Online Appendix A. Derivations and proofs

A.1. Proof of Lemma 1

Suppose a threshold $x^* \in \mathbb{R}$ exists such that each agent invests if and only if $x \leq x^*$. The measure of agents who invest is thus

$$A(\theta) = \Pr\left(x \leq x^* \middle| \theta\right) = \begin{cases} 0 & \text{if } \theta > x^* + \delta \\ \frac{x^* - (\theta - \delta)}{2\delta} & \text{if } x^* - \delta \leq \theta \leq x^* + \delta \\ 1 & \text{if } \theta < x^* - \delta. \end{cases} \tag{A.1}$$

It follows that the investment succeeds if and only if $\theta \leq \theta^*$, where θ^* solves

$$A(\theta^*) + m = \theta^*. \tag{A.2}$$

By standard Bayesian updating, the posterior distribution about θ conditional on the private signal is also a uniform distribution with bandwidth 2δ. Therefore, the posterior probability of investment success is

$$\Pr\left(s = S \middle| x\right) = \Pr\left(\theta \leq \theta^* \middle| x\right) = \begin{cases} 0 & \text{if } x > \theta^* + \delta \\ \frac{\theta^* - (x - \delta)}{2\delta} & \text{if } \theta^* - \delta \leq x \leq \theta^* + \delta \\ 1 & \text{if } x < \theta^* - \delta. \end{cases} \tag{A.3}$$

For the marginal investor who is indifferent between investing or not, his signal x^* satisfies

$$\Pr\left(s = S \middle| x^*\right) = c. \tag{A.4}$$

Jointly solving Eqs. (A.2) and (A.4), we obtain the two thresholds:

$$\begin{cases} \theta^* = 1 + m - c \\ x^* = 1 - c + \delta - 2c\delta + m. \end{cases} \tag{A.5}$$

A.2. Proof of Lemma 2

Proof. "if" ⇐

If $m_2 > m_1 - c$, and if all agents know other agents will adopt a threshold strategy $x_2^* = \infty$,

$$A_2 + m_2 = 1 + m_2 > 1 + m_1 - c = \theta_1^* > \theta. \tag{A.6}$$

Therefore, the investment succeeds with probability 1. Therefore, it is individually rational for each agent to set $x_2^* = \infty$.

Proof. "only if" ⇒

We prove by contradiction. Suppose an equilibrium exists in which all agents adopt a threshold $x_2^* = 1 + m_1 - c + \delta$ when $(m_1 - c) - m_2 = \Delta > 0$. Therefore, any agent with a signal $x_2 < \theta_1^* + \delta$ will invest. In other words,

$$\Pr\left(\theta < 1 + m_2 \middle| x_2, \theta < \theta_1^*\right) \geq c \tag{A.7}$$

holds for any x_2.

Consider an agent who observes $\hat{x}_2 = m_1 + 1 - c + \delta - \frac{\Delta}{2}$. Such an agent exists when

$\theta \in \left(m_1 + 1 - c + \delta - \frac{\Delta}{2}, m_1 + 1 - c + \delta\right)$. Apparently,

$$\Pr\left(\theta < 1 + m_2 \middle| x_2 = \hat{x}_2, \theta < \theta_1^*\right) \geq c = 0 < c, \tag{A.8}$$

which violates the assumption that all agents invest irrespective of their signals.

□

A.3. Proof of Lemmas 3 and 4, Propositions 1, 2, and 12

Here we solve the equilibrium in period 2 under both $s_1 = S$ and $s_1 = F$ and under all parameter values. The solutions directly prove the lemmas and propositions.

Our solutions take two steps. First, we assume a solution pair (θ_2^*, x_2^*) exists and derive the equilibrium values. Second, we check the conditions these solutions must satisfy and, thus, derive the parameter ranges such that they constitute a solution.

Case 1. The period 1 fund survives: $s_1 = S$.

(1) If $\theta_1^* - 2\delta < \theta_2^* < \theta_1^*$, then, in equilibrium,

$$\theta_2^* - m_2 = A\left(\theta_2^*\right) = \begin{cases} 1 & \text{if } x_2^* - \theta_2^* > \delta \\ \frac{x_2^* - \left(\theta_2^* - \delta\right)}{2\delta} & \text{if -}\delta\text{¡}x_2^* - \theta_2^* < \delta \\ 0 & \text{if } x_2^* - \theta_2^* < -\delta \end{cases} \tag{A.9}$$

and

$$c=\begin{cases}1 & \text{if } x_2^* < \theta_2^* - \delta < \theta_2^* < \theta_1^* \\ \frac{\theta_2^* - \left(x_2^* - \delta\right)}{2\delta} & \text{if } \theta_2^* - \delta < x_2^* < \theta_1^* - \delta \\ \frac{\theta_2^* - \left(x_2^* - \delta\right)}{\theta_1^* - \left(x_2^* - \delta\right)} & \text{if } \theta_1^* - \delta < x_2^* < \theta_2^* + \delta \\ 0 & \text{if } \theta_2^* + \delta < x_2^*. \end{cases} \tag{A.10}$$

Jointly solving the above equations, the solutions are as follows.

(a) $\theta_2^* = 1 + m_2 - c$ and $x_2^* = 1 + m_2 - c + \delta\,(1 - 2c)$. The solution exists if $m_1 - 2\delta < m_2 < m_1 - 2\delta\,(1 - c)$.

(b) $\theta_2^* = 1 + m_2 - c + \frac{c[m_2 - m_1 + 2\delta(1-c)]}{2\delta - c(1+2\delta)}$ and $x_2^* = 1 + m_2 - c + \delta\,(1 - 2c) + \frac{c(1+2\delta)[m_2 - m_1 + 2\delta(1-c)]}{2\delta - c(1+2\delta)}$. The solution exists in two cases: (1) $m_1 - 2\delta\,(1-c) < m_2 < m_1 - c$ if $0 < c < \frac{2\delta}{1+2\delta}$ and (2) $m_1 - c < m_2 < m_1 - 2\delta\,(1-c)$ if $\frac{2\delta}{1+2\delta} < c < 1$

(2) If $\theta_2^* < \theta_1^* - 2\delta$, then, in equilibrium,

$$\theta_2^* - m_2 = A\left(\theta_2^*\right) = \begin{cases} 1 & \text{if } x_2^* - \theta_2^* > \delta \\ \frac{x_2^* - \left(\theta_2^* - \delta\right)}{2\delta} & \text{if -}\delta\text{¡}x_2^* - \theta_2^* < \delta \\ 0 & \text{if } x_2^* - \theta_2^* < -\delta \end{cases} \tag{A.11}$$

and

$$c = \begin{cases} 1 & \text{if } x_2^* < \theta_2^* - \delta \\ \frac{\theta_2^* - (x_2^* - \delta)}{2\delta} & \text{if } \theta_2^* - \delta < x_2^* < \theta_2^* + \delta \\ 0 & \text{if } \theta_2^* + \delta < x_2^*. \end{cases} \tag{A.12}$$

Jointly solving the above equations, the solutions are $\theta_2^* = 1 + m_2 - c$ and $x_2^* = 1 + m_2 - c + \delta(1 - 2c)$. The solution exists if $m_2 < m_1 - 2\delta$.

Combining the above results, we prove Lemmas 3 and 4, Proposition 1, and half of Proposition 12. Case 2 finishes the rest of the proof.

Case 2. The period 1 fund fails: $s_1 = F$.

The analysis is identical.

(a) $\theta_2^* = 1 + m_2 - c - \frac{(1-c)(m_1+2c\delta-m_2)}{c(1+2\delta)-1}$ and $x_2^* = 1 + m_2 - c + \delta\,(1 - 2c) - \frac{(1-c)(1+2\delta)(m_1+2c\delta-m_2)}{c(1+2\delta)-1}$.
The solution exists in two cases: (1) $m_1 + 2c\delta < m_2 < m_1 + (1 - c)$ if $0 < c < \frac{1}{1+2\delta}$ and (2) $m_1 + (1 - c) < m_2 < m_1 + 2c\delta$ if $\frac{1}{1+2\delta} < c < 1$.

(b) $\theta_2^* = 1+m_2-c$ and $x_2^* = 1+m_2-c+\delta\,(1 - 2c)$. The solution exists if $m_1+2c\delta < m_2 < m_1+2\delta$.

(c) $\theta_2^* = 1 + m_2 - c$ and $x_2^* = 1 + m_2 - c + \delta\,(1 - 2c)$. The solution exists if $m_2 > m_1 + 2\delta$.

Combining this result, Proposition 2 and the other parts of 12 naturally follow.

A.4. Proof of Proposition 5

Plugging in the government's budget constraint, we are able to obtain the aggregate social welfare as a function of m_1. As a by-product, we are also able to calculate the net benefit of initial intervention. Lemma 5 summarizes the results.

Lemma 5. *Aggregate social welfare W and net benefit of initial intervention* $\left.\frac{\partial W}{\partial m_1}\right|_{m_1+m_2=M}$.

(1) If $m_1 > \frac{M+2\delta(1-c)}{2}$,

$$\begin{aligned} W &= W_1 + \Pr(s_1 = S)\, W_{2S}^{nc} + \Pr(s_1 = F)\, W_{2F}^{c} \\ &= \frac{1-c}{2B}\left[2 + 2B - 2c(1+\delta) + M\right] \end{aligned}$$

and

$$\frac{\partial W}{\partial m_1} = 0.$$

This case exists only for $M > 2\delta(1-c)$.

(2) If $\frac{M+c}{2} < m_1 < \frac{M+2\delta(1-c)}{2}$,

$$\begin{aligned} W &= W_1 + \Pr(s_1 = S)\, W_{2S}^{pc} + \Pr(s_1 = F)\, W_{2F}^{c} \\ &= \frac{1-c}{2B}\Big[2 + 2B - c(2+\delta) + 2m_1 \\ &\quad + \frac{\delta c(c+M-2m_1)^2 - 2\delta\left[c - 2(1-c)\delta\right](c+M-2m_1)}{\left[c-2(1-c)\delta\right]^2}\Big] \end{aligned}$$

and

$$\frac{\partial W}{\partial m_1} = \frac{(1-c)\left[2c(1+2\delta)\left[c-2(1-c)\delta\right] - 4c\delta(c+M-2m_1)\right]}{2B\left[c-2(1-c)\delta\right]^2} < 0.$$

This case exists only for $M > c$.

(3) If $\frac{M-(1-c)}{2} < m_1 < \frac{M+c}{2}$,

$$\begin{aligned} W &= W_1 + \Pr(s_1 = S) W_{2S}^{c} + \Pr(s_1 = F) W_{2F}^{c} \\ &= \frac{1-c}{2B}\left[2 + 2B - c(2+\delta) + 2m_1\right] \end{aligned}$$

and

$$\frac{\partial W}{\partial m_1} = \frac{1-c}{B} > 0.$$

This case always exists.

(4) If $\frac{M-2c\delta}{2} < m_1 < \frac{M-(1-c)}{2}$,

$$\begin{aligned} W &= W_1 + \Pr(s_1 = S) W_{2S}^{c} + \Pr(s_1 = F) W_{2F}^{pc} \\ &= \frac{1-c}{2B}\left[2 + 2B - c(2+\delta) + 2m_1 + \frac{c\delta\left(-1+c+M-2m_1\right)^2}{\left(-1+c+2c\delta\right)^2}\right] \end{aligned}$$

and

$$\frac{\partial W}{\partial m_1} = \frac{1-c}{2B}\left[2 - \frac{4c\delta\left(c-2m_1+M-1\right)}{(2c\delta+c-1)^2}\right].$$

This case exists only for $M > 1-c$*. The derivative changes sign from negative to positive exactly once in this region.*

(5) If $m_1 < \frac{M-2c\delta}{2}$,

$$\begin{aligned} W &= W_1 + \Pr(s_1 = S) W_{2S}^{c} + \Pr(s_1 = F) W_{2F}^{nc} \\ &= \frac{1-c}{2B}\left[2 + 2B - 2c(1+\delta) + M\right] \end{aligned}$$

and

$$\frac{\partial W}{\partial m_1} = 0.$$

This case exists only for $M > 2c\delta$.

Given that the welfare function is continuous, the maximum welfare in case 3 is higher than case 1 and case 5, and how welfare varies with respect to m_1 in region 2 and 4, the result in the Proposition 6 follows.

A.5. *Proof of Proposition 6*

Proof. Suppose the optimal $m_1 < m_2$. We show this leads to a contradiction. Welfare $W_1 + E[W_2] - K(m_1, m_2)$ is

$$L = -K(m_1, m_2) + \frac{1-c}{2B}\begin{cases} 2(m_1 + 1 - c + B) - c\delta & \text{if } m_2 < m_1 + (1-c) \\ 2(m_1 + 1 - c + B) - c\delta + \frac{c\delta(-1+c-m_1+m_2)^2}{(-1+c+2c\delta)^2} & \text{if } m_1 + (1-c) < m_2 < m_1 + 2c\delta \\ m_1 + m_2 + 2(1 - c + B - c\delta) & \text{if } m_2 > m_1 + 2c\delta. \end{cases} \tag{A.13}$$

We want to show the above is not optimal, because it is strictly dominated by the welfare at $\left(m_1^{'}, m_2^{'}\right) = \left(\frac{1}{2}[m_1 + m_2], \frac{1}{2}[m_1 + m_2 - 2c]\right)$, which equals

$$L^{'} = \frac{1-c}{2B}\left[m_1 + m_2 + 2(1 - c + B) - c\delta\right] - K\left(\frac{1}{2}(m_1 + m_2), \frac{1}{2}[m_1 + m_2 - 2c]\right). \tag{A.14}$$

The cases in which $m_2 < m_1 + (1-c)$ and $m_2 > m_1 + 2c\delta$ are straightforward. It remains to be shown that $W^{'} > W$ when $m_1 + (1-c) < m_2 < m_1 + 2c\delta$. Note that

$$\begin{aligned} sgn\left(L^{'} - L\right) &= sgn\left[(m_2 - m_1) - \frac{c\delta\left(-1 + c - m_1 + m_2\right)^2}{\left(-1 + c + 2c\delta\right)^2}\right] \\ &= sgn\left\{-c\delta\left[(-1+c)^2 + (m_2 - m_1)^2 + 2\left(-1 + c\right)(m_2 - m_1)\right] + (m_2 - m_1)\left(-1 + c + 2c\delta\right)^2\right\} \\ &= sgn\left\{-c\delta\left(m_2 - m_1\right)^2 + \left[2c\delta\left(1 - c\right) + \left(-1 + c + 2c\delta\right)^2\right](m_2 - m_1) - c\delta\left(-1 + c\right)^2\right\}. \end{aligned}$$

It suffices to show $sgn\left\{-c\delta\left(m_2-m_1\right)^2+\left[2c\delta\left(1-c\right)+\left(-1+c+2c\delta\right)^2\right]\left(m_2-m_1\right)-c\delta\left(-1+c\right)^2\right\}>0$ at both $m_2=m_1+2c\delta$ and $m_2=m_1+(1-c)$, which is straightforward algebra.

When the cost is separable and symmetric,

$$\begin{aligned} & K(m_1,m_2)-K\left(\frac{1}{2}[m_1+m_2],\frac{1}{2}[m_1+m_2-2c]\right) \\ = & \left[K(m_2)-K\left(\frac{1}{2}[m_1+m_2]\right)\right]-\left[K\left(\frac{1}{2}[m_1+m_2-2c]\right)-K(m_1)\right]>0 \end{aligned} \tag{A.15}$$

due to K being weakly convex and increasing. The above also holds when $\frac{1}{2}[m_1+m_2-2c]$ is replaced by $\frac{1}{2}(m_1+m_2)$ and K depends only on m_1+m_2 and $|m_1-m_2|$ and is increasing in $|m_1-m_2|$. Therefore, initial intervention is strictly emphasized under those conditions. □

A.6. Proof of Proposition 7 and conditions for under- and over-interventions

Proof. Write the explicit expression for $Y(m_1;\chi)$:

$$\begin{aligned} Y(m_1;\chi) = & W_1-K(m_1,0)+\chi\max_{m_2}\left[\frac{B+m_1+1-c}{2B}[W_{2S}-(K(m_1,m_2)-K(m_1,0))]\right. \\ & \left.+\frac{B-m_1-1+c}{2B}[W_{2F}-(K(m_1,m_2)-K(m_1,0))]\right]. \end{aligned} \tag{A.16}$$

The following identity holds:

$$\begin{aligned} \frac{\partial}{\partial\chi}Y(m_1;\chi) = & \max_{m_2}\left[\mathbb{E}[W_2(m_1,m_2)-(K(m_1,m_2)-K(m_1,0))]\right] \\ = & \left[\mathbb{I}_{\{m_2^*>m_1+1-c\}}+\mathbb{I}_{\{m_1>c\}}\mathbb{I}_{\{0<m_2^*<m_1-c\}}\right]\left(\mathbb{E}[W_2(m_1,m_2^*)]-(K(m_1,m_2^*)-K(m_1,0))\right) \\ & +\mathbb{I}_{\{m_1\geq c\}}\mathbb{I}_{\{m_2^*=m_1-c\}}\left(\mathbb{E}[W_2(m_1,m_1-c)]-(K(m_1,m_1-c)-K(m_1,0))\right) \\ & +\mathbb{I}_{\{m_1\leq c\}}\mathbb{I}_{\{m_2^*=0\}}\mathbb{E}[W_2(m_1,0)]. \end{aligned} \tag{A.17}$$

Fixing m_1, increasing m_2 does not increase welfare $\mathbb{E}[W_2]$ in $[m_1 - c, m_1 + 1 - c]$, thus, the four indicator products sum to one. This is seen in Fig. A1, with the four indicators corresponding to $m_2 > m_1 + 1 - c$, $m_2 < m_1 - c$, $m_2 = m_1 - c$, and $m_1 - c < m_2 \leq m_1 + 1 - c$.

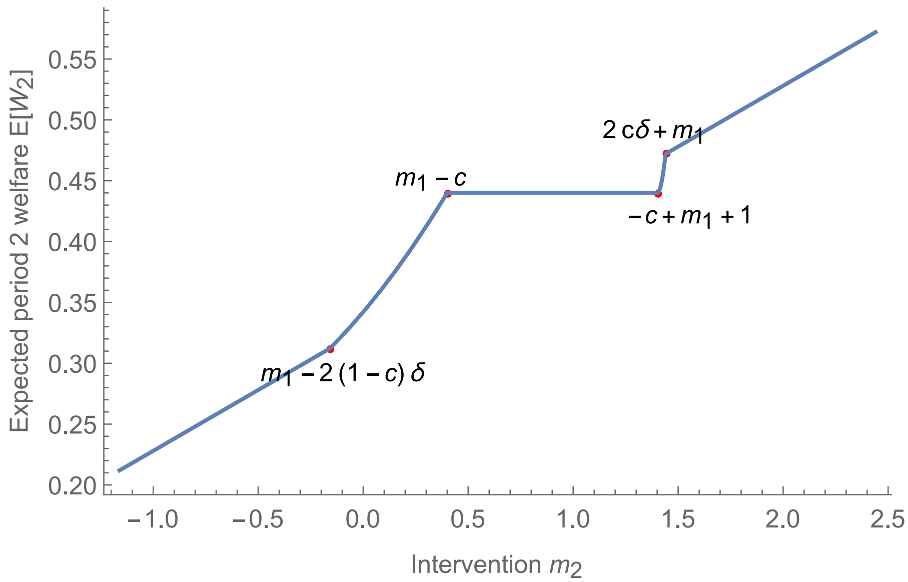

Fig. A1. $E[W_2]$ as a function of m_2. This figure plots investors expected welfare in period 2 $E[W_2]$ as a function of m_2, the size of government intervention in that period. The plot shows that while $E[W_2]$ is in general increasing in m_2, it is flat when $m_2 \in [m_1 - c, 1 + m_1 - c]$. Parameters are $\delta = 0.8,\ c = 0.4,\ B = 3,$ and $m_1 = 0.8$.

The first term is nonincreasing in m_1, and the last two terms are nondecreasing. In general, the overall expression is non-monotone in m_1 because its value could jump up or down when the indicators change values. However, if parameters are such that one indicator function is always one, the expression is monotone in m_1 and we could draw robust comparative statics. If one of the first two indicators is always 1 as we vary m_1, Y has decreasing differences in m_1 and χ; if one of the last two indicators is always one, Y has increasing differences. The conclusions then follow from Theorem 2.1 in Athey, Milgrom, and Roberts (1998). $\square$

Corollaries 2 and 3 in Online Appendix Section A illustrate sufficient conditions on exogenous parameters under which over- or under-interventions take place. These conditions are neither unique nor restrictive. For simplicity in exposition, we assume for the remainder of the discussion that K is twice-differentiable in a continuous feasible range of intervention $\mathcal{I} = [0, \bar{m}_1] \times [0, \bar{m}_2]$. This specification includes cases of budget constraint and separable quadratic intervention costs. Let K_i denote the partial derivative with respect to m_i.

Corollary 2. *A myopic government under-intervenes initially if one of the two following conditions holds.*

(1) $K_1(c, \cdot) > \frac{1-c}{B}$ *and* $K_2(\cdot, 1-c) \geq \frac{1-c}{B}\frac{c\delta}{2c\delta+c-1}$.

(2) For some $b > c$, *it holds that* $K_1(b, \cdot) > \frac{1-c}{B}$, $K_1(c, \cdot) < \frac{1-c}{B}\frac{\delta - c(1+2\delta)}{2\delta - c(1+2\delta)}$, $K_2(\cdot, b-c) \leq \frac{1-c}{2B}$, *and* $K_2(\cdot, 1) \geq \frac{(1-c)c\delta}{B(2c\delta+c-1)}$.

Corollary 3. *A myopic government over-intervenes initially if one of the following conditions holds.*

(1) For some $b \geq 0$, *it holds* $K_1(c, \cdot) > \frac{1-c}{B}$ *and* $K_2(\cdot, b + 2c\delta) < \frac{1-c}{2B}\frac{\delta}{1+2\delta}$.

(2) $K_1(b, \cdot) > \frac{1-c}{B}$, $K_1(c, \cdot) < \frac{1-c}{B}\frac{\delta - c(1+2\delta)}{2\delta - c(1+2\delta)}$, $K_2(\cdot, 0) > \frac{1-c}{2B}\frac{2\delta}{2\delta - c(1+2\delta)}$.

We now prove them but note that other sufficient conditions exist, especially ones on cost parameters defined through levels, not derivatives.

Proof. If $K_1(c, \cdot) > \frac{1-c}{B}$, we have $m_1^* < c$ because the maximum marginal benefit of m_1 on investors' welfare is $\frac{1-c}{B}$, $K_1(c, \cdot) \geq \frac{1-c}{B}$ implies $m_1^* \leq c$. Therefore, $m_2^* > m_1 - c$ for sure and $W_{2S} = (1-c)$. Given $K_2(\cdot, c(1+2\delta)) < \frac{1-c}{2B}\frac{\delta}{1+2\delta}$, we have $K(\cdot, c(1+2\delta)) < \frac{c(1-c)\delta}{2B}$, then $\frac{B-m_1-1+c}{2B}W_{2F} > K(m_1, m_1 + 2\delta c) - K(m_1, 0)$. Thus, the increase in W_{2F} exceeds the intervention cost at $m_2 = m_1 + 2\delta c$, $m_2^* > m_1 + 1 - c$. When this happens, we know $\frac{\partial Y}{\partial \chi}$ equals the first term with the first indicator product being one and is nonincreasing in m_1. Thus, Y has decreasing differences in m_1 and χ.

If $K_1(b,\cdot) > \frac{1-c}{B}$, we have $m_1^* < b$ for b potentially bigger than c. If the cost in the second intervention is sufficiently small such that $m_2^* > b + 2c\delta > 2c\delta + m_1^*$, we also have the first indicator product being one, and Y has decreasing differences in m_1 and χ. One sufficient condition is $\left[W_2(m_1^*, m_1^* + 2c\delta) - W_2(m_1^*, m_1^* - c)\right]/c(1+2\delta) > K_2(b+2c\delta)$, that is, $K_2(b+2c\delta) < \frac{(1-c)\delta}{2B(1+2\delta)}$.

Next, for the last indicator to always be one, $K_1(c,\cdot) > \frac{1-c}{B}$. In addition, $K_2(\cdot, 1-c) \geq \frac{1-c}{B}\frac{c\delta}{2c\delta+c-1}$ is a sufficient condition for $m_2^* = 0$ because this implies the cost exceeds the benefit at both $m_2 = 1-c+m_1$ and $m_2 = m_1 + 2c\delta$, and the convexity of K in m_2 excludes $m_2^* > 1-c+m_1$. Then, $\frac{\partial Y}{\partial \chi}$ equals the last term and is non-decreasing in m_1. We could alternatively use $K(\cdot, 1-c) \geq \frac{c(1-c)\delta}{2B}$ as a sufficient condition on cost, not on the derivative. The same goes for other sufficient conditions that we provide in this proposition.

If $K_1(b,\cdot) > \frac{1-c}{B}$ and $K_1(c,\cdot) < \frac{1-c}{B}\frac{\delta - c(1+2\delta)}{2\delta - c(1+2\delta)} \leq \frac{\partial Y}{\partial m_1}$ for some $b > c$, we have $b > m_1^* > c$. On the one hand, if, in addition, the minimum marginal benefit in region $m_2 \leq m_1^* - c$ is bigger than the maximum marginal cost, that is, $\min\left\{\frac{1-c}{2B}, \frac{1-c}{2B}\frac{2\delta(1-c)}{2\delta-c(1+2\delta)}\right\} \geq \frac{1-c}{2B} \geq K_2(\cdot, b-c) > K_2(\cdot, m_1 - c)$, we have $\mathbb{E}[W_2] - (K(m_1, m_2) - K(m_1, 0))$ increasing in the entire region of $[0, m_1 - c]$. Moreover, if $K_2(m_1^*, m_1^* + 1 - c) > K_2(\cdot, 1) \geq \frac{(1-c)c\delta}{B(2c\delta+c-1)}$, the lower bound on the marginal cost is weakly bigger than the maximum marginal benefit (right-hand side) in the region $m_2 \geq 1 - c + m_1^*$. Therefore, $m_2^* = m_1^* - c$ and the third indicator product is always one. Y has increasing differences in m_1 and χ.

On the other hand, $K_2(\cdot, 0) > \frac{1-c}{2B}\frac{2\delta}{2\delta-c(1+2\delta)}$ implies $\frac{1-c}{2B}\frac{2\delta}{2\delta-c(1+2\delta)} < K_2(m_1, m_1 - c)$, which means the maximum marginal benefit in the region $m_2 \leq m_1^* - c$ taken at equality is less than the marginal cost. Thus, $m_2^* < m_1^* - c$ and the second indicator product is always one. Y has decreasing differences in m_1 and χ.

These sufficient conditions are stated in the corollaries. Instead of directly computing the derivatives for the first term in $\frac{\partial Y}{\partial \chi}$, when m_2^* is interior, we can apply the envelope theorem to compute the partial derivative in m_1 of $\mathbb{E}[W_2(m_1, m_2)] - (K(m_1, m_2) - K(m_1, 0))$. Because $K_{12} = 0$, we know the partial derivative must be negative in the corresponding regions from Fig. 3. □

A.7. Proof of Proposition 8

Proof. Define $n_1 = m_1$ and $n_2 = \frac{m_2}{\lambda}$. The two inequalities on funds' survival translate into

$$A_1 + n_1 \geq \theta \tag{A.18}$$

and

$$A_2 + n_2 \geq \theta. \tag{A.19}$$

For the remaining analysis, we show the optimal intervention plan by a benevolent government that faces a hard budget constraint.

The proof is conducted in two steps. In the first step, we assume q, the probability of fund 1's outcome being realized first, is equal to one or zero and study the optimal intervention plan (n_1, n_2) when λ varies. The budget constraint shows as $n_1 + n_2\lambda = M$. We show that, for both $\lambda > 1$ and $\lambda \in (0, 1)$, the government intervenes up to $n_2 = n_1 - c$. The case in which $\lambda \in (0, 1)$ is simply isomorphic to $q = 0$. Thus, we establish the result that the government always induces perfectly correlated intervention outcomes. In the second step, we solve the optimal intervention with general q.

Lemma 6. *Suppose $q = 1$, $\forall \lambda > 0$. The optimal intervention plan is*

$$n_1^* = \frac{M + c\lambda}{1 + \lambda}$$

and

$$n_2^* = \frac{M - c}{1 + \lambda}.$$

Under (n_1^*, n_2^*), intervention always leads to correlated outcomes: $s_1 = s_2$.

Proof. With heterogeneous fund sizes, the aggregate social welfare naturally follows.

(1) If $n_1 > \frac{M+2\delta\lambda(1-c)}{1+\lambda}$,

$$W = \frac{1-c}{2B} \left\{ \left[1 + B - c(1+\delta) \right] (1+\lambda) + M \right\}$$

and

$$\frac{\partial W}{\partial n_1} = 0.$$

(2) If $\frac{M+c\lambda}{1+\lambda} < n_1 < \frac{M+2\delta\lambda(1-c)}{1+\lambda}$,

$$\begin{aligned} W &= \frac{1-c}{2B} \left[1 + B - c(1+\delta) + n_1 \right] \\ &\quad + \lambda \frac{1-c}{2B} \left[1 + n_1 - c + B + \frac{\delta c (c - n_1 + n_2)^2 + 2\delta (c - n_1 + n_2) \left[2\delta - c(1+2\delta) \right]}{\left[2\delta - c(1+2\delta) \right]^2} \right] \end{aligned}$$

and

$$\frac{\partial W}{\partial n_1} < 0.$$

(3) If $\frac{M-(1-c)\lambda}{1+\lambda} < n_1 < \frac{M+c\lambda}{1+\lambda}$,

$$W = \frac{1-c}{2B}\left[\left(1+B-c+n_1\right)\left(1+\lambda\right)-c\delta\right]$$

and

$$\frac{\partial W}{\partial n_1} > 0.$$

(4) If $\frac{M-2c\delta\lambda}{1+\lambda} < n_1 < \frac{M-(1-c)\lambda}{1+\lambda}$,

$$\begin{aligned} W &= \frac{1-c}{2B}\left[1+B-c\left(1+\delta\right)+n_1\right]+\lambda\frac{1-c}{2B}\left[1+B-c+n_1\right] \\ &\quad +\lambda\frac{1-c}{2B}\frac{c\delta\left(-1+c-n_1+n_2\right)^2}{\left(-1+c+2c\delta\right)^2} \end{aligned}$$

and

$$\frac{\partial W}{\partial n_1} \quad \text{changes from negative to positive exactly once.}$$

(5) If $n_1 < \frac{M-2c\delta\lambda}{1+\lambda}$,

$$W = \frac{1-c}{2B}\left\{\left[1+B-c\left(1+\delta\right)\right]\left(1+\lambda\right)+M\right\}$$

and

$$\frac{\partial W}{\partial n_1} = 0.$$

It easily establishes that $\frac{\partial W}{\partial n_1} = 0$ in Case 1 and Case 5, $\frac{\partial W}{\partial n_1} > 0$ in Case 3, and $\frac{\partial W}{\partial n_1} < 0$ in Case 2. Similar to the proof of Proposition 5, the aggregate welfare in Case 1 equals that in Case 5. The maximal welfare is attained at the right boundary of Case 3, that is, when $n_1 = \frac{M+c\lambda}{1+\lambda}$ □

Next, we can compare the social welfare under general q.

When n_1 and n_2 are such that interventions are correlated, the overall welfare to be maximized is

$$q\frac{1-c}{2B}\left[(1+B-c+n_1)(1+\lambda)-c\delta\right]+(1-q)\frac{1-c}{2B}\left[(1+B-c+n_2)(1+\lambda)-c\lambda\delta\right]. \quad \text{(A.20)}$$

Essentially, the government maximizes $qn_1+(1-q)\,n_2$ subject to the following constraints

$$n_1 \geq n_2-c, \quad \text{(A.21)}$$

$$n_2 \geq n_1-c, \quad \text{(A.22)}$$

$$n_1+n_2\lambda = M. \quad \text{(A.23)}$$

The solution is

$$\begin{cases} n_1^*=\frac{M+c\lambda}{1+\lambda},\ n_2^*=\frac{M-c}{1+\lambda} & \text{if } q>\frac{1}{\lambda+1} \\ n_1^*=\frac{M-c\lambda}{1+\lambda},\ n_2^*=\frac{M+c}{1+\lambda} & \text{if } q<\frac{1}{\lambda+1}. \end{cases} \quad \text{(A.24)}$$

Therefore, the government puts more resources into the small fund if q is very high and vice versa if q is very low. Because $\lambda>1$, the government puts more resources into the small fund when $q=\frac{1}{2}$. □

Online Appendix B. Full analysis of sub-section 3.2.2

Does any equilibrium exist that agents choose to run irrespective of their signals? In other words, the threshold x_2^* that agents in period 2 adopt satisfies $x_2^* \leq \theta_1^* - \delta$. Such an equilibrium exists if and only if $m_2 < m_1 + 1 - c$. In this type of equilibrium, government intervention in the first period has a dominant effect on coordination among investors in the second period. Therefore, we name it the *Stage Game Equilibrium with Dynamic Coordination.*

Lemma 7 describes this type of equilibrium. Because it is common knowledge that $\theta > \theta_1^*$, any equilibrium with $(\theta_2^* < \theta_1^*, x_2^* < \theta_1^* - \delta)$ is equivalent to $(\theta_2^*, x_2^*) = (-\infty, -\infty)$.

Lemma 7 (Stage Game Equilibrium with Dynamic Coordination). . *If* $s_1 = F$, $(\theta_2^*, x_2^*) = (-\infty, -\infty)$ *constitutes an equilibrium if and only if* $m_2 < m_1 + 1 - c$.

Next, we turn to threshold equilibria with $\theta_2^* > \theta_1^*$ so that the fate of the fund in period 2 still has uncertainty. Similar to the analysis when $s_1 = S$, we consider two types of equilibria, depending on whether the marginal investor finds the public news useful.

Lemma 8 (Stage Game Equilibrium without Dynamic Coordination). . *If* $s_1 = F$ *and* $m_2 > m_1 + 2c\delta$, *an equilibrium exists with thresholds*

$$\begin{cases} \theta_2^* = 1 + m_2 - c \\ x_2^* = 1 + m_2 - c + \delta\left(1 - 2c\right). \end{cases} \tag{B.1}$$

Lemma 9 (Stage Game Equilibrium with Partial Dynamic Coordination). . *If* $s_1 = F$ *and* $\min\{m_1 + 2c\delta, m_1 + 1 - c\} < m_2 < \max\{m_1 + 2c\delta, m_1 + 1 - c\}$, *there exists an equilibrium with* *thresholds*

$$\begin{cases} \theta_2^* = 1 + m_2 - c - \frac{(1-c)(m_1+2c\delta-m_2)}{c(1+2\delta)-1} \\ x_2^* = 1 + m_2 - c + \delta\,(1-2c) - \frac{(1-c)(1+2\delta)(m_1+2c\delta-m_2)}{c(1+2\delta)-1}. \end{cases} \tag{B.2}$$

Given any (m_1, m_2) and $s_1 = F$, Proposition 2 clearly follows Lemmas 7, 8, and 9.

Online Appendix C. Contingent interventions

Proposition 9 (emphasis on initial intervention; contingent case)**.** *When* $K_2(0, 1-c) > \frac{(1-c)^2}{B-1}$*, then contingent interventions strictly emphasizes initial intervention:* $m_1^* > m_{2S_1}^*$.

Proof. If $s_1 = S$, increasing m_{2S} beyond $m_1^* - c$ incurs additional cost without increasing $\mathbb{E}[W_2]$, as is clear in Fig. 1. Thus, $m_{2S}^* \leq m_1^* - c$. When $s_1 = F$, the condition on the parameter means the marginal cost of increasing m_{2F} at $1-c$ exceeds the marginal benefit, which is bounded above by $\frac{(1-c)^2}{B-1}$. Thus, $m_{2F}^* < 1-c$. Subsequently, $m_{2F}^* = 0$ because increasing m_{2F} does not increase $\mathbb{E}[W_2]$, also clearly seen in Fig. 1. □

This is just one example of the sufficient conditions under which the endogenous correlation effect dominates the conditional inference effect. This result applies to situations in which the initial intervention is more costly than the subsequent intervention.

Proposition 10 (myopic intervention; contingent case)**.** *The government's initial intervention is weakly increasing in the extent it considers dynamic coordination, i.e.,* $\frac{\partial m_1^*}{\partial \chi} \geq 0$ *if one of the two following conditions holds.*

(1) $K(\cdot, 1-c) - K(\cdot, 0) > \min\{\frac{(1-c)^2}{2c\delta-1+c}, \frac{c\delta(1-c)}{B-(1-c)}\}$ *and* $K_1(c, \cdot) \geq \frac{1-c}{B}$.

(2) $K(\cdot, 1-c) - K(\cdot, 0) > \min\{\frac{(1-c)^2}{2c\delta-1+c}, \frac{c\delta(1-c)}{B-(1-c)}\}$ *and* $1 - c - K(\cdot, 1-c) + K(\cdot, 0) - (2 + B - c)K_2(\cdot, 1-c) > 0$.

The government's contingent intervention is weakly decreasing in the extent it considers dynamic coordination, i.e., $\frac{\partial m_1^*}{\partial \chi} \leq 0$*, when* $K_2(\cdot, 0) > \frac{1-c}{B+c-2}\frac{2\delta}{2\delta-c(1+2\delta)}$ *and* $K_1(c, \cdot) < \frac{1-c}{B}\frac{\delta-c(1+2\delta)}{2\delta-c(1+2\delta)}$.

In general, the government chooses $\{m_1, m_{2S}, m_{2F}\}$ to maximize welfare. For a given m_1, define the objective as

$$Y(m_1;\chi) = W_1 - K(m_1,0) + \chi\left[\frac{B+m_1+1-c}{2B}\max_{m_{2S}}[W_{2S} - (K(m_1,m_{2S}) - K(m_1,0))] + \frac{B-m_1-1+c}{2B}\max_{m_{2F}}[W_{2F} - (K(m_1,m_{2F}) - K(m_1,0))]\right]. \quad \text{(C.1)}$$

Here, $\chi \in [0,1]$ measures how much the government cares about the fate of the second period's fund.

Proof. The proof is similar to that in Proposition 7 and Corollaries 2 and 3, albeit algebraically more involved. We start with the first half of the proposition. Because the maximum marginal benefit of m_1 on the investors' total welfare is $\frac{1-c}{B}$, $K_1(c,\cdot) \geq \frac{1-c}{B}$ implies $m_1^* \leq c$. Fig. 1 implies that when $m_{2S} > m_1 - c$, welfare is weakly decreasing in m_2. Thus, $m_{2S}^* = 0$.

$$\begin{aligned}
&\frac{\partial}{\partial m_1}\frac{\partial}{\partial \chi}Y(m_1;\chi) \\
= \ & \frac{d}{dm_1}\left[\frac{B+m_1+1-c}{2B}\max_{\{m_{2S}\}}[W_{2S} - (K(m_1,m_{2S}) - K(m_1,0))]\right. \\
&\left.+\frac{B-m_1-1+c}{2B}\max_{\{m_{2F}\}}[W_{2F} - (K(m_1,m_{2F}) - K(m_1,0))]\right] \\
= \ & \frac{1-c}{2B} + \frac{\partial}{\partial m_1}\left[\frac{B-m_1-1+c}{2B}\max_{\{m_{2F}\}}[W_{2F} - (K(m_1,m_{2F}) - K(m_1,0))]\mathbb{I}_{\{m_{2F}>m_1+1-c\}}\right] \\
= \ & \frac{1-c}{2B} + \frac{1}{2B}[K(m_1,m_{2F}) - K(m_1,0)]\mathbb{I}_{\{m_{2F}^*>m_1+1-c\}} - \frac{1-c}{2B}\mathbb{I}_{\{m_{2F}^*>m_1+2c\delta\}} \\
&-\frac{1-c}{2B}\frac{2c\delta(-1+c-m_1+m_{2F}^*)}{(-1+c+2c\delta)^2}\mathbb{I}_{\{m_{2F}^*\in(m_1+1-c,m_1+2c\delta]\}} \\
\geq \ & 0 \qquad \text{(C.2)}
\end{aligned}$$

The second equality holds by the envelope theorem and by the fact that if $m_{2F} \leq m_1 + 1 - c$, taking $m_{2F} = 0$ dominates, as seen in Fig. 1. When $K(\cdot,1-c) - K(\cdot,0) > \frac{c\delta(1-c)}{B-(1-c)}$, $W_{2F}(m_2 = m_1 + 2\delta c) < K(m_1, M_1 + 1 - c)$, and thus $m_{2F}^* = 0$. When $K(\cdot,1-c) - K(\cdot,0) > \frac{(1-c)^2}{2c\delta-1+c}$, the last two terms on the RHS of the third equality are dominated by the first two terms as $m_{2F}^* = 0$. In either case, we have the whole expression being non-negative.

From $1-c-K(\cdot,1-c)+K(\cdot,0)-(2+B-c)K_2(\cdot,1-c) > 0$, we have $K_2(\cdot,1-c) < \frac{1-c}{B+2-c}\frac{2\delta}{2\delta-(1+2\delta)c}$. The marginal benefit for increasing m_{2S} in W_{2S} exceeds the cost as long as $m_{2S} < m_1 - c$. Therefore, $m_{2S}^* = [m_1 - c]^+$. When $m_1 \leq c$, $m_{2S}^* = 0$, the local derivative is the same as above and, thus, is positive. When $m_1 \geq c$, $m_{2S}^* = m_1 - c$, $m_{2F}^* = 0$, the local derivative is

$$\begin{aligned}
& \frac{\partial}{\partial m_1}\frac{\partial}{\partial \chi}Y(m_1;\chi) \\
= \; & \frac{1-c}{2B} - \frac{1}{2B}[K(m_1,m_{2S}) - K(m_1,0)] - \frac{m_1+B+1-c}{2B}\frac{\partial}{\partial m_1}[K(m_1,m_1-c) - K(m_1,0)] \\
= \; & \frac{1-c}{2B} - \frac{1}{2B}[K(m_1,m_{2S}) - K(m_1,0)] - \frac{m_1+B+1-c}{2B}[K_2(m_1,m_1-c) \\
& +K_1(m_1,m_1-c) - K_1(m_1,0)], \quad \textit{note } K_1(m_1,m_1-c) = K_1(m_1,0) \\
= \; & \frac{1-c}{2B} - \frac{1}{2B}[K(m_1,m_{2S}) - K(m_1,0)] - \frac{m_1+B+1-c}{2B}[K_2(m_1,m_1-c)] \\
\geq \; & \frac{1}{2B}[1-c-K(m_1,1-c)+K(m_1,0)-(2+B-c)K_2(m_1,1-c)] \geq 0. \qquad \text{(C.3)}
\end{aligned}$$

The last two inequalities come from the fact that $m_1 \leq 1$ and that $1-c-K(\cdot,1-c)+K(\cdot,0)-(2+B-c)K_2(\cdot,1-c) > 0$. Therefore, we have that Y has increasing differences in (m_1,χ).

To prove the second half of the theorem, note that $K_2(\cdot,0) > \frac{1-c}{B+c-2}\frac{2\delta}{2\delta-c(1+2\delta)}$ and, thus, $K(\cdot,1-c) > \frac{1-c}{B+c-2}\frac{1-c}{1-c(1+\frac{1}{2\delta})} > \frac{1-c}{B-2+c}$ and W_{2F} at $m_2 = m_1+2c\delta$ is still less than $K(m_1,m_1+1-c)-K(m_1,0)$. Consequently, $m_{2F}^* = 0$. $K_2(\cdot,0) > \frac{1-c}{B+c-2}\frac{2\delta}{2\delta-c(1+2\delta)}$ also implies $\frac{1-c}{B+m_1+1-c}\frac{2\delta}{2\delta-c(1+2\delta)} <$

$K_2(m_1, m_1 - c)$, which means $m_{2S}^* < m_1 - c$.

$$\begin{aligned}
& \frac{\partial}{\partial m_1}\frac{\partial}{\partial \chi}Y(m_1;\chi) \\
= & \frac{d}{dm_1}\left[\frac{B+m_1+1-c}{2B}\max_{\{m_{2S}\}}[W_{2S}-(K(m_1,m_{2S})-K(m_1,0))]\right. \\
& \left.+\frac{B-m_1-1+c}{2B}\max_{\{m_{2F}\}}[W_{2F}-(K(m_1,m_{2F})-K(m_1,0))]\right] \\
= & \frac{\partial}{\partial m_1}\left[\frac{B-m_1-1+c}{2B}\max_{\{m_{2F}\}}[W_{2S}-(K(m_1,m_{2S})-K(m_1,0))]\right] \\
= & \frac{\partial}{\partial m_1}W_{2S}(m_{2S}^*)+\frac{B-m_1-1+c}{2B}\frac{\partial}{\partial m_1}[K(m_1,m_{2S}^*)-K(m_1,0)] \\
& -\frac{1}{2B}(K(m_1,m_{2S})-K(m_1,0))<0 \qquad \text{(C.4)}
\end{aligned}$$

The first term is negative because $m_{2S}^* < m_1 - c$. The second term is non-positive because K is weakly increasing in the second argument. Finally, the third term is zero because K has zero cross-partials.

Finally, the above argument would not work if $m_1^* \leq c$. But this scenario can be ruled out in that the minimum $\frac{\partial Y}{\partial m_1} = \frac{1-c}{2B}\left[1 - \frac{c(1+2\delta)}{2\delta - c(1+2\delta)}\right] = \frac{1-c}{B}\frac{\delta - c(1+2\delta)}{2\delta - c(1+2\delta)}$. We have used the fact that $m_{2F}^* = 0$. This is bigger than the marginal cost $K_1(c, \cdot)$ and, thus, $m_1^* > c$. We have an interior m_{2S}^*. □

These sufficient conditions for under- and over-interventions simply correspond to Corollaries 2 and 3.

Proposition 11 (too big to save first; contingent case)**.** *If interventions always lead to perfectly correlated outcomes ($s_1 = s_2$), it is socially efficient to save the small fund first when the government can determine the order of the realization of outcomes.*

Proof. We prove the case under separable cost functions to focus on the information channel: $K(m_1, m_2) = k(m_1) + k(m_2)$. Let L (L') be the total welfare net the intervention cost if the smaller (larger) fund is saved first.

Consider when $m_1^* \leq c$ and $m_1^{'*} \leq c$. In this case,

$$L = \max_{m_1} \frac{1-c}{2B}\left[\left(1+B-c+m_1\right)(1+\lambda)-c\delta\right]-k\left(m_1\right) \tag{C.5}$$

and

$$L^{'} = \max_{m_1^{'}} \frac{1-c}{2B}\left[\left(1+B-c+\frac{m_1^{'}}{\lambda}\right)(1+\lambda)-c\lambda\delta\right]-k\left(m_1^{'}\right). \tag{C.6}$$

Because $\lambda > 1$, obviously $L > L^{'}$.

Next, consider the case when $m_1^* > c$ and $m_1^{'*} > c$:

$$L = \max_{m_1} \frac{1-c}{2B}\left[\left(1+B-c+m_1\right)(1+\lambda)-c\delta\right]-k\left(m_1\right)-\frac{B+m_1+1-c}{2B}k\left(\left(m_1-c\right)\lambda\right) \tag{C.7}$$

and

$$L^{'} = \max_{m_1^{'}} \frac{1-c}{2B}\left[\left(1+B-c+m_1^{'}\right)(1+\lambda)-c\lambda\delta\right]-k\left(\lambda m_1^{'}\right)-\frac{B+m_1^{'}+1-c}{2B}k\left(m_1\lambda-c\right). \tag{C.8}$$

In this case, even if $m_1^* = \frac{m_1^{'*}}{\lambda}$, $L\big|_{m_1^*=\frac{m_1^{'*}}{\lambda}} - L^{'}_{m_1^{'}=m_1^{'*}}$ equals

$$\begin{aligned} L\big|_{m_1^*-\frac{m_1^{'*}}{\lambda}} - L^{'}_{m_1^{'}=m_1^{'*}} &= c\delta(\lambda-1) + \left[k\left(m_1^{'*}\right)-k\left(\frac{m_1^{'*}}{\lambda}\right)\right] \\ &+ \frac{B+\frac{m_1^{'*}}{\lambda}+1-c}{2B}\left[k\left(\frac{m_1^{'*}}{\lambda}-c\right)-k\left(\left(\frac{m_1^{'*}}{\lambda}-c\right)\lambda\right)\right]. \end{aligned} \tag{C.9}$$

Because $k(\cdot)$ is convex,

$$\begin{aligned} L\Big|_{m_1^*=\frac{m_1^{'*}}{\lambda}} - L^{'}_{m_1^{'}=m_1^{'*}} &> \left[k\left(m_1^{'*}\right) - k\left(\frac{m_1^{'*}}{\lambda}\right)\right] && \text{(C.10)} \\ &- \left[k\left(\left(\frac{m_1^{'*}}{\lambda} - c\right)\lambda\right) - k\left(\frac{m_1^{'*}}{\lambda} - c\right)\right] && \text{(C.11)} \\ &> 0. && \text{(C.12)} \end{aligned}$$

□

When the order of outcome realizations are exogenous, we can similarly show that the smaller fund gets more interventions that disproportional to its size and probability of realizing the outcome first.

Online Appendix D. Other discussions

D.1. General δ and c, and equilibrium multiplicity

We now relate our paper to Angeletos, Hellwig, and Pavan (2007) and explain why our baseline model yields unique equilibrium. We show how equilibrium multiplicity is restored via a mechanism isomorphic to the one in their paper. More importantly, we highlight how equilibrium multiplicity could be endogenized by intervention policy.

Angeletos, Hellwig, and Pavan (2007) show that multiple equilibria emerge under the same conditions that guarantee uniqueness in static global games. The results rely on endogenous learning from regime survivals and exogenous learning from private news that arrives over time. Two elements are necessary for this multiplicity result. First, private information interacts with endogenous learning from earlier coordination outcomes. Second, the private information gets very precise as agents continuously receive private signals about the fundamental. Without the first element, the game is equivalent to one in which agents receive only one precise private signal.[8] Our paper shows the equilibrium results without the second element. We show that multiple equilibria may exist when private signals are very precise. That is, when δ gets very small. Likewise, Angeletos, Hellwig, and Pavan (2007) show that there always exists an equilibrium in which no attack occurs after the first period, and this would be the unique equilibrium if agents did not receive any private information after the first period.[9]

To see this, note that the set of parameters we have examined corresponds to imprecise signals ($2\delta > 1$ and $\frac{1}{1+2\delta} < c < \frac{2\delta}{1+2\delta}$. Moreover, the signal does not get more precise because agents are non-overlapping. If we relax the parameter assumptions, or allow agents' signals to become more precise over time, multiplicity follows. Proposition 12 complements Propositions 1 and 2.[10]

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[8] The variance of the signal is $Var\left(\frac{\sigma^2}{n}\right)$ with n signals.

[9] One can easily write a two-period version of Angeletos, Hellwig, and Pavan (2007) and show this is the only equilibrium if the private signal is sufficiently imprecise.

[10] Technically, multiple equilibria resurface because we can apply the argument of iterated deletion of dominated regions only from one end of θ space. Despite this, with slight modifications on the intervention cost functions, the main intuitions for the results from earlier sections still apply as long as we are consistent

Proposition 12 (Equilibria with general δ and c)**.**

1. *If* $s_1 = S$ *and* $\frac{2\delta}{2\delta+1} < c < 1$,

   (a) *If* $m_2 < m_1 - c$, *the unique equilibrium is the Subgame Equilibrium without Dynamic Coordination.*

   (b) *If* $m_1 - c < m_2 < m_1 - 2\delta(1-c)$, *all three types of equilibria exist. However, in the Equilibrium with Partial Dynamic Coordination, the threshold* θ_2^* *decreases with* m_2,

   (c) *If* $m_1 - 2\delta(1-c) < m_2$, *the unique equilibrium is the Subgame Equilibrium with Dynamic Coordination.*

2. *If* $s_1 = F$ *and* $0 < c < \frac{1}{2\delta+1}$

   (a) *If* $m_2 < m_1 + 2c\delta$,*the unique equilibrium is the Subgame Equilibrium with Dynamic Coordination.*

   (b) *If* $m_1 + 2c\delta < m_2 < m_1 + 1 - c$, *all three types of equilibria exist. However, in the Equilibrium with Partial Dynamic Coordination, the threshold* θ_2^* *decreases with* m_2.

   (c) *If* $m_2 > m_1 + 1 - c$, *the unique equilibrium is the Subgame Equilibrium without Dynamic Coordination.*

Our model also differs from Angeletos, Hellwig, and Pavan (2007) in two additional ways: the government's action is endogenous and the private signal is bounded.[11] Government's action therefore affects equilibrium selection and learning. In particular, when the government's intervention induces equilibria with full or no dynamic coordination, it shuts down the interaction between private signal and public learning. Consequently, the equilibrium is unique even if the signal is infinitely precise. In this regard, the government's endogenous intervention can determine the equilibrium multiplicity.

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with equilibrium selection.

[11](Uniform $[-\delta, \delta]$) in our model but unbounded support in their model ($N\left(z, \frac{1}{\alpha}\right)$).

D.2. Moral hazard

Moral hazard is a big concern in government bailouts and interventions. Indeed, fund managers may divert the capital injected by the government, or gamble by investing in projects with risk profiles different from the pre-specification. We now demonstrate that our general cost specification already encompasses many forms of moral hazard. In particular, we show managerial stealing and risk shifting provide micro-foundations for the intervention cost. Moreover, by modeling moral hazard, we enrich the model with fund managers' utility function and endogenous actions, making the analysis more realistic and relevant.

Cash diversion

First consider the case in which the fund manager is able to divert a fraction $\eta \in [0,1]$ for any amount of liquidity μ injected by the government. Suppose the fund manager gets compensated a fraction of π of the surplus she generates for investors, and among the diverted capital, she can consume $f(\eta)\mu \leq \eta\mu$, where $f : [0,1] \mapsto [0,1)$ satisfies $f'(\cdot) > 0$ and $f''(\cdot) \leq 0$. The rest $\mu[\eta - f(\eta)]$ is inefficiently lost (iceberg costs), consistent with the standard assumption in the literature that cash diversion is increasingly inefficient in the amount diverted.[12] Because the fund manager only cares about her own fund, she does not internalize the intervention externality. Thus, to pin down the unique η, she equalizes the marginal benefit of keeping more injected liquidity in the fund, $\frac{\pi(1-c)\mu}{2B}$, to the marginal benefit of stealing more $f'(\eta)\mu$.

Under this setup, the optimal intervention problem is isomorphic to the problem solved earlier, where intervention incurs a cost $k(m)$ in the period. To see this, note the government is aware of the diverting technology. Therefore, to effectively inject m into the fund, the government needs to spend μ such that $(1-\eta)\mu = m$. Equivalently, injecting m into the fund costs the government $k(m) = k_o\left(\frac{m}{1-\eta}\right) - \frac{f(\eta)m}{1-\eta}$, where $k_o(\mu)$ is other social costs not associated with cash diversion. If k_o is increasing and convex in m, adding the moral hazard cost preserves these properties, consistent with our cost specification.

[12]This specification captures the fact that the fraction of diversion matters for the efficiency loss. Alternatively, one could use $f(\eta\mu)$, where $f : [0,\mu] \mapsto [0,\mu)$ satisfies $f'(\cdot) > 0$ and $f''(\cdot) \leq 0$, which implies the diversion efficiency depends on the total amount. This alternative specification does not affect our conclusion.

Risk shifting

Now consider the case in which the fund manager could secretly choose projects with survival threshold $\theta + \Delta$ with a corresponding private payoff $\alpha\Delta$ conditional on' success after paying the promised payoffs to investors (or some asymmetric split of the additional payoff). The project is thus more illiquid and risky (failure probability is higher), but the fund manager has an incentive to shift the risk, because she captures the upside (limited liability means she does not incur additional loss upon failure). Let the fixed cost of risk shifting be $c_o \geq 0$, and then given the liquidity injection m, the optimal risk shifting is

$$\Delta^* = argmax_\Delta \left[\alpha\Delta\frac{1-c+m-\Delta}{2B} - c_o\mathbb{I}_{\{\Delta>0\}}\right] = \frac{1-c+m}{2}\mathbb{I}_{\left\{c_o<\frac{\alpha(1-c+m)^2}{8B}\right\}}. \tag{C.13}$$

The greater the intervention, the greater the distortion in investment by the manager. Compared to the case in which moral hazard is absent, the welfare is reduced by

$$\left[c_o + \frac{\Delta^*(1-c)}{2B} - \frac{1-c+m-\Delta^*}{2B}\alpha\Delta^*\right]\mathbb{I}_{\left\{c_o<\frac{\alpha\Delta^*(1-c+m-\Delta^*)^2}{2B}\right\}}, \tag{C.14}$$

where the first term is the reduction in welfare due to a lower probability of fund survival, and the second term is the private benefit to the fund manager. For simplicity, we assume $c_o > \frac{\alpha(1-c)^2}{8B}$ and α is sufficiently small (e.g., $\alpha < \frac{1-c}{1-c+\bar{m}}$) so that moral hazard is only induced by the intervention. Then the moral hazard cost can be nested in $k(m) = k_o(m) + \left[c_o + \frac{-\alpha m^2+2m(1-c)(1-\alpha)+(2-\alpha)(1-c)^2}{8B}\right]\mathbb{I}_{\left\{c_o<\frac{\alpha(1-c+m)^2}{8B}\right\}}$. Once again, $k(m)$ is weakly increasing and convex in m.

Because the sum of increasing and convex functions is still increasing and convex, our general cost function accommodates multiple types of moral hazard. In other words, moral hazard considerations constitute and motivate the general cost function we use. For example, when the moral hazard of risk shifting and stealing are both present, their costs can still be represented by the general cost function in our model, and such moral hazard costs motivate the cost specification.

D.3. θ-dependent payoff structures

In this section, we extend the analysis in Section 3 to the case whereby the fundamental θ impacts not only the probability of the fund's survival but also investors' payoff structure. The government's intervention $\{m_1, m_2\}$ then impacts both the probability that the fund survives and the actual amount investors receive. We extend our model in three alternative specifications and show that most results and economic intuition remain. The Equilibrium without Dynamic Coordination may not exist under certain conditions in the extensions, but it is a sub-game equilibrium in which neither endogenous correlation nor conditional inference has effect, and do not matter for our results. Below we elaborate.

Extensions 1 and 2

In this section, we introduce an alternative payoff structure in which θ directly impacts the eventual profitability of investment. We follow Goldstein and Pauzner (2005) by assuming that each fund's investment succeeds with probability $p(\theta)$ and fails with probability $1 - p(\theta)$, where $\theta \sim Unif[-B, B]$ is interpreted as the fundamental. To be consistent, we assume $p' \leq (\theta) < 0$ so that higher θ reduces the possibility of investment success. Conditional on fund's investment success, investors receive payoff $1 - c(\theta)$ if and only if the fund survives a run. Let A continue to be the measure of investors who choose to stay. The condition for the fund to survive a run is $A \geq \theta$. If the fund fails, either due to fundamental investment failure which occurs with probability $1 - p(\theta)$ or due to more endogenous investor runs ($A < \theta$), each investor receives payoff $-d(\theta)$.

The government can intervene by increasing each investor's return by m even if the fund fails. In other words, each investor's return increase to $m - d(\theta)$ with the intervention. Under this setup, the fundamental θ affects the NPV of investment through its effect on both the probability of investment success and the return conditional on success. The intervention therefore requires the government to allocate rather than just promise real resources. Following the baseline model, we continue to assume that θ is unobservable and each investor has a private signal about it.

Since investors in period 2 can observe the outcome of period 1—including the payoff $1-c(\theta)$ conditional on survival and $m_1 - d(\theta)$ conditional on failure, they could perfectly infer the realization of θ if there were a one-to-one mapping between either $c(\theta)$ and θ, or $d(\theta)$ and θ. In that case, the game in the second period no longer features incomplete information, and both the coordination issue and equilibrium multiplicity rise again. To avoid such perfect inference and to introduce fewer deviations from the baseline model, we assume $c(\theta)$ is flat and $d(\theta)$ is piecewise linear: $c(\theta) \equiv c$ and $d(\theta) \equiv \begin{cases} -c & \text{if } \theta \leq \bar{\theta} \\ -c-d & \text{if } \theta > \bar{\theta} \end{cases}$, where $d \geq 0$. $\bar{\theta}$ is a threshold which can be thought of as barriers in the fund's investment technology. Table 2 shows the payoff under this setup.

Payoffs in the Extended Setup

	Stay	Run
Survive	$1-c$	0
Fail	$\begin{cases} m-c & \text{if } \theta \leq \bar{\theta} \\ m-c-d & \text{if } \theta > \bar{\theta} \end{cases}$	0

For the remaining analysis, we study two special cases. In the first case, we assume $p(\theta) \equiv 1$ so that the fund can only fail due to investor runs. In this case, the fundamental θ only directly affects $c(\theta)$ and $d(\theta)$, the payoff conditional on survival/failure. We solve the model and show that the results from the baseline model go through. In the second case, we assume $d \equiv 0$ so that θ only directly affects $p(\theta)$, the probability of fund's fundamental investment success. We will see the results differ slightly in the sense that the sub-game equilibrium without coordination disappears in the latter case. In both cases, θ directly affects investors' payoff through its effect on the NPV of investment. Moreover, θ also indirectly affect payoff through coordination. Finally, to make the game non-trivial, we assume that both m_1 and m_2 are less than $\bar{m}$, which in turn is strictly less than c.

Extension 1: θ affects conditional investment payoff

Period 1 The equilibrium in period 1 is still characterized by two thresholds (θ_1^*, x_1^*): the fund survives a run if and only $\theta < \theta_1^*$; investor i stays if and only if the observed signal $x_{1i} < x_1^*$. However, the equilibrium is no longer unique; investors' beliefs on the comparison between θ_1^* and $\bar{\theta}$ generate another source of self-fulfilling multiplicity. Intuitively, when investors expect $\theta_1^* < \bar{\theta}$, they behave more cautiously by choosing a lower threshold x_1^*. When investors are more cautious about the staying decision, fewer of them choose to stay for the same set of signals $\{x_{1i}\}_{i\in[0,1]}$, and A_1, the aggregate measure of investors who stay, indeed becomes smaller, leading to a lower survival threshold for the fund θ_1^* and thus fulfilling investors' expectation that $\theta_1^* < \bar{\theta}$. On the other hand, when investors expect $\theta_1^* > \bar{\theta}$, they behave less cautiously by choosing a higher threshold x_1^*. As a result, more of them choose to stay for the same set of signals $\{x_{1i}\}_{i\in[0,1]}$, and A_1 will indeed get higher, leading to a higher survival threshold for the fund θ_1^* and thus fulfilling investors' expectation that $\theta_1^* > \bar{\theta}$. More generally, if the payoff conditional on success or failure experiences jumps, the equilibrium multiplicity may come back. Proposition 13 summarizes the equilibrium in the first period.

Proposition 13. *The equilibrium in the first period is characterized by two thresholds (θ_1^*, x_1^*): the fund survives a run if and only if $\theta < \theta_1^*$; investor i stays if and only if the observed signal $x_{1i} < x_1^*$.*

1. *If $m_1 < \frac{\bar{\theta}-(1-c)}{\bar{\theta}}$, equilibrium is unique:*

$$\begin{cases} \theta_1^* = \frac{1-c+\frac{\bar{\theta}}{2\delta}d}{1+\frac{1+2\delta}{2\delta}d-m_1} \\ x_1^* = \frac{1-c+\frac{\bar{\theta}}{2\delta}d}{1+\frac{1+2\delta}{2\delta}d-m_1}(1+2\delta)-\delta. \end{cases}$$

   Moreover, $\theta_1^* < \bar{\theta}$.

2. *If $\frac{\bar{\theta}-(1-c)}{\bar{\theta}} < m_1 < \frac{(1+d)\bar{\theta}-(1-c)}{\bar{\theta}}$, there are two equilibria.*

$$\begin{cases} \theta_1^* = \frac{1-c}{1-m_1} \\ x_1^* = \frac{1-c}{1-m_1}(1+2\delta)-\delta \end{cases} \qquad \begin{cases} \theta_1^* = \frac{1-c+\frac{\bar{\theta}}{2\delta}d}{1+\frac{1+2\delta}{2\delta}d-m_1} \\ x_1^* = \frac{1-c+\frac{\bar{\theta}}{2\delta}d}{1+\frac{1+2\delta}{2\delta}d-m_1}(1+2\delta)-\delta. \end{cases}$$

   In the first equilibrium, $\theta_1^* > \bar{\theta}$, where as in the second equilibrium, $\theta_1^* < \bar{\theta}$.

3. *If* $m_1 > \frac{(1+d)\bar{\theta}-(1-c)}{\bar{\theta}}$, *equilibrium is unique*

$$\begin{cases} \theta_1^* = \frac{1-c}{1-m_1} \\ x_1^* = \frac{1-c}{1-m_1}\left(1+2\delta\right)-\delta. \end{cases}$$

Moreover, $\theta_1^* > \bar{\theta}$.

Proof. When investors expect $\theta_1^* < \bar{\theta}$, the two equations determining equilibria are

$$\begin{aligned} A\left(\theta_1^*\right) = \theta_1^* \Rightarrow \frac{x_1^* - \left(\theta_1^* - \delta\right)}{2\delta} &= \theta_1^* \\ \Pr\left(\theta < \theta_1^* | x_1^*\right)(1-c) + \Pr\left(\theta_1^* < \theta < \bar{\theta}_1 | x_1^*\right)(m_1 - c) + \Pr\left(\theta > \bar{\theta}_1 | x_1^*\right)(m_1 - c - d) &= 0. \end{aligned}$$

Solving the two equations, we get

$$\begin{cases} \theta_1^* = \frac{1-c+\frac{\bar{\theta}}{2\delta}d}{1+\frac{1+2\delta}{2\delta}d-m_1} \\ x_1^* = \frac{1-c+\frac{\bar{\theta}}{2\delta}d}{1+\frac{1+2\delta}{2\delta}d-m_1}\left(1+2\delta\right)-\delta. \end{cases}$$

By imposing the requirement $\theta_1^* < \bar{\theta}$, we get $m_1 < \frac{(1+d)\bar{\theta}-(1-c)}{\bar{\theta}}$.

When investors expect $\theta_1^* > \bar{\theta}$, the two equations determining equilibria are

$$\begin{aligned} A\left(\theta_1^*\right) = \theta_1^* \Rightarrow \frac{x_1^* - \left(\theta_1^* - \delta\right)}{2\delta} &= \theta_1^* \\ \Pr\left(\theta < \theta_1^* | x_1^*\right)(1-c) + \Pr\left(\theta > \theta_1^* | x_1^*\right)(m_1 - c) &= 0. \end{aligned}$$

Solving the two equations, we get

$$\begin{cases} \theta_1^* = \frac{1-c}{1-m_1} \\ x_1^* = \frac{1-c}{1-m_1}\left(1+2\delta\right)-\delta. \end{cases}$$

By imposing the requirement $\theta_1^* > \bar{\theta}$, we get $m_1 > \frac{\bar{\theta}-(1-c)}{\bar{\theta}}$. □

Period 2 We now turn to period 2. Obviously, the analysis differs in whether θ_1^*, the equilibrium fund survival threshold in first period, is higher or lower than $\bar{\theta}$, the cutoff for more negative payment if the fund fails.

1. $\theta_1^* < \bar{\theta}$. In this case, the intervention in period 1 truncates the prior belief on θ into three regions: $[-B, \theta_1^*]$, $\left[\theta_1^*, \bar{\theta}\right]$, and $\left[\bar{\theta}, B\right]$, respectively corresponding to the case that the fund in the first period has succeeded, the fund in the first period has failed and investors who have stayed receive $m_1 - c$, the fund in the first period has failed and investors who have stayed receive $m_1 - c - d$.

2. $\theta_1^* \geq \bar{\theta}$. In this case, the intervention in period 1 again truncates the prior belief on θ into two regions: $[-B, \theta_1^*]$ and $[\theta_1^*, B]$.

Below we will study the first case $\theta_1^* < \bar{\theta}$ which is more general. The second case is a subcase of the first one.

If $s_1 = S$, then it becomes publicly known that $\theta \in [-B, \theta_1^*]$. As in the baseline model, there are also three types of equilibria: Equilibrium Without Dynamic Coordination, with Partial Coordination, and with Coordination.

Proposition 14. *Equilibrium in period 2 if $s_1 = S$ and $\theta_1^* < \bar{\theta}$*

1. *If $m_2 < 1 - \frac{1+\frac{1+2\delta}{2\delta}d - m_1}{1-c+\frac{\theta}{2\delta}d}\left(1-c\right)\left(1+2\delta\right)$, the equilibrium in period 2 has no dynamic coordination:*

$$\begin{cases} \theta_2^* = \frac{1-c}{1-m_2} \\ x_2^* = \frac{1-c}{1-m_2}\left(1+2\delta\right) - \delta. \end{cases}$$

2. *If $1 - \frac{1+\frac{1+2\delta}{2\delta}d - m_1}{1-c+\frac{\theta}{2\delta}d}\left(1-c\right)\left(1+2\delta\right) < m_2 < c - 2\left(1-c\right)\delta$,[13] the equilibrium in period 2 has partial*

[13] This condition either requires us to relax the assumption in the baseline model that $c < \frac{2\delta}{1+2\delta}$ or the assumption that $m_2 > 0$.

dynamic coordination:

$$\begin{cases} \theta_2^* = \frac{\theta_1^*(c-m_2)-2(1-c)\delta}{c-m_2-2(1-c)\delta} \\ x_2^* = \theta_2^*(1+2\delta)+\delta. \end{cases}$$

3. *If $m_2 > c - 2(1-c)\delta$, the equilibrium in period 1 has dynamic coordination.*

Proof. Consider first that there is no dynamic coordination. Since it is publicly known that $\theta \in [-B, \theta_1^*]$ and that $\theta_1^* < \bar{\theta}$, investors in the second period can never receive payoff $m_2 - c - d$. Therefore, ignoring the public news that $\theta < \theta_1^*$, the thresholds that determine equilibrium are similar to Case 3 of Proposition 13

$$\begin{cases} \theta_2^* = \frac{1-c}{1-m_2} \\ x_2^* = \frac{1-c}{1-m_2}(1+2\delta)-\delta. \end{cases}$$

The sufficient and necessary condition for the equilibrium to have no dynamic coordination is

$$x_2^* + \delta < \theta_1^*,$$

which leads to the condition $m_2 < 1 - \frac{1+\frac{1+2\delta}{2\delta}d-m_1}{1-c+\frac{\theta}{2\delta}d}(1-c)(1+2\delta)$.

Consider next the Equilibrium with Dynamic Coordination, the thresholds are now determined by

$$\begin{aligned} A(\theta_2^*) = \frac{x_2^* - (\theta_2^* - \delta)}{2\delta} &= \theta_2^* \\ \Pr(\theta < \theta_2^* | x_2^*, \theta < \theta_1^*)(1-c) + \Pr(\theta > \theta_2^* | x_2^*, \theta < \theta_1^*)(m_2 - c) &= 0. \end{aligned}$$

The solutions are

$$\begin{cases} \theta_2^* = \frac{\theta_1^*(c-m_2)-2(1-c)\delta}{c-m_2-2(1-c)\delta} \\ x_2^* = \theta_2^*(1+2\delta)+\delta. \end{cases}$$

The sufficient and necessary condition for the equilibrium to have dynamic coordination is

$$\theta_2^* > \theta_1^*,$$

which lead to the condition $m_2 > c - 2(1-c)\delta$. $\square$

The expressions for conditional payoffs W_2^{nc}, W_2^{pc}, and W_2^{c} get lengthier and are omitted to avoid complication. However, it is clear that θ_1^* increases with m_1 and in the case with dynamic coordination, investors continue to receive $1-c$ if $s_1 = S$ and 0 if $s_1 = F$. Therefore, the *Endogenous Coordination Effect* and *Conditional Inference Effect* continue to exist.

Corollary 4. *The* Endogenous Coordination Effect *and* Conditional Inference Effect *continue to exist.*

The remaining cases $\theta \in \left[\theta_1^*, \bar{\theta}\right]$, $\theta \in \left[\bar{\theta}, B\right]$ under $s_1 = S$ and $\theta_1^* < \bar{\theta}$ can be analyzed in a similar way. So are the cases when $\theta_1^* > \bar{\theta}$ and $s_1 = F$. To avoid clunky and repeated expressions and statements, we omit the details. However, we would like to point out one exception when $\theta \in \left[\theta_1^*, \bar{\theta}\right]$. In this case, the length of the support of the updated belief on θ becomes $\bar{\theta} - \theta_1^*$, which can be smaller than 2δ. In this case, the Equilibrium without Dynamic Coordination may disappear. Indeed, this result depends on the prior belief on θ to be sufficiently wide. In the case $\theta \in \left[\theta_1^*, \bar{\theta}\right]$, the truncations $\theta > \theta_1^*$ and $\theta < \bar{\theta}$ at least partially helpsupdate the marginal investor's belief in additional to his private signal x_2^*. In Extension 2 and also Section A.7, we will show that the result of the Equilibrium without Dynamic Coordination further depends on the assumption that private noises follow uniform distribution, and the payoff conditional on success or failure is sufficiently flat and thus non-informative of the fundamental θ.

However, since the other two types of sub-game equilibria continue to exist, we would like to emphasize that the two key channels in this paper, *Conditional Inference Effect* and *Endogenous Coordination Effect*, remain in the current extension. Therefore, the interactions with the intervention cost function lead to implications similar to those in Section 4.

Extension 2: θ affects investment success probability

Setup In this section, we assume $p(\theta) \in (0,1)$ and $c(\theta) = d(\theta) \equiv c$. Table 3 shows the payoff under this extension. Note that survival requires two conditions. First, the fund's investment needs to succeed, which occurs with probability $p(\theta) < 1$. Second, enough investors must stay and the fund thus survives a run: $A > \theta$. We continue to assume that $m \in [0, \bar{m}]$ and $\bar{m} < c$. Otherwise, the government can guarantee higher payoff for investors who choose to stay.

Payoffs in Case 2

	Stay	Run
Survive	$1-c$	0
Fail	$m-c$	0

Period 1 The equilibrium in the first period is again captured by two thresholds (θ_1^*, x_1^*). If $\theta = \theta_1^*$, then $A(\theta_1^*) = \theta_1^*$ so that if the fund's investment succeeds, the extent of coordination among investors is just enough to guarantee the fund to survive. Moreover, each investor adopts a threshold strategy $a_i = \mathbb{1}\{x_i \leq x^*\}$: he stays if and only if the probability of receiving $1-c$ exceeds $\frac{c-m}{1-m}$. Therefore, the two equations that characterize the two thresholds are

$$\begin{cases} A(\theta_1^*) = \frac{x_1^* - (\theta_1^* - \delta)}{2\delta} = \theta_1^* \\ \int_{x_1^*-\delta}^{\theta_1^*} \frac{p(\theta)}{2\delta} d\theta = \frac{c-m}{1-m}. \end{cases}$$

Note that if $p(\theta) \equiv 1$ and $m = 0$, the solutions are identical to that in standard regime shifting games: $(\theta_1^*, x_1^*) = (1-c, 1-c+\delta(1-2c))$.

In general, θ_1^* is the solution to the following equation

$$\int_{\theta_1^*(1+2\delta)-2\delta}^{\theta_1^*} \frac{p(\theta)}{2\delta} = \frac{c-m}{1-m}.$$

To get closed-form solutions, one needs to specify a functional form for $p(\theta)$. Without narrowing us to any ad hoc functional form, we can still proceed with the qualitative analysis to show that the conditional inference effect remains: since $p(\theta)$ is a decreasing function of θ and the length of the interval $[\theta_1^*(1+2\delta) - 2\delta, \theta_1^*]$ decreases with θ_1^*, there exists a unique solution to θ_1^*. Moreover, θ_1^* increases with m_1— the conditional inference effect. Finally, $x_1^* = \theta_1^*(1+2\delta) - 2\delta$.

Next, we move on to the analysis in the second period. Upon observing that investors have received

$1-c$ in the first period, investors again get more optimistic about the distribution of θ. Their optimism can be decomposed into two effects.

1. Truncation. After observing a payoff $1-c$, investors can safely conclude that $\theta \in [-B, \theta_1^*]$. In other words, the regions where θ is very high gets excluded. Moreover, θ_1^* increases with m_1 so that the conditional inference effect carries over.

2. More optimistic on the untruncated region. While the support of θ is truncated from above: $\theta \in [-B, \theta_1^*]$, the belief on the distribution of θ on $[-B, \theta_1^*]$ is also updated. In particular, while the prior distribution of θ is uniform on $[-B, B]$, the updated distribution is no longer uniform. It involves updating on $p(\theta)$ as well.

   To see this, let's assume $p(\theta) = \frac{\theta+B}{2B} \sim Unif\,[0,1]$ for simplicity. Let us temporarily ignore the truncation effect so that upon observing investors have received $1-c$, the support of θ remains $[-B, B]$. Even so, observing $s_1 = S$ increases the probability density on high $p(\theta)$ (low θ) and decreases the probability density on low $p(\theta)$ (high θ). The updated belief distribution should entail this optimism. In particular, the updated belief becomes $p(\theta) \sim \beta(2,1)$ after $s_1 = S$.[14] In other words, the belief on θ gets skewed towards lower realizations. With the truncation effect, the updated distribution of θ becomes a conditional beta distribution on $[-B, \theta_1^*]$.

Combining both effects, the second-period stage equilibrium can never be one without dynamic coordination. In other words, the equilibrium in period 2 is either the *Stage Game Equilibrium with Partial Coordination* when the conditional inference effect more likely dominates; or *Stage Game Equilibrium with Coordination* when the endogenous correlation effect dominates. Consequently, since the updated belief of θ on $[-B, \theta_1^*]$ is no longer uniform, the second-period stage equilibrium may no longer be unique. However, the conditional inference effect also remains. The remaining analysis, which includes solving equilibrium cutoffs (numerically) for different equilibria, follows the analysis in Section O.5.

[14]The updated belief becomes $p(\theta) \sim \beta(1,2)$ after $s_1 = F$

Extension 3

In this subsection, we introduce a different extension in which the fundamental θ and government intervention m directly affect investors' payoff. Specifically, we keep the setup in Section 2.2 except for the following payoff structure for each investor:

$$V_{ti} = \begin{cases} 1-c & \text{if } A_t + m_t \geq \theta + \psi \\ -c + \frac{1}{2\psi}\left[A_t + m_t - (\theta - \psi)\right] & \text{if } \theta - \psi < A_t + m_t < \theta + \psi \\ -c & \text{if } A_t + m_t \leq \theta - \psi. \end{cases}$$

Each investor receives $1-c$ only if the fund fully survives: $A_t + m_t \geq \theta + \psi$. In contrast, each investor receives $-c$ if the fund fully fails: $A_t + m_t \leq \theta - \psi$. If the fund partially survives (or fails), however, i.e., if $\theta - \psi < A_t + m_t < \theta + \psi$, the investor's payoff is linear in the fundamental θ. Note that the baseline model refers to one when $\psi \to 0$. We continue to assume that investors in period 2 can observe the outcome of period 1.

This payoff structure can be interpreted as debts or deposits, and generally applies to cases where an investor's payoff is capped above when the fund is very successful, but is zero if the fund fails. $A_t + m_t \geq \theta + \psi$ can be interpreted as that the project is successful enough that every staying investor gets the promised principal and interest (or promised return as in the case of wealth management products some funds offer); if the situation is really bad $A_t + m_t \leq \theta - \psi$, the fund has nothing left for the investors; if the fund does poorly but does not completely fail $\theta - \psi < A_t + m_t < \theta + \psi$, the investors get some payoff less than what is originally promised (think of them as debts who have senior claims on the limited revenue the fund generates). ψ is then a parameter capturing the range of fundamental states that leads to the fund to survive but fails to deliver to the investors the full promised return.

Lemma 10 summarizes the equilibrium in period 1.

Lemma 10. *In the stage game, there exists a unique symmetric PBE in monotone strategies* $\left(\theta_1^L, \theta_1^H, x_1^*\right)$

where

$$\begin{cases} \theta_1^L &= 1 + m_1 - c - \frac{2\delta\psi}{1+2\delta\psi} \\ \theta_1^H &= 1 + m_1 - c + \frac{2\delta\psi}{1+2\delta\psi} \\ x_1^* &= 1 + m_1 - c + \delta(1-2c). \end{cases}$$

Each investor's strategy follows $a_i = \mathbb{1}\{x_i \leq x_1^*\}$. *The fund pays off each investor* $1-c$ *if* $\theta \leq \theta_1^L$, $-c + \frac{1}{2\psi}[A_1 + m_1 - (\theta - \psi)]$ *if* $\theta \in (\theta_1^L, \theta_1^H)$ *and* $-c$ *if* $\theta \geq \theta_1^H$. *Moreover,* $A_1 = \frac{x_1^* - (\theta - \delta)}{2\delta}$.

Lemma 10 is the extended version of Lemma 1. Note that if $\psi = 0$, both thresholds are identical to θ_1^*: $\theta_1^L = \theta_1^H = \theta_1^*$, and the equilibrium is identical to one when θ only impacts the fund's survival. When θ also directly enters investor's payoff, the equilibrium is characterized by three thresholds. Specifically, θ_1^L is the threshold of θ under which the fund fully succeeds, and θ_1^H is the threshold above which the fund fully fails. If $\theta \in (\theta_1^L, \theta_1^H)$, the fund succeeds (or equivalently fails) only partially and repays between $-c$ and $1-c$.

Proof. The payoff structure satisfies A.1 to A.5 in Section 2.2.1 of Morris and Shin (2003). According to Proposition 2.1 there, each investor's strategy is a threshold strategy. We conjecture that the equilibrium is characterized by three thresholds that satisfy three equations

$$A(\theta_1^H) + m_1 - (\theta_1^H - \psi) = 0 \quad \text{(C.15)}$$

$$A(\theta_1^L) + m_1 - (\theta_1^L + \psi) = 0 \quad \text{(C.16)}$$

$$\Pr(\theta < \theta_1^L | x_1^*)(1-c) + \int_{\theta_1^L}^{\theta_1^H} \left\{-c + \frac{1}{2\psi}[A(\theta) + m_1 - \theta + \psi]\right\} d\theta + \Pr(\theta > \theta_1^H | x_2^*)(-c) = 0 \quad \text{(C.17)}$$

Solving the three equations, we get the solutions as below

$$\begin{cases} \theta_1^L &= 1 + m_1 - c - \frac{2\delta\psi}{1+2\delta\psi} \\ \theta_1^H &= 1 + m_1 - c + \frac{2\delta\psi}{1+2\delta\psi} \\ x_1^* &= 1 + m_1 - c + \delta(1-2c). \end{cases} \quad \text{(C.18)}$$

□

Lemma 11 shows the total welfare.

Lemma 11. *The total welfare is* $W_1 = \frac{(1-c)[1+B-c(1+\delta)+m_1]}{2B} - \frac{\delta\psi^2}{6B(1+2\delta)^2}$.

Note that when $\psi = 0$, the social welfare is identical to what we have in Section 2.1.3. As ψ increases, the total social welfare decreases. Intuitively, larger ψ increases the uncertainty on payoff, the difficulty in coordination, and thus the overall welfare.

Next, we turn to the equilibrium in period 2. After the outcome of period-1 intervention gets publicly known, the belief on θ is either partitioned or precisely known. Specifically, if investors receive $1-c$, it becomes public knowledge that $\theta \in \left[-B, \theta_1^L\right]$. In contrast, it becomes publicly known that $\theta \in \left[\theta_1^H, B\right]$ if investors receive $-c$. If investors receive $V_1 \in (1-c, c)$, they can perfectly infer the true realization θ, which is the solution to the following equation:

$$V_1 = -c + \frac{1}{2\psi}\left[A_1(\theta) + m_1 - (\theta - \psi)\right],$$

where $A_1(\theta) = \frac{x_1^* - (\theta - \delta)}{2\delta}$. Since m_1 is publicly announced, and $A_1(\theta)$ decreases with θ, the above equation admits a unique solution when $V_1 \in (-c, 1-c)$. In this case, θ becomes public information in period 2, and the equilibrium may not be unique. For the remainder of this section, we focus on the case that $\theta \in \left[-B, \theta_1^L\right]$ and $\theta \in \left[\theta_1^H, B\right]$ to avoid the issue of equilibrium selection.

If investors have received $1-c$ in period 1, the prior knowledge on θ is updated as $\left[-B, \theta_1^L\right]$. Similar to the baseline model, the equilibrium in period 2 can occur with, without or with partial dynamic coordination, summarized as follows. We assume ψ is sufficiently small such that $\frac{\psi}{1+2\delta} < 1-c$.

Proposition 15. *Equilibrium in Period 2 when* $s_1 = S$

1. *If* $m_2 < m_1 - 2\delta(1-c) - \frac{2\delta\psi}{1+2\delta}$, *the unique equilibrium is the Stage Game Equilibrium without Dynamic Coordination. The total social welfare is* $W_{2S}^{nc} = \frac{(1-c)[1+B-c(1+\delta)+m_2]}{B+\theta_1^L} - \frac{\delta\psi^2}{3\left(B+\theta_1^L\right)(1+2\delta)^2}$.
2. *If* $m_1 - 2\delta(1-c) - \frac{2\delta\psi}{1+2\delta} < m_2 < m_1 - c - \frac{c\psi}{(1-c)(1+2\delta)}$, *the unique equilibrium is the Stage Game Equilibrium with Partial Dynamic Coordination.*

(a) *If* $m_1 - 2\delta(1-c) - \frac{2\delta\psi}{1+2\delta} < m_2 < m_1 - c - \frac{\psi[4(1-c)\delta - c]}{(1-c)(1+2\delta)}$, *in the unique equilibrium, the fund may succeed, fail or partially fail. The total social welfare is*

$$\begin{aligned} W_{2S}^{pc} &= \frac{(1-c)}{B+\theta_1^L}\left[\theta_1^L + B + \frac{\delta c(c-m_1+m_2)^2 + 2\delta(c-m_1+m_2)[2\delta - c(1+2\delta)]}{[2\delta - c(1+2\delta)]^2}\right] \\ &- \frac{\delta\psi\left\{12(c-1)\delta(1+2\delta)[c(-1+c-m_1+m_2-2\delta)+2\delta] + \psi\left[c^2 - 4(1-c)\delta(c-\delta+4c\delta)\right]\right\}}{3(1+2\delta)^2[2\delta - c(1+2\delta)]^2\left(\theta_1^L + B\right)}. \end{aligned}$$

(b) *If* $m_1 - c - \frac{\psi[4(1-c)\delta - c]}{(1-c)(1+2\delta)} < m_2 < m_1 - c - \frac{c\psi}{(1-c)(1+2\delta)}$, *in the unique equilibrium, the fund can never fail. The total social welfare is* $W_{2S}^{pc} = \frac{(1-c)}{B+\theta_1^L}\left[\theta_1^L + B + \frac{\delta c(c-m_1+m_2)^2 + 2\delta(c-m_1+m_2)[2\delta - c(1+2\delta)]}{[2\delta - c(1+2\delta)]^2}\right] - H_1(\psi)$ *where* $H_1(\psi) > 0$ *and converges to* 0 *as* $\psi \to 0$.[15]

3. *If* $m_2 > m_1 - c - \frac{c\psi}{(1-c)(1+2\delta)}$, *the unique equilibrium is the Stage Game Equilibrium with Dynamic Coordination. In this equilibrium, the fund will always succeed and repay* $1-c$. *The social welfare is* $W_{2S}^c = 1 - c$.

Proof. Again, we solve the model respectively in three cases: equilibrium with dynamic coordination, with partial coordination and without coordination.

1. Equilibrium without dynamic coordination. We solve the equilibrium assuming no dynamic coordination. The solutions are similar to those in period 1:

$$\begin{cases} \theta_2^L &= 1 + m_2 - c - \frac{2\delta\psi}{1+2\delta\psi} \\ \theta_2^H &= 1 + m_2 - c + \frac{2\delta\psi}{1+2\delta\psi} \\ x_2^* &= 1 + m_2 - c + \delta(1-2c). \end{cases} \tag{C.19}$$

A necessary condition for the equilibrium to feature no dynamic coordination is the equilibrium x_2^* satisfies $x_2^* + \delta < \theta_1^L$ so that the marginal investor finds the period-1 information useless. Moreover, $\theta_2^H < \theta_1^L$ so that it is indeed possible that the fund in period 2 still fails. Combining both inequalities and the assumption $\frac{\psi}{1+2\delta} < 1 - c$, the condition for this case is derived as $m_2 < m_1 - 2\delta(1-c) - \frac{2\delta\psi}{1+2\delta}$.

[15] We leave out the complicated closed-form expression for $H_1(\psi)$ which is available upon request.

2. Equilibrium with partial coordination. In this case, the marginal investor finds the information in period 1 being useful. As a result, the equation which shows the marginal investor is indifferent between two actions is

$$\begin{aligned}\Pr\left(\theta < \theta_2^L \middle| x_2^*, \theta < \theta_1^L\right)(1-c) + \int_{\theta_2^L}^{\theta_2^H} \left\{-c + \frac{1}{2\psi}\left[A(\theta) + m_2 - \theta + \psi\right]\right\} d\theta + \Pr\left(\theta > \theta_2^H \middle| x_2^*, \theta < \theta_1^L\right)(-c) &= 0 \\ \Rightarrow \frac{\theta_2^L - (x_2^* - \delta)}{\theta_1^L - (x_2^* - \delta)} \times (1-c) + \frac{\theta_1^L - \theta_2^H}{\theta_1^L - (x_2^* - \delta)} \times (-c) + \frac{\theta_2^H - \theta_2^L}{\theta_1^L - (x_2^* - \delta)} \times \left(\frac{1}{2} - c\right) &= 0.\end{aligned}$$

Note that the denominator is replaced by $\theta_1^L - (x_2^* - \delta)$ which is less than 2δ since the information in period 1 truncates the range of θ coming from the marginal investor's private signal alone. The new thresholds are

$$\begin{cases} \theta_2^L &= 1 + m_2 - c - \frac{c[m_2 - (m_1 - c)]}{c - 2(1-c)\delta} - \frac{4\delta\psi}{1+2\delta} \frac{c - (1-c)\delta}{c - 2(1-c)\delta} \\ \theta_2^H &= 1 + m_2 - c - \frac{c[m_2 - (m_1 - c)]}{c - 2(1-c)\delta} - \frac{4\delta^2\psi}{1+2\delta} \frac{1-c}{c - 2(1-c)\delta} \\ x_2^* &= 1 + m_2 - c + \delta(1-2c) - \frac{(1-c)(1+2\delta)(m_1 + 2c\delta - m_2)}{c(1+2\delta) - 1} - \frac{2c\delta\psi}{c - 2(1-c)\delta}. \end{cases} \tag{C.20}$$

Note that if $\psi = 0$, the thresholds are exactly equal to Lemma 4. In this case, if $\theta_2^H > \theta_1^L$, the fund can never fully fail in the equilibrium with partial dynamic coordination. If it further holds that $\theta_2^L > \theta_1^L$, the fund always fully succeeds, and the equilibrium has dynamic coordination. The condition in 2(a) guarantees that $\theta_2^H < \theta_1^L$ so that the fund may still fail, whereas the condition in 2(b) enables $\theta_2^L < \theta_1^L < \theta_2^H$. Case 3 lists the condition for $\theta_2^L > \theta_1^L$.

□

Proposition 15 is the extended version of Proposition 1. Note that when $\psi \to 0$, the two propositions are identical. In case 1 when m_2 is small relative to m_1, the initial intervention is in vain. In case 3 when m_2 is large relative to m_1, the initial intervention guarantees success for the second fund. When m_2 is in between, the initial intervention has partial coordination effect. Case 2(a) describes a case similar to the equilibrium with partial coordination in the baseline setup. Case 2(b) describes a case when the initial intervention eliminates full failure but not partial failure (success) in the second period.

Proposition 16 summarizes the result when $s_1 = F$. We assume ψ is sufficiently small such that $\psi < \frac{c}{1+2\delta c}$.

Proposition 16. *Equilibrium in Period 2 when* $s_1 = F$

1. *If* $m_2 < m_1 + 1 - c + \frac{(1-c)\psi}{c(1+2\delta)}$, *the unique equilibrium is the Stage Game Equilibrium with Dynamic Coordination. The total social welfare is* $W_{2F}^{c} = 0$.

2. *If* $m_1 + 1 - c + \frac{(1-c)\psi}{c(1+2\delta)} < m_2 < m_1 + 2c\delta + \frac{2\delta\psi}{1+2\delta\psi}$, *the unique equilibrium is the Stage Game Equilibrium with Partial Dynamic Coordination.*

   (a) *If* $m_1 + 1 - c + \frac{(1-c)\psi}{c(1+2\delta)} < m_2 < m_1 + 1 - c + \frac{\psi[c(1+4\delta)-1]}{c(1+2\delta)}$, *in the unique equilibrium, the fund can never fully succeed. The total social welfare is* $W_{2F}^{pc} = \frac{1-c}{B-\theta_1^L}\frac{(-1+c-m_1+m_2)^2}{(-1+c+2c\delta)^2} + H_2(\psi)$ *where* $H_2(\psi) \to 0$ *as* $\psi \to 0$.[16]

   (b) *If* $m_1 + 1 - c + \frac{\psi[c(1+4\delta)-1]}{c(1+2\delta)} < m_2 < m_1 + 2c\delta + \frac{2\delta\psi}{1+2\delta\psi}$, *in the unique equilibrium, the fund may succeed, fail or partially fail. The total social welfare is* $W_{2F}^{pc} = \frac{1-c}{B-\theta_1^L}\frac{(-1+c-m_1+m_2)^2}{(-1+c+2c\delta)^2} - \frac{\delta\psi\left\{12(-1+c)(-1+c-m_1+m_2)(1+2\delta)(-1+c+c\delta)+\left[-11-24\delta+c\left(34+4\delta(23+9\delta)+12c^2(1+2\delta)^2-c(35+4\delta(29+20\delta))\right)\right]\psi\right\}}{3(1+2\delta)^2(-1+c+2c\delta)^2\left(B-\theta_1^L\right)}$.

3. *If* $m_2 > m_1 + 2c\delta + \frac{2\delta\psi}{1+2\delta\psi}$, *the unique equilibrium is the Stage Game Equilibrium without Dynamic Coordination. The total social welfare is* $W_{2F}^{nc} = \frac{1-c}{B-\theta_1^L}(m_2 - m_1 - c\delta) - \frac{\psi\delta[\psi-6(1-c)(1+2\delta)]}{3(1+2\delta)^2\left(B-\theta_1^L\right)}$.

Proof. We solve the model respectively in three cases: equilibrium with dynamic coordination, with partial coordination and without coordination.

1. Equilibrium without dynamic coordination. Assuming no dynamic coordination. The solutions are:

$$\begin{cases} \theta_2^L & = 1 + m_2 - c - \frac{2\delta\psi}{1+2\delta\psi} \\ \theta_2^H & = 1 + m_2 - c + \frac{2\delta\psi}{1+2\delta\psi} \\ x_2^* & = 1 + m_2 - c + \delta(1-2c). \end{cases}$$

[16]The detailed expression for $H_2(\psi)$ is available upon request.

The conditions for no dynamic coordination are:

$$\begin{cases} x_2^* - \delta > \theta_1^H \\ \theta_2^L > \theta_2^H. \end{cases}$$

Then, as long as ψ is sufficiently small such that $\psi < \frac{c}{1+2\delta c}$, the equilibrium without dynamic coordination exists if and only if $m_2 > m_1 + 2c\delta + \frac{2\delta\psi}{1+2\delta\psi}$. Note that if $\psi \to 0$, the condition is the same as Proposition 2.

2. Equilibrium with partial coordination. In this case, the equation which characterizes the marginal investor's indifference is:

$$\begin{aligned} \Pr\left(\theta < \theta_2^L \middle| x_2^*, \theta > \theta_2^H\right)(1-c) + \int_{\theta_2^L}^{\theta_2^H} \left\{-c + \frac{1}{2\psi}\left[A(\theta) + m_2 - \theta + \psi\right]\right\} d\theta + \Pr\left(\theta > \theta_2^H \middle| x_2^*, \theta > \theta_2^H\right)(-c) &= 0 \\ \Rightarrow \frac{\theta_2^L - \theta_1^H}{(x_2^* + \delta) - \theta_1^H} \times (1-c) + \frac{(x_2^* + \delta) - \theta_2^H}{(x_2^* + \delta) - \theta_1^H} \times (-c) + \frac{\theta_2^H - \theta_2^L}{(x_2^* + \delta) - \theta_1^H} \times \left(\frac{1}{2} - c\right) &= 0. \end{aligned}$$

The solutions are

$$\begin{cases} \theta_2^L &= 1 + m_2 - c - \frac{(1-c)(m_1+2c\delta-m_2)}{c(1+2\delta)-1} - \frac{4c\delta^2\psi}{(1+2\delta)[c(1+2\delta)-1]} \\ \theta_2^H &= 1 + m_2 - c - \frac{(1-c)(m_1+2c\delta-m_2)}{c(1+2\delta)-1} + \frac{4\delta(-1+c+c\delta)\psi}{(1+2\delta)[c(1+2\delta)-1]} \\ x_2^* &= 1 + m_2 - c + \delta(1-2c) - \frac{(1-c)(1+2\delta)(m_1+2c\delta-m_2)}{c(1+2\delta)-1} - \frac{2(1-c)\delta\psi}{c(1+2\delta)-1}. \end{cases}$$

In this case, if $\theta_2^L < \theta_1^H$, the fund can never fully repay $1-c$ in equilibrium. If it further holds that $\theta_2^H < \theta_1^H$, then the fund fails for sure and there is full dynamic coordination. The condition for $\theta_2^L < \theta_1^H$ to hold is $m_2 < m_1 + 1 - c + \frac{\psi[c(1+4\delta)-1]}{c(1+2\delta)}$. The condition for $\theta_2^H < \theta_1^H$ is $m_2 < m_1 + 1 - c + \frac{(1-c)\psi}{c(1+2\delta)}$

□

Given the results in Propositions 15 and 16, one can easily verify:

1. Conditional Inference Effect. Given s_1 and m_2, both W_{2S} and W_{2F} decrease in m_1.

2. Endogenous Correlation Effect. Investor's welfare $E\left[W_2\right]$ increases in m_1 when $m_1 - c - \frac{c\psi}{(1-c)(1+2\delta)} < m_2 < m_1 + 1 - c + \frac{(1-c)\psi}{c(1+2\delta)}$.

With this current payoff structure, one can also think about how much m is actually used up ex post. For example, it could be the case that if the fund is very successful, m is not used; if the fund survives and cannot deliver to the investors the full-promise of return, it has to use m to cover that; if the fund fails, m is just lost or taken away by the manager who runs away. This would result in an intervention cost $\tilde{k}(m, \theta)$ that is contingent on the true state. However, as the government makes the intervention decision ex ante, all it cares about is $\tilde{k}(m, \theta)$ integrated over the prior on θ — an expected cost that can be nested into our general cost specification.

Because the key economic channels remain, their interactions with the intervention cost function lead to implications similar to those in Section 4. What is integral to our model is that the investor payoff cannot be mapped one-to-one with the fundamental state, lest it is fully revealing and the private signal in the second period becomes completely useless, and the intervention loses the coordination effect.

D.4. Normally distributed signals

In this subsection, we discuss the results when the private signals follow Normal distribution, i.e., $\varepsilon_i \sim N\left(0, \delta\right)$.[17] We still assume that investors are non-overlapping to keep matters comparable with our baseline model. The case when investors perfectly overlap can be identically analyzed as one in which $\varepsilon_i \sim N\left(0, \frac{\delta}{2}\right)$. We characterize the equilibrium in each period and emphasize that government intervention in period 1 still has a dynamic coordination effect in period 2 through altering the informational environment.

Lemma 12 below summarizes equilibrium outcomes in two periods.

Lemma 12. *Equilibrium when signals follow Normal distribution*

[17] We use uninformative prior belief which is common in global games literature.

1. *Given* m_1*, there exists unique equilibrium thresholds in period 1:*

$$\theta_1^* = 1 + m_1 - c$$
$$x_1^* = 1 + m_1 - c - \delta\Phi^{-1}(c).$$

2. *Given* (m_1, m_2) *and* $s_1 = S$*,*

   (a) *When* $m_2 > m_1 - c$*,* $(\theta_2^* = \theta_1^*, x_2^* = \infty)$ *consists a threshold equilibrium.*

   (b) *Equilibrium strategies* (θ_2^*, x_2^*) *which satisfy* $\theta_2^* < \theta_1^*$ *and* $x_2^* < \infty$ *may or may not exist. If they exist, they can be non-unique.*

To see this, note that the equilibrium outcome in period 1 is characterized by two thresholds (θ_1^*, x_1^*) that satisfy

$$A_1(\theta_1^*) + m_1 = \theta_1^*$$
$$\Pr\left(\theta < \theta_1^* \middle| x_1 = x_1^*\right) = c.$$

where $A_1(\theta_1^*) = \Pr\left(x_1 < x_1^* \middle| \theta = \theta_1^*\right)$ is the measure of investors who choose to roll over. Simple calculation shows that

$$\theta_1^* = 1 + m_1 - c$$
$$x_1^* = 1 + m_1 - c - \delta\Phi^{-1}(c).$$

The equilibrium in period 2 is again state-independent. We discuss the outcomes when $s_1 = S$ here and the case of $s_1 = F$ is similar. When the intervention in the first period has succeeded, equilibrium in the second period will be either a stage-game equilibrium with full dynamic coordination (similar to Lemma 2), or one with partial dynamic coordination (similar to Lemma 4). The case without dynamic coordination vanishes as the support of the noise now spans between (∞, ∞). The first type of equilibrium is denoted as

$(\theta_2^*, x_2^*) = (\infty, \infty)$ and any equilibrium with $(\theta_2^* > \theta_1^*, x_2^* = \infty)$ is equivalent. The necessary conditions that $(\theta_2^*, x_2^*) = (\infty, \infty)$ constitutes an equilibrium are

$$\begin{aligned} \Pr\left(1 + m_2 > \theta \middle| \theta < \theta_1^*\right) &= 1 \\ \Rightarrow m_2 &> m_1 - c. \end{aligned}$$

Likewise, the necessary conditions that an equilibrium with partial dynamic coordination exists is that the solution (θ_2^*, x_2^*) to the equation system

$$\begin{aligned} A_2\left(\theta_2^*\right) + m_2 &= \theta_2^* \\ \Pr\left(\theta < \theta_2^* \middle| x_2^*, \theta < \theta_1^*\right) &= c. \end{aligned}$$

exists and satisfies $\theta_2^* < \theta_1^*$. Equivalently, we are looking for θ_2^* that solves

$$1 - (\theta_2^* - m_2) = c\Phi\left(\frac{\theta_1^* - \theta_2^* - \delta\Phi^{-1}\left(\theta_2^* - m_2\right)}{\delta}\right) \tag{C.21}$$

We can numerically solve Eq. (C.21), and importantly, Table 4 presents the local comparative statics when there exists a unique equilibrium strategy. When m_1 increases from 0.7 to 0.9, both θ_2^* and x_2^* decrease, validating the dynamic coordination effect.

θ_2^* as a function of m_1 $(s_1 = S)$

m_1	0.7	0.75	0.8	0.85	0.9
θ_2^*	0.7602	0.6981	0.6693	0.6511	0.6384
x_2^*	0.9667	0.8222	0.7565	0.7152	0.6866

Other parameters are $c = 0.5, \delta = 0.5, m_2 = 0.1$.

O.5. General distributions of bounded noise

In this section, we relax the assumption that each agent's private noise follows uniform distribution on $[-\delta, \delta]$. Instead, we assume that ε_i follows a general distribution with CDF $G(\cdot)$ on $[-\delta, \delta]$. In addition, we continue to assume that ε_i is i.i.d. across investors. The analysis is meant to show that the three-type of equilibria that we shown in Section 3 robust to more general noise distributions. Indeed, it is the assumption that noises have bounded support that drive the results. The assumption of uniform distribution helps us establish equilibrium uniqueness and derive the closed-form expressions for cutoffs and welfare.

The equilibrium in period 1 is still characterized by two equations

$$\begin{aligned} A_1(\theta_1^*) + m_1 &= \theta_1^* \\ \Pr\left(\theta < \theta_1^* \middle| x_1 = x_1^*\right) &= c. \end{aligned}$$

Given the noise distribution, $A_1(\theta_1^*) = G(x_1^* - \theta_1^*)$ and $\Pr\left(\theta < \theta_1^* \middle| x_1 = x_1^*\right) = 1 - G(x_1^* - \theta_1^*)$. Therefore, we can easily reach the solutions:

$$\begin{aligned} \theta_1^* &= 1 + m_1 - c \\ x_1^* &= 1 + m_1 - c + G^{-1}(1-c). \end{aligned}$$

Next, we turn to equilibrium in period 2. We will consider the case when $s_1 = S$, and the other case can be analyzed similarly.

If the equilibrium does not feature any dynamic coordination, the thresholds in the second period are

$$\begin{aligned} \theta_2^* &= 1 + m_2 - c \\ x_2^* &= 1 + m_2 - c + G^{-1}(1-c) \end{aligned}$$

In this case, if $x_2^* + \delta < \theta_1^* \Rightarrow m_2 < m_1 - \delta - G^{-1}(1-c)$ so that the marginal investor indeed finds the public information useless, then such an equilibrium without dynamic coordination exists.

Likewise, the equilibrium with partial coordination can also be characterized by two equations:

$$\begin{aligned} A_1\left(\theta_2^*\right)+m_2 &= \theta_2^* \Rightarrow G\left(x_2^*-\theta_2^*\right)+m_2=\theta_2^* \\ \Pr\left(\theta<\theta_2^* \middle| x_2=x_2^*, \theta<\theta_1^*\right) &= c \Rightarrow \frac{1-\left(\theta_2^*-m_2\right)}{G\left(\theta_1^*-x_2^*+\delta\right)}=c. \end{aligned}$$

Any solution to the equation system comprises an equilibrium. Note that the second equation is non-monotonic in x_2^* so that the solution in general is not unique. However, for each solution, as m_1 increases, θ_1^* increases and so is θ_2^*. Therefore, the conditional inference effect continues to exist.

Finally, for any solution (θ_2^*, x_2^*) with partial coordination, the condition $\theta_2^* > \theta_1^*$ pins down the relation between m_1 and m_2 such that the equilibrium features full dynamic coordination.