Persistent Government Debt and Aggregate Risk Distribution

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Online Appendix

Appendix A. Econometric Methods

Data sources. Most of our empirical results are based on data provided in the NIPA tables, for the sample 1947:Q1–2016:Q4. Quarterly consumption growth is constructed from real per capita nondurables and services expenditure. Government-spending-to-GDP ($Govt/GDP$) comprises current government expenditures. GDP is real gross domestic product per the Bureau of Economic Analysis (BEA) (Series ID: GDPC96).

Debt-to-GDP ($DGDP$) is defined as gross public debt divided by lagged real gross domestic product from the BEA (Series ID: GDPC96). The gross public debt series is concatenated from two different sources. For the period 1947:Q1–1965:Q4 we use "Total gross public debt" from the monthly statement of the public debt files maintained by the US Treasury,

$$https://www.treasurydirect.gov/govt/reports/pd/mspd/mspd.htm,$$

and from 1966:Q1–2016:Q4 we use the "Federal Debt: Total Public Debt" (GFDEBTN) series from the Federal Reserve Bank of St. Louis.

Total factor productivity (TFP) comes from the Federal Reserve Bank of San Francisco and John Fernald's quarterly TFP series. We use the business-sector TFP ($dtfp$) variable as our measure of TFP. Labor tax rate data are from the NBER's US Federal Marginal Income Tax Rates data (http://users.nber.org/ taxsim/conrate/), which are annual data from 1960–2016. Data on labor hours worked are from the series "Average Weekly Hours of Production and Nonsupervisory Employees: Manufacturing" (AWHMAN) from the Federal Reserve Bank of St. Louis.

The wealth-consumption ratio returns were graciously provided by Lustig et al. (2013).

Estimating $\rho_{B,t}$. We estimate the speed of repayment of debt-to-output, $\rho_{B,t}$, in several ways in order to assess the reliability of our results across methods. We start by adopting a quasi-maximum-likelihood approach in order to estimate the following system of equations:

$$DGDP_t = a + \rho_{B,t} DGDP_{t-1} + \epsilon_t \tag{A.1}$$

$$\rho_{B,t} = b + \rho_{\rho,B} \cdot \rho_{B,t-1} + v_t \tag{A.2}$$

We estimate these equations either by leaving $\rho_{\rho,B}$ unrestricted (SSM-AR(1)), or by setting this parameter at a predetermined value for sensitivity analysis purposes. We also consider the extreme case, $\rho_{\rho,B} = 1$ (SSM-RW(1)). We define these estimates respectively as $\hat{\rho}_t(DGDP)^{AR}$ and $\hat{\rho}_t(DGDP)^{RW}$.

As an alternative to the parametric approach described above, we also estimate rolling-window AR(1) models on $DGDP$ for various window lengths. This estimation framework inherently contains a large degree of model uncertainty around the choice of window size; therefore, we adopt a forecast combination mindset and use equally weighted averages of the autocorrelation estimates across a grid of window sizes. We look at an equally spaced grid of window sizes from 10 to 50 quarters, implying that the maximum window size represents approximately 20% of our sample size. We define this estimate as $\hat{\rho}_t(DGDP)^{RollWind}$.

As a way to aggregate these different estimates, we also construct a model ensemble using equally weighted averages across the nonparametric and parametric estimates:

$$\hat{\rho}_t(DGDP)^{Combo} = \frac{1}{2}\hat{\rho}_t(DGDP)^{RollWind} + \frac{1}{2}\left(\frac{1}{2}\hat{\rho}_t(DGDP)^{AR} + \frac{1}{2}\hat{\rho}_t(DGDP)^{RW}\right). \tag{A.3}$$

In figure A1, we depict $\hat{\rho}_t(DGDP)^{Combo}$ along with its bootstrapped 95% confidence interval.

Fig. A1. Autocorrelation of Debt-to-Output

Notes: This figure shows our estimate of $\hat{\rho}_t(DGDP)^{Combo}$ (solid line) along with the associated 95% bootstrap confidence interval.

Construction of long- and short-run conditional moments. Let y represent either real per capita consumption growth or the consumption claim excess return series. We construct cumulative growth rates over different quarterly horizons as follows:

$$y_{t+1\to t+J} = \sum_{j=1}^{J} y_{t+j} \quad \text{for} \quad J \in \{1, 2, 4, 8, 20\}.$$

Across different horizons, we estimate the conditional expectation functions through a regression approach that uses the market price-dividend ratio (PD_t), a proxy for market volatiliy (MV_t), and Debt/DGDP ($DGDP_t$) as forecasting variables:

$$y_{t+1\to t+J} = \underbrace{\beta_0 + \beta_1 PD_t + \beta_2 MV_t + \beta_3 DGDP_t}_{y^{LR}_{t+1\to t+J}} + \underbrace{\epsilon_{t+J}}_{y^{SR}_{t+1\to t+J}}, \tag{A.4}$$

and denote the long-run and short-run components of y at horizon J as $y^{LR}_{t+1\to t+J}$ and $y^{SR}_{t+1\to t+J}$, respectively.

Conditional variances are estimated using a GJR-GARCH(1,1) model for each horizon. We construct conditional variances for both the overall series, $\text{Var}_t(y_{t+1\to t+J})$, and the their long-run components, $\text{Var}_t(y^{LR}_{t+1\to t+J}) \equiv \text{Var}_t[\hat{E}_{t+1}(y_{t+2\to t+1+J})]$.

The conditional skewness for the long-run component, as well as the overall series, are defined as follows:

$$\text{Skew}_t\left(y^{LR}_{t+1\to t+J}\right) \equiv E_t\left[\left(\frac{y^{LR}_{t+2\to t+1+J} - E_{t+1}[y^{LR}_{t+2\to t+1+J}]}{\text{Var}_t(y^{LR}_{t+1\to t+J})}\right)^3\right] \tag{A.5}$$

$$\text{Skew}_t\left(y^{SR}_{t+1\to t+J}\right) \equiv E_t\left[\left(\frac{y^{SR}_{t+1\to t+J} - E_t[\hat{\epsilon}_{t+J}]}{Var_t(y^{SR}_{t+1\to t+J})}\right)^3\right] \tag{A.6}$$

$$\text{Skew}_t\left(y_{t+1\to t+J}\right) \equiv E_t\left[\left(\frac{y_{t+1\to t+J} - E_t[y_{t+1\to t+J}]}{\text{Var}_t(y_{t+1\to t+J})}\right)^3\right]. \tag{A.7}$$

We also estimate the conditional autocorrelation of the long-run component $y^{LR}_{t+1\to t+J}$, $\rho_t(y^{LR}_{t+1\to t+J})$, using a rolling-window method with 40 quarters.

Equations of interest. Given the methods described above, we study the impact of $\rho_t(DGDP)$ on the conditional distribution of $y_{t+1\to t+J}$ and its long-run component by estimating the following system of equations:

$$y_{t+1\to t+J} = \gamma_0 + \gamma_1 \cdot \rho_t(DGDP) + v_{t+J} \tag{A.8}$$

$$y^{LR}_{t+1\to t+J} = \gamma^{LR}_0 + \gamma^{LR}_1 \cdot \rho_t(DGDP) + v^{LR}_{t+J} \tag{A.9}$$

$$\text{Var}_t(y_{t+1\to t+J}) = \gamma_{0,V} + \gamma_{1,V} \cdot \rho_t(DGDP) + v_{t+J,V} \tag{A.10}$$

$$\text{Var}_t(y^{LR}_{t+1\to t+J}) = \gamma^{LR}_{0,V} + \gamma^{LR}_{1,V} \cdot \rho_t(DGDP) + v^{LR}_{t+J,V} \tag{A.11}$$

$$\text{Skew}_t(y_{t+1\to t+J}) = \gamma_{0,S} + \gamma_{1,S} \cdot \rho_t(DGDP) + v_{t+J,S} \tag{A.12}$$

$$\text{Skew}_t(y^{LR}_{t+1\to t+J}) = \gamma^{LR}_{0,S} + \gamma^{LR}_{1,S} \cdot \rho_t(DGDP) + v^{LR}_{t+J,S}. \tag{A.13}$$

Appendix B. Robustness

In table B1, we report the results for the case in which $\rho_t(DGDP)$ is estimated with its persistence fixed to 0.9, that is, by fixing $\rho_{\rho,B} = 0.9$ in equation equation (A.2) when estimating the persistence.

Table B1. Impact of $\rho_t(DGDP)$ on conditional moments (I)

Horizons (J)	1	2	4	8	20
			Consumption growth		
$E_t[\Delta c^{LR}_{t+J}]$	-0.46^{***}	-0.53^{***}	-0.70^{***}	-0.72^{***}	-0.69^{***}
	(0.10)	(0.08)	(0.07)	(0.15)	(0.18)
$\text{Var}_t[\Delta c_{t+J}]$	-0.28^{*}	-0.26^{**}	-0.46^{***}	-0.50^{**}	-0.44^{*}
	(0.16)	(0.13)	(0.18)	(0.21)	(0.23)
$\text{Var}_t[\Delta c^{LR}_{t+J}]$	0.50^{***}	0.31^{***}	0.49^{***}	0.30^{***}	0.43^{***}
	(0.13)	(0.08)	(0.14)	(0.08)	(0.10)
$\text{Skew}_t[\Delta c^{LR}_{t+J}]$	-0.27^{***}	-0.28^{***}	-0.29^{***}	-0.44^{***}	-0.48^{***}
	(0.06)	(0.06)	(0.06)	(0.07)	(0.09)
$\rho_t[\Delta c^{LR}_{t+1}]$	0.26^{*}				
	(0.15)				
			Consumption claim excess returns		
$E_t[R^{W,LR}_{ex,t+J}]$	0.67^{***}	0.68^{***}	0.76^{***}	0.72^{***}	0.62^{**}
	(0.08)	(0.10)	(0.15)	(0.20)	(0.24)
$\text{Var}_t[R^{W}_{ex,t+J}]$	-0.04	0.04	-0.05	-0.18	-0.19
	(0.22)	(0.22)	(0.25)	(0.24)	(0.27)
$\text{Var}_t[R^{W,LR}_{ex,t+J}]$	0.31^{***}	0.31^{***}	0.33^{***}	0.35^{***}	0.01
	(0.08)	(0.09)	(0.09)	(0.09)	(0.10)
$\text{Skew}_t[R^{W,LR}_{ex,t+J}]$	0.33^{***}	0.33^{***}	0.45^{***}	0.58^{***}	0.53^{***}
	(0.07)	(0.07)	(0.06)	(0.11)	(0.18)
$\rho_t[R^{W,LR}_{ex,t+1}]$	0.33^{*}				
	(0.17)				

Notes: This table shows estimated coefficients from regressions of conditional moments for consumption growth (Δc_t) and consumption-wealth excess returns ($R^{W}_{ex,t}$) at various cumulative horizons on $\rho_t(DGDP)$, the time-varying autocorrelation of debt-to-GDP. See Appendix A for details on variable constructions. Quarterly data are from 1947:Q2–2016:Q4. Newey and West (1987) standard errors are in parentheses. One, two, and three asterisks denote significance at the 10%, 5%, and 1% levels, respectively.

In table B2, we report the results for the case in which $\rho_t(DGDP)$ is estimated with its persistence fixed to 0.95, that is, by fixing $\rho_{\rho,B} = 0.95$ in equation equation (A.2) when estimating the persistence.

Table B2. Impact of $\rho_t(DGDP)$ on conditional moments(II)

Horizons (J)	1	2	4	8	20
			Consumption growth		
$E_t[\Delta c^{LR}_{t+J}]$	-0.36^{***}	-0.46^{***}	-0.72^{***}	-0.91^{***}	-0.89^{***}
	(0.12)	(0.11)	(0.10)	(0.09)	(0.10)
$\text{Var}_t[\Delta c_{t+J}]$	-0.44^{***}	-0.39^{***}	-0.58^{***}	-0.66^{***}	-0.63^{***}
	(0.11)	(0.09)	(0.13)	(0.14)	(0.17)
$\text{Var}_t[\Delta c^{LR}_{t+J}]$	0.43^{***}	0.21^{**}	0.42^{**}	0.20^{**}	0.33^{***}
	(0.16)	(0.08)	(0.17)	(0.08)	(0.11)
$\text{Skew}_t[\Delta c^{LR}_{t+J}]$	-0.17^{***}	-0.18^{***}	-0.19^{***}	-0.57^{***}	-0.64^{***}
	(0.05)	(0.06)	(0.06)	(0.10)	(0.10)
$\rho_t[\Delta c^{LR}_{t+1}]$	0.33^{*}				
	(0.18)				
			Consumption claim excess returns		
$E_t[R^{W,LR}_{ex,t+J}]$	0.67^{***}	0.70^{***}	0.86^{***}	0.89^{***}	0.86^{***}
	(0.07)	(0.08)	(0.08)	(0.12)	(0.15)
$\text{Var}_t[R^{W}_{ex,t+J}]$	-0.24	-0.19	-0.25	-0.39^{*}	-0.43^{*}
	(0.21)	(0.21)	(0.25)	(0.22)	(0.26)
$\text{Var}_t[R^{W,LR}_{ex,t+J}]$	0.22^{**}	0.22^{**}	0.24^{**}	0.25^{**}	-0.05
	(0.09)	(0.10)	(0.10)	(0.10)	(0.09)
$\text{Skew}_t[R^{W,LR}_{ex,t+J}]$	0.24^{***}	0.25^{***}	0.39^{***}	0.61^{***}	0.67^{***}
	(0.07)	(0.07)	(0.07)	(0.08)	(0.12)
$\rho_t[R^{W,LR}_{ex,t+1}]$	0.34^{*}				
	(0.19)				

Notes: This table shows estimated coefficients from regressions of conditional moments for consumption growth (Δc_t) and consumption-wealth excess returns ($R^W_{ex,t}$) at various cumulative horizons on $\rho_t(DGDP)$, the time-varying autocorrelation of debt-to-GDP. See Appendix A for details on variable constructions. Quarterly data are from 1947:Q2–2016:Q4. Newey and West (1987) standard errors are in parentheses. One, two, and three asterisks denote significance at the 10%, 5%, and 1% levels, respectively.

In table B3, we report the results for the case in which $\rho_t(DGDP)$ is estimated with its persistence fixed to 0.98, that is, by fixing $\rho_{\rho,B} = 0.98$ in equation equation (A.2) when estimating the persistence.

Table B3. Impact of $\rho_t(DGDP)$ on conditional moments (III)

Horizons (J)	1	2	4	8	20
			Consumption growth		
$E_t[\Delta c^{LR}_{t+J}]$	-0.27^{**}	-0.37^{***}	-0.67^{***}	-0.94^{***}	-0.94^{***}
	(0.11)	(0.11)	(0.11)	(0.08)	(0.08)
$\text{Var}_t[\Delta c_{t+J}]$	-0.51^{***}	-0.44^{***}	-0.60^{***}	-0.71^{***}	-0.69^{***}
	(0.08)	(0.06)	(0.11)	(0.11)	(0.13)
$\text{Var}_t[\Delta c^{LR}_{t+J}]$	0.34^{**}	0.12	0.34^{**}	0.12^{*}	0.24^{**}
	(0.16)	(0.08)	(0.17)	(0.07)	(0.10)
$\text{Skew}_t[\Delta c^{LR}_{t+J}]$	-0.09^{**}	-0.10^{**}	-0.11^{**}	-0.60^{***}	-0.68^{***}
	(0.04)	(0.05)	(0.05)	(0.12)	(0.11)
$\rho_t[\Delta c^{LR}_{t+1}]$	0.34^{*}				
	(0.19)				
			Consumption claim excess returns		
$E_t[R^{W,LR}_{ex,t+J}]$	-0.27^{**}	-0.37^{***}	-0.67^{***}	-0.94^{***}	-0.94^{***}
	(0.11)	(0.11)	(0.11)	(0.08)	(0.08)
$\text{Var}_t[R^{W}_{ex,t+J}]$	-0.51^{***}	-0.44^{***}	-0.60^{***}	-0.71^{***}	-0.69^{***}
	(0.08)	(0.06)	(0.11)	(0.11)	(0.13)
$\text{Var}_t[R^{W,LR}_{ex,t+J}]$	0.34^{**}	0.12	0.34^{**}	0.12^{*}	0.24^{**}
	(0.16)	(0.08)	(0.17)	(0.07)	(0.10)
$\text{Skew}_t[R^{W,LR}_{ex,t+J}]$	-0.09^{**}	-0.10^{**}	-0.11^{**}	-0.60^{***}	-0.68^{***}
	(0.04)	(0.05)	(0.05)	(0.12)	(0.11)
$\rho_t[R^{W,LR}_{ex,t+1}]$	0.34^{*}				
	(0.19)				

Notes: This table shows estimated coefficients from regressions of conditional moments for consumption growth (Δc_t) and consumption-wealth excess returns ($R^W_{ex,t}$) at various cumulative horizons on $\rho_t(DGDP)$, the time-varying autocorrelation of debt-to-GDP. See Appendix A for details on variable constructions. Quarterly data are from 1947:Q2–2016:Q4. Newey and West (1987) standard errors are in parentheses. One, two, and three asterisks denote significance at the 10%, 5%, and 1% levels, respectively.

In table B4, we report the results for the case in which $\rho_t(DGDP)$ is estimated using rolling-window regressions.

Table B4. Impact of $\rho_t(DGDP)$ on conditional moments (IV)

Horizons (J)	1	2	4	8	20
			Consumption growth		
$E_t[\Delta c^{LR}_{t+J}]$	-0.34^{***}	-0.35^{***}	-0.36^{***}	-0.32^{*}	-0.31
	(0.11)	(0.11)	(0.12)	(0.18)	(0.19)
$\text{Var}_t[\Delta c_{t+J}]$	-0.04	-0.04	-0.10	-0.23	-0.35^{*}
	(0.24)	(0.17)	(0.22)	(0.19)	(0.21)
$\text{Var}_t[\Delta c^{LR}_{t+J}]$	0.31^{***}	0.33^{***}	0.42^{***}	0.40^{***}	0.40^{***}
	(0.07)	(0.08)	(0.12)	(0.08)	(0.09)
$\text{Skew}_t[\Delta c^{LR}_{t+J}]$	-0.31^{***}	-0.32^{***}	-0.32^{***}	-0.20^{**}	-0.18^{**}
	(0.07)	(0.06)	(0.06)	(0.09)	(0.08)
$\rho_t[\Delta c^{LR}_{t+1}]$	0.29				
	(0.23)				
			Consumption claim excess returns		
$E_t[R^{W,LR}_{ex,t+J}]$	0.52^{***}	0.51^{***}	0.51^{***}	0.42^{**}	0.22
	(0.12)	(0.13)	(0.16)	(0.18)	(0.21)
$\text{Var}_t[R^{W}_{ex,t+J}]$	-0.11	0.03	-0.05	-0.17	-0.22
	(0.21)	(0.21)	(0.23)	(0.23)	(0.25)
$\text{Var}_t[R^{W,LR}_{ex,t+J}]$	0.33^{***}	0.33^{***}	0.35^{***}	0.37^{***}	0.06
	(0.08)	(0.08)	(0.08)	(0.07)	(0.13)
$\text{Skew}_t[R^{W,LR}_{ex,t+J}]$	0.35^{***}	0.36^{***}	0.41^{***}	0.36^{***}	0.12
	(0.07)	(0.07)	(0.08)	(0.13)	(0.19)
$\rho_t[R^{W,LR}_{ex,t+1}]$	0.57^{***}				
	(0.13)				

Notes: This table shows estimated coefficients from regressions of conditional moments for consumption growth (Δc_t) and consumption-wealth excess returns ($R^{W}_{ex,t}$) at various cumulative horizons on $\rho_t(DGDP)$, the time-varying autocorrelation of debt-to-GDP. See Appendix A for details on variable constructions. Quarterly data are from 1947:Q2–2016:Q4. Newey and West (1987) standard errors are in parentheses. One, two, and three asterisks denote significance at the 10%, 5%, and 1% levels, respectively.

In table B5, we report the results for the case in which $\rho_t(DGDP)$ is estimated under the assumption that it follows a random walk.

Table B5. Impact of $\rho_t(DGDP)$ on conditional moments (V)

Horizons (J)	1	2	4	8	20
			Consumption growth		
$E_t[\Delta c^{LR}_{t+J}]$	-0.27^{***}	-0.38^{***}	-0.67^{***}	-0.94^{***}	-0.92^{***}
	(0.11)	(0.12)	(0.11)	(0.08)	(0.10)
$\text{Var}_t[\Delta c_{t+J}]$	-0.51^{***}	-0.44^{***}	-0.59^{***}	-0.70^{***}	-0.64^{***}
	(0.08)	(0.06)	(0.11)	(0.11)	(0.14)
$\text{Var}_t[\Delta c^{LR}_{t+J}]$	0.24^{**}	0.09^{*}	0.34^{**}	0.11^{*}	0.31^{**}
	(0.13)	(0.06)	(0.17)	(0.07)	(0.16)
$\text{Skew}_t[\Delta c^{LR}_{t+J}]$	-0.09^{**}	-0.10^{**}	-0.11^{***}	-0.63^{***}	-0.71^{***}
	(0.04)	(0.05)	(0.04)	(0.12)	(0.10)
$\rho_t[\Delta c^{LR}_{t+1}]$	0.34^{**}				
	(0.19)				
			Consumption claim excess returns		
$E_t[R^{W,LR}_{ex,t+J}]$	0.60^{***}	0.64^{***}	0.83^{***}	0.91^{***}	0.94^{***}
	(0.07)	(0.07)	(0.06)	(0.07)	(0.08)
$\text{Var}_t[R^{W}_{ex,t+J}]$	-0.36^{**}	-0.33^{**}	-0.37^{*}	-0.50^{***}	-0.60^{***}
	(0.20)	(0.19)	(0.23)	(0.20)	(0.21)
$\text{Var}_t[R^{W,LR}_{ex,t+J}]$	0.13^{**}	0.14^{**}	0.15^{**}	0.16^{**}	-0.05
	(0.07)	(0.08)	(0.08)	(0.08)	(0.09)
$\text{Skew}_t[R^{W,LR}_{ex,t+J}]$	0.16^{***}	0.17^{***}	0.31^{***}	0.59^{***}	0.73^{***}
	(0.04)	(0.04)	(0.05)	(0.06)	(0.08)
$\rho_t[R^{W,LR}_{ex,t+1}]$	0.33^{**}				
	(0.20)				

Notes: This table shows estimated coefficients from regressions of conditional moments for consumption growth (Δc_t) and consumption-wealth excess returns ($R^W_{ex,t}$) at various cumulative horizons on $\rho_t(DGDP)$, the time-varying autocorrelation of debt-to-GDP. See Appendix A for details on variable constructions. Quarterly data are from 1947:Q2–2016:Q4. Newey and West (1987) standard errors are in parentheses. Hypothesis tests are associated with the null that the signs are inconsistent with those predicted by our model. One, two, and three asterisks denote significance at the 10%, 5%, and 1% levels, respectively.

In table B6, we report our panel regression results based on the system of equations (5)-(6).

Table B6. International Panel Regressions

	$E_t[\Delta y^{LR}_{t+J}]$		$\text{Var}_t[\Delta y_{t+J}]$	
β_1	-0.057^{**}	-0.059^{**}	-0.007^{**}	-0.005
	(0.025)	(0.026)	(0.004)	(0.006)
β_2	-0.112	-0.057	-0.198^{**}	-0.192^{**}
	(0.807)	(0.842)	(0.107)	(0.111)
β_3	-0.015	-0.026	0.034^{**}	0.034^{**}
	(0.159)	(0.159)	(0.017)	(0.017)
Time FE	No	Yes	No	Yes
Adj R^2	0.008	0.057	0.006	0.025

Notes: This table shows estimated coefficients from the system of equations (5)-(6). "Time FE" indicates whether the model includes time fixed effects or not. Annual data is for 15 countries from 1978-2014. Newey and West (1987) standard errors are in parentheses. Hypothesis tests are associated with the null that the signs are inconsistent with those predicted by our model. One, two, and three asterisks denote significance at the 10%, 5%, and 1% levels, respectively.

Appendix C. Auxiliary Regressions for Calibration

The following estimations are used to calibrate the government's debt policy functions in the model:

$$DGDP_t = \underset{(0.0022)}{0.0009} + \underset{(0.0036)}{\overset{***}{0.9985}} DGDP_{t-1} + \hat{\epsilon}_t^{DGDP} \tag{C1}$$

$$GY_t = \underset{(0.0022)}{\overset{*}{0.0042}} + \underset{(0.0076)}{\overset{***}{0.9870}} GY_{t-1} + \hat{\epsilon}_t^{GY} \tag{C2}$$

$$TFP_t = \underset{(0.0005)}{\overset{***}{0.0025}} + \underset{(0.0593)}{\overset{***}{0.1829}} TFP_{t-1} + \hat{\epsilon}_t^{TFP} \tag{C3}$$

$$\hat{\epsilon}_t^{DGDP} = \underset{(0.0007)}{0.0000} + \underset{(0.1358)}{\overset{***}{0.5075}} \hat{\epsilon}_t^{GY} + \underset{(0.0833)}{\overset{***}{-0.4542}} \hat{\epsilon}_t^{TFP} + \hat{v}_t, \tag{C4}$$

where numbers in parentheses are heteroscedasticity-adjusted standard errors, and implied p-values of 1%, 5%, and 10% are denoted by ***, **, and *, respectively.

The estimated volatilities of the residuals are

$$\hat{\sigma}(\epsilon_t^{DGDP}) = 0.0126 \tag{C5}$$

$$\hat{\sigma}(\epsilon_t^{GY}) = 0.0053 \tag{C6}$$

$$\hat{\sigma}(\epsilon_t^{TFP}) = 0.0086 \tag{C7}$$

$$\hat{\sigma}(v_t) = 0.0114. \tag{C8}$$

We then use the following equations to calibrate the dynamics of the autocorrelation of $DGDP$, where we estimate a restricted model such that the unconditional autocorrelation of $DGDP$ aligns with the unconditional AR(1) estimated in equation equation (C1). In particular, we estimate

$$\rho_t(DGDP) = e_0 + e_1\rho_{t-1}(DGDP) + e_2\hat{\epsilon}_t^{TFP} + u_t$$
$$\text{s.t. } \frac{e_0}{(1-e_1)} = a_1,$$

where a_1 is the persistence in equation (C1).

The estimated version follows:

$$\rho_t(DGDP) = \underset{(0.02)}{0.03} + \underset{(0.022)}{\overset{***}{0.96}}\,\rho_{t-1}(DGDP) - \underset{(0.30)}{\overset{**}{0.61}}\,\hat{\epsilon}_t^{TFP} + \hat{u}_t,$$

where $\hat{\sigma}(u_t) = 0.019$.

C.1. Simulation-based Estimates

Given our estimated model for $DGDP$ in Appendix C, we can simulate artificial data and compare our estimated time-varying persistence of $DGDP$ with the true simulated series. This exercise is important because it establishes the link between our empirical and theoretical analysis and it justifies our empirical procedure.

Specifically, in table C1 we report the correlations between the true persistence, $\rho_t(DGDP)$, and the filtered one, $\hat{\rho}_t(DGDP)$, across different estimation methods. We report our results for both a long-sample simulation and repetitions of small samples whose length is equal to the one in our data set.

In both long and short samples, our rolling window procedure produces a less precise proxy for the actual process $\rho_{B,t}$. In contrast, when we parameterize $\rho_{B,t}$ either as a stationary AR(1) or as a random walk, the quality of our inference increases sharply. In our long sample, our correlations rise to very high levels greater or equal to 80%. Across repetitions of small sample, $Avg[corr(\hat{\rho}_{B,t}, \rho_{B,t})]$ is

Table C1. Filtered vs Actual Persistence Process: $corr(\hat{\rho}_{B,t}, \rho_{B,t})$

	$\hat{\rho}_{B,t}^{Combo}$	$\hat{\rho}_{B,t}^{AR}$	$\hat{\rho}_{B,t}^{RW}$	$\hat{\rho}_{B,t}^{RollWind}$	$\hat{\rho}_{B,t}^{FixedAR}$
Long sample	0.8	0.9	0.9	0.3	0.9
Short sample repetitions	0.5	0.7	0.6	0.4	0.7

Notes: This table shows correlations between actual and filtered time-varying persistence for $DGDP$. The long sample simulation features 5000 quarterly observations. In the case of short sample repetitions, we report averages across 100 samples with 280 quarters (our sample length in the empirical analysis). The model is calibrated using the estimates reported in Appendix C.

lower than in long sample, but it remains above 60%.

In small sample, our model ensemble (Combo) delivers a satisfactory correlation of 50%. Since our estimators are consistent, in long sample this correlation becomes as large as 80%, a very high level.

In figure C1 we plot the various series from our long sample simulation. This plot shows that our empirical estimators are consistent and hence they produce filtered series very close to the actual simulated process.

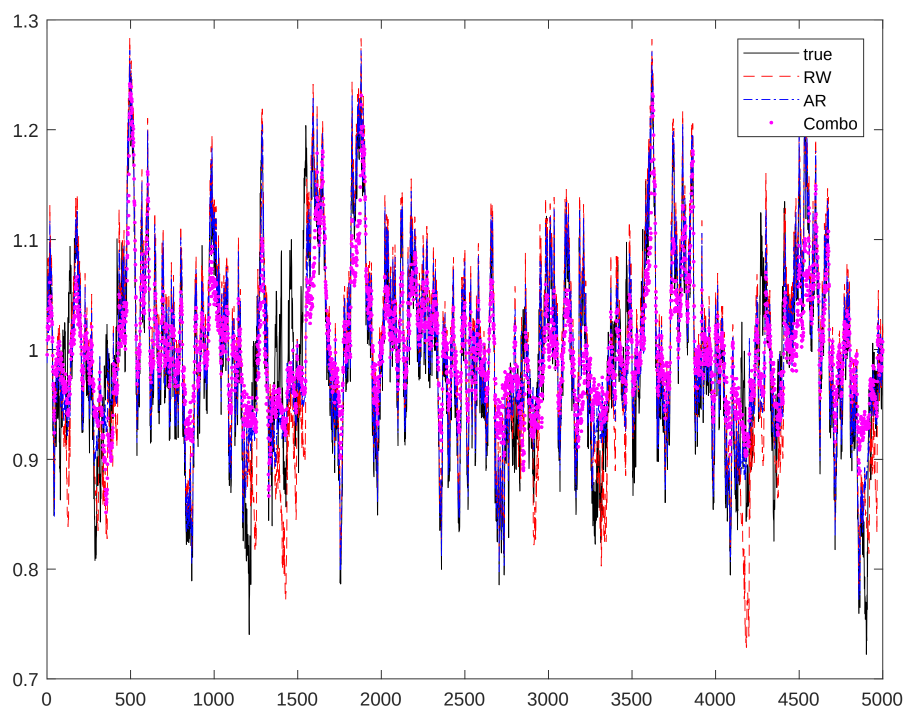

Fig. C1. Actual vs Filtered $\rho_{B,t}$

Notes: This figure plots both the actual and the filtered $DGDP$ persistence processes along a long sample simulation.

Appendix D. Solution Method and Welfare Costs

Solution method and computations. We solve the model in `dynare++4.2.1` using a fourth-order approximation. The policies are centered about a fixed point that takes into account the effects of volatility on decision rules. In the **.mat** file generated by `dynare++`, the vector with the fixed point for all our endogenous variables is denoted as `dyn_ss`. All conditional moments are computed by means of simulations with a fixed seed to facilitate the comparison across fiscal policies.

Welfare costs. Consider two consumption-bundle processes, $\{u^1\}$ and $\{u^2\}$. We express welfare costs as the additional fraction λ of the lifetime consumption bundle required to make the representative agent indifferent between $\{u^1\}$ and $\{u^2\}$:

$$U_0(\{u^1\}) = U_0(\{u^2\}(1+\lambda)).$$

Since we specify U so that it is homogenous of degree one with respect to u, the following holds:

$$\frac{U_0(\{u^1\})}{u_0^1} \cdot u_0^1 = \frac{U_0(\{u^2\})}{u_0^2} \cdot u_0^2 \cdot (1+\lambda).$$

This shows that the welfare costs depend on both the utility-consumption ratio and the initial level of our two consumption profiles. In our production economy, the initial level of consumption is endogenous, so we cannot choose it. The initial level of patents, $A_0^i \quad i \in \{1,2\}$, in contrast, is exogenous:

$$\frac{U_0(\{u^1\})}{u_0^1} \cdot \frac{u_0^1}{A_0^1} \cdot A_0^1 = \frac{U_0(\{u^2\})}{u_0^2} \cdot \frac{u_0^2}{A_0^2} \cdot A_0^2 \cdot (1+\lambda).$$

We compare economies with different tax regimes but the same initial condition for the stock of patents: $A_0^1 = A_0^2$. After taking logs, evaluating utility- and consumption-productivity ratios at their unconditional mean, and imposing $A_0^1 = A_0^2$, we obtain the following expression:

$$\lambda \approx \overline{\ln U^1/A} - \overline{\ln U^2/A},$$

where the bar denotes the unconditional average, which is computed using the `dyn_ss` variable in `dynare++`.

Appendix E. Counterfactual Analysis

Stabilizing Consumption. The fiscal policy functions that we estimate from the data are similar to ones designed to stabilize short-run consumption fluctuations. Here we make this point more formal by focusing on a modified version of our model in which we have

$$\frac{B_t}{Y_t} = \rho_{B,t} \cdot \frac{B_{t-1}}{Y_{t-1}} + \phi_B(\Delta c_{ss} - \Delta c_t)$$

$$\rho_{B,t} = \rho_B(1 - \rho_{\rho,B}) + \rho_{\rho,B}\rho_{B,t-1} + A_\rho \epsilon_t,$$

and $\phi_B > 0$ so that the government increases public debt when consumption growth is subdued.

Figure E1 shows patterns for both welfare and patents value qualitatively similar to the ones in figure 5(a). These results are consistent with Croce et al. (2012) as they have been the first one to document that short-run stabilization can often be achieved only by allowing more long-run tax rate uncertainty. In an economy in which long-run uncertainty is priced, this trade off is relevant for welfare.

Appendix F. Term Structure Insights

To better illustrate how our long-term-oriented tax policy affects profit risk premia across different horizons, in figure F1 we depict the variation of the whole term structure of profit excess returns across the long-term-oriented and the benchmark fiscal policies (for a detailed analysis of the term structure of equities see, among others, Binsbergen et al. (2012), Binsbergen et al. (2013), Binsbergen and Koijen (2016), and Binsbergen and Koijen (2017)). Specifically, let $P^{\pi,C}_{n,t}$ ($P^{\pi,BM}_{n,t}$) denote the time t value of profits realized at time $t+n$ under our long-run-oriented (benchmark) fiscal policy. The one-period excess return of a zero-coupon claim to profits with maturity n is

$$R^{\pi,j}_{n,t} = E_t[P^{\pi,j}_{n-1,t+1}/P^{\pi,j}_{n,t}] - r^f_t, \quad j \in \{C, BM\}.$$

Under recursive preferences, the fiscal system becomes a vehicle by which to significantly alter the shape of the term structure of profits. Specifically, under our long-run-oriented policy, the

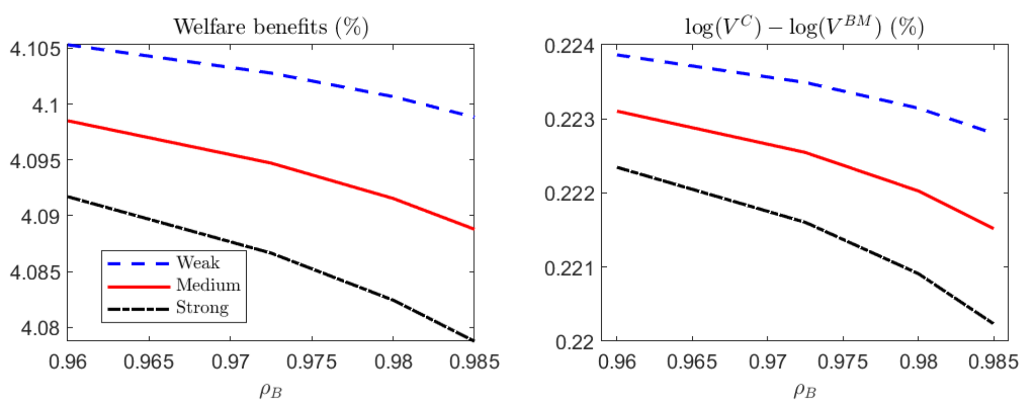

Fig. E1. Consumption Stabilization with No Commitment policy

Notes: C refers to the consumption stabilization policy with no commitment described below:

$$\frac{B_t}{Y_t} = \rho_{B,t} \cdot \frac{B_{t-1}}{Y_{t-1}} + \phi_B(\Delta c_{ss} - \Delta c_t)$$
$$\rho_{B,t} = \rho_B(1 - \rho_{\rho,B}) + \rho_{\rho,B}\rho_{B,t-1} + A_\rho \epsilon_t.$$

This economy is calibrated for different level of average speed of repayment ρ_B as shown in the x-axis. Weak, medium, and strong cyclical policies are generated by calibrating ϕ_B to 0.2%, 0.3% and 0.4%, respectively. BM refers to our benchmark model and calibration.

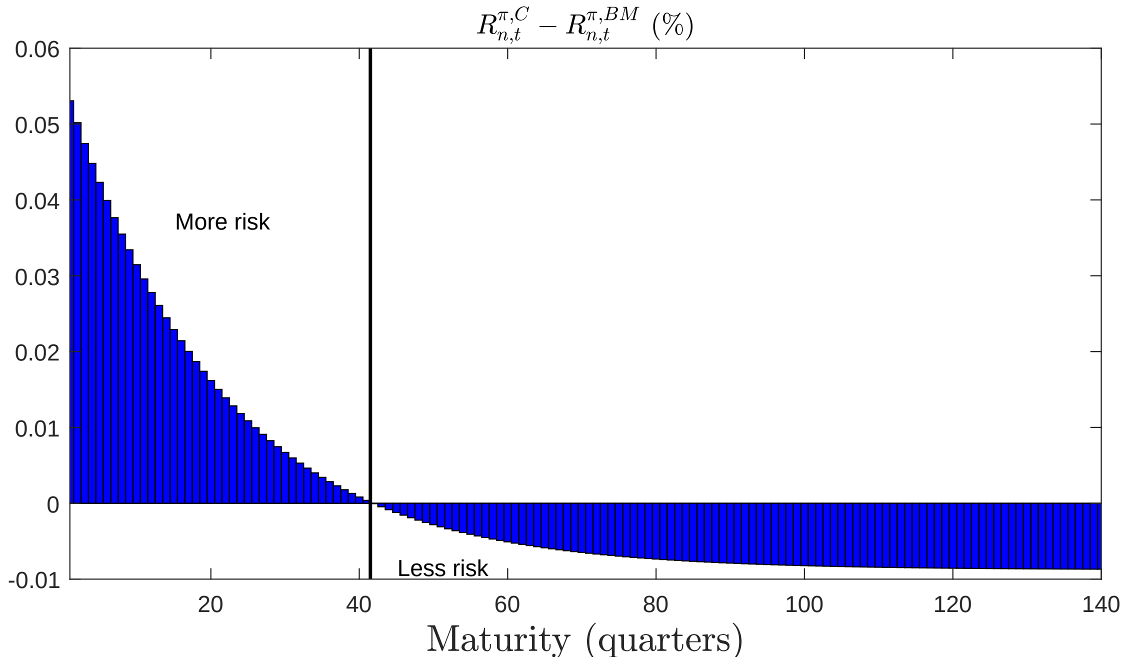

Fig. F1. Fiscal Policies and Term Structure of Profits

Notes: This figure depicts the difference in the term structure of profit average excess returns across our long-run-oriented policy defined in equation (27) (C) and our benchmark policy (BM). Let $P^{\pi,C}_{n,t}$ ($P^{\pi,BM}_{n,t}$) denote the time t value of profits realized at time $t+n$ under our long-run-oriented (benchmark) fiscal policy. The one-period excess return of a zero-coupon claim to profits with maturity n is defined as:

$$R^{\pi,j}_{n,t} = E_t[P^{\pi,j}_{n-1,t+1}/P^{\pi,j}_{n,t}] - r^f_t, \quad j \in \{C, BM\}.$$

Excess returns are annualized and in percentages.

value-weighted return of a strip of dividends paid over a maturity of up to 42 periods (about 10 years) is riskier than under the benchmark policy. This increase in short-term risk, however, comes with the benefit of significantly reduced risk over the long horizon. Since our representative agent is very patient and averse to long-run risk, the reduction in long-term risk premia compounded over the infinite horizon dominates and enhances patent values and growth.

More broadly, this analysis demonstrates that government financing policies, along with their associated sources of uncertainty, are important determinants of the distribution of consumption and profits fluctuations. Financing policies that increase long-term uncertainty may disrupt innovation and growth and be welfare inferior even though they are effective at stabilizing the economy in the short run.

Appendix G. Additional Simulation Results

In table G1, we report both unconditional and conditional moments for both consumption and the consumption-claim excess returns. We consider both our benchmark configuration without corporate taxation and the configuration suggested by the referee. In all cases, our general findings are confirmed: a more sluggish debt-to-output ratio is a leading indicator of lower long-run growth, higher consumption long-run risk, and more severe downside risk in long-run consumption growth. Simultaneously, the consumption claim risk premium increases and features more adverse (positive) skewness. In many cases, the coefficients implied by our model configurations are also quantitatively consistent with our empirical confidence intervals.

In the model, this is true both when we consider the actual simulated conditional persistence of debt-to-output ($\rho_{B,t}$), and when we estimate it from simulated data as we do in the true empirical data ($\hat{\rho}_{B,t}$). For parsimony, we report only the results for $J = 8$, but similar considerations apply to the other horizons that we have considered.

Table G1. The Role of $A_\rho = -1.4$

Moment	Data		No Corp. Tax	with Corp. Tax
	Estimate	SE		
Panel A: Unconditional moments				
$E(\Delta c)$	2.09	0.25	2.04	2.08
$\sigma(\Delta c)$	1.63	0.18	1.93	1.87
$ACF_1(\Delta c)$	0.09	0.16	0.24	0.29
$E(L/3)$	40.63	0.13	34.01	33.41
$E(\tau)\,(\%)$	32.93	0.99	33.5	25
$\sigma(\tau)\,(\%)$	7.56	0.64	11.29	6.3
$E(r_f)$	1.01	0.12	2.44	2.72
$\sigma(m)\,(\%)$			21.96	19.4
$E(r^C)$	3.57	1.16	3.51	3.51
Panel B: Conditional moments with $\rho_{B,t}$ ($J = 8$)				
$E_t[\Delta c^{LR}_{t+J}]$	-0.76	0.16	-0.93	-0.92
$\text{Var}_t[\Delta c^{LR}_{t+J}]$	0.36	0.09	0.07	0.01
$\text{Skew}_t[\Delta c^{LR}_{t+J}]$	-0.51	0.11	-0.74	-0.70
$E_t[R^{W,LR}_{ex,t+J}]$	0.79	0.17	0.80	0.58
$\text{Var}_t[R^{W,LR}_{ex,t+J}]$	0.32	0.09	0.08	0.02
$\text{Skew}_t[R^{W,LR}_{ex,t+J}]$	0.58	0.11	0.58	0.39
Panel C: Conditional moments with estimated $\hat{\rho}_{B,t}$ ($J = 8$)				
$E_t[\Delta c^{LR}_{t+J}]$	-0.76	0.16	-0.24	-0.14
$\text{Var}_t[\Delta c^{LR}_{t+J}]$	0.36	0.09	0.02	0.01
$\text{Skew}_t[\Delta c^{LR}_{t+J}]$	-0.51	0.11	-0.25	-0.14
$E_t[R^{W,LR}_{ex,t+J}]$	0.79	0.17	0.25	0.16
$\text{Var}_t[R^{W,LR}_{ex,t+J}]$	0.32	0.09	0.05	0.03
$\text{Skew}_t[R^{W,LR}_{ex,t+J}]$	0.58	0.11	0.32	0.19

Notes: In panel A, all moments are annualized and multiplied by 100, except the first-order autocorrelation of consumption growth, $ACF_1(\Delta c)$. The log discount factor is denoted by m, and τ is the labor tax rate. r^C and r_f are the return of the consumption claim and the risk-free bond, respectively. $E(L/3)$ is the fraction of hours worked. The entries for the data moments are based on (i) aggregate data provided in the NIPA tables, for the sample 1947:Q1–2016:Q4, and (ii) the estimates in Lustig et al. (2013). In panel B and C, we report estimated coefficients from regressions of conditional moments for consumption growth (Δc_t) and consumption claim excess returns ($R^W_{ex,t}$) for the cumulative forward time horizon of 8 quarters ($J = 8$) on time-varying autocorrelation of debt-to-GDP ($\rho_t(DGDP)$). The long-run (LR) components are constructed from predictive regressions detailed in Appendix A. Standard errors are computed as in Newey and West (1987). In panel B, the entries from the model are based on the actual $\rho_{B,t}$ simulates series. In panel C, we use the model-implied $\hat{\rho}_{B,t}$, which is obtained by estimating $\rho_t(DGDP)$ from simulated data by employing the same procedure adopted for the true data. The column 'No Corp. Tax' ('With Corp. Tax') refers to the benchmark model (extended model in section 3.8) with $A_\rho = -1.4$.

Appendix H. Estimation details

Estimation. Let $m(Y_T, \phi)$ be a vector consisting of moment conditions which depend on the auxiliary parameters vector ϕ and the data Y_T, with T denoting the sample size. We adopt the following procedure to estimate the vector of model's parameters θ.

1. Moment conditions. Let AVG_T denote a sample average over T observations. Our vector of moments conditions, $m(Y_T, \phi) = 0$, comprises the following list of moments for $m(Y_T, \phi)$:

$$AVG_T\left\{DGDP_{t-1} \cdot (DGDP_t - \rho^A_B DGDP_{t-1})\right\} \quad \text{(M.1)}$$

$$AVG_T\left\{\rho_t(DGDP) - (\rho^A_B(1-\rho^A_{\rho,B}) + \rho^A_{\rho,B}\rho_{t-1}(DGDP) + A^A_\rho \epsilon^{TFP}_t)\right\} \quad \text{(M.2)}$$

$$AVG_T\left\{\rho_{t-1}(DGDP) \cdot (\rho_t(DGDP) - \rho^A_B(1-\rho^A_{\rho,B}) - \rho^A_{\rho,B}\rho_{t-1}(DGDP) - A^A_\rho \epsilon^{TFP}_t)\right\} \quad \text{(M.3)}$$

$$AVG_T\left\{\epsilon^{TFP}_t \cdot (\rho_t(DGDP) - \rho^A_B(1-\rho^A_{\rho,B}) - \rho^A_{\rho,B}\rho_{t-1}(DGDP) - A^A_\rho \epsilon^{TFP}_t)\right\} \quad \text{(M.4)}$$

$$AVG_T\left\{\rho_t(DGDP) \cdot (E_t[\Delta c^{LR}_{t+1}] - \gamma^{C,LR}_{1,E}\rho_t(DGDP))\right\} \quad \text{(M.5)}$$

$$AVG_T\left\{\rho_t(DGDP) \cdot (Skew_t[\Delta c^{LR}_{t+1}] - \gamma^{C,LR}_{1,S}\rho_t(DGDP))\right\} \quad \text{(M.6)}$$

$$AVG_T\left\{\rho_t(DGDP) \cdot (E_t[R^{W,LR}_{ex,t+1}] - \gamma^{W,LR}_{1,E}\rho_t(DGDP))\right\} \quad \text{(M.7)}$$

$$AVG_T\left\{\rho_t(DGDP) \cdot (Skew_t[R^{W,LR}_{ex,t+1}] - \gamma^{W,LR}_{1,S}\rho_t(DGDP))\right\} \quad \text{(M.8)}$$

$$AVG_T\left\{\Delta c_{t+1}\right\} - \mu^A_C \quad \text{(M.9)}$$

$$AVG_T\left\{(\Delta c_{t+1} - \mu^A_C)^2\right\} - (\sigma^A_C)^2 \quad \text{(M.10)}$$

$$AVG_T\left\{(\Delta c_{t+1} - \mu^A_C)(\Delta c_t - \mu^A_C)\right\} - \rho^A_c \cdot (\sigma^A_C)^2 \quad \text{(M.11)}$$

$$AVG_T\left\{R^{W,LR}_{t+1}\right\} - \mu^A_{rC} \quad \text{(M.12)}$$

$$AVG_T\left\{(R^{W,LR}_{t+1} - \mu^A_{rC})^2\right\} - (\sigma^A_{rC})^2 \quad \text{(M.13)}$$

where (i) constants are omitted when we use demeaned data, and (ii) the series ϵ^{TFP}_t is obtained from the residuals of a standard OLS estimation of an AR(1) on TFP_t.

2. Estimation using the actual data. Using the observations in the sample Y_T of actual data, we obtain an estimate of the vector ϕ as

$$\hat{\phi}_T = \arg\min_{\phi} \left[m\left(Y_T, \phi\right)' S_T^{-1}(\theta) m\left(Y_T, \phi\right) \right]$$

where $S_T^{-1}(\theta)$ is the continuous GMM optimal weighting matrix. In this case, $T = 279$, that is, our empirical sample size.

3. Simulations and estimation from the model. For a given value of the model's parameters θ, consider a simulated path $Y_{T_S}(\theta)$ based on independent drawings of the shocks in our DSGE model. We set $T_S = 10 \times T$ in the spirit of Michaelides and Ng (2000).

   Using the observations in the sample $Y_{T_S}(\theta)$, we obtain an estimate of the vector $\phi(\theta)$ similarly to what we have done with real data, that is, as

$$\tilde{\phi}(\theta) = \arg\min_{\phi} Q(Y_{T_S}(\theta), \phi)$$

   where

$$Q(Y_{T_S}(\theta), \phi) = \left[m\left(Y_{T_S}(\theta), \phi\right)' S_{T_S}^{-1}(\theta) m\left(Y_{T_S}(\theta), \phi\right) \right],$$

   and $S_{T_S}^{-1}(\theta)$ is the associated continuous GMM optimal weighting matrix.

4. Estimation of the structural parameters. Obtain an indirect estimator of θ as the solution of a minimum distance problem:

$$\min_{\theta} \left[\hat{\phi}_T - \tilde{\phi}(\theta) \right]' W \left[\hat{\phi}_T - \tilde{\phi}(\theta) \right],$$

   where W is a positive definite weighting matrix, which we set equal to the identity matrix,

$W = I$, in all of our estimations. We denote the estimated vector as $\hat{\theta}$.

Distribution of the estimated structural parameters. The estimator of the vector of structural parameters converges in distribution to

$$\sqrt{T}\left(\hat{\theta} - \theta_0\right) \xrightarrow{d} N\left[0, \Omega\left(H, W\right)\right],$$

where the covariance matrix is:

$$\Omega(H, W) = \left(1 + \frac{1}{H}\right)\left(K' J^{-1} W J^{-1} K\right)^{-1}\left(K' J^{-1} W J^{-1}\right) M \left(J^{-1} W J^{-1} K\right)\left(K' J^{-1} W J^{-1} K\right)^{-1}.$$

We discuss the choice of the scalar H later. We specify that:

$$K = \frac{\partial^2 Q}{\partial \phi\, \partial \theta'}, \quad J = -\frac{\partial^2 Q}{\partial \phi\, \partial \phi'}, \quad M = \lim_{T \to \infty} Var\left[\sqrt{T}\frac{\partial Q}{\partial \phi} - E\left(\sqrt{T}\frac{\partial Q}{\partial \phi}\right)\right].$$

Gourieroux et al. (1993) propose to estimate the matrix K as

$$\frac{\partial^2 Q}{\partial \phi\, \partial \theta'}\left(y^s\left(\hat{\theta}\right), \hat{\phi}\right).$$

This amounts to taking the numerical derivative of

$$\frac{\partial Q}{\partial \phi}\left(y^s\left(\hat{\theta}\right), \hat{\phi}\right)$$

with respect to the vector of structural parameters θ and evaluating it at $\hat{\theta}$, where $y^s\left(\hat{\theta}\right)$ is a simulated path of y based on the parameter θ (see page S113 of Gourieroux et al. (1993)). We use the simulated path $Y_{T_S}(\hat{\theta})$.

The matrix J can be obtained by taking the negative of the second derivative of the auxiliary model objective function and evaluating it at the observed sample and the associated estimated coefficient, i.e.,

$$\frac{\partial^2 Q}{\partial \phi \, \partial \phi'} \left(Y_T, \hat{\phi} \right).$$

Finally, using the methodology outlined by Gourieroux et al. (1993) (page S112) we consistently estimate the matrix M as

$$\frac{T}{H} \sum_{h=1}^{H} \left(S_h - \bar{S} \right) \left(S_h - \bar{S} \right)$$

with

$$S_h = \frac{\partial Q}{\partial \phi} \left(y_h \left(\hat{\theta} \right) \right), \quad \bar{S} = \frac{1}{H} \sum_h S_h,$$

where $y_h \left(\hat{\theta} \right)$ is a simulation from the model using the estimated $\hat{\theta}$ based on T observations and $h = 1, 2, .., H$ is a particular repetition. In order to have enough simulation repetitions, we set $H = 20$.

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