}$$

with an SDF linear in  $k$  pricing factors  $f_{t+1}$ , given by

$$M_{t+1} = 1 - b'(f_{t+1} - \mu) \tag{IA.6}$$

where  $b$  is the vector of factor loadings, and  $\mu$  denotes the factor means. Equation (IA.6) is referred to as an unconditional asset pricing model because the SDF factor loadings in  $b$  are assumed to be time-invariant. The vast majority of papers in the currency asset pricing literature, starting from Lustig and Verdelhan (2007), focus on Equation (IA.6). However, in a more general setting the

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<sup>2</sup>The hedge fund risk factors returns are available on David A. Hsieh’s website at <http://faculty.fuqua.duke.edu/~dah7/DataLibrary/TF-FAC.xls>. We collect liquidity data from Lubos Pastor’s website at <http://faculty.chicagobooth.edu/lubos.pastor/research/> and market data from Kenneth French’s website at [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). We thank each author for making their data publicly available.

Euler equation (IA.5) implies SDF parameters that are time-varying, which means the covariance between excess returns and the SDF is conditional (Hansen and Richard, 1987; Boguth et al., 2011). If this is the case, for example, it could be that excess returns from high output gap currencies (strong economies) are more risky in bad times, in which case the unconditional version of the model with constant  $b$  is misspecified. We analyze this more general case of conditional asset pricing tests in Section 5.3 of the paper.

The SDF specification (IA.6) implies a beta pricing model in which the expected excess return on portfolio  $j$  is equal to the factor risk price  $\lambda$  times the risk quantities  $\beta^j$ . The beta pricing model is defined as

$$E[RX^j] = \lambda' \beta^j \quad (\text{IA.7})$$

where the market price of risk  $\lambda = \Sigma_f b$  can be obtained via the factor loadings  $b$ .  $\Sigma_f = E[(f_t - \mu)(f_t - \mu)']$  is the variance-covariance matrix of the risk factors, and  $\beta^j$  are the regression coefficients of each portfolio's excess return  $RX_{t+1}^j$  on the risk factors  $f_{t+1}$ .

We estimate the SDF parameters, including estimates of factor loadings  $b$  and the market prices of risk  $\lambda$ , via the Generalized Method of Moments (*GMM*) of Hansen (1982). To implement *GMM*, we use the pricing errors as a set of moments and the identity weighting matrix. Since the objective is to test whether the model can explain the cross section of expected currency excess returns, we only rely on unconditional moments and do not employ instruments. With an identity matrix, *GMM* attempts to price all currency portfolios equally well. Factor means and the individual elements of the covariance matrix of risk factors  $\Sigma_f$  are estimated simultaneously with the SDF parameters by adding the corresponding moment conditions to the asset pricing moment conditions implied by the Euler condition (IA.5). This one-step approach ensures that we adequately incorporate estimation uncertainty associated with the fact that factor means and the factor covariance matrix need to be estimated (see, for example, Burnside, 2011).

Formally, the Euler equation (IA.5) implies the following moment conditions for the  $N$ -dimensional vector of test asset excess returns  $RX_{t+1}$

$$E\{[1 - b'(f_{t+1} - \mu)] RX_{t+1}\} = 0. \quad (\text{IA.8})$$

In addition to these  $N$  moment restrictions, our set of *GMM* moment conditions also includes  $k$  moment conditions  $E[f_t - \mu] = 0$  accounting for the fact that factor means  $\mu$  have to be estimated.<sup>3</sup> Factor risk prices  $\lambda$  can be easily obtained from our *GMM* estimates via the relation  $\lambda = \Sigma_f b$ , where  $\Sigma_f = E[(f_t - \mu)(f_t - \mu)']$  is the factor covariance matrix. Following Burnside (2011), the covariance matrix  $\Sigma_f$  is estimated along with the other model parameters by including an additional set of corresponding moment conditions. Hence, the estimating function takes the following form

$$g(z_t, \theta) = \begin{bmatrix} [1 - b'(f_t - \mu)] RX_t \\ f_t - \mu \\ \text{vec}((f_t - \mu)(f_t - \mu)') - \text{vec}(\Sigma_f) \end{bmatrix} \quad (\text{IA.9})$$

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<sup>3</sup>This applies because the risk factors considered here are not mean zero by construction.

where  $\theta$  contains the parameters  $(b', \mu', \text{vec}(\Sigma_f)')$  and  $z_t$  represents the data  $(RX_t, f_t)$ . By exploiting the moment conditions  $E[g(z_t, \theta)] = 0$  defined by (IA.9), estimation uncertainty – due to the fact that factor means and the covariance matrix of factors have to be estimated – is incorporated in the standard errors of factor risk prices. The one-step *GMM* estimation uses a pre-specified weighting matrix  $W_T$  based on the identity matrix  $I_N$  for the first  $N$  asset pricing moment conditions and a large weight assigned to the additional moment conditions (for precise estimation of factor means and the factor covariance matrix). Standard errors are computed based on a HAC estimate of the long-run covariance matrix  $S = \sum_{j=-\infty}^{\infty} E[g(z_t, \theta)g(z_{t-j}, \theta)']$  by the Newey and West (1987) procedure with the number of lags in the Bartlett kernel determined optimally by the data-driven approach of Andrews (1991).

The model's performance is evaluated using the cross-sectional  $R^2$ , the Root Mean Squared Error (RMSE), and the *HJ* distance measure of Hansen and Jagannathan (1997), which quantifies the mean-squared distance between the SDF of a proposed model and the set of admissible SDFs. Put simply, this measure gives the least squares distance between the model's SDF and the nearest point to such SDF in the space of all SDFs that price the test assets correctly. The *HJ* distance measure for a given model's SDF applied to the data  $z_t$  and with parameters  $\theta$ ,  $M_t(z_t, \theta)$  is defined as

$$HJ = \sqrt{\min_{\theta} g_T(z_t, \theta)' G_T^{-1} g_T(z_t, \theta)} \quad (\text{IA.10})$$

where  $G_T \equiv T^{-1} \sum_{t=1}^T RX_t RX_t'$ , and  $g_T(z_t, \theta) \equiv T^{-1} \sum_{t=1}^T [M_t(z_t, \theta) RX_t - I_N]$ . The *HJ* calculation is essentially a *GMM* application with the (non-optimal) weighting matrix equal to the inverse of the second moment matrix of asset returns; setting  $W_T = G_T^{-1}$  is important because this weighting matrix does not depend on the model parameters  $\theta$ , and hence the *HJ* distance metric is comparable across different candidate SDF specifications (see Hansen and Jagannathan, 1997; Ludvigson, 2013). To test whether the *HJ* distance is statistically significant, we simulate  $p$ -values using a weighted sum of  $\chi_1^2$  distributed random variables (see, Jagannathan and Wang, 1996; Ren and Shimotsu, 2009).

## Details of a model for the GAP premium

In what follows, we provide the derivations for the model presented in the main text. For compactness, we will denote  $\theta = \frac{1}{1-\gamma}$ .

**Lemma 1** (Equilibrium Utility). *The equilibrium utility takes the form:*

$$U_{i,t} = \log C_{i,t} + A_i + B \cdot z_{i,t} + D_i \cdot \sigma_{i,t},$$

where

$$A_i = \frac{\delta}{1-\delta} \left[ \mu_c + (1-\rho_\sigma) D_i \bar{\sigma} + \frac{1}{2\theta} (B \varphi_z \sqrt{\bar{\sigma}} + D_i \sqrt{\sigma_\sigma})^2 + \frac{\alpha^2}{2\theta} D_i^2 \sigma_\sigma \right],$$

$$B = \frac{\delta}{1-\delta \rho_z}, \quad D_i = \frac{\delta}{2\theta (1-\delta \rho_\sigma)}.$$

*Proof.* We shall solve for  $V_{i,t} = U_{i,t} - \log C_{i,t}$ :

$$\begin{aligned} V_{i,t} &= \delta \theta \log E_t \exp \left\{ \frac{U_{i,t+1} - \log C_{i,t+1} + \Delta c_{i,t+1}}{\theta} \right\} \\ &= \delta \theta \log E_t \exp \left\{ \frac{V_{i,t+1} + \mu_c + z_{i,t} + \sqrt{\sigma_{i,t}} \varepsilon_{i,t+1}^c}{\theta} \right\}. \end{aligned}$$

Guess that the solution is of the type:

$$V_{i,t} = A_i + B_i z_{i,t} + D_i \sigma_{i,t}.$$

For compactness, we are going to suppress all the subscript  $i$  in the remainder of these derivations. Then:

$$\begin{aligned} V_t &= \delta \theta \log E_t \exp \left\{ \frac{A + B z_t + D \sigma_{t+1} + \mu_c + z_t + \sqrt{\sigma_t} \varepsilon_{t+1}^c}{\theta} \right\} \\ &= \delta A + \delta \mu_c + \delta z_t + \frac{\delta}{2\theta} \sigma_t + \delta \theta \log E_t \exp \left\{ \frac{B}{\theta} (\rho_z z_t + \phi_z \sqrt{\sigma} \varepsilon_{t+1}^z) + \right. \\ &\quad \left. + \frac{D}{\theta} [(1 - \rho_\sigma \bar{\sigma} + \rho_\sigma \sigma_t + \sqrt{\sigma_\sigma} (\varepsilon_{t+1}^z + \alpha \varepsilon_{t+1}^\nu)] \right\} \\ &= \delta [A + \mu_c + (1 - \rho_\sigma (D \bar{\sigma}))] + \delta (1 + B \rho_z) z_t + \delta \left[ \frac{1}{2\theta} + \rho_\sigma D \right] \sigma_t + \\ &\quad + \delta \theta \log E_t \exp \left\{ \frac{1}{\theta} [B \phi_z \sqrt{\sigma} + D \sqrt{\sigma_\sigma}] \varepsilon_{t+1}^z + \frac{\alpha}{\theta} (D \sqrt{\sigma_\sigma}) \varepsilon_{t+1}^\nu \right\} \\ &= \delta \left[ A + \mu_c + (1 - \rho_\sigma (D \bar{\sigma})) + \frac{1}{2\theta} (B \phi_z \sqrt{\sigma} + D \sqrt{\sigma_\sigma})^2 + \frac{\alpha^2}{2\theta} D^2 \sigma_\sigma \right] + \\ &\quad + \delta (1 + B \rho_z) z_t + \delta \left( \frac{1}{2\theta} + \rho_\sigma D \right) \sigma_t. \end{aligned}$$

Matching coefficients concludes the proof. □

**Lemma 2** (Equilibrium SDFs). *The equilibrium of the logarithm of the SDF in each country is*

$$m_{i,t+1} = \log \delta - \Delta c_{i,t+1} + \frac{\tilde{V}_{i,t+1}}{\theta} - \frac{1}{\theta} E_t \tilde{V}_{i,t+1} - \frac{1}{2\theta^2} \text{Var}_t \tilde{V}_{i,t+1},$$

where  $\tilde{V}_{i,t} = V_t + \Delta c_{i,t}$ .

*Proof.* The log-SDF in each country is

$$\begin{aligned} m_{t+1} &= \log \delta - \Delta c_{t+1} + \frac{1}{\theta} U_{t+1} - \log E_t \exp \left\{ \frac{U_{t+1}}{\theta} \right\} \\ &= \log \delta - \Delta c_{t+1} + \frac{U_{t+1} - \log C_t}{\theta} - \log E_t \exp \left\{ \frac{U_{t+1} - \log C_{t+1} + \log C_{t+1} - \log C_t}{\theta} \right\} \\ &= \log \delta - \Delta c_{t+1} + \frac{V_{t+1} + \Delta c_{t+1}}{\theta} - \log E_t \exp \left\{ \frac{V_{t+1} + \Delta c_{t+1}}{\theta} \right\}. \end{aligned}$$

Using the definition of  $\tilde{V}_{i,t}$  and noting that  $\tilde{V}_{i,t}$  is conditionally normally distributed conclude the proof.  $\square$

**Lemma 3** (Risk-free rates). *In each country, the logarithm of the risk-free rate is equal to*

$$r_{i,t} = \mu_c - \log \delta - \left( \gamma - \frac{1}{2} \right) (\bar{\sigma} + \tilde{\sigma}_{i,t}),$$

where  $\tilde{\sigma}_{i,t} = \rho_\sigma \tilde{\sigma}_{i,t-1} + \alpha \rho_\sigma \sqrt{\sigma_\sigma} \varepsilon_t^\nu$ .

*Proof.* By definition, the log-risk-free rate in each country is equal to

$$r_{i,t} = -\log E_t \exp \{m_{i,t+1}\}.$$

Using the equilibrium SDF and dropping the subscripts  $i$  to simplify notation, we get

$$\begin{aligned} r_t &= -\log E_t \exp \left\{ \log \delta - \Delta c_{t+1} + \frac{\tilde{V}_{i,t+1}}{\theta} - \frac{1}{\theta} E_t \tilde{V}_{i,t+1} - \frac{1}{2\theta^2} \text{Var}_t \tilde{V}_{i,t+1} \right\} \\ &= -\log \delta + E_t \Delta c_{t+1} - \frac{1}{2} \text{Var}_t \Delta c_{t+1} + \frac{1}{\theta} \text{cov}_t [\Delta c_{t+1}, \tilde{V}_{i,t+1}] \\ &= -\log \delta + \mu_c + z_t - \frac{1}{2} \sigma_t + \frac{1}{\theta} \sigma_t \\ &= -(\log \delta - \mu_c) + z_t - \left( \frac{1}{2} - \frac{1}{\theta} \right) \sigma_t. \end{aligned}$$

$\square$

**Lemma 4** (Currency excess return). *Let  $\gamma > 1$  and take currency of country  $k$  as the base currency. The excess return of currency  $i \in \{1, 2, \dots, N\}$  over the base currency is*

$$\begin{aligned} \log E_t [RX_{t+1}^i] &= \text{Var}_t [m_{t+1}^k] - \text{cov}_t [m_{t+1}^k, m_{t+1}^i] \\ &= \frac{\delta^2 \alpha^2 (1-\gamma)^4}{4(1-\delta\rho_\sigma)^2} \sigma_\sigma + \gamma^2 \sigma_t^k + \kappa (1 - \rho_{ik,t}^z), \end{aligned} \tag{IA.11}$$

where the coefficient of proportionality  $\kappa$  is equal to

$$\kappa = \delta \left[ \frac{\rho_z \sqrt{\bar{\sigma}}}{1 - \delta \rho_z} + \frac{1}{2} \frac{(1-\gamma) \sqrt{\sigma_\sigma}}{1 - \delta \rho_\sigma} \right]^2 > 0.$$

*Proof.* We shall start by computing the conditional variance and covariance needed for the conditional currency risk premium. Based on the equilibrium SDF, it is easy to show that

$$m_{t+1} - E_t[m_{t+1}] = - \left( 1 - \frac{1}{\theta} \right) \sqrt{\sigma_t} \varepsilon_{t+1}^c + \frac{1}{\theta} [B \varphi_z \sqrt{\bar{\sigma}} + D \sqrt{\sigma_\sigma}] \varepsilon_{t+1}^z + \frac{D}{\theta} \sqrt{\sigma_\sigma} (\alpha \varepsilon_{t+1}^\nu),$$



where we dropped the country subscript  $i$  to simplify notation. It follows that the conditional variance of the SDF is equal to:

$$V_t[m_{t+1}] = \left(1 - \frac{1}{\theta}\right)^2 \sigma_t + \frac{1}{\theta^2} [B\varphi_z\sqrt{\bar{\sigma}} + D\sqrt{\sigma_\sigma}]^2 + \frac{D^2\alpha^2}{\theta^2}\sigma_\sigma, \quad (\text{IA.12})$$

while the conditional covariance is:

$$\text{cov}_t[m_{i,t+1}, m_{j,t+1}] = \left(1 - \frac{1}{\theta}\right)^2 \sqrt{\sigma_{i,t}}\sqrt{\sigma_{j,t}}\rho_c + \frac{1}{\theta^2} [B\varphi_z\sqrt{\bar{\sigma}} + D\sqrt{\sigma_\sigma}]^2 \rho_{ij,t}^z. \quad (\text{IA.13})$$

Combining (IA.12) and (IA.13) into the formula for the currency excess return in (IA.11) and using the expressions for  $B$ ,  $D$ , and  $\theta$  concludes the proof.  $\square$

*Proof of Proposition 1 in main text.* The proof follows directly by taking the difference of the currency risk premiums in Lemma 4 for countries  $i$  and  $j$  and by using the assumed process for the conditional correlation of shocks in Equation (16) of the main text.  $\square$

## Relationship between predictive component and output gap in the model

Consider the following data generating process:

$$\begin{aligned} y_t &= \rho_y y_{t-1} + x_{t-1} + \sigma_y \varepsilon_{y,t}, \\ x_t &= \rho_x x_{t-1} + \sigma_x \varepsilon_{x,t}, \end{aligned} \quad (\text{IA.14})$$

where all shocks are *i.i.d.* distributed as standard normals and  $\varepsilon_{y,t} \perp \varepsilon_{x,t}$ . In what follows, we will consider the limit for  $\rho_y \rightarrow 1$ , in which case the long-run risk  $x_{t-1}$  is the conditional expectation of  $\Delta y_t$ . Additionally, we will assume stationarity of the  $x_t$  process, i.e. the assumption of  $\rho_x < 1$  is retained throughout the derivations. This data generating process maps directly into the one used in the model when  $y_t = \log C_{i,t}$  and  $x_t = z_{i,t}$  and where, for simplicity, we set the unconditional growth rate of consumption to zero and abstract away from time-varying volatility.

Consider the following regression:

$$y_t = \sum_{j=0}^l \beta_j \cdot y_{t-k-l} + \xi_t. \quad (\text{IA.15})$$

which we use in the empirical analysis to estimate the output gap. We want to sign and quantify the magnitude of the correlation between:

1. the fitted residual,  $\widehat{\xi}_t$  (our measure of the output gap)
2. the conditional expectation of  $\Delta y_t$ , i.e.  $x_{t-1}$  (the long-run risk).

In what follows, we shall consider both the univariate case ( $l = 0$  in equation (IA.15)) and the multivariate case ( $l > 0$  in equation (IA.15)).

**Univariate case.** In the univariate case of regression (IA.15), i.e.  $l = 0$ , we can characterize the correlation in closed form. The following lemma contains the analytical expression for several moments that are useful to characterize the regression coefficients in equation (IA.15).

**Lemma 5.** *The following unconditional moments can be obtained from the system of equations (IA.14):*

$$E[x_t^2] = \frac{\sigma_x^2}{1 - \rho_x^2}, \quad (\text{IA.16})$$

$$E[x_t y_t] = \frac{\rho_x \sigma_x^2}{(1 - \rho_x \rho_y)(1 - \rho_x^2)}, \quad (\text{IA.17})$$

$$E[y_t^2] = \frac{\sigma_y^2}{1 - \rho_y^2} + \frac{(1 + \rho_x \rho_y) \sigma_x^2}{(1 - \rho_x \rho_y)(1 - \rho_x^2)(1 - \rho_y^2)}, \quad (\text{IA.18})$$

$$E[y_t y_{t-k}] = \rho_y^k E[y_t^2] + \left( \sum_{i=0}^{k-1} \rho_x^i \rho_y^{(k-1)-i} \right) E[x_t y_t]. \quad (\text{IA.19})$$

*Proof.* The proof of equation (IA.16) is trivial and will be omitted in the interest of space. For equation (IA.17), we note that:

$$\begin{aligned} E[x_t y_t] &= E[(\rho_x x_{t-1} + \sigma_x \varepsilon_{x,t})(\rho_y y_{t-1} + x_{t-1} + \sigma_y \varepsilon_{y,t})] \\ &= \rho_y \sigma_x E[y_{t-1} \varepsilon_{x,t}] + \rho_x \rho_y E[x_{t-1} y_{t-1}] + \rho_x E[x_{t-1}^2] \\ &\quad + \sigma_x E[x_{t-1} \varepsilon_{x,t}] + \rho_x \sigma_y E[x_{t-1} \varepsilon_{y,t}] + \sigma_x \sigma_y E[\varepsilon_{x,t} \varepsilon_{y,t}]. \end{aligned}$$

Equation (IA.17) follows immediately by noticing that  $E[y_{t-1} \varepsilon_{x,t}] = 0$ ,  $E[x_{t-1} \varepsilon_{x,t}] = 0$ ,  $E[x_{t-1} \varepsilon_{y,t}] = 0$ ,  $E[\varepsilon_{x,t} \varepsilon_{y,t}] = 0$ , and  $E[x_t y_t] = E[x_{t-1} y_{t-1}]$ . Equation (IA.18) is obtained from

$$\begin{aligned} E[y_t^2] &= E[(\rho_y y_{t-1} + x_{t-1} + \sigma_y \varepsilon_{y,t})^2] \\ &= \rho_y^2 \underbrace{E[y_{t-1}^2]}_{E[y_t^2]} + \underbrace{E[x_{t-1}^2]}_{E[x_t^2]} + \underbrace{\sigma_y^2 E[\varepsilon_{y,t}^2]}_1 + 2\rho_y \underbrace{E[x_{t-1} y_{t-1}]}_{E[x_t y_t]} + 2\sigma_y \underbrace{E[x_{t-1} \varepsilon_{y,t}]}_0 + 2\rho_y \sigma_y \underbrace{E[y_{t-1} \varepsilon_{y,t}]}_0, \end{aligned}$$

after using equations (IA.16)-(IA.17). In order to obtain equation (IA.19), we first note that the autocovariance of  $y_t$  can be written recursively:

$$\begin{aligned} E[y_t y_{t-1}] &= E[(\rho_y y_{t-1} + x_{t-1} + \sigma_y \varepsilon_{y,t}) y_{t-1}] \\ &= \rho_y \underbrace{E[y_{t-1}^2]}_{\text{eq. (IA.18)}} + \underbrace{E[x_{t-1} y_{t-1}]}_{\text{eq. (IA.17)}} + \underbrace{\sigma_y E[y_{t-1} \varepsilon_{y,t}]}_0 = \rho_y E[y_t^2] + E[x_t y_t], \\ E[y_t y_{t-2}] &= E[(\rho_y y_{t-1} + x_{t-1} + \sigma_y \varepsilon_{y,t}) y_{t-2}] \\ &= \rho_y \underbrace{E[y_{t-1} y_{t-2}]}_{E[y_t y_{t-1}]} + E[x_{t-1} y_{t-2}] = \rho_y E[y_t y_{t-1}] + E[(\rho_x x_{t-2} + \sigma_x \varepsilon_{x,t-1}) y_{t-2}] \\ &= \rho_y E[y_t y_{t-1}] + \rho_x E[x_{t-2} y_{t-2}] = \rho_y E[y_t y_{t-1}] + \rho_x E[x_t y_t], \end{aligned}$$

$$\begin{aligned}
E[y_t y_{t-3}] &= \rho_y E[y_t y_{t-2}] + \rho_x^2 E[x_t y_t] \\
&\dots \\
E[y_t y_{t-k}] &= \rho_y E[y_t y_{t-k+1}] + \rho_x^{(k-1)} E[x_t y_t].
\end{aligned}$$

Equation (IA.19) obtains by recursive substitution of each lag  $k - 1$  autocovariance into each lag  $k$  autocovariance. □

The following proposition summarizes the main finding.

**Proposition 2.** *Let  $l = 0$  in equation (IA.15). When  $\rho_y \rightarrow 1$ :*

1. *the estimated regression coefficient is  $\widehat{\beta}_0 = 1$ ,*
2. *the covariance between  $\widehat{\xi}_t$  and  $x_{t-1}$  is positive and equal to*

$$\text{cov}(\widehat{\xi}_t, x_{t-1}) = \frac{(1 - \rho_x^k)}{(1 - \rho_x)} \frac{\sigma_x^2}{1 - \rho_x^2} > 0,$$

3. *the correlation between  $\widehat{\xi}_t$  and  $x_{t-1}$  is*

$$\text{corr}(\widehat{\xi}_t, x_{t-1}) = \frac{(1 - \rho_x^k) / (1 - \rho_x)}{\sqrt{(1 - \rho_x^2) \left[ \frac{\sigma_y^2}{\sigma_x^2} k + \frac{k}{(1 - \rho_x)^2} - 2\rho_x \frac{1 - \rho_x^k}{1 - \rho_x^2} \right]}}.$$

*Proof.* The proof consists of three parts.

1. We show that  $\lim_{\rho_y \rightarrow 1} \widehat{\beta}_0 = 1$ . Note that:

$$\lim_{\rho_y \rightarrow 1} \widehat{\beta}_0 = \lim_{\rho_y \rightarrow 1} \frac{E[y_t y_{t-k}]}{E[y_t^2]}.$$

Using the moments obtained in equations (IA.18) and (IA.19) of lemma 5, we can rewrite this as

$$\lim_{\rho_y \rightarrow 1} \frac{E[y_t y_{t-k}]}{E[y_t^2]} = \lim_{\rho_y \rightarrow 1} \frac{\rho_y^k E[y_t^2] + \left( \sum_{i=0}^{k-1} \rho_x^i \rho_y^{(k-1)-i} \right) E[x_t y_t]}{E[y_t^2]} = 1,$$

where the last equality follows from the fact that  $E[x_t y_t]$  in equation (IA.17) of lemma 2 converges to a finite number.

2. We show that  $\text{cov}(\widehat{\xi}_t, x_{t-1}) = \frac{(1-\rho_x^k) \sigma_x^2}{(1-\rho_x) 1-\rho_x^2} > 0$ . Since  $\widehat{\beta}_0 = 1$ , the residual of regression (IA.15) is equal to

$$\begin{aligned}\widehat{\xi}_t &= y_t - y_{t-k} = \sum_{j=0}^{k-1} x_{t-1-j} + \sigma_y \sum_{j=0}^{k-1} \varepsilon_{y,t-j} \\ &= \left( \sum_{j=0}^{k-1} \rho_x^j \right) x_{t-k} + \sigma_x \left[ \sum_{j=0}^{k-2} \left( \sum_{i=0}^k \rho_x^i \right) \varepsilon_{x,t-1-j} \right] + \sigma_y \sum_{j=0}^{k-1} \varepsilon_{y,t-j} \\ &= \left( \frac{1-\rho_x^k}{1-\rho_x} \right) x_{t-k} + \sigma_x \left[ \sum_{j=0}^{k-2} \left( \frac{1-\rho_x^{j+1}}{1-\rho_x} \right) \varepsilon_{x,t-1-j} \right] + \sigma_y \sum_{j=0}^{k-1} \varepsilon_{y,t-j}.\end{aligned}$$

Using the autoregressive structure of  $x_t$ , it is possible to show that

$$x_{t-1} = \rho_x^{k-1} x_{t-k} + \sigma_x \sum_{j=0}^{k-2} \rho_x^j \varepsilon_{x,t-1-j}.$$

We can now compute the covariance between the estimated output gap and the conditional expectation of  $y_t$ :

$$\begin{aligned}\text{cov}(\widehat{\xi}_t, x_{t-1}) &= \rho_x^{k-1} \left( \frac{1-\rho_x^k}{1-\rho_x} \right) \frac{\sigma_x^2}{1-\rho_x^2} + \sigma_x^2 \sum_{j=0}^{k-2} \frac{\rho_x^j - \rho_x^{2j+1}}{1-\rho_x} \\ &= \rho_x^{k-1} \left( \frac{1-\rho_x^k}{1-\rho_x} \right) \frac{\sigma_x^2}{1-\rho_x^2} + \frac{\sigma_x^2}{1-\rho_x} \sum_{j=0}^{k-2} \rho_x^j - \frac{\sigma_x^2}{1-\rho_x} \sum_{j=0}^{k-2} \rho_x^{2j+1} \\ &= \rho_x^{k-1} \left( \frac{1-\rho_x^k}{1-\rho_x} \right) \frac{\sigma_x^2}{1-\rho_x^2} + \frac{\sigma_x^2}{1-\rho_x} \frac{1-\rho_x^{k-1}}{1-\rho_x} - \sigma_x^2 \frac{\rho_x}{1-\rho_x} \frac{1-\rho_x^{2k-2}}{1-\rho_x^2} \\ &= \frac{\sigma_x^2}{1-\rho_x^2} \left[ \rho_x^{k-1} \left( \frac{1-\rho_x^k}{1-\rho_x} \right) + \frac{(1-\rho_x^{k-1})(1-\rho_x^2)}{(1-\rho_x)^2} - \frac{\rho_x(1-\rho_x^{2k-2})}{1-\rho_x} \right] \\ &= \frac{\sigma_x^2}{1-\rho_x^2} \left[ \frac{\rho_x^{k-1} - \rho_x^{2k-1} - \rho_x + \rho_x^{2k-1}}{1-\rho_x} + \frac{1-\rho_x^{k-1} + \rho_x - \rho_x^k}{1-\rho_x} \right] \\ &= \frac{\sigma_x^2}{1-\rho_x^2} \left[ \frac{1-\rho_x^k}{1-\rho_x} \right].\end{aligned}$$

As long as  $\rho_x < 1$ , this covariance is positive.

3. We show that  $\text{corr}(\widehat{\xi}_t, x_{t-1}) = \frac{(1-\rho_x^k)/(1-\rho_x)}{\sqrt{(1-\rho_x^2) \left[ \frac{\sigma_y^2}{\sigma_x^2} k + \frac{k}{(1-\rho_x)^2} - 2\rho_x \frac{1-\rho_x^k}{1-\rho_x} \right]}}$ . Since we have already com-

puted the covariance between output gap and long-run risk in the previous step of this proof and the variance of  $x_t$  in lemma 5, we shall focus on the variance of  $\widehat{\xi}_t$  to obtain the expression for the correlation:

$$V(\widehat{\xi}_t) = \left( \frac{1-\rho_x^k}{1-\rho_x} \right)^2 V(x_t) + \sigma_x^2 \sum_j^{k-2} \left( \frac{1-\rho_x^{j+1}}{1-\rho_x} \right)^2 + k\sigma_y^2. \quad (\text{IA.20})$$

Focus on the middle term of the above equation:

$$\begin{aligned}
\sigma_x^2 \sum_j^{k-2} \left( \frac{1 - \rho_x^{j+1}}{1 - \rho_x} \right)^2 &= \frac{\sigma_x^2}{(1 - \rho_x)^2} \sum_{j=0}^{k-2} (1 + \rho_x^{2j+2} - 2\rho_x^{j+1}) \\
&= \frac{\sigma_x^2}{(1 - \rho_x)^2} \left[ (k-1) + \rho_x^2 \sum_{j=0}^{k-2} (\rho_x^2)^j - 2\rho_x \sum_{j=0}^{k-2} \rho_x^j \right] \\
&= \frac{\sigma_x^2}{(1 - \rho_x)^2} \left[ (k-1) + \rho_x^2 \frac{1 - (\rho_x^2)^{k-1}}{1 - \rho_x^2} - 2\rho_x \frac{1 - \rho_x^{k-1}}{1 - \rho_x} \right] \\
&= \frac{\sigma_x^2 [(k-1)(1 - \rho_x^2) + \rho_x^2 - \rho_x^{2k} - (2\rho_x - 2\rho_x^k)(1 + \rho_x)]}{(1 - \rho_x)^2 (1 - \rho_x^2)} \\
&= \frac{\sigma_x^2 [k(1 - \rho_x^2) - 1 - \rho_x^{2k} - 2\rho_x + 2\rho_x^k + 2\rho_x^{k+1}]}{(1 - \rho_x)^2 (1 - \rho_x^2)}. \tag{IA.21}
\end{aligned}$$

By plugging (IA.21) inside (IA.20), we get:

$$\begin{aligned}
V(\widehat{\xi}_t) &= \frac{(1 + \rho_x^{2k} - 2\rho_x^k) \sigma_x^2}{(1 - \rho_x)^2 (1 - \rho_x^2)} + \frac{\sigma_x^2 [k(1 - \rho_x^2) - 1 - \rho_x^{2k} - 2\rho_x + 2\rho_x^k + 2\rho_x^{k+1}]}{(1 - \rho_x)^2 (1 - \rho_x^2)} + k\sigma_y^2 \\
&= \frac{\sigma_x^2}{(1 - \rho_x)^2 (1 - \rho_x^2)} [k(1 - \rho_x^2) - 2\rho_x(1 - \rho_x^k)] + k\sigma_y^2.
\end{aligned}$$

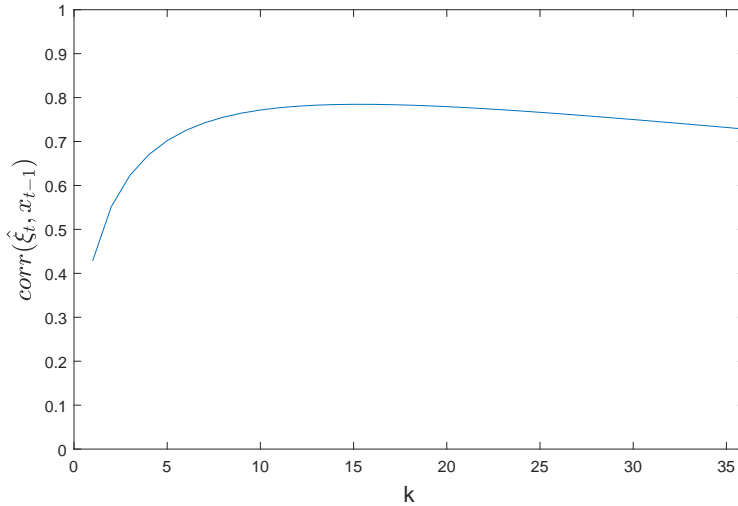
Using the definition of correlation concludes the proof. □

The explanation for why the covariance (and thus correlation) is positive is straightforward. Consider the case in which  $k = 1$ . If this is the case, then  $\widehat{\xi}_t = x_{t-1} + \sigma_y \varepsilon_t^y$ , which is clearly positively correlated with  $x_{t-1}$ .

To get an assessment of how large this positive correlation is, we set the model's parameters to  $\rho_x = 0.98$ ,  $\sigma_y = 0.02/\sqrt{12}$ , and  $\sigma_x = 0.1\sigma_y$ . This choice of parameters results in an annual volatility of  $\Delta y_t$  of about 2% and in a moderate autocorrelation of  $\Delta y_t$ . The figure below shows that the correlation can be as high as 0.7 – 0.8, for values of  $k$  in the range of 24 – 36, which correspond to the lags that we use in the empirical investigation.

**Multivariate case.** In the multivariate case ( $l > 0$  in regression (IA.15)) we lose analytical tractability. To quantify the correlation between the estimated output gap and the long-run risk, we conduct a simulation exercise. We set the parameters of the model in the system of equations (IA.14) to  $\rho_y = 1$ ,  $\rho_x = 0.98$ ,  $\sigma_y = 0.02/\sqrt{12}$ , and  $\sigma_x = 0.1\sigma_y$  and simulate  $T = 25000$  observations. We then estimate regression (IA.15) for various combinations of  $k$  and  $l$ , obtain the fitted residual  $\widehat{\xi}_t$ , and compute its correlation with  $x_{t-1}$ .

The results are reported in Panel A of the table below. The correlation between estimated output gap and long-run risk is positive in all cases and it can be as high as 0.77. For the



**Correlation between output gap and long-run risk (univariate case).** The correlation is displayed for values of  $k$  ranging from 1 to 36 and  $l = 0$ . The parameters of the data generating process are set to  $\rho_y = 1$ ,  $\rho_x = 0.98$ ,  $\sigma_y = 0.02/\sqrt{12}$ , and  $\sigma_x = 0.1\sigma_y$ .

specification that we consider in the main empirical exercise in the paper,  $k = 24$  and  $l = 11$ , the correlation is 0.62.

**Non-integrated case.** We also consider the case in which the autocorrelation of  $y_t$  does not converge to 1. We replicate the same simulation exercises described for the multivariate setup and vary the autocorrelation coefficient  $\rho_y$  between 0.9 and 1. When we run the regressions to estimate the output gap, we set the parameter  $k$  to 24 and vary the parameter  $l$  that governs the number of lags in the regression between 0 and 11. The results are tabulated in Panel B of the table below and show that the correlation between output gap and long-run risk is still positive and large.

<b>Panel A: Output gap and long-run risk correlation (multivariate case)</b>												
	0	1	2	3	4	5	6	7	8	9	10	11
k=1	0.42	0.35	0.31	0.27	0.25	0.23	0.22	0.21	0.20	0.19	0.19	0.18
k=12	0.77	0.71	0.66	0.62	0.60	0.58	0.56	0.55	0.54	0.53	0.53	0.52
k=24	0.77	0.72	0.70	0.68	0.66	0.65	0.64	0.63	0.63	0.62	0.62	0.62
k=36	0.72	0.70	0.68	0.67	0.67	0.66	0.66	0.65	0.65	0.65	0.65	0.65

<b>Panel B: Output gap and long-run risk correlation (non-integrated case)</b>												
	0	1	2	3	4	5	6	7	8	9	10	11
$\rho_y = 1.00$	0.77	0.72	0.70	0.68	0.66	0.65	0.64	0.63	0.63	0.62	0.62	0.62
$\rho_y = 0.95$	0.67	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.65	0.65	0.65	0.65
$\rho_y = 0.90$	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.66

Notes: The parameters of the data generating process are set to  $\rho_x = 0.98$ ,  $\sigma_y = 0.02/\sqrt{12}$ , and  $\sigma_x = 0.1\sigma_y$ . In Panel A, we set  $\rho_y = 1$ , and in Panel B, the regression parameter  $k$  is set to 24.

## Further Empirical Evidence

In this section we provide further empirical evidence on the validity of the model's assumptions. An assumption of the model is that shocks to country  $i$ 's conditional variance are positively correlated with shocks to country  $i$ 's output gap ( $\rho_{i,t}^{z,\sigma} > 0$ ). Related to this assumption, if the correlation is positive but *imperfect* (i.e.  $\alpha \neq 0$ ), then sorting currencies by interest rates is different to sorting currencies by output gaps. We explore this assumption empirically. To do so, we extract the conditional variance of industrial production growth using a GARCH(1,1) model and proxy for the shocks to conditional variances and output gaps using the residuals from AR(1) models fitted to the series. We test the sign and magnitude of the relationship by estimating a pooled OLS regression of conditional variance shocks on output gap shocks (including country-level dummy variables). We find the estimated coefficient is positive and highly statistically significant ( $\beta = 0.25$ ,  $p$ -value = 0.00) but we can strongly reject the hypothesis that the coefficient equals unity ( $p$ -value = 0.00).

A necessary condition for the output gap premia to be positive is linked to the behaviour of the time-varying correlation between output gap shocks in countries  $i$  and  $j$ . The condition requires that the correlation falls as the output gap in country  $i$  rises (here country  $j$  refers to the base currency, i.e. the US in our empirical setup). We test the condition empirically by forming output gap shocks as the residuals from AR(1) models and construct dynamic conditional correlations using a DCC(1,1) model. We estimate the relationship between these conditional correlations and the model implied transformation of output gaps  $(1 + \exp\{z_{i,t}\})^{-1}$ , via a pooled OLS regression (with country-level dummy variables). The condition is supported if the slope coefficient is positive. We estimate the coefficient to equal 0.32, with associated  $p$ -value = 0.01, and thus we find clear empirical support for the condition.

**Table A1: Foreign Exchange Rate Data**

DataStream Codes							
Country	Code	Currency	Spot	1M Forward	Source	Start Date	End Date
Austria	ATS	schilling	AUSTSC\$	USATS1F	Reuters	31/12/1996	31/12/1998
Australia	AUD	dollar	BBAUDSP	BBAUD1F	Barclays	31/12/1984	31/01/2016
Belgium	BEF	franc	BELGLU\$	USBEF1F	Reuters	31/12/1996	31/12/1998
Brazil	BRL	real	BRACRU\$	USBRL1F	Reuters	31/03/2004	31/01/2016
Canada	CAD	dollar	BBCADSP	BBCAD1F	Barclays	31/12/1984	31/01/2016
Switzerland	CHF	franc	BBCHFSP	BBCHF1F	Barclays	31/10/1983	31/01/2016
Chile	CLP	peso	CHILPE\$	USCLP1F	Reuters	31/03/2004	31/01/2016
Czech Republic	CZK	koruna	TDCZKSP	TDCZK1M	Reuters	31/12/1996	31/01/2016
Germany*	DEM	deutschemark	BBDEMSP	BBDEM1F	Barclays	31/10/1983	31/01/2016
Spain	ESP	peseta	SPANPE\$	USESP1F	Reuters	31/12/1996	31/12/1998
Finland	FIM	markka	FINMAR\$	USFIM1F	Reuters	31/12/1996	31/12/1998
France	FRF	franc	BBFRFSP	BBFRF1F	Barclays	31/10/1983	31/12/1998
UK	GBP	pound	BBGBPSP	BBGBP1F	Barclays	31/10/1983	31/01/2016
Ireland	IEP	punt	BBIEPSP	BBIEP1F	Barclays	31/10/1993	31/12/1998
Iceland	ISK	krona	ICEKRO\$	USISK1F	Reuters	31/03/2004	31/01/2016
Italy	ITL	lira	BBITLSP	BBITL1F	Barclays	31/03/1984	31/12/1998
Japan	JPY	yen	BBJPYSP	BBJPY1F	Barclays	31/10/1983	31/01/2016
South Korea	KRW	won	KORSWO\$	USKRW1F	Reuters	28/02/2002	31/01/2016
Mexico	MXN	peso	MEXPES\$	USMXN1F	Reuters	31/12/1996	31/01/2016
Netherlands	NLG	guilder	BBNLGSP	BBNLG1F	Barclays	31/10/1983	31/12/1998
Norway	NOK	krona	BBNOKSP	BBNOK1F	Barclays	31/12/1984	31/01/2016
New Zealand	NZD	dollar	BBNZDSP	BBNZD1F	Barclays	31/12/1984	31/01/2016
Poland	PLN	zloty	TDPLNSP	TDPLN1M	Reuters	31/08/1996	31/01/2016
Portugal	PTE	escudo	PORTES\$	USPTE1F	Reuters	31/12/1996	31/12/1998
Sweden	SEK	krona	BBSEKSP	BBSEK1F	Barclays	31/12/1984	31/01/2016
Turkey** (first lira)	TRY	lira	TURKLI\$	USTRY1F	Reuters	31/12/1996	31/10/2000
Turkey (second lira)	TRY	lira	TURKLI\$	USTRY1F	Reuters	31/03/2004	31/01/2016

\* We replace the German deutschemark with the euro after 1998.

\*\* We remove the period of hyperinflation in Turkey due to large deviations from CIP.



**Table A2: Carry-Trade Currency Portfolios**

The table presents descriptive statistics for five currency portfolios sorted by forward premia. Portfolios are rebalanced monthly with high (low) interest rate currencies entering  $P_5$  ( $P_1$ ). We report summary statistics for the annualized excess mean return and its decomposition between the exchange rate ( $fx$ ) and interest rate ( $ir$ ) components. We also report the Sharpe ratio ( $Sharpe$ ), standard deviation ( $std$ ), skewness ( $skew$ ), kurtosis ( $kurt$ ), maximum drawdown ( $mdd$ ), average turnover ( $t/o$ ), average forward premium ( $fp$ ), and average output gap ( $gap$ ) for each portfolio. The *Cross Section* portfolio is long  $P_5$  and short  $P_1$ . The *Time Series* portfolio takes a  $1/N$  position in currencies, going long (short) currencies issued by countries with an interest rate above (below) the US interest rate. The superscripts \*, \*\*, \*\*\* represent significance of the *Cross Section* and *Time Series* portfolios at the 10%, 5%, and 1% level using Newey and West (1987) standard errors. The sample is from October 1983 to January 2016.

	<i>Forward Premia</i>					<b>Cross Section</b>	<b>Time Series</b>
	<b>P<sub>1</sub></b>	<b>P<sub>2</sub></b>	<b>P<sub>3</sub></b>	<b>P<sub>4</sub></b>	<b>P<sub>5</sub></b>		
<i>mean (%)</i>	-0.63	1.02	3.88	2.83	7.17	7.80***	4.43***
<i>fx (%)</i>	1.58	1.35	2.54	-0.40	-3.05	-4.63	0.85
<i>ir (%)</i>	-2.20	-0.33	1.34	3.22	10.22	12.43	3.59
<i>Sharpe</i>	-0.06	0.11	0.42	0.29	0.68	0.72	0.79
<i>std</i>	9.80	9.30	9.23	9.72	10.49	10.87	5.60
<i>skew</i>	0.26	-0.09	-0.29	-0.48	-0.63	-0.93	-1.14
<i>kurt</i>	3.80	3.73	5.12	4.85	5.56	5.30	9.32
<i>mdd (%)</i>	54.0	32.6	23.2	27.8	19.9	19.8	8.2
<i>t/o (%)</i>	18.5	25.6	29.6	24.1	13.5		
<i>fp (t, %)</i>	-2.15	-0.32	1.25	3.25	10.94		
<i>gap (t, %)</i>	-0.04	0.02	0.08	-0.05	0.30		

**Table A3: Correlation and Factor Structure of Output Gap Measures**

The table presents the average cross-sectional correlation and factor structure across measures of countries' output gap. The output gap is estimated as (log) industrial production minus the (log) trend in industrial production. The trend is estimated in four ways using a (i) Hodrick-Prescott filter; (ii) Baxter-King filter, (iii) linear projection, and (iv) quadratic time trend. In Panel A, entries below the diagonal are linear Pearson correlations, calculated by taking the time-series average of monthly cross-sectional correlations for all available currencies. The entries above the diagonal are Spearman rank correlations, also calculated as the time-series average of monthly cross-sectional correlations. In Panel B, we report the average proportion of cross-sectional variation accounted for by each principal component ( $PC$ ). To calculate, we estimate the variation explained by each  $PC$  every month and report the average across the sample. The sample is from October 1983 to January 2016.

<b>Panel A: Output-Gap Correlations</b>				
	<b>HP</b>	<b>BK</b>	<b>LP</b>	<b>QT</b>
<i>Hodrick-Prescott Filter (HP)</i>		0.63	0.51	0.41
<i>Baxter-King Filter (BK)</i>	0.65		0.55	0.53
<i>Linear Projection (LP)</i>	0.54	0.56		0.47
<i>Quadratic Time-trend (QT)</i>	0.45	0.58	0.48	

<b>Panel B: Output-Gap Factor Structure</b>				
	<b>PC<sub>1</sub></b>	<b>PC<sub>2</sub></b>	<b>PC<sub>3</sub></b>	<b>PC<sub>4</sub></b>
<i>var explained</i>	86%	10%	3%	1%

**Table A4: Currency Portfolios Sorted on Deviations from Taylor-Rule-Implied Interest Rates**

The table presents descriptive statistics for five currency portfolios sorted by their deviation from a Taylor-rule implied interest rate. The Taylor rule is calibrated to equal  $1.5\pi_t + 0.5y_t$ , where  $\pi_t$  is inflation and  $y_t$  is the in-sample output gap calculated using a Hodrick-Prescott filter. Portfolios are rebalanced monthly with the highest (lowest) interest-rate deviation currencies entering  $P_5$  ( $P_1$ ). We report summary statistics for the annualized excess mean return and its decomposition between the exchange rate ( $fx$ ) and interest rate ( $ir$ ) components. We also report the Sharpe ratio ( $Sharpe$ ), standard deviation ( $std$ ), skewness ( $skew$ ), kurtosis ( $kurt$ ), maximum drawdown ( $mdd$ ), average turnover ( $t/o$ ), average forward premium ( $fp$ ), and average output gap ( $gap$ ) for each portfolio. The *Cross Section* portfolio is long  $P_5$  and short  $P_1$ . The *Time Series* portfolio takes a 1/N position in currencies, going long (short) currencies issued by countries with a positive (negative) deviation from the Taylor-rule interest rate. The superscripts \*, \*\*, \*\*\* represent significance of the *Cross Section* and *Time Series* portfolios at the 10%, 5%, and 1% level using Newey and West (1987) standard errors. We also report the correlation of the *Cross Section* and *Time Series* portfolios with the equivalent portfolios sorted on Hodrick-Prescott filtered output gaps ( $\rho_{GAP}$ ), and interest rates ( $\rho_{HML_{FX}}$ ). The sample is from October 1983 to January 2016.

	<i>Deviations from Taylor Rule</i>					<b>Cross Section</b>	<b>Time Series</b>
	<b>P<sub>1</sub></b>	<b>P<sub>2</sub></b>	<b>P<sub>3</sub></b>	<b>P<sub>4</sub></b>	<b>P<sub>5</sub></b>		
<i>mean</i> (%)	3.95	2.76	0.74	3.17	3.06	-0.89	1.82**
<i>fx</i> (%)	0.12	1.92	-0.53	1.03	-1.17	-1.29	0.98
<i>ir</i> (%)	3.83	0.84	1.28	2.14	4.23	0.40	0.84
<i>Sharpe</i>	0.40	0.30	0.08	0.33	0.30	-0.11	0.34
<i>std</i>	9.83	9.29	9.27	9.64	10.27	8.44	5.30
<i>skew</i>	0.06	-0.10	-0.27	-0.49	-0.29	-0.26	-0.39
<i>kurt</i>	5.98	4.17	4.40	5.21	4.45	8.09	5.93
<i>mdd</i> (%)	25.2	26.8	24.1	25.0	34.0	63.3	15.0
<i>t/o</i> (%)	29.8	49.2	59.1	54.4	34.9		
<i>fp</i> (t, %)	4.41	0.89	1.27	2.20	4.25		
<i>gap</i> (t, %)	1.42	0.63	0.10	-0.35	-1.58		
$\rho_{GAP}$						-0.33	-0.05
$\rho_{HML_{FX}}$						0.20	0.61

**Table A5: Portfolio Weights**

The table presents summary statistics on the portfolio weights in the  $GAP_{CS}$ ,  $LIN$ , and  $RNK$  output-gap-sorted portfolios. We report the average maximum ( $\overline{max}$ ), minimum ( $\overline{min}$ ), and standard deviation ( $\overline{std}$ ), calculated as the time-series mean of the maximum, minimum, and standard deviation of weights each month. The sample is from December 1999 to January 2016.

	<b>GAP<sub>CS</sub></b>	<b>LIN</b>	<b>RNK</b>
$\overline{max}$	0.20	0.18	0.25
$\overline{min}$	-0.20	-0.16	-0.25
$\overline{std}$	0.28	0.09	0.16

**Table A6: Real-Time Business Cycle Currency Portfolios During US Booms and Recessions**

The table presents investment performance for output-gap-based currency trading strategies during booms and recessions in the US. The output gap is estimated using monthly ‘vintages’ of real-time industrial production data from the OECD’s *Real-Time Data and Revisions Database*. To estimate the output-gap we follow the linear projection procedure in Hamilton (2018) by running the regression,  $y_{i,t} = \alpha_i + \sum_{s=0}^{11} \beta_{i,s} y_{i,t-24-s} + \varepsilon_{i,t}$  each month, in which  $y$  is (log) industrial production. Periods of recession are consistent with those defined by the NBER’s Business Cycle Dating Committee. The output gap is constructed as the difference between the most recently available data point at time  $t$  ( $y_t$ ) and the fitted value from the regression.  $GAP_{CS}$  is a *high-minus-low* portfolio formed as  $P_5 - P_1$ , after sorting currencies into five portfolios ranging from the lowest ( $P_1$ ) to the highest ( $P_5$ ) output gap.  $GAP_{TS}$  is a 1/N time-series strategy long (short) currencies issued by countries with an output gap above (below) the US output gap. We report summary statistics for the annualized mean, which is then further split between the exchange rate ( $fx$ ) and interest rate ( $ir$ ) components, we also report the Sharpe ratio ( $Sharpe$ ), skewness ( $skew$ ), kurtosis ( $kurt$ ), maximum drawdown ( $mdd$ ), and the exposure of the strategy to the US dollar ( $\$ exposure$ ). The superscripts \*, \*\*, \*\*\* represent significance of the strategies’ mean excess returns at the 10%, 5%, and 1% significance levels using Newey and West (1987) corrected standard errors. The sample runs from December 1999 to January 2016.

	Expansions		Recessions	
	GAP <sub>CS</sub>	GAP <sub>TS</sub>	GAP <sub>CS</sub>	GAP <sub>TS</sub>
<i>mean (%)</i>	4.75**	3.90***	5.94*	-3.99
<i>fx (%)</i>	3.97	4.09	5.65	-4.61
<i>ir (%)</i>	0.78	-0.19	0.29	0.62
<i>Sharpe</i>	0.71	1.00	0.73	-0.69
<i>skew</i>	0.27	-0.20	0.40	-1.08
<i>kurt</i>	2.78	4.23	2.71	3.69
<i>mdd (%)</i>	8.18	6.92	5.99	16.68
<i>\$ exposure</i>	0.00	0.26	0.00	-0.27

**Table A7: Real-Time Taylor-rule Currency Portfolios**

The table presents investment performance for Taylor-rule-based trading strategies. The Taylor rule is calibrated to equal  $1.5\pi_t + 0.5y_t$ , where  $\pi_t$  is inflation and  $y_t$  is the out-of-sample output gap calculated using monthly ‘vintages’ of real-time industrial production data from the OECD’s *Real-Time Data and Revisions Database*. To estimate the output-gap we follow the linear projection procedure in Hamilton (2018) by running the regression,  $y_{i,t} = \alpha_i + \sum_{s=0}^{11} \beta_{i,s}y_{i,t-24-s} + \varepsilon_{i,t}$  each month, in which  $y$  is (log) industrial production. The output gap is constructed as the difference between the most recently available data point at time  $t$  ( $y_t$ ) and the fitted value from the regression. *CS* is a *high-minus-low* portfolio formed as  $P_5 - P_1$ , after sorting currencies into five portfolios ranging from the lowest ( $P_1$ ) to the highest ( $P_5$ ) implied interest rate. *LIN* and *RNK* take a position in all currencies with the weight determined by either the magnitude or relative size of the implied interest rate. *TS* is a 1/N time-series strategy long (short) currencies issued by countries with an implied rate above (below) the US implied rate. The three *COM* portfolios take 50–50 weights in *TS* and the *CS*, *LIN*, and *RNK* strategies. We report summary statistics for the annualized mean, which is then further split between the exchange rate (*fx*) and interest rate (*ir*) components, we also report the Sharpe ratio (*Sharpe*), skewness (*skew*), kurtosis (*kurt*), and maximum drawdown (*mdd*). The superscripts \*, \*\*, \*\*\* represent significance of the strategy mean excess returns at the 10%, 5%, and 1% significance levels using Newey and West (1987) corrected standard errors. We also report the correlation of the portfolios with the equivalent strategies sorted on output gaps as in Table 4 ( $\rho_{GAP}$ ) and forward premia ( $\rho_{HML_{FX}}$ ). The sample runs from December 1999 to January 2016.

Panel A: Investment Performance <i>Excluding</i> Bid-Ask Spreads							
	CS	LIN	RNK	GAP <sub>TS</sub>	COM <sub>GAP</sub>	COM <sub>LIN</sub>	COM <sub>RNK</sub>
<i>mean</i> (%)	6.05**	3.24***	4.68**	3.40**	4.73***	3.33***	4.04***
<i>fx</i> (%)	-2.34	-1.23	-1.49	1.48	-0.41	0.15	0.02
<i>ir</i> (%)	8.39	4.47	6.17	1.92	5.13	3.18	4.02
<i>Sharpe</i>	0.63	0.73	0.64	0.79	0.77	0.87	0.78
<i>skew</i>	-0.25	-0.70	-0.25	-0.19	-0.22	-0.52	-0.34
<i>kurt</i>	3.35	6.06	3.34	5.70	3.27	4.51	3.61
<i>mdd</i> (%)	9.57	4.47	7.34	4.31	6.17	3.84	5.21
$\rho_{GAP}$	0.24	0.18	0.17	0.21	0.26	0.23	0.23
$\rho_{HML_{FX}}$	0.79	0.87	0.83	0.44	0.77	0.69	0.74

Panel B: Investment Performance <i>Including</i> Bid-Ask Spreads							
	CS	LIN	RNK	GAP <sub>TS</sub>	COM <sub>GAP</sub>	COM <sub>LIN</sub>	COM <sub>RNK</sub>
<i>mean</i> (%)	4.75*	2.52**	3.37*	2.78***	3.77**	2.65***	3.08**
<i>fx</i> (%)	-3.34	-1.78	-2.49	1.02	-1.14	-0.36	-0.71
<i>ir</i> (%)	8.09	4.30	5.85	1.77	4.91	3.01	3.79
<i>Sharpe</i>	0.50	0.57	0.46	0.65	0.61	0.69	0.59
<i>skew</i>	-0.26	-0.73	-0.27	-0.21	-0.23	-0.54	-0.35
<i>kurt</i>	3.37	6.15	3.36	5.74	3.28	4.55	3.64
<i>mdd</i> (%)	9.55	4.45	7.32	4.30	6.16	3.83	5.20
$\rho_{GAP}$	0.24	0.18	0.17	0.21	0.26	0.23	0.23
$\rho_{HML_{FX}}$	0.79	0.87	0.83	0.44	0.77	0.69	0.74

**Table A8: Real-Time Business Cycle Portfolios across Different Home Investors**

The table presents investment performance for output-gap-based currency trading strategies from the perspective of German, Japanese, British, and Swiss investors. The output gap is estimated using monthly ‘vintages’ of real-time industrial production data from the OECD’s *Real-Time Data and Revisions Database*. To estimate the output-gap we follow the linear projection procedure in Hamilton (2018) by running the regression,  $y_{i,t} = \alpha_i + \sum_{s=0}^{11} \beta_{i,s} y_{i,t-24-s} + \varepsilon_{i,t}$  each month, in which  $y$  is (log) industrial production. The output gap is constructed as the difference between the most recently available data point at time  $t$  ( $y_t$ ) and the fitted value from the regression.  $GAP_{CS}$  is a *high-minus-low* portfolio formed as  $P_5 - P_1$ , after sorting currencies into five portfolios ranging from the lowest ( $P_1$ ) to the highest ( $P_5$ ) output gap.  $LIN$  and  $RNK$  take a position in all currencies with the weight determined by either the magnitude or relative size of the output gap.  $GAP_{TS}$  is a  $1/N$  time-series strategy long (short) currencies issued by countries with an output gap above (below) the US output gap. The three  $COM$  portfolios take 50–50 weights in  $GAP_{TS}$  and the  $GAP_{CS}$ ,  $LIN$ , and  $RNK$  strategies. We report summary statistics for the annualized mean, which is then further split between the exchange rate ( $fx$ ) and interest rate ( $ir$ ) components, we also report the Sharpe ratio (*Sharpe*), skewness (*skew*), kurtosis (*kurt*), and maximum drawdown (*mdd*). The superscripts \*, \*\*, \*\*\* represent significance of the strategies’ mean excess returns at the 10%, 5%, and 1% significance levels using Newey and West (1987) corrected standard errors. The sample runs from December 1999 to January 2016.

Panel A: German Investor							
	GAP <sub>CS</sub>	LIN	RNK	GAP <sub>TS</sub>	COM <sub>GAP</sub>	COM <sub>LIN</sub>	COM <sub>RNK</sub>
<i>mean (%)</i>	5.61***	2.24***	3.88***	1.76**	3.73***	2.05***	2.87***
<i>fx (%)</i>	4.84	1.72	3.26	2.25	3.57	2.01	2.78
<i>ir (%)</i>	0.77	0.52	0.62	-0.49	0.16	0.03	0.09
<i>Sharpe</i>	0.79	0.76	0.72	0.50	0.80	0.74	0.73

Panel B: Japanese Investor							
	GAP <sub>CS</sub>	LIN	RNK	GAP <sub>TS</sub>	COM <sub>GAP</sub>	COM <sub>LIN</sub>	COM <sub>RNK</sub>
<i>mean (%)</i>	4.49***	1.93***	3.40***	4.19**	4.41***	3.13***	3.86***
<i>fx (%)</i>	4.08	1.61	3.07	3.68	3.92	2.68	3.41
<i>ir (%)</i>	0.41	0.32	0.33	0.51	0.49	0.44	0.45
<i>Sharpe</i>	0.66	0.68	0.67	0.61	0.82	0.77	0.80

Panel C: British Investor							
	GAP <sub>CS</sub>	LIN	RNK	GAP <sub>TS</sub>	COM <sub>GAP</sub>	COM <sub>LIN</sub>	COM <sub>RNK</sub>
<i>mean (%)</i>	4.98***	2.27***	4.12***	1.51**	3.26***	1.91***	2.83***
<i>fx (%)</i>	4.51	1.87	3.70	1.24	2.87	1.55	2.47
<i>ir (%)</i>	0.47	0.41	0.41	0.27	0.39	0.36	0.36
<i>Sharpe</i>	0.72	0.77	0.77	0.40	0.69	0.66	0.71

Panel D: Swiss Investor							
	GAP <sub>CS</sub>	LIN	RNK	GAP <sub>TS</sub>	COM <sub>GAP</sub>	COM <sub>LIN</sub>	COM <sub>RNK</sub>
<i>mean (%)</i>	6.35***	2.32***	4.25***	0.66	3.55***	1.54**	2.50***
<i>fx (%)</i>	5.43	1.76	3.59	1.21	3.35	1.51	2.42
<i>ir (%)</i>	0.92	0.56	0.66	-0.55	0.21	0.03	0.08
<i>Sharpe</i>	0.91	0.81	0.81	0.14	0.73	0.48	0.59

**Table A9: Pricing Currency Portfolios Sorted on Output Gaps with Fama-MacBeth Estimation**

The table presents cross-sectional asset pricing results. We construct various two-factor linear SDF's that include the *DOL* factor plus a second pricing factor, including 'slope' risk (*HML<sub>FX</sub>*), global imbalance risk (*IMB*), volatility risk (*VOL*), and the *GAP<sub>CS</sub>* factor. In each model, we price five currency portfolios sorted on output gaps using real-time information. We report Fama-MacBeth estimates of factor loadings on the pricing kernel (*b*'s) and prices of factor risk ( $\lambda$ 's). The superscripts \*, \*\*, \*\*\* represent significance of the coefficients at the 10%, 5%, and 1% significance levels using Newey and West (1987) corrected standard errors (reported in parentheses). We also report goodness-of-fit statistics for each model including the adjusted  $R^2$  statistic, Root Mean Squared Pricing Error (*RMSE*), and the Hansen-Jagannathan distance statistic ( $HJ_{dist}$ ) with simulated  $p$ -values in brackets. The  $HJ_{dist}$  statistic measures the distance between the estimated pricing kernel and the efficient set of permissible pricing kernels. A  $p$ -value less than 5% indicates the null hypothesis that the pricing kernel is efficient can be rejected at the 95% confidence level. We provide full details of the pricing factors in Section 5. The sample runs from December 1999 to January 2016.

	SDF		Risk		Model Fit		
	Loadings ( <i>b</i> )		Prices ( $\lambda$ )		Adj.R <sup>2</sup>	RMSE	HJ <sub>dist</sub>
	DOL	FAC	DOL	FAC			
DOL + HML <sub>FX</sub>	0.22 (0.26)	0.19 (0.72)	0.02 (0.02)	0.03 (0.10)	-0.78	1.69	0.22 [0.03]
DOL + IMB	-1.39 (1.43)	7.52 (5.78)	0.04 (0.04)	0.26 (0.20)	-0.15	1.59	0.19 [0.61]
DOL + VOL	-3.21 (2.86)	-40.5 (31.6)	0.03 (0.03)	-0.03 (0.02)	-0.19	1.51	0.21 [0.46]
DOL + GAP <sub>CS</sub>	0.08 (0.26)	0.83*** (0.29)	0.02 (0.02)	0.05*** (0.02)	0.44	0.95	0.13 [0.34]



**Table A10: Asset Pricing using DOL, HML<sub>FX</sub>, and GAP<sub>CS</sub> as Pricing Factors with Fama-MacBeth Estimation**

The table presents cross-sectional asset pricing results for two sets of test portfolios. The SDF is constructed as a linear combination of *DOL* and *HML<sub>FX</sub>* (2 pricing factors, left-side) and *DOL*, *HML<sub>FX</sub>*, and *GAP<sub>CS</sub>* (3 pricing factors, right-side). In Panel B, we also include *HML<sub>FX</sub>* and *GAP<sub>CS</sub>* as test assets. We report Fama-MacBeth estimates of factor loadings on the pricing kernel (*b*'s) and prices of factor risk (*λ*'s). The superscripts \*, \*\*, \*\*\* represent significance of the coefficients at the 10%, 5%, and 1% significance levels using Newey and West (1987) corrected standard errors (reported in parentheses). In addition, we report goodness-of-fit statistics for each model including the adjusted *R*<sup>2</sup> statistic, Root Mean Squared Pricing Error (*RMSE*), and the Hansen-Jagannathan distance statistic (*HJ<sub>dist</sub>*) with simulated *p*-values in brackets. The *HJ<sub>dist</sub>* statistic measures the distance between the estimated pricing kernel and the efficient set of permissible pricing kernels. A *p*-value less than 5% indicates the null hypothesis that the pricing kernel is efficient can be rejected at the 95% confidence level. The sample runs from December 1999 to January 2016.

<b>Panel A: Excluding Pricing Factors as Test Portfolios</b>																	
25	2 Pricing Factors (DOL + HML <sub>FX</sub> )							3 Pricing Factors (DOL + HML <sub>FX</sub> + GAP <sub>CS</sub> )									
	Loadings ( <i>b</i> )		Risk Prices ( <i>λ</i> )		Model Fit			Loadings ( <i>b</i> )			Risk Prices ( <i>λ</i> )			Model Fit			
	DOL	HML <sub>FX</sub>	DOL	HML <sub>FX</sub>	Adj.R <sup>2</sup>	DOL	HJ <sub>dist</sub>	DOL	HML <sub>FX</sub>	GAP <sub>CS</sub>	DOL	HML <sub>FX</sub>	GAP <sub>CS</sub>	Adj.R <sup>2</sup>	RMSE	HJ <sub>dist</sub>	
10 TPs (val, mom)	0.19 (0.26)	0.08 (0.32)	0.02 (0.02)	0.01 (0.04)	-0.35	1.32	0.22 [0.81]	-0.31 (0.36)	0.43 (0.41)	2.57** (1.01)	0.02 (0.03)	0.07 (0.05)	0.14** (0.06)	0.59	0.67	0.16 [0.96]	
20 TPs (gap, car, val, mom)	0.18 (0.27)	0.36 (0.22)	0.02 (0.02)	0.05* (0.03)	0.26	1.36	0.33 [0.99]	-0.02 (0.28)	0.35 (0.22)	1.05*** (0.32)	0.02 (0.02)	0.06* (0.03)	0.06*** (0.02)	0.62	0.94	0.30 [0.99]	
<b>Panel B: Including Pricing Factors as Test Portfolios</b>																	
25	2 Pricing Factors (DOL + HML <sub>FX</sub> )							3 Pricing Factors (DOL + HML <sub>FX</sub> + GAP <sub>CS</sub> )									
	Loadings ( <i>b</i> )		Risk Prices ( <i>λ</i> )		Model Fit			Loadings ( <i>b</i> )			Risk Prices ( <i>λ</i> )			Model Fit			
	DOL	HML <sub>FX</sub>	DOL	HML <sub>FX</sub>	Adj.R <sup>2</sup>	DOL	HJ <sub>dist</sub>	DOL	HML <sub>FX</sub>	GAP <sub>CS</sub>	DOL	HML <sub>FX</sub>	GAP <sub>CS</sub>	Adj.R <sup>2</sup>	RMSE	HJ <sub>dist</sub>	
10 TPs (val, mom)	0.16 (0.28)	0.41* (0.22)	0.02 (0.02)	0.06** (0.03)	0.31	1.37	0.23 [0.80]	-0.02 (0.28)	0.40* (0.21)	0.97*** (0.30)	0.02 (0.02)	0.06** (0.03)	0.06*** (0.02)	0.65	0.96	0.19 [0.91]	
20 TPs (gap, car, val, mom)	0.17 (0.28)	0.41* (0.22)	0.02 (0.02)	0.06** (0.03)	0.46	1.34	0.69 [0.91]	0.00 (0.28)	0.40* (0.21)	0.91*** (0.30)	0.02 (0.02)	0.06** (0.03)	0.05*** (0.02)	0.74	0.92	0.69 [0.98]	

**Table A11: Asset Pricing using DOL, IMB, and GAP<sub>CS</sub> as Pricing Factors**

The table presents cross-sectional asset pricing results for two sets of test portfolios. The SDF is constructed as a linear combination of *DOL* and *IMB* (2 pricing factors, left-side) and *DOL*, *IMB*, and *GAP<sub>CS</sub>* (3 pricing factors, right-side). In Panel B, we also include *IMB* and *GAP<sub>CS</sub>* as test assets. We report Generalized Method of Moments (GMM) one-step estimates of factor loadings on the pricing kernel (*b*'s) and prices of factor risk ( $\lambda$ 's). The superscripts \*, \*\*, \*\*\* represent significance of the coefficients at the 10%, 5%, and 1% significance levels using Newey and West (1987) corrected standard errors (reported in parentheses). In addition, we report goodness-of-fit statistics for each model including the adjusted  $R^2$  statistic, Root Mean Squared Pricing Error (*RMSE*), and the Hansen-Jagannathan distance statistic ( $HJ_{dist}$ ) with simulated *p*-values in brackets. The  $HJ_{dist}$  statistic measures the distance between the estimated pricing kernel and the efficient set of permissible pricing kernels. A *p*-value less than 5% indicates the null hypothesis that the pricing kernel is efficient can be rejected at the 95% confidence level. The sample runs from December 1999 to January 2016.

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<b>Panel A: Excluding Pricing Factors as Test Portfolios</b>																	
	2 Pricing Factors (DOL + IMB)							3 Pricing Factors (DOL + IMB + GAP <sub>CS</sub> )									
	Loadings ( <i>b</i> )		Risk Prices ( $\lambda$ )		Model Fit			Loadings ( <i>b</i> )			Risk Prices ( $\lambda$ )			Model Fit			
	DOL	IMB	DOL	IMB	Adj.R <sup>2</sup>	DOL	HJ <sub>dist</sub>	DOL	IMB	GAP <sub>CS</sub>	DOL	IMB	GAP <sub>CS</sub>	Adj.R <sup>2</sup>	RMSE	HJ <sub>dist</sub>	
10 TPs (val, mom)	-0.13 (0.45)	2.04* (1.23)	0.03 (0.02)	0.08** (0.04)	-0.03	1.20	0.20 [0.92]	-0.27 (0.42)	1.26 (1.48)	1.85* (1.12)	0.03 (0.02)	0.05 (0.05)	0.11** (0.04)	0.47	0.80	0.14 [0.98]	
20 TPs (gap, car, val, mom)	-0.20 (0.41)	2.43** (1.18)	0.03 (0.02)	0.09*** (0.03)	0.43	1.35	0.41 [0.98]	-0.28 (0.39)	2.03* (1.19)	0.93** (0.38)	0.03 (0.02)	0.08** (0.02)	0.06*** (0.03)	0.64	1.04	0.37 [0.99]	
<b>Panel B: Including Pricing Factors as Test Portfolios</b>																	
	2 Pricing Factors (DOL + IMB)							3 Pricing Factors (DOL + IMB + GAP <sub>CS</sub> )									
	Loadings ( <i>b</i> )		Risk Prices ( $\lambda$ )		Model Fit			Loadings ( <i>b</i> )			Risk Prices ( $\lambda$ )			Model Fit			
	DOL	IMB	DOL	IMB	Adj.R <sup>2</sup>	DOL	HJ <sub>dist</sub>	DOL	IMB	GAP <sub>CS</sub>	DOL	IMB	GAP <sub>CS</sub>	Adj.R <sup>2</sup>	RMSE	HJ <sub>dist</sub>	
10 TPs (val, mom)	0.05 (0.31)	1.36*** (0.43)	0.03 (0.02)	0.05*** (0.02)	0.04	1.18	0.20 [0.96]	-0.10 (0.31)	1.24*** (0.43)	0.96*** (0.31)	0.03 (0.02)	0.05*** (0.02)	0.06*** (0.02)	0.48	0.85	0.16 [0.99]	
20 TPs (gap, car, val, mom)	-0.01 (0.32)	1.65*** (0.56)	0.04 (0.02)	0.06*** (0.02)	0.36	1.41	0.41 [0.99]	-0.13 (0.32)	1.47*** (0.56)	0.91*** (0.32)	0.04 (0.02)	0.06*** (0.02)	0.06*** (0.02)	0.63	1.05	0.47 [0.99]	

**Table A12: Asset Pricing using DOL, VOL, and GAP<sub>CS</sub> as Pricing Factors**

The table presents cross-sectional asset pricing results for two sets of test portfolios. The SDF is constructed as a linear combination of *DOL* and *VOL* (2 pricing factors, left-side) and *DOL*, *VOL*, and *GAP<sub>CS</sub>* (3 pricing factors, right-side). In Panel B, we also include *VOL* and *GAP<sub>CS</sub>* as test assets. We report Generalized Method of Moments (GMM) one-step estimates of factor loadings on the pricing kernel (*b*'s) and prices of factor risk ( $\lambda$ 's). The superscripts \*, \*\*, \*\*\* represent significance of the coefficients at the 10%, 5%, and 1% significance levels using Newey and West (1987) corrected standard errors (reported in parentheses). In addition, we report goodness-of-fit statistics for each model including the adjusted  $R^2$  statistic, Root Mean Squared Pricing Error (*RMSE*), and the Hansen-Jagannathan distance statistic ( $HJ_{dist}$ ) with simulated *p*-values in brackets. The  $HJ_{dist}$  statistic measures the distance between the estimated pricing kernel and the efficient set of permissible pricing kernels. A *p*-value less than 5% indicates the null hypothesis that the pricing kernel is efficient can be rejected at the 95% confidence level. The sample runs from December 1999 to January 2016.

<b>Panel A: Excluding Pricing Factors as Test Portfolios</b>																	
27	2 Pricing Factors (DOL + VOL)							3 Pricing Factors (DOL + VOL + GAP <sub>CS</sub> )									
	Loadings ( <i>b</i> )		Risk Prices ( $\lambda$ )		Model Fit			Loadings ( <i>b</i> )			Risk Prices ( $\lambda$ )			Model Fit			
	DOL	VOL	DOL	VOL	Adj.R <sup>2</sup>	DOL	HJ <sub>dist</sub>	DOL	VOL	GAP <sub>CS</sub>	DOL	VOL	GAP <sub>CS</sub>	Adj.R <sup>2</sup>	RMSE	HJ <sub>dist</sub>	
10 TPs (val, mom)	-0.02 (0.58)	-3.37 (6.16)	0.02 (0.02)	-0.00 (0.01)	-0.37	1.29	0.21 [0.85]	-0.89 (0.75)	-7.93 (7.52)	2.40** (1.01)	0.02 (0.02)	-0.01* (0.01)	0.14*** (0.05)	0.57	0.67	0.17 [0.95]	
20 TPs (gap, car, val, mom)	-0.56 (0.49)	-9.64* (5.05)	0.03 (0.02)	-0.01** (0.00)	0.26	1.42	0.37 [0.99]	-0.65 (0.49)	-8.38* (5.00)	1.04*** (0.34)	0.02 (0.02)	-0.01** (0.00)	0.07*** (0.02)	0.57	1.05	0.34 [0.99]	
<b>Panel B: Including Pricing Factors as Test Portfolios</b>																	
27	2 Pricing Factors (DOL + VOL)							3 Pricing Factors (DOL + VOL + GAP <sub>CS</sub> )									
	Loadings ( <i>b</i> )		Risk Prices ( $\lambda$ )		Model Fit			Loadings ( <i>b</i> )			Risk Prices ( $\lambda$ )			Model Fit			
	DOL	VOL	DOL	VOL	Adj.R <sup>2</sup>	DOL	HJ <sub>dist</sub>	DOL	VOL	GAP <sub>CS</sub>	DOL	VOL	GAP <sub>CS</sub>	Adj.R <sup>2</sup>	RMSE	HJ <sub>dist</sub>	
10 TPs (val, mom)	-0.09 (0.55)	-4.10 (5.74)	0.02 (0.02)	-0.01 (0.00)	0.22	1.25	0.30 [0.57]	-0.35 (0.54)	-5.03 (5.86)	0.95*** (0.31)	0.02 (0.02)	-0.01 (0.00)	0.06*** (0.02)	0.74	0.89	0.25 [0.73]	
20 TPs (gap, car, val, mom)	-0.57 (0.48)	-9.70* (4.94)	0.03 (0.02)	-0.01** (0.00)	0.39	1.39	0.38 [0.99]	-0.63 (0.48)	-8.45* (4.88)	0.88*** (0.32)	0.02 (0.02)	-0.01** (0.00)	0.06*** (0.02)	0.67	1.02	0.43 [0.99]	

**Table A13: Asset Pricing using DOL and GAP<sub>CS</sub> as Pricing Factors**

The table presents cross-sectional asset pricing results for two sets of test portfolios. The SDF is constructed as a linear combination of *DOL* and *GAP<sub>CS</sub>*. In Panel B, we include *GAP<sub>CS</sub>* as a test asset. We report Generalized Method of Moments (GMM) one-step estimates of factor loadings on the pricing kernel (*b*'s) and prices of factor risk ( $\lambda$ 's). The superscripts \*, \*\*, \*\*\* represent significance of the coefficients at the 10%, 5%, and 1% significance levels using Newey and West (1987) corrected standard errors. In addition, we report goodness-of-fit statistics for each model including the  $R^2$  statistic and the Hansen-Jagannathan distance statistic (*HJ*) with simulated *p*-values in brackets. The *HJ* statistic measures the distance between the estimated pricing kernel and the efficient set of permissible pricing kernels. A *p*-value less than 5% indicates the null hypothesis that the pricing kernel is efficient can be rejected at the 95% confidence level. The sample runs from December 1999 to January 2016.

<b>Panel A: Excluding Pricing Factor as Test Portfolio</b>							
<b>2 Pricing Factors (DOL + GAP<sub>CS</sub>)</b>							
	<b>Loadings (<i>b</i>)</b>		<b>Risk Prices (<math>\lambda</math>)</b>		<b>Model Fit</b>		
	DOL	GAP <sub>CS</sub>	DOL	GAP <sub>CS</sub>	Adj.R <sup>2</sup>	RMSE	HJ <sub>dist</sub>
10 TPs (val, mom)	-0.15 (0.33)	1.96** (0.87)	0.02 (0.02)	0.11** (0.05)	0.35	0.91	0.17 [0.95]
20 TPs (gap, car, val, mom)	0.02 (0.26)	1.06*** (0.29)	0.02 (0.02)	0.06*** (0.02)	0.31	1.31	0.33 [0.99]
<b>Panel B: Including Pricing Factor as Test Portfolio</b>							
<b>2 Pricing Factors (DOL + GAP<sub>CS</sub>)</b>							
	<b>Loadings (<i>b</i>)</b>		<b>Risk Prices (<math>\lambda</math>)</b>		<b>Model Fit</b>		
	DOL	GAP <sub>CS</sub>	DOL	GAP <sub>CS</sub>	Adj.R <sup>2</sup>	RMSE	HJ <sub>dist</sub>
10 TPs (val, mom)	0.03 (0.26)	0.94*** (0.29)	0.02 (0.02)	0.05*** (0.02)	0.46	1.00	0.19 [0.89]
20 TPs (gap, car, val, mom)	0.04 (0.26)	0.93*** (0.28)	0.02 (0.02)	0.05*** (0.02)	0.40	1.29	0.42 [0.99]