Online appendix

O.1. General δ and c, and equilibrium multiplicity

We now relate our paper to Angeletos et al. (2007) and explain why our baseline model yields unique equilibrium. We show how equilibrium multiplicity is restored via a mechanism isomorphic to the one in their paper. More importantly, we highlight how equilibrium multiplicity could be endogenized by intervention policy.

Angeletos et al. (2007) show that multiple equilibria emerge under the same conditions that guarantee uniqueness in static global games. The results rely on endogenous learning from regime survivals and exogenous learning from private news that arrives over time. Two elements are necessary for this multiplicity result. First, private information interacts with endogenous learning from earlier coordination outcomes. Second, the private information gets very precise as agents continuously receive private signals about the fundamental. Without the first element, the game is equivalent to one in which agents receive only one precise private signal.[30] Our paper shows the equilibrium results without the second element. We show that multiple equilibria may exist when private signals are very precise. That is, when δ gets very small. Likewise, Angeletos et al. (2007) show that there always exists an equilibrium in which no attack occurs after the first period, and this would be the unique equilibrium if agents did not receive any private information after the first period.[31]

To see this, note that the set of parameters we have examined corresponds to imprecise signals ($2\delta > 1$ and $\frac{1}{1+2\delta} < c < \frac{2\delta}{1+2\delta}$. Moreover, the signal does not get more precise because agents are non-overlapping. If we relax the parameter assumptions, or allow agents' signals to become more precise over time, multiplicity follows. Proposition 12 complements Propositions 1 and 2.[32]

[30] The variance of the signal is $Var\left(\frac{\sigma^2}{n}\right)$ with n signals.

[31] One can easily write a two-period version of Angeletos et al. (2007) and show this is the only equilibrium if the private signal is sufficiently imprecise.

[32] Technically, multiple equilibria resurface because we can apply the argument of iterated deletion of dominated regions only from one end of θ space. Despite this, with slight modifications on the intervention cost functions, the main intuitions for the results from earlier sections still apply as long as we are consistent

Proposition 12 (Equilibria with general δ and c)

1. *If* $s_1 = S$ *and* $\frac{2\delta}{2\delta+1} < c < 1$,

   (a) *If* $m_2 < m_1 - c$, *the unique equilibrium is the Subgame Equilibrium without Dynamic Coordination.*

   (b) *If* $m_1 - c < m_2 < m_1 - 2\delta(1-c)$, *all three types of equilibria exist. However, in the Equilibrium with Partial Dynamic Coordination, the threshold* θ_2^* *decreases with* m_2,

   (c) *If* $m_1 - 2\delta(1-c) < m_2$, *the unique equilibrium is the Subgame Equilibrium with Dynamic Coordination.*

2. *If* $s_1 = F$ *and* $0 < c < \frac{1}{2\delta+1}$

   (a) *If* $m_2 < m_1 + 2c\delta$, *the unique equilibrium is the Subgame Equilibrium with Dynamic Coordination.*

   (b) *If* $m_1 + 2c\delta < m_2 < m_1 + 1 - c$, *all three types of equilibria exist. However, in the Equilibrium with Partial Dynamic Coordination, the threshold* θ_2^* *decreases with* m_2.

   (c) *If* $m_2 > m_1 + 1 - c$, *the unique equilibrium is the Subgame Equilibrium without Dynamic Coordination.*

Our model also differs from Angeletos et al. (2007) in two additional ways: the government's action is endogenous and the private signal is bounded.[33] Government's action therefore affects equilibrium selection and learning. In particular, when the government's intervention induces equilibria with full or no dynamic coordination, it shuts down the interaction between private signal and public learning. Consequently, the equilibrium is unique even if the signal is infinitely precise. In this regard, the government's endogenous intervention can determine the equilibrium multiplicity.

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with equilibrium selection.

[33](Uniform $[-\delta, \delta]$) in our model but unbounded support in their model ($N\left(z, \frac{1}{\alpha}\right)$).

O.2. Moral hazard

Moral hazard is a big concern in government bailouts and interventions. Indeed, fund managers may divert the capital injected by the government, or gamble by investing in projects with risk profiles different from the pre-specification. We now demonstrate that our general cost specification already encompasses many forms of moral hazard. In particular, we show managerial stealing and risk shifting provide micro-foundations for the intervention cost. Moreover, by modeling moral hazard, we enrich the model with fund managers' utility function and endogenous actions, making the analysis more realistic and relevant.

Cash diversion

First consider the case in which the fund manager is able to divert a fraction $\eta \in [0,1]$ for any amount of liquidity μ injected by the government. Suppose the fund manager gets compensated a fraction of π of the surplus she generates for investors, and among the diverted capital, she can consume $f(\eta)\mu \leq \eta\mu$, where $f : [0,1] \mapsto [0,1)$ satisfies $f'(\cdot) > 0$ and $f''(\cdot) \leq 0$. The rest $\mu[\eta - f(\eta)]$ is inefficiently lost (iceberg costs), consistent with the standard assumption in the literature that cash diversion is increasingly inefficient in the amount diverted.[34] Because the fund manager only cares about her own fund, she does not internalize the intervention externality. Thus, to pin down the unique η, she equalizes the marginal benefit of keeping more injected liquidity in the fund, $\frac{\pi(1-c)\mu}{2B}$, to the marginal benefit of stealing more $f'(\eta)\mu$.

Under this setup, the optimal intervention problem is isomorphic to the problem solved earlier, where intervention incurs a cost $k(m)$ in the period. To see this, note the government is aware of the diverting technology. Therefore, to effectively inject m into the fund, the government needs to spend μ such that $(1-\eta)\mu = m$. Equivalently, injecting m into the fund costs the government $k(m) = k_o\left(\frac{m}{1-\eta}\right) - \frac{f(\eta)m}{1-\eta}$, where $k_o(\mu)$ is other social costs not associated with cash diversion. If k_o is increasing and convex in m, adding the moral hazard cost preserves these properties, consistent with our cost specification.

[34] This specification captures the fact that the fraction of diversion matters for the efficiency loss. Alternatively, one could use $f(\eta\mu)$, where $f : [0,\mu] \mapsto [0,\mu)$ satisfies $f'(\cdot) > 0$ and $f''(\cdot) \leq 0$, which implies the diversion efficiency depends on the total amount. This alternative specification does not affect our conclusion.

Risk shifting

Now consider the case in which the fund manager could secretly choose projects with survival threshold $\theta + \Delta$ with a corresponding private payoff $\alpha\Delta$ conditional on' success after paying the promised payoffs to investors (or some asymmetric split of the additional payoff). The project is thus more illiquid and risky (failure probability is higher), but the fund manager has an incentive to shift the risk, because she captures the upside (limited liability means she does not incur additional loss upon failure). Let the fixed cost of risk shifting be $c_o \geq 0$, and then given the liquidity injection m, the optimal risk shifting is

$$\Delta^* = argmax_{\Delta} \left[\alpha\Delta \frac{1-c+m-\Delta}{2B} - c_o \mathbb{I}_{\{\Delta>0\}} \right] = \frac{1-c+m}{2} \mathbb{I}_{\left\{c_o < \frac{\alpha(1-c+m)^2}{8B}\right\}}. \tag{54}$$

The greater the intervention, the greater the distortion in investment by the manager. Compared to the case in which moral hazard is absent, the welfare is reduced by

$$\left[c_o + \frac{\Delta^*(1-c)}{2B} - \frac{1-c+m-\Delta^*}{2B} \alpha\Delta^* \right] \mathbb{I}_{\left\{c_o < \frac{\alpha\Delta^*(1-c+m-\Delta^*)^2}{2B}\right\}}, \tag{55}$$

where the first term is the reduction in welfare due to a lower probability of fund survival, and the second term is the private benefit to the fund manager. For simplicity, we assume $c_o > \frac{\alpha(1-c)^2}{8B}$ and α is sufficiently small (e.g., $\alpha < \frac{1-c}{1-c+\bar{m}}$) so that moral hazard is only induced by the intervention. Then the moral hazard cost can be nested in $k(m) = k_o(m) + \left[c_o + \frac{-\alpha m^2 + 2m(1-c)(1-\alpha) + (2-\alpha)(1-c)^2}{8B} \right] \mathbb{I}_{\left\{c_o < \frac{\alpha(1-c+m)^2}{8B}\right\}}$. Once again, $k(m)$ is weakly increasing and convex in m.

Because the sum of increasing and convex functions is still increasing and convex, our general cost function accommodates multiple types of moral hazard. In other words, moral hazard considerations constitute and motivate the general cost function we use. For example, when the moral hazard of risk shifting and stealing are both present, their costs can still be represented by the general cost function in our model, and such moral hazard costs motivate the cost specification.

O.3. θ-dependent payoff structures

In this section, we extend the analysis in Section 3 to the case whereby the fundamental θ impacts not only the probability of the fund's survival but also investors' payoff structure. The government's intervention $\{m_1, m_2\}$ then impacts both the probability that the fund survives and the actual amount investors receive. We extend our model in three alternative specifications and show that most results and economic intuition remain. The Equilibrium without Dynamic Coordination may not exist under certain conditions in the extensions, but it is a sub-game equilibrium in which neither endogenous correlation nor conditional inference has effect, and do not matter for our results. Below we elaborate.

Extensions 1 and 2

In this section, we introduce an alternative payoff structure in which θ directly impacts the eventual profitability of investment. We follow Goldstein and Pauzner (2005) by assuming that each fund's investment succeeds with probability $p(\theta)$ and fails with probability $1 - p(\theta)$, where $\theta \sim Unif[-B, B]$ is interpreted as the fundamental. To be consistent, we assume $p' \leq (\theta) < 0$ so that higher θ reduces the possibility of investment success. Conditional on fund's investment success, investors receive payoff $1 - c(\theta)$ if and only if the fund survives a run. Let A continue to be the measure of investors who choose to stay. The condition for the fund to survive a run is $A \geq \theta$. If the fund fails, either due to fundamental investment failure which occurs with probability $1 - p(\theta)$ or due to more endogenous investor runs ($A < \theta$), each investor receives payoff $-d(\theta)$.

The government can intervene by increasing each investor's return by m even if the fund fails. In other words, each investor's return increase to $m - d(\theta)$ with the intervention. Under this setup, the fundamental θ affects the NPV of investment through its effect on both the probability of investment success and the return conditional on success. The intervention therefore requires the government to allocate rather than just promise real resources. Following the baseline model, we continue to assume that θ is unobservable and each investor has a private signal about it.

Since investors in period 2 can observe the outcome of period 1—including the payoff $1-c(\theta)$ conditional on survival and $m_1 - d(\theta)$ conditional on failure, they could perfectly infer the realization of θ if there were a one-to-one mapping between either $c(\theta)$ and θ, or $d(\theta)$ and θ. In that case, the game in the second period no longer features incomplete information, and both the coordination issue and equilibrium multiplicity rise again. To avoid such perfect inference and to introduce fewer deviations from the baseline model, we assume $c(\theta)$ is flat and $d(\theta)$ is piecewise linear: $c(\theta) \equiv c$ and $d(\theta) \equiv \begin{cases} -c & \text{if } \theta \leq \bar{\theta} \\ -c-d & \text{if } \theta > \bar{\theta} \end{cases}$, where $d \geq 0$. $\bar{\theta}$ is a threshold which can be thought of as barriers in the fund's investment technology. Table 2 shows the payoff under this setup.

Table 2: Payoffs in the Extended Setup

	Stay	Run
Survive	$1-c$	0
Fail	$\begin{cases} m-c & \text{if } \theta \leq \bar{\theta} \\ m-c-d & \text{if } \theta > \bar{\theta} \end{cases}$	0

For the remaining analysis, we study two special cases. In the first case, we assume $p(\theta) \equiv 1$ so that the fund can only fail due to investor runs. In this case, the fundamental θ only directly affects $c(\theta)$ and $d(\theta)$, the payoff conditional on survival/failure. We solve the model and show that the results from the baseline model go through. In the second case, we assume $d \equiv 0$ so that θ only directly affects $p(\theta)$, the probability of fund's fundamental investment success. We will see the results differ slightly in the sense that the sub-game equilibrium without coordination disappears in the latter case. In both cases, θ directly affects investors' payoff through its effect on the NPV of investment. Moreover, θ also indirectly affect payoff through coordination. Finally, to make the game non-trivial, we assume that both m_1 and m_2 are less than $\bar{m}$, which in turn is strictly less than c.

Extension 1: θ affects conditional investment payoff

Period 1 The equilibrium in period 1 is still characterized by two thresholds (θ_1^*, x_1^*): the fund survives a run if and only $\theta < \theta_1^*$; investor i stays if and only if the observed signal $x_{1i} < x_1^*$. However, the equilibrium is no longer unique; investors' beliefs on the comparison between θ_1^* and $\bar{\theta}$ generate another source of self-fulfilling multiplicity. Intuitively, when investors expect $\theta_1^* < \bar{\theta}$, they behave more cautiously by choosing a lower threshold x_1^*. When investors are more cautious about the staying decision, fewer of them choose to stay for the same set of signals $\{x_{1i}\}_{i\in[0,1]}$, and A_1, the aggregate measure of investors who stay, indeed becomes smaller, leading to a lower survival threshold for the fund θ_1^* and thus fulfilling investors' expectation that $\theta_1^* < \bar{\theta}$. On the other hand, when investors expect $\theta_1^* > \bar{\theta}$, they behave less cautiously by choosing a higher threshold x_1^*. As a result, more of them choose to stay for the same set of signals $\{x_{1i}\}_{i\in[0,1]}$, and A_1 will indeed get higher, leading to a higher survival threshold for the fund θ_1^* and thus fulfilling investors' expectation that $\theta_1^* > \bar{\theta}$. More generally, if the payoff conditional on success or failure experiences jumps, the equilibrium multiplicity may come back. Proposition 13 summarizes the equilibrium in the first period.

Proposition 13

The equilibrium in the first period is characterized by two thresholds (θ_1^*, x_1^*): the fund survives a run if and only if $\theta < \theta_1^*$; investor i stays if and only if the observed signal $x_{1i} < x_1^*$.

1. *If $m_1 < \frac{\bar{\theta}-(1-c)}{\bar{\theta}}$, equilibrium is unique:*

$$\begin{cases} \theta_1^* = \frac{1-c+\frac{\bar{\theta}}{2\delta}d}{1+\frac{1+2\delta}{2\delta}d-m_1} \\ x_1^* = \frac{1-c+\frac{\bar{\theta}}{2\delta}d}{1+\frac{1+2\delta}{2\delta}d-m_1}(1+2\delta)-\delta. \end{cases}$$

   Moreover, $\theta_1^* < \bar{\theta}$.

2. *If $\frac{\bar{\theta}-(1-c)}{\bar{\theta}} < m_1 < \frac{(1+d)\bar{\theta}-(1-c)}{\bar{\theta}}$, there are two equilibria.*

$$\begin{cases} \theta_1^* = \frac{1-c}{1-m_1} \\ x_1^* = \frac{1-c}{1-m_1}(1+2\delta)-\delta \end{cases} \qquad \begin{cases} \theta_1^* = \frac{1-c+\frac{\bar{\theta}}{2\delta}d}{1+\frac{1+2\delta}{2\delta}d-m_1} \\ x_1^* = \frac{1-c+\frac{\bar{\theta}}{2\delta}d}{1+\frac{1+2\delta}{2\delta}d-m_1}(1+2\delta)-\delta. \end{cases}$$

   In the first equilibrium, $\theta_1^* > \bar{\theta}$, where as in the second equilibrium, $\theta_1^* < \bar{\theta}$.

3. *If* $m_1 > \frac{(1+d)\bar{\theta}-(1-c)}{\bar{\theta}}$*, equilibrium is unique*

$$\begin{cases} \theta_1^* = \frac{1-c}{1-m_1} \\ x_1^* = \frac{1-c}{1-m_1}(1+2\delta) - \delta. \end{cases}$$

Moreover, $\theta_1^* > \bar{\theta}$*.*

Proof. When investors expect $\theta_1^* < \bar{\theta}$, the two equations determining equilibria are

$$\begin{aligned} A(\theta_1^*) = \theta_1^* \Rightarrow \frac{x_1^* - (\theta_1^* - \delta)}{2\delta} &= \theta_1^* \\ \Pr(\theta < \theta_1^* | x_1^*)(1-c) + \Pr\left(\theta_1^* < \theta < \bar{\theta}_1 | x_1^*\right)(m_1 - c) + \Pr\left(\theta > \bar{\theta}_1 | x_1^*\right)(m_1 - c - d) &= 0. \end{aligned}$$

Solving the two equations, we get

$$\begin{cases} \theta_1^* = \frac{1-c+\frac{\bar{\theta}}{2\delta}d}{1+\frac{1+2\delta}{2\delta}d-m_1} \\ x_1^* = \frac{1-c+\frac{\bar{\theta}}{2\delta}d}{1+\frac{1+2\delta}{2\delta}d-m_1}(1+2\delta) - \delta. \end{cases}$$

By imposing the requirement $\theta_1^* < \bar{\theta}$, we get $m_1 < \frac{(1+d)\bar{\theta}-(1-c)}{\bar{\theta}}$.

When investors expect $\theta_1^* > \bar{\theta}$, the two equations determining equilibria are

$$\begin{aligned} A(\theta_1^*) = \theta_1^* \Rightarrow \frac{x_1^* - (\theta_1^* - \delta)}{2\delta} &= \theta_1^* \\ \Pr(\theta < \theta_1^* | x_1^*)(1-c) + \Pr(\theta > \theta_1^* | x_1^*)(m_1 - c) &= 0. \end{aligned}$$

Solving the two equations, we get

$$\begin{cases} \theta_1^* = \frac{1-c}{1-m_1} \\ x_1^* = \frac{1-c}{1-m_1}(1+2\delta) - \delta. \end{cases}$$

By imposing the requirement $\theta_1^* > \bar{\theta}$, we get $m_1 > \frac{\bar{\theta}-(1-c)}{\bar{\theta}}$. □

Period 2 We now turn to period 2. Obviously, the analysis differs in whether θ_1^*, the equilibrium fund survival threshold in first period, is higher or lower than $\bar{\theta}$, the cutoff for more negative payment if the fund fails.

1. $\theta_1^* < \bar{\theta}$. In this case, the intervention in period 1 truncates the prior belief on θ into three regions: $[-B, \theta_1^*]$, $\left[\theta_1^*, \bar{\theta}\right]$, and $\left[\bar{\theta}, B\right]$, respectively corresponding to the case that the fund in the first period has succeeded, the fund in the first period has failed and investors who have stayed receive $m_1 - c$, the fund in the first period has failed and investors who have stayed receive $m_1 - c - d$.

2. $\theta_1^* \geq \bar{\theta}$. In this case, the intervention in period 1 again truncates the prior belief on θ into two regions: $[-B, \theta_1^*]$ and $[\theta_1^*, B]$.

Below we will study the first case $\theta_1^* < \bar{\theta}$ which is more general. The second case is a subcase of the first one.

If $s_1 = S$, then it becomes publicly known that $\theta \in [-B, \theta_1^*]$. As in the baseline model, there are also three types of equilibria: Equilibrium Without Dynamic Coordination, with Partial Coordination, and with Coordination.

Proposition 14

Equilibrium in period 2 if $s_1 = S$ and $\theta_1^* < \bar{\theta}$

1. *If $m_2 < 1 - \frac{1+\frac{1+2\delta}{2\delta}d - m_1}{1-c+\frac{\bar{\theta}}{2\delta}d}(1-c)(1+2\delta)$, the equilibrium in period 2 has no dynamic coordination:*

$$\begin{cases} \theta_2^* = \frac{1-c}{1-m_2} \\ x_2^* = \frac{1-c}{1-m_2}(1+2\delta) - \delta. \end{cases}$$

2. *If $1 - \frac{1+\frac{1+2\delta}{2\delta}d - m_1}{1-c+\frac{\bar{\theta}}{2\delta}d}(1-c)(1+2\delta) < m_2 < c - 2(1-c)\delta$,[35] the equilibrium in period 2 has partial*

[35] This condition either requires us to relax the assumption in the baseline model that $c < \frac{2\delta}{1+2\delta}$ or the assumption that $m_2 > 0$.

dynamic coordination:

$$\begin{cases} \theta_2^* = \frac{\theta_1^*(c-m_2)-2(1-c)\delta}{c-m_2-2(1-c)\delta} \\ x_2^* = \theta_2^*(1+2\delta)+\delta. \end{cases}$$

3. *If $m_2 > c - 2(1-c)\delta$, the equilibrium in period 1 has dynamic coordination.*

Proof. Consider first that there is no dynamic coordination. Since it is publicly known that $\theta \in [-B, \theta_1^*]$ and that $\theta_1^* < \bar{\theta}$, investors in the second period can never receive payoff $m_2 - c - d$. Therefore, ignoring the public news that $\theta < \theta_1^*$, the thresholds that determine equilibrium are similar to Case 3 of Proposition 13

$$\begin{cases} \theta_2^* = \frac{1-c}{1-m_2} \\ x_2^* = \frac{1-c}{1-m_2}(1+2\delta)-\delta. \end{cases}$$

The sufficient and necessary condition for the equilibrium to have no dynamic coordination is

$$x_2^* + \delta < \theta_1^*,$$

which leads to the condition $m_2 < 1 - \frac{1+\frac{1+2\delta}{2\delta}d-m_1}{1-c+\frac{\theta}{2\delta}d}(1-c)(1+2\delta)$.

Consider next the Equilibrium with Dynamic Coordination, the thresholds are now determined by

$$\begin{aligned} A(\theta_2^*) = \frac{x_2^* - (\theta_2^* - \delta)}{2\delta} &= \theta_2^* \\ \Pr(\theta < \theta_2^* | x_2^*, \theta < \theta_1^*)(1-c) + \Pr(\theta > \theta_2^* | x_2^*, \theta < \theta_1^*)(m_2 - c) &= 0. \end{aligned}$$

The solutions are

$$\begin{cases} \theta_2^* = \frac{\theta_1^*(c-m_2)-2(1-c)\delta}{c-m_2-2(1-c)\delta} \\ x_2^* = \theta_2^*(1+2\delta)+\delta. \end{cases}$$

The sufficient and necessary condition for the equilibrium to have dynamic coordination is

$$\theta_2^* > \theta_1^*,$$

which lead to the condition $m_2 > c - 2(1-c)\delta$. □

The expressions for conditional payoffs W_2^{nc}, W_2^{pc}, and W_2^{c} get lengthier and are omitted to avoid complication. However, it is clear that θ_1^* increases with m_1 and in the case with dynamic coordination, investors continue to receive $1-c$ if $s_1 = S$ and 0 if $s_1 = F$. Therefore, the *Endogenous Coordination Effect* and *Conditional Inference Effect* continue to exist.

Corollary 4

The Endogenous Coordination Effect *and* Conditional Inference Effect *continue to exist.*

The remaining cases $\theta \in \left[\theta_1^*, \bar{\theta}\right]$, $\theta \in \left[\bar{\theta}, B\right]$ under $s_1 = S$ and $\theta_1^* < \bar{\theta}$ can be analyzed in a similar way. So are the cases when $\theta_1^* > \bar{\theta}$ and $s_1 = F$. To avoid clunky and repeated expressions and statements, we omit the details. However, we would like to point out one exception when $\theta \in \left[\theta_1^*, \bar{\theta}\right]$. In this case, the length of the support of the updated belief on θ becomes $\bar{\theta} - \theta_1^*$, which can be smaller than 2δ. In this case, the Equilibrium without Dynamic Coordination may disappear. Indeed, this result depends on the prior belief on θ to be sufficiently wide. In the case $\theta \in \left[\theta_1^*, \bar{\theta}\right]$, the truncations $\theta > \theta_1^*$ and $\theta < \bar{\theta}$ at least partially helpsupdate the marginal investor's belief in additional to his private signal x_2^*. In Extension 2 and also Section C, we will show that the result of the Equilibrium without Dynamic Coordination further depends on the assumption that private noises follow uniform distribution, and the payoff conditional on success or failure is sufficiently flat and thus non-informative of the fundamental θ.

However, since the other two types of sub-game equilibria continue to exist, we would like to emphasize that the two key channels in this paper, *Conditional Inference Effect* and *Endogenous Coordination Effect*, remain in the current extension. Therefore, the interactions with the intervention cost function lead to implications similar to those in Section 4.

Extension 2: θ affects investment success probability

Setup In this section, we assume $p(\theta) \in (0,1)$ and $c(\theta) = d(\theta) \equiv c$. Table 3 shows the payoff under this extension. Note that survival requires two conditions. First, the fund's investment needs to succeed, which occurs with probability $p(\theta) < 1$. Second, enough investors must stay and the fund thus survives a run: $A > \theta$. We continue to assume that $m \in [0, \bar{m}]$ and $\bar{m} < c$. Otherwise, the government can guarantee higher payoff for investors who choose to stay.

Table 3: Payoffs in Case 2

	Stay	Run
Survive	$1 - c$	0
Fail	$m - c$	0

Period 1 The equilibrium in the first period is again captured by two thresholds (θ_1^*, x_1^*). If $\theta = \theta_1^*$, then $A(\theta_1^*) = \theta_1^*$ so that if the fund's investment succeeds, the extent of coordination among investors is just enough to guarantee the fund to survive. Moreover, each investor adopts a threshold strategy $a_i = \mathbb{1}\{x_i \leq x^*\}$: he stays if and only if the probability of receiving $1 - c$ exceeds $\frac{c-m}{1-m}$. Therefore, the two equations that characterize the two thresholds are

$$\begin{cases} A(\theta_1^*) = \frac{x_1^* - (\theta_1^* - \delta)}{2\delta} = \theta_1^* \\ \int_{x_1^* - \delta}^{\theta_1^*} \frac{p(\theta)}{2\delta} d\theta = \frac{c-m}{1-m}. \end{cases}$$

Note that if $p(\theta) \equiv 1$ and $m = 0$, the solutions are identical to that in standard regime shifting games: $(\theta_1^*, x_1^*) = (1 - c, 1 - c + \delta(1 - 2c))$.

In general, θ_1^* is the solution to the following equation

$$\int_{\theta_1^*(1+2\delta)-2\delta}^{\theta_1^*} \frac{p(\theta)}{2\delta} = \frac{c-m}{1-m}.$$

To get closed-form solutions, one needs to specify a functional form for $p(\theta)$. Without narrowing us to any ad hoc functional form, we can still proceed with the qualitative analysis to show that the conditional inference effect remains: since $p(\theta)$ is a decreasing function of θ and the length of the interval $[\theta_1^*(1 + 2\delta) - 2\delta, \theta_1^*]$ decreases with θ_1^*, there exists a unique solution to θ_1^*. Moreover, θ_1^* increases with m_1— the conditional inference effect. Finally, $x_1^* = \theta_1^*(1 + 2\delta) - 2\delta$.

Next, we move on to the analysis in the second period. Upon observing that investors have received $1-c$ in the first period, investors again get more optimistic about the distribution of θ. Their optimism can be decomposed into two effects.

1. Truncation. After observing a payoff $1-c$, investors can safely conclude that $\theta \in [-B, \theta_1^*]$. In other words, the regions where θ is very high gets excluded. Moreover, θ_1^* increases with m_1 so that the conditional inference effect carries over.

2. More optimistic on the untruncated region. While the support of θ is truncated from above: $\theta \in [-B, \theta_1^*]$, the belief on the distribution of θ on $[-B, \theta_1^*]$ is also updated. In particular, while the prior distribution of θ is uniform on $[-B, B]$, the updated distribution is no longer uniform. It involves updating on $p(\theta)$ as well.

   To see this, let's assume $p(\theta) = \frac{\theta+B}{2B} \sim Unif[0,1]$ for simplicity. Let us temporarily ignore the truncation effect so that upon observing investors have received $1-c$, the support of θ remains $[-B, B]$. Even so, observing $s_1 = S$ increases the probability density on high $p(\theta)$ (low θ) and decreases the probability density on low $p(\theta)$ (high θ). The updated belief distribution should entail this optimism. In particular, the updated belief becomes $p(\theta) \sim \beta(2,1)$ after $s_1 = S$.[36] In other words, the belief on θ gets skewed towards lower realizations. With the truncation effect, the updated distribution of θ becomes a conditional beta distribution on $[-B, \theta_1^*]$.

Combining both effects, the second-period stage equilibrium can never be one without dynamic coordination. In other words, the equilibrium in period 2 is either the *Stage Game Equilibrium with Partial Coordination* when the conditional inference effect more likely dominates; or *Stage Game Equilibrium with Coordination* when the endogenous correlation effect dominates. Consequently, since the updated belief of θ on $[-B, \theta_1^*]$ is no longer uniform, the second-period stage equilibrium may no longer be unique. However, the conditional inference effect also remains. The remaining analysis, which includes solving equilibrium cutoffs (numerically) for different equilibria, follows the analysis in Section O.5.

[36] The updated belief becomes $p(\theta) \sim \beta(1,2)$ after $s_1 = F$

Extension 3

In this subsection, we introduce a different extension in which the fundamental θ and government intervention m directly affect investors' payoff. Specifically, we keep the setup in Section 2.2 except for the following payoff structure for each investor:

$$V_{ti} = \begin{cases} 1-c & \text{if } A_t + m_t \geq \theta + \psi \\ -c + \frac{1}{2\psi}\left[A_t + m_t - (\theta - \psi)\right] & \text{if } \theta - \psi < A_t + m_t < \theta + \psi \\ -c & \text{if } A_t + m_t \leq \theta - \psi. \end{cases}$$

Each investor receives $1 - c$ only if the fund fully survives: $A_t + m_t \geq \theta + \psi$. In contrast, each investor receives $-c$ if the fund fully fails: $A_t + m_t \leq \theta - \psi$. If the fund partially survives (or fails), however, i.e., if $\theta - \psi < A_t + m_t < \theta + \psi$, the investor's payoff is linear in the fundamental θ. Note that the baseline model refers to one when $\psi \to 0$. We continue to assume that investors in period 2 can observe the outcome of period 1.

This payoff structure can be interpreted as debts or deposits, and generally applies to cases where an investor's payoff is capped above when the fund is very successful, but is zero if the fund fails. $A_t + m_t \geq \theta + \psi$ can be interpreted as that the project is successful enough that every staying investor gets the promised principal and interest (or promised return as in the case of wealth management products some funds offer); if the situation is really bad $A_t + m_t \leq \theta - \psi$, the fund has nothing left for the investors; if the fund does poorly but does not completely fail $\theta - \psi < A_t + m_t < \theta + \psi$, the investors get some payoff less than what is originally promised (think of them as debts who have senior claims on the limited revenue the fund generates). ψ is then a parameter capturing the range of fundamental states that leads to the fund to survive but fails to deliver to the investors the full promised return.

Lemma 10 summarizes the equilibrium in period 1.

Lemma 10

In the stage game, there exists a unique symmetric PBE in monotone strategies $\left(\theta_1^L, \theta_1^H, x_1^*\right)$ *where*

$$\begin{cases} \theta_1^L & = 1 + m_1 - c - \frac{2\delta\psi}{1+2\delta\psi} \\ \theta_1^H & = 1 + m_1 - c + \frac{2\delta\psi}{1+2\delta\psi} \\ x_1^* & = 1 + m_1 - c + \delta\left(1 - 2c\right). \end{cases}$$

Each investor's strategy follows $a_i = \mathbb{1}\left\{x_i \leq x_1^*\right\}$. *The fund pays off each investor* $1 - c$ *if* $\theta \leq \theta_1^L$, $-c + \frac{1}{2\psi}\left[A_1 + m_1 - (\theta - \psi)\right]$ *if* $\theta \in \left(\theta_1^L, \theta_1^H\right)$ *and* $-c$ *if* $\theta \geq \theta_1^H$. *Moreover,* $A_1 = \frac{x_1^* - (\theta - \delta)}{2\delta}$.

Lemma 10 is the extended version of Lemma 1. Note that if $\psi = 0$, both thresholds are identical to θ_1^*: $\theta_1^L = \theta_1^H = \theta_1^*$, and the equilibrium is identical to one when θ only impacts the fund's survival. When θ also directly enters investor's payoff, the equilibrium is characterized by three thresholds. Specifically, θ_1^L is the threshold of θ under which the fund fully succeeds, and θ_1^H is the threshold above which the fund fully fails. If $\theta \in \left(\theta_1^L, \theta_1^H\right)$, the fund succeeds (or equivalently fails) only partially and repays between $-c$ and $1 - c$.

Proof. The payoff structure satisfies A.1 to A.5 in Section 2.2.1 of Morris and Shin (2003). According to Proposition 2.1 there, each investor's strategy is a threshold strategy. We conjecture that the equilibrium is characterized by three thresholds that satisfy three equations

$$A\left(\theta_1^H\right) + m_1 - \left(\theta_1^H - \psi\right) = 0 \quad (56)$$

$$A\left(\theta_1^L\right) + m_1 - \left(\theta_1^L + \psi\right) = 0 \quad (57)$$

$$\Pr\left(\theta < \theta_1^L \middle| x_1^*\right)(1 - c) + \int_{\theta_1^L}^{\theta_1^H} \left\{-c + \frac{1}{2\psi}\left[A\left(\theta\right) + m_1 - \theta + \psi\right]\right\} d\theta + \Pr\left(\theta > \theta_1^H \middle| x_2^*\right)(-c) = 0 \quad (58)$$

Solving the three equations, we get the solutions as below

$$\begin{cases} \theta_1^L & = 1 + m_1 - c - \frac{2\delta\psi}{1+2\delta\psi} \\ \theta_1^H & = 1 + m_1 - c + \frac{2\delta\psi}{1+2\delta\psi} \\ x_1^* & = 1 + m_1 - c + \delta\left(1 - 2c\right). \end{cases} \quad (59)$$

□

Lemma 11 shows the total welfare.

Lemma 11

The total welfare is $W_1 = \frac{(1-c)[1+B-c(1+\delta)+m_1]}{2B} - \frac{\delta\psi^2}{6B(1+2\delta)^2}$.

Note that when $\psi = 0$, the social welfare is identical to what we have in Section 2.1.3. As ψ increases, the total social welfare decreases. Intuitively, larger ψ increases the uncertainty on payoff, the difficulty in coordination, and thus the overall welfare.

Next, we turn to the equilibrium in period 2. After the outcome of period-1 intervention gets publicly known, the belief on θ is either partitioned or precisely known. Specifically, if investors receive $1-c$, it becomes public knowledge that $\theta \in \left[-B, \theta_1^L\right]$. In contrast, it becomes publicly known that $\theta \in \left[\theta_1^H, B\right]$ if investors receive $-c$. If investors receive $V_1 \in (1-c, c)$, they can perfectly infer the true realization θ, which is the solution to the following equation:

$$V_1 = -c + \frac{1}{2\psi}\left[A_1(\theta) + m_1 - (\theta - \psi)\right],$$

where $A_1(\theta) = \frac{x_1^* - (\theta - \delta)}{2\delta}$. Since m_1 is publicly announced, and $A_1(\theta)$ decreases with θ, the above equation admits a unique solution when $V_1 \in (-c, 1-c)$. In this case, θ becomes public information in period 2, and the equilibrium may not be unique. For the remainder of this section, we focus on the case that $\theta \in \left[-B, \theta_1^L\right]$ and $\theta \in \left[\theta_1^H, B\right]$ to avoid the issue of equilibrium selection.

If investors have received $1-c$ in period 1, the prior knowledge on θ is updated as $\left[-B, \theta_1^L\right]$. Similar to the baseline model, the equilibrium in period 2 can occur with, without or with partial dynamic coordination, summarized as follows. We assume ψ is sufficiently small such that $\frac{\psi}{1+2\delta} < 1-c$.

Proposition 15

Equilibrium in Period 2 when $s_1 = S$

1. *If* $m_2 < m_1 - 2\delta(1-c) - \frac{2\delta\psi}{1+2\delta}$*, the unique equilibrium is the Stage Game Equilibrium without Dynamic Coordination. The total social welfare is* $W_{2S}^{nc} = \frac{(1-c)[1+B-c(1+\delta)+m_2]}{B+\theta_1^L} - \frac{\delta\psi^2}{3\left(B+\theta_1^L\right)(1+2\delta)^2}$.

2. *If* $m_1 - 2\delta(1-c) - \frac{2\delta\psi}{1+2\delta} < m_2 < m_1 - c - \frac{c\psi}{(1-c)(1+2\delta)}$*, the unique equilibrium is the Stage Game Equilibrium with Partial Dynamic Coordination.*

   (a) *If* $m_1 - 2\delta(1-c) - \frac{2\delta\psi}{1+2\delta} < m_2 < m_1 - c - \frac{\psi[4(1-c)\delta - c]}{(1-c)(1+2\delta)}$*, in the unique equilibrium, the fund may succeed, fail or partially fail. The total social welfare is*

$$
\begin{aligned}
W_{2S}^{pc} &= \frac{(1-c)}{B+\theta_1^L}\left[\theta_1^L + B + \frac{\delta c(c-m_1+m_2)^2 + 2\delta(c-m_1+m_2)[2\delta - c(1+2\delta)]}{[2\delta - c(1+2\delta)]^2}\right] \\
&- \frac{\delta\psi\left\{12(c-1)\delta(1+2\delta)[c(-1+c-m_1+m_2-2\delta)+2\delta] + \psi\left[c^2 - 4(1-c)\delta(c-\delta+4c\delta)\right]\right\}}{3(1+2\delta)^2[2\delta - c(1+2\delta)]^2\left(\theta_1^L + B\right)}.
\end{aligned}
$$

   (b) *If* $m_1 - c - \frac{\psi[4(1-c)\delta-c]}{(1-c)(1+2\delta)} < m_2 < m_1 - c - \frac{c\psi}{(1-c)(1+2\delta)}$*, in the unique equilibrium, the fund can never fail. The total social welfare is* $W_{2S}^{pc} = \frac{(1-c)}{B+\theta_1^L}\left[\theta_1^L + B + \frac{\delta c(c-m_1+m_2)^2 + 2\delta(c-m_1+m_2)[2\delta-c(1+2\delta)]}{[2\delta-c(1+2\delta)]^2}\right] - H_1(\psi)$ *where* $H_1(\psi) > 0$ *and converges to* 0 *as* $\psi \to 0$.[37]

3. *If* $m_2 > m_1 - c - \frac{c\psi}{(1-c)(1+2\delta)}$*, the unique equilibrium is the Stage Game Equilibrium with Dynamic Coordination. In this equilibrium, the fund will always succeed and repay* $1-c$*. The social welfare is* $W_{2S}^{c} = 1-c$*.*

Proof. Again, we solve the model respectively in three cases: equilibrium with dynamic coordination, with partial coordination and without coordination.

1. Equilibrium without dynamic coordination. We solve the equilibrium assuming no dynamic coordination. The solutions are similar to those in period 1:

$$
\begin{cases}
\theta_2^L &= 1 + m_2 - c - \frac{2\delta\psi}{1+2\delta\psi} \\
\theta_2^H &= 1 + m_2 - c + \frac{2\delta\psi}{1+2\delta\psi} \\
x_2^* &= 1 + m_2 - c + \delta(1-2c).
\end{cases}
\tag{60}
$$

   A necessary condition for the equilibrium to feature no dynamic coordination is the equilibrium x_2^* satisfies $x_2^* + \delta < \theta_1^L$ so that the marginal investor finds the period-1 information useless. Moreover,

[37] We leave out the complicated closed-form expression for $H_1(\psi)$ which is available upon request.

$\theta_2^H < \theta_1^L$ so that it is indeed possible that the fund in period 2 still fails. Combining both inequalities and the assumption $\frac{\psi}{1+2\delta} < 1-c$, the condition for this case is derived as $m_2 < m_1 - 2\delta(1-c) - \frac{2\delta\psi}{1+2\delta}$.

2. Equilibrium with partial coordination. In this case, the marginal investor finds the information in period 1 being useful. As a result, the equation which shows the marginal investor is indifferent between two actions is

$$\begin{aligned} \Pr\left(\theta < \theta_2^L \middle| x_2^*, \theta < \theta_1^L\right)(1-c) + \int_{\theta_2^L}^{\theta_2^H} \left\{-c + \frac{1}{2\psi}\left[A(\theta) + m_2 - \theta + \psi\right]\right\} d\theta + \Pr\left(\theta > \theta_2^H \middle| x_2^*, \theta < \theta_1^L\right)(-c) &= 0 \\ \Rightarrow \frac{\theta_2^L - (x_2^* - \delta)}{\theta_1^L - (x_2^* - \delta)} \times (1-c) + \frac{\theta_1^L - \theta_2^H}{\theta_1^L - (x_2^* - \delta)} \times (-c) + \frac{\theta_2^H - \theta_2^L}{\theta_1^L - (x_2^* - \delta)} \times \left(\frac{1}{2} - c\right) &= 0. \end{aligned}$$

Note that the denominator is replaced by $\theta_1^L - (x_2^* - \delta)$ which is less than 2δ since the information in period 1 truncates the range of θ coming from the marginal investor's private signal alone. The new thresholds are

$$\begin{cases} \theta_2^L &= 1 + m_2 - c - \frac{c[m_2-(m_1-c)]}{c-2(1-c)\delta} - \frac{4\delta\psi}{1+2\delta}\frac{c-(1-c)\delta}{c-2(1-c)\delta} \\ \theta_2^H &= 1 + m_2 - c - \frac{c[m_2-(m_1-c)]}{c-2(1-c)\delta} - \frac{4\delta^2\psi}{1+2\delta}\frac{1-c}{c-2(1-c)\delta} \\ x_2^* &= 1 + m_2 - c + \delta(1-2c) - \frac{(1-c)(1+2\delta)(m_1+2c\delta-m_2)}{c(1+2\delta)-1} - \frac{2c\delta\psi}{c-2(1-c)\delta}. \end{cases} \quad (61)$$

Note that if $\psi = 0$, the thresholds are exactly equal to Lemma 4. In this case, if $\theta_2^H > \theta_1^L$, the fund can never fully fail in the equilibrium with partial dynamic coordination. If it further holds that $\theta_2^L > \theta_1^L$, the fund always fully succeeds, and the equilibrium has dynamic coordination. The condition in 2(a) guarantees that $\theta_2^H < \theta_1^L$ so that the fund may still fail, whereas the condition in 2(b) enables $\theta_2^L < \theta_1^L < \theta_2^H$. Case 3 lists the condition for $\theta_2^L > \theta_1^L$.

□

Proposition 15 is the extended version of Proposition 1. Note that when $\psi \to 0$, the two propositions are identical. In case 1 when m_2 is small relative to m_1, the initial intervention is in vain. In case 3 when m_2 is large relative to m_1, the initial intervention guarantees success for the second fund. When m_2 is in between, the initial intervention has partial coordination effect. Case 2(a) describes a case similar to the equilibrium with partial coordination in the baseline setup. Case 2(b) describes a case when the initial intervention eliminates full failure but not partial failure (success) in the second period.

Proposition 16 summarizes the result when $s_1 = F$. We assume ψ is sufficiently small such that $\psi < \frac{c}{1+2\delta c}$.

Proposition 16

Equilibrium in Period 2 when $s_1 = F$

1. *If* $m_2 < m_1 + 1 - c + \frac{(1-c)\psi}{c(1+2\delta)}$, *the unique equilibrium is the Stage Game Equilibrium with Dynamic Coordination. The total social welfare is* $W_{2F}^{c} = 0$.

2. *If* $m_1 + 1 - c + \frac{(1-c)\psi}{c(1+2\delta)} < m_2 < m_1 + 2c\delta + \frac{2\delta\psi}{1+2\delta\psi}$, *the unique equilibrium is the Stage Game Equilibrium with Partial Dynamic Coordination.*

   (a) *If* $m_1 + 1 - c + \frac{(1-c)\psi}{c(1+2\delta)} < m_2 < m_1 + 1 - c + \frac{\psi[c(1+4\delta)-1]}{c(1+2\delta)}$, *in the unique equilibrium, the fund can never fully succeed. The total social welfare is* $W_{2F}^{pc} = \frac{1-c}{B-\theta_1^L}\frac{(-1+c-m_1+m_2)^2}{(-1+c+2c\delta)^2} + H_2(\psi)$ *where* $H_2(\psi) \to 0$ *as* $\psi \to 0$.[38]

   (b) *If* $m_1 + 1 - c + \frac{\psi[c(1+4\delta)-1]}{c(1+2\delta)} < m_2 < m_1 + 2c\delta + \frac{2\delta\psi}{1+2\delta\psi}$, *in the unique equilibrium, the fund may succeed, fail or partially fail. The total social welfare is* $W_{2F}^{pc} = \frac{1-c}{B-\theta_1^L}\frac{(-1+c-m_1+m_2)^2}{(-1+c+2c\delta)^2} - \frac{\delta\psi\left\{12(-1+c)(-1+c-m_1+m_2)(1+2\delta)(-1+c+c\delta)+\left[-11-24\delta+c\left(34+4\delta(23+9\delta)+12c^2(1+2\delta)^2-c(35+4\delta(29+20\delta))\right)\right]\psi\right\}}{3(1+2\delta)^2(-1+c+2c\delta)^2\left(B-\theta_1^L\right)}$.

3. *If* $m_2 > m_1 + 2c\delta + \frac{2\delta\psi}{1+2\delta\psi}$, *the unique equilibrium is the Stage Game Equilibrium without Dynamic Coordination. The total social welfare is* $W_{2F}^{nc} = \frac{1-c}{B-\theta_1^L}(m_2 - m_1 - c\delta) - \frac{\psi\delta[\psi-6(1-c)(1+2\delta)]}{3(1+2\delta)^2\left(B-\theta_1^L\right)}$.

Proof. We solve the model respectively in three cases: equilibrium with dynamic coordination, with partial coordination and without coordination.

[38] The detailed expression for $H_2(\psi)$ is available upon request.

1. Equilibrium without dynamic coordination. Assuming no dynamic coordination. The solutions are:

$$\begin{cases} \theta_2^L & = 1 + m_2 - c - \frac{2\delta\psi}{1+2\delta\psi} \\ \theta_2^H & = 1 + m_2 - c + \frac{2\delta\psi}{1+2\delta\psi} \\ x_2^* & = 1 + m_2 - c + \delta\left(1 - 2c\right). \end{cases}$$

The conditions for no dynamic coordination are:

$$\begin{cases} x_2^* - \delta > \theta_1^H \\ \theta_2^L > \theta_2^H. \end{cases}$$

Then, as long as ψ is sufficiently small such that $\psi < \frac{c}{1+2\delta c}$, the equilibrium without dynamic coordination exists if and only if $m_2 > m_1 + 2c\delta + \frac{2\delta\psi}{1+2\delta\psi}$. Note that if $\psi \to 0$, the condition is the same as Proposition 2.

2. Equilibrium with partial coordination. In this case, the equation which characterizes the marginal investor's indifference is:

$$\begin{aligned} \Pr\left(\theta < \theta_2^L \middle| x_2^*, \theta > \theta_2^H\right)(1-c) + \int_{\theta_2^L}^{\theta_2^H} \left\{-c + \frac{1}{2\psi}\left[A\left(\theta\right) + m_2 - \theta + \psi\right]\right\} d\theta + \Pr\left(\theta > \theta_2^H \middle| x_2^*, \theta > \theta_2^H\right)(-c) &= 0 \\ \Rightarrow \frac{\theta_2^L - \theta_1^H}{(x_2^* + \delta) - \theta_1^H} \times (1-c) + \frac{(x_2^* + \delta) - \theta_2^H}{(x_2^* + \delta) - \theta_1^H} \times (-c) + \frac{\theta_2^H - \theta_2^L}{(x_2^* + \delta) - \theta_1^H} \times \left(\frac{1}{2} - c\right) &= 0. \end{aligned}$$

The solutions are

$$\begin{cases} \theta_2^L & = 1 + m_2 - c - \frac{(1-c)(m_1+2c\delta-m_2)}{c(1+2\delta)-1} - \frac{4c\delta^2\psi}{(1+2\delta)[c(1+2\delta)-1]} \\ \theta_2^H & = 1 + m_2 - c - \frac{(1-c)(m_1+2c\delta-m_2)}{c(1+2\delta)-1} + \frac{4\delta(-1+c+c\delta)\psi}{(1+2\delta)[c(1+2\delta)-1]} \\ x_2^* & = 1 + m_2 - c + \delta\left(1 - 2c\right) - \frac{(1-c)(1+2\delta)(m_1+2c\delta-m_2)}{c(1+2\delta)-1} - \frac{2(1-c)\delta\psi}{c(1+2\delta)-1}. \end{cases}$$

In this case, if $\theta_2^L < \theta_1^H$, the fund can never fully repay $1-c$ in equilibrium. If it further holds that

$\theta_2^H < \theta_1^H$, then the fund fails for sure and there is full dynamic coordination. The condition for $\theta_2^L < \theta_1^H$ to hold is $m_2 < m_1 + 1 - c + \frac{\psi[c(1+4\delta)-1]}{c(1+2\delta)}$. The condition for $\theta_2^H < \theta_1^H$ is $m_2 < m_1 + 1 - c + \frac{(1-c)\psi}{c(1+2\delta)}$

□

Given the results in Propositions 15 and 16, one can easily verify:

1. Conditional Inference Effect. Given s_1 and m_2, both W_{2S} and W_{2F} decrease in m_1.
2. Endogenous Correlation Effect. Investor's welfare $E[W_2]$ increases in m_1 when $m_1 - c - \frac{c\psi}{(1-c)(1+2\delta)} < m_2 < m_1 + 1 - c + \frac{(1-c)\psi}{c(1+2\delta)}$.

With this current payoff structure, one can also think about how much m is actually used up ex post. For example, it could be the case that if the fund is very successful, m is not used; if the fund survives and cannot deliver to the investors the full-promise of return, it has to use m to cover that; if the fund fails, m is just lost or taken away by the manager who runs away. This would result in an intervention cost $\tilde{k}(m, \theta)$ that is contingent on the true state. However, as the government makes the intervention decision ex ante, all it cares about is $\tilde{k}(m, \theta)$ integrated over the prior on θ — an expected cost that can be nested into our general cost specification.

Because the key economic channels remain, their interactions with the intervention cost function lead to implications similar to those in Section 4. What is integral to our model is that the investor payoff cannot be mapped one-to-one with the fundamental state, lest it is fully revealing and the private signal in the second period becomes completely useless, and the intervention loses the coordination effect.

O.4. Normally distributed signals

In this subsection, we discuss the results when the private signals follow Normal distribution, i.e., $\varepsilon_i \sim N(0, \delta)$.[39] We still assume that investors are non-overlapping to keep matters comparable with our baseline model. The case when investors perfectly overlap can be identically analyzed as one in which $\varepsilon_i \sim N\left(0, \frac{\delta}{2}\right)$. We characterize the equilibrium in each period and emphasize that government intervention in period 1 still has a dynamic coordination effect in period 2 through altering the informational environment.

[39] We use uninformative prior belief which is common in global games literature.

Lemma 12 below summarizes equilibrium outcomes in two periods.

Lemma 12

Equilibrium when signals follow Normal distribution

1. *Given m_1, there exists unique equilibrium thresholds in period 1:*

$$\begin{aligned} \theta_1^* &= 1 + m_1 - c \\ x_1^* &= 1 + m_1 - c - \delta\Phi^{-1}(c). \end{aligned}$$

2. *Given (m_1, m_2) and $s_1 = S$,*

   (a) *When $m_2 > m_1 - c$, $(\theta_2^* = \theta_1^*, x_2^* = \infty)$ consists a threshold equilibrium.*

   (b) *Equilibrium strategies (θ_2^*, x_2^*) which satisfy $\theta_2^* < \theta_1^*$ and $x_2^* < \infty$ may or may not exist. If they exist, they can be non-unique.*

To see this, note that the equilibrium outcome in period 1 is characterized by two thresholds (θ_1^*, x_1^*) that satisfy

$$\begin{aligned} A_1(\theta_1^*) + m_1 &= \theta_1^* \\ \Pr\left(\theta < \theta_1^* \middle| x_1 = x_1^*\right) &= c. \end{aligned}$$

where $A_1(\theta_1^*) = \Pr\left(x_1 < x_1^* \middle| \theta = \theta_1^*\right)$ is the measure of investors who choose to roll over. Simple calculation shows that

$$\begin{aligned} \theta_1^* &= 1 + m_1 - c \\ x_1^* &= 1 + m_1 - c - \delta\Phi^{-1}(c). \end{aligned}$$

The equilibrium in period 2 is again state-independent. We discuss the outcomes when $s_1 = S$ here and the case of $s_1 = F$ is similar. When the intervention in the first period has succeeded, equilibrium in the second period will be either a stage-game equilibrium with full dynamic coordination (similar to Lemma 2), or one with partial dynamic coordination (similar to Lemma 4). The case without dynamic coordination vanishes as the support of the noise now spans between (∞, ∞). The first type of equilibrium is denoted as $(\theta_2^*, x_2^*) = (\infty, \infty)$ and any equilibrium with $(\theta_2^* > \theta_1^*, x_2^* = \infty)$ is equivalent. The necessary conditions that $(\theta_2^*, x_2^*) = (\infty, \infty)$ constitutes an equilibrium are

$$\begin{aligned} \Pr\left(1 + m_2 > \theta \middle| \theta < \theta_1^*\right) &= 1 \\ \Rightarrow m_2 &> m_1 - c. \end{aligned}$$

Likewise, the necessary conditions that an equilibrium with partial dynamic coordination exists is that the solution (θ_2^*, x_2^*) to the equation system

$$\begin{aligned} A_2\left(\theta_2^*\right) + m_2 &= \theta_2^* \\ \Pr\left(\theta < \theta_2^* \middle| x_2^*, \theta < \theta_1^*\right) &= c. \end{aligned}$$

exists and satisfies $\theta_2^* < \theta_1^*$. Equivalently, we are looking for θ_2^* that solves

$$1 - (\theta_2^* - m_2) = c\Phi\left(\frac{\theta_1^* - \theta_2^* - \delta\Phi^{-1}\left(\theta_2^* - m_2\right)}{\delta}\right) \tag{62}$$

We can numerically solve Eq. (62), and importantly, Table 4 presents the local comparative statics when there exists a unique equilibrium strategy. When m_1 increases from 0.7 to 0.9, both θ_2^* and x_2^* decrease, validating the dynamic coordination effect.

Table 4: θ_2^* as a function of m_1 ($s_1 = S$)

m_1	0.7	0.75	0.8	0.85	0.9
θ_2^*	0.7602	0.6981	0.6693	0.6511	0.6384
x_2^*	0.9667	0.8222	0.7565	0.7152	0.6866

Other parameters are $c = 0.5, \delta = 0.5, m_2 = 0.1$.

O.5. General distributions of bounded noise

In this section, we relax the assumption that each agent's private noise follows uniform distribution on $[-\delta, \delta]$. Instead, we assume that ε_i follows a general distribution with CDF $G(\cdot)$ on $[-\delta, \delta]$. In addition, we continue to assume that ε_i is i.i.d. across investors. The analysis is meant to show that the three-type of equilibria that we shown in Section 3 robust to more general noise distributions. Indeed, it is the assumption that noises have bounded support that drive the results. The assumption of uniform distribution helps us establish equilibrium uniqueness and derive the closed-form expressions for cutoffs and welfare.

The equilibrium in period 1 is still characterized by two equations

$$\begin{aligned} A_1(\theta_1^*) + m_1 &= \theta_1^* \\ \Pr\left(\theta < \theta_1^* \middle| x_1 = x_1^*\right) &= c. \end{aligned}$$

Given the noise distribution, $A_1(\theta_1^*) = G(x_1^* - \theta_1^*)$ and $\Pr\left(\theta < \theta_1^* \middle| x_1 = x_1^*\right) = 1 - G(x_1^* - \theta_1^*)$. Therefore, we can easily reach the solutions:

$$\begin{aligned} \theta_1^* &= 1 + m_1 - c \\ x_1^* &= 1 + m_1 - c + G^{-1}(1 - c). \end{aligned}$$

Next, we turn to equilibrium in period 2. We will consider the case when $s_1 = S$, and the other case can be analyzed similarly.

If the equilibrium does not feature any dynamic coordination, the thresholds in the second period are

$$\begin{aligned} \theta_2^* &= 1 + m_2 - c \\ x_2^* &= 1 + m_2 - c + G^{-1}(1 - c) \end{aligned}$$

In this case, if $x_2^* + \delta < \theta_1^* \Rightarrow m_2 < m_1 - \delta - G^{-1}(1 - c)$ so that the marginal investor indeed finds the public information useless, then such an equilibrium without dynamic coordination exists.

Likewise, the equilibrium with partial coordination can also be characterized by two equations:

$$\begin{aligned} A_1(\theta_2^*) + m_2 &= \theta_2^* \Rightarrow G(x_2^* - \theta_2^*) + m_2 = \theta_2^* \\ \Pr\left(\theta < \theta_2^* \middle| x_2 = x_2^*, \theta < \theta_1^*\right) &= c \Rightarrow \frac{1 - (\theta_2^* - m_2)}{G(\theta_1^* - x_2^* + \delta)} = c. \end{aligned}$$

Any solution to the equation system comprises an equilibrium. Note that the second equation is non-monotonic in x_2^* so that the solution in general is not unique. However, for each solution, as m_1 increases, θ_1^* increases and so is θ_2^*. Therefore, the conditional inference effect continues to exist.

Finally, for any solution (θ_2^*, x_2^*) with partial coordination, the condition $\theta_2^* > \theta_1^*$ pins down the relation between m_1 and m_2 such that the equilibrium features full dynamic coordination.