Internet Appendix to: "Idea Sharing and the Performance of Mutual Funds"

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Contents

IA.1. Proof of Lemma A.1

Denote by $p(\Pi|\mathscr{F}_t^i)$ the conditional density of Π with respect to $\mathscr{F}_t^i$. Fixing a time $t = \tau_k$ and applying Bayes' rule, the conditional density $p(\Pi|\mathscr{F}_t^i)$ satisfies the recursive relation

$$p(\Pi|\mathscr{F}_t^i) = \frac{p(\Pi|\mathscr{F}_{t-}^i) f(\mathbf{S}|\Pi)}{\int_{\mathbb{R}} p(x|\mathscr{F}_{t-}^i) f(\mathbf{S}|x) \mathrm{d}x} \tag{IA.1}$$

where $f(\mathbf{S}|\Pi)$ denotes the density of a vector of signals $\mathbf{S}$ conditional on Π and where $p(\Pi|\mathscr{F}_{t-}^i)$ satisfies

$$p(\Pi|\mathscr{F}_{t-}^i) = \left(2\pi o_{t-}^i\right)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\frac{(\widehat{\Pi}_{t-}^i - \Pi)^2}{o_{t-}^i}\right) \tag{IA.2}$$

since, from Theorem A.1, $p(\Pi|\mathscr{F}_t^i)$ is conditionally Gaussian for any $t \in (\tau_{k-1}, \tau_k)$ (Liptser and Shiryaev (2001), Theorem 12.6). First, let $\mathbf{S} = \begin{bmatrix} S_{n_{t-}^i+1}^i & S_{n_{t-}^i+2}^i & \cdots & S_{n_{t-}^i+\Delta n_t^i}^i \end{bmatrix}^\top$ be the vector of signals in the sequence $(S_{j+n_{t-}^i}^i : 1 \leq j \leq \Delta n_t^i)$. Conditional on Π, these signals are independent and thus

$$f(\mathbf{S}|\Pi) = (2\pi\sigma_S^2)^{-\frac{\Delta n_t^i}{2}} \prod_{j=1}^{\Delta n_t^i} \exp\left(-\frac{1}{2}\left(\frac{S_{j+n_{t-}^i}^i - \Pi}{\sigma_S}\right)^2\right). \tag{IA.3}$$

After substituting (IA.3) in (IA.1) and integrating, the conditional density $p(\Pi|\mathscr{F}_t^i)$ is explicitly given by

$$p(\Pi|\mathscr{F}_t^i) = \sqrt{\frac{1}{2\pi}\left(\frac{1}{o_{t-}^i} + \frac{\Delta n_t^i}{\sigma_S^2}\right)} \exp\left(-\frac{1}{2}\left(\frac{1}{o_{t-}^i} + \frac{\Delta n_t^i}{\sigma_S^2}\right)\left(\left(\frac{\widehat{\Pi}_{t-}^i}{o_{t-}^i} + \frac{\sum_{j=1}^{\Delta n_t^i} S_{j+n_{t-}^i}^i}{\sigma_S^2}\right)\left(\frac{1}{o_{t-}^i} + \frac{\Delta n_t^i}{\sigma_S^2}\right)^{-1} - \Pi\right)^2\right). \tag{IA.4}$$

Second, let $\mathbf{S} = Y_t^i$ be the aggregate signal, in which case

$$f(Y_t^i|\Pi) = \left(2\pi \frac{\sigma_S^2}{\Delta n_t^i}\right)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\left(Y_t^i - \Pi\right)^2 \frac{\Delta n_t^i}{\sigma_S^2}\right). \tag{IA.5}$$

Substituting (IA.5) in (IA.1) and integrating, the conditional density in (IA.4) becomes

$$p(\Pi|\mathscr{F}_t^i) = \sqrt{\frac{1}{2\pi}\left(\frac{1}{o_{t-}^i} + \frac{\Delta n_t^i}{\sigma_S^2}\right)} \exp\left(-\frac{1}{2}\left(\frac{1}{o_{t-}^i} + \frac{\Delta n_t^i}{\sigma_S^2}\right)\left(\left(\frac{\widehat{\Pi}_{t-}^i}{o_{t-}^i} + \frac{Y_t^i}{\sigma_S^2/\Delta n_t^i}\right)\left(\frac{1}{o_{t-}^i} + \frac{\Delta n_t^i}{\sigma_S^2}\right)^{-1} - \Pi\right)^2\right). \tag{IA.6}$$

Clearly, the expressions in (IA.4) and (IA.6) are equivalent if and only if $Y_t^i = (\Delta n_t^i)^{-1} \sum_{j=1}^{\Delta n_t^i} S_{j+n_{t-}^i}^i$ and, by induction, this result must be true for all $t = \tau_k$ and $k \in \{0, 1, ..., N_T^i\}$.

IA.2. Proof of Lemma A.3

Observe that the Brownian motion $\widehat{B}^c$ is adapted to $\mathscr{F}^c$ and, therefore, to $\mathscr{F}^i \supset \mathscr{F}^c$. Second, combine (57) and (74) and obtain

$$\mathrm{d}\widehat{B}_t^i = \mathrm{d}\widehat{B}_t^c - \frac{1}{\lambda_{2,t}\sigma_\Theta}(\lambda'_{1,t}\Delta_t^i + (\lambda'_{2,t} - a_\Theta \lambda_{2,t})(\widehat{\Theta}_t^i - \widehat{\Theta}_t^c))\mathrm{d}t. \tag{IA.7}$$

Since $\xi_t \in \mathscr{F}_t^c \subset \mathscr{F}_t^i$, it follows that $\widehat{\Theta}_t^i - \widehat{\Theta}_t^c = -\frac{\lambda_{1,t}}{\lambda_{2,t}}\Delta_t^i$, which, substituted in (IA.7) and using (61) gives (86). Third, for $E^{\widehat{\mathbb{P}}^c}[Z] = 1$ to hold, i.e., for $\widehat{\mathbb{P}}^i$ to be absolutely continuous with respect to $\widehat{\mathbb{P}}^c$ under $\mathscr{F}^i$, the Radon-Nikodym derivative in (85) must be a martingale. A sufficient condition under which Z is a martingale is the Novikov condition (Karatzas and Shreve (1988), Proposition 5.12):

$$\mathbb{E}^{\widehat{\mathbb{P}}^c}\left[\exp\left(\frac{1}{2}\int_0^T (k_t\Delta_t^i)^2 \mathrm{d}t\right)\right] < \infty, \quad 0 \leq T \leq \infty \tag{IA.8}$$

where the process Δ^i under $\widehat{\mathbb{P}}^c$ satisfies

$$\mathrm{d}\Delta_t^i = -o_t(n^i)k_t^2\Delta_t^i \mathrm{d}t + k_t(o_t(n^i) - o_t^c)\mathrm{d}\widehat{B}_t^c. \tag{IA.9}$$

Observing that the process in (IA.9) is Gaussian, Example 3 (a) (Liptser and Shiryaev (2000), p. 233) shows that the Novikov condition in (IA.8) boils down to

$$\sup_{t\leq T} |k_t|\mathbb{E}^{\widehat{\mathbb{P}}^c}\left[\left|\Delta_t^i\right|\right] < \infty, \quad \sup_{t\leq T} k_t^2 \mathbb{V}^{\widehat{\mathbb{P}}^c}\left[\Delta_t^i\right] < \infty. \tag{IA.10}$$

In this setup, the conditions in (IA.10) simplify to

$$\sup_{t\leq T} |k_t| \textstyle\int_0^t k_s^2 \mathrm{d}s < \infty, \quad \sup_{t\leq T} k_t^2 \int_0^t k_s^2 \mathrm{d}s < \infty. \tag{IA.11}$$

Using (61) and anticipating on the equilibrium result of Lemma C.2, the two conditions in (IA.11) are equivalent to requiring that the function $\phi(\cdot)$ be continuous, which it is by assumption. Theorem A.3 then follows from Girsanov theorem (Karatzas and Shreve (1988), Theorem 5.1).

IA.3. Proof of Lemma B.2

Consider first the right-hand side of (138). From (134), I obtain

$$\Sigma_t(n,m)\frac{A_{Q,t}^\top A_{Q,t}}{B_{Q,t}^2}\Sigma_t(n,m) = \omega_t^\top \omega_t \left(k_t \frac{m}{\sigma_S^2} o_t(n) o_t(n+m)\right)^2 = \Omega_t \left(k_t \frac{m}{\sigma_S^2} o_t(n) o_t(n+m)\right)^2. \tag{IA.12}$$

Furthermore, from (135), I can write

$$\left(\frac{B_{\Psi,t}(n+m)A_{Q,t}}{B_{Q,t}} - A_{\Psi,t}\right)\Sigma_t(n,m) = \left(\frac{B_{\Psi,t}(n)A_{Q,t}}{B_{Q,t}} - A_{\Psi,t}\right)\Sigma_t(n,m) - \Omega_t \left(k_t \frac{m}{\sigma_S^2} o_t(n) o_t(n+m)\right)^2, \tag{IA.13}$$

which, substituted in (IA.12) yields

$$\dot{\Sigma}_t(n,m) = \Omega_t \left(k_t \frac{m}{\sigma_S^2} o_t(n) o_t(n+m)\right)^2 - \Sigma_t(n,m)\left(\frac{B_{\Psi,t}(n)A_{Q,t}}{B_{Q,t}} - A_{\Psi,t}\right)^\top - \left(\frac{B_{\Psi,t}(n)A_{Q,t}}{B_{Q,t}} - A_{\Psi,t}\right)\Sigma_t(n,m) \tag{IA.14}$$

$$= \Omega_t \left(k_t \frac{m}{\sigma_S^2} o_t(n) o_t(n+m)\right)^2 + \left(\dot{\Omega}_t - 2\Omega_t k_t^2 o_t(n)\right)\left(\frac{m}{\sigma_S^2} o_t(n) o_t(n+m)\right) \tag{IA.15}$$

where the second equality follows from simplifications based on (61). Second, consider the left-hand side of (138) and directly differentiate Σ in (107) using (69) to obtain

$$\dot{\Sigma}_t(n,m) = \dot{\Omega}_t \frac{m}{\sigma_S^2} o_t(n) o_t(n+m) - \Omega_t \frac{m}{\sigma_S^2} k_t^2 o_t(n) o_t(n+m)(o_t(n) + o_t(n+m)). \tag{IA.16}$$

Finally, regrouping terms in (IA.14) and using Eq. 68 shows that (IA.14) and (IA.16) coincide and the differential equation in (138) must therefore hold.

IA.4. Population dynamics in the general setting of Section 2.1

In this appendix, I derive the dynamics of information sharing for the general setting of Section 2.1 (Appendix IA.4). In the general setting of Section 2.1, the dynamics of a manager i's number n^i of ideas satisfy

$$\mathrm{d}n_t^i = \Delta n_t^i \mathrm{d}N_t^i, \quad n_0^i \sim \pi_0, \quad \Delta n_t^i \sim \pi_t(\cdot; n_{t-}^i) \tag{IA.17}$$

where $(N^i)_{t\geq 0}$ denotes a Poisson process with intensity $\eta(n_{t-}^i)$. These dynamics imply a certain cross-sectional distribution, μ, of number of ideas, i.e., the distribution μ must satisfy a certain Kolmogorov Forward Equation (KFE), which I derive using the result formulated in Lemma IA.4.1.

Lemma IA.4.1. *Define the expectation*

$$g_t \equiv \mathbb{E}[f(n_t)] = \sum_{k\in\mathbb{N}} f(k)\mu_t(k) \tag{IA.18}$$

for an arbitrary function $f(\cdot)$. Then, the function g must satisfy the differential equation

$$\frac{d}{dt}g_t = \sum_{n\in\mathbb{N}} \eta_t(n)\mu_t(n) \sum_{m\in\mathbb{N}} f(n+m)\pi_t(m;n) - \sum_{n\in\mathbb{N}} \eta_t(n)\mu_t(n)f(n) \tag{IA.19}$$

with initial condition $g_0 = \sum_{k\in\mathbb{N}} f(k)\pi_0(k)$.

Proof. Observe that the generator of the process in (IA.17) satisfies

$$Af(n) = \eta_t(n) \sum_{m\in\mathbb{N}} \pi_t(m;n)(f(n+m) - f(n)) \tag{IA.20}$$

and rewrite the expectation in (IA.18) as

$$g_t = \mathbb{E}[f(n_0)] + \int_0^t \mathbb{E}[Af(n_s)]\mathrm{d}s = \mathbb{E}[f(n_0)] + \int_0^t \sum_{n\in\mathbb{N}} Af(n)\mu_s(n)\mathrm{d}s. \tag{IA.21}$$

Differentiating Eq. (IA.21) with respect to time and rearranging yields (IA.19). □

To obtain the KFE in (3), I then change the summation order in (IA.19). Specifically, introduce the change of variable $k \equiv n+m$ and rewrite the first term in (IA.19) as

$$\sum_{n\in\mathbb{N}}\sum_{m\in\mathbb{N}} f(n+m)\eta_t(n)\pi_t(m;n)\mu_t(n) = \sum_{k\in\mathbb{N}}\sum_{m=1}^{k-1} f(k)\eta_t(k-m)\pi_t(m;k-m)\mu_t(k-m). \tag{IA.22}$$

Plugging (IA.22) into (IA.19) and rearranging, I obtain

$$\sum_{n\in\mathbb{N}} f(n)\frac{\mathrm{d}}{\mathrm{d}t}\mu_t(n) = \sum_{n\in\mathbb{N}} f(n)\left(\eta\sum_{m=1}^{n-1}\eta_t(n-m)\pi_t(m;n-m)\mu_t(n-m) - \eta_t(n)\mu_t(n)\right). \tag{IA.23}$$

Observing that the function $f(\cdot)$ is arbitrary, the KFE in (3) follows.

IA.5. Average trajectories of number of ideas

In this appendix, I derive a manager's average trajectory of number of ideas, which I use for the main analysis in Section 5 (Appendix IA.5). In particular, I obtain the average trajectory of a manager i's number of ideas conditional on manager i holding $n_T^i = k$ ideas at the horizon date, i.e., $\mathbb{E}\left[n_t^i \middle| n_T^i = k\right]$. Applying Bayes' rule, first observe that

$$\mathbb{P}\left[n_t^i = m \middle| n_T^i = k\right] = \frac{\mathbb{P}[n_t^i = m]\mathbb{P}[n_T^i = k | n_t^i = m]}{\mathbb{P}[n_T^i = k]} = \frac{\mu_t(m)\rho_{T-t}(k;m)}{\mu_T(k)} \tag{IA.24}$$

where ρ is the probability that manager i gets $k-m$ ideas by the horizon date conditional on holding m ideas at time t. To compute this probability, apply the result of Lemma IA.4.1 to $g_s \equiv \sum_{n\in\mathbb{N}} f(n)\rho_{T-s}(n;m)$ for $s > t$ with initial condition, $g_t = m$, which yields:

$$\rho_{T-t}(k;m) = \mathbf{1}_{\{k\geq m\}} e^{t-(k-m+1)T}\left(\left(e^{\eta T}-1\right)^{k-m-1}\left(e^{\eta(T-t)}-1\right)\right)^{\mathbf{1}_{\{k-m\geq 1\}}} \tag{IA.25}$$

under idea sharing, whereas under idea origination:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_s(k;m) = -\eta\rho_s(k;m) + \eta\rho_s(k;m-1), \tag{IA.26}$$

with initial condition, $\rho_t(n;m) = \delta_{n=m}$, where $\delta_{n=m}$ is a Dirac mass at $n = m$. It then follows that the average trajectory of manager i's number of ideas is given by

$$\mathbb{E}\left[n_t^i \middle| n_T^i = k\right] = \frac{1}{\mu_T(k)} \sum_{m=1}^{k} m\mu_t(m)\rho_{T-t}(k;m), \tag{IA.27}$$

which has a closed-form solution under idea sharing:

$$\mathbb{E}\left[n_t^i \middle| n_T^i = k\right] = \frac{1 - e^{\eta T} + e^{k\eta(T-t)}\left(e^{\eta t}-1\right)^k \left(e^{\eta T}-1\right)^{1-k}}{1-e^{\eta(T-t)}}. \tag{IA.28}$$

IA.6. Proof of Proposition 9 (distribution shift relative to luck)

In this appendix I show that the cross-sectional distribution of $t-$statistics is shifted to the left when the distribution of number of ideas is symmetric or satisfies Corollary 1.

I start by computing the probability that a manager i's $t-$statistic is negative:

$$\mathbb{P}[t_{\alpha,t}^i \leq 0] = \int_{-\infty}^{0} l_t(x)\left(1 + \sum_{n\in\mathbb{N}^*} \mu_t(n)\mathrm{erf}\left(\frac{x}{\sqrt{2}}R_t(n)\right)\right)\mathrm{d}x \tag{IA.29}$$

$$= \frac{1}{2} - \frac{1}{\pi}\sum_{n\in\mathbb{N}^*}\mu_t(n)\tan^{-1}\left(\frac{n-\phi_t}{\sqrt{n}}\frac{\sqrt{\mathbb{E}[SR_t^2]}}{\sigma_S|k_t|}\right). \tag{IA.30}$$

Proving that Eq. (43) holds is thus equivalent to proving:

$$\sum_{n\in\mathbb{N}^*}\mu_t(n)\tan^{-1}\left(\frac{n-\phi_t}{\sqrt{n}}\varphi_t\right) \equiv \Upsilon_t < 0, \tag{IA.31}$$

where $\varphi_t > 0$ is positive at all finite times.

Assuming the functional form for μ_t in Corollary 1, I start by bounding $\tan^{-1}(\cdot)$ in Eq. (IA.31) by a piecewise linear function:

$$\tan^{-1}\left(\varphi_t\tfrac{n-\phi_t}{\sqrt{n}}\right) \leq \mathbf{1}_{n\leq\lfloor\phi_t\rfloor}\tfrac{\phi_t-n}{\phi_t-1}\tan^{-1}(\varphi_t(1-\phi_t)) + \mathbf{1}_{n>\lfloor\phi_t\rfloor}\min\left\{\tfrac{n-\phi_t}{\sqrt{\phi_t}}, \tfrac{\pi}{2}\right\} \equiv g(n), \quad n\in\mathbb{N}^*\ , \tag{IA.32}$$

where the first term exploits that the support is bounded at 1 and the second term exploits both that the function is concave for all $n \geq \phi_t$ and thus bounded above by its first-order Taylor expansion and that $\tan^{-1}(\cdot)$ is bounded above at $\pi/2$. Although this bound can be tightened, it is sufficient to prove Eq. (IA.31) and simple enough to compute explicit expressions: rewriting μ_t in Eq. (27) as

$$\mu_t(n) = \phi_t^{-n}(\phi_t-1)^{n-1}, \tag{IA.33}$$

and taking expectations over the function $g(\cdot)$ in Eq. (IA.32) yields the explicit bound:

$$\Upsilon_t < \left(\frac{\phi_t - 1}{\phi_t}\right)^{\lfloor\phi_t\rfloor} \frac{\varphi_t \lfloor\phi_t\rfloor}{\sqrt{\phi_t}} \left(\begin{array}{c} \left(1 + \frac{\pi\sqrt{\phi_t}}{2\varphi_t} - \left\lceil \phi_t + \frac{\pi\sqrt{\phi_t}}{2\varphi_t} \right\rceil\right) \left(\frac{\phi_t - 1}{\phi_t}\right)^{\left\lceil \phi_t + \frac{\pi\sqrt{\phi_t}}{2\varphi_t} \right\rceil - (\lfloor\phi_t\rfloor + 1)} \\ + \sqrt{\phi_t}\left(\frac{\tan^{-1}(\varphi_t(1-\phi_t))}{\varphi_t(\phi_t - 1)} + \frac{1}{\sqrt{\phi_t}}\right) \end{array} \right). \quad \text{(IA.34)}$$

By assumption, $\eta > 0$, which implies that $\phi_t > 1$ and thus the term inside the bracket is nonpositive, from which the inequality in Eq. (IA.31) follows.

Note that the result also obtains when μ_t is strictly symmetric at all times. Since the function $\tan^{-1}(\cdot)$ in Eq. (IA.31) is odd and negative for all $n \leq \lfloor\phi_t\rfloor$, I define a new function $f : [\lfloor\phi_t\rfloor + 1, 2\lfloor\phi_t\rfloor] \to \mathbb{R}_+$ by reflecting $\tan^{-1}(\cdot)$ in Eq. (IA.31) first horizontally about zero and then vertically about ϕ_t to obtain:

$$f(n) = \tan^{-1}\left(\varphi_t \frac{n - (2\lfloor\phi_t\rfloor + 1 - \phi_t)}{\sqrt{2\lfloor\phi_t\rfloor + 1 - n}}\right), \quad n = \lfloor\phi_t\rfloor + 1, \ldots, 2\lfloor\phi_t\rfloor. \quad \text{(IA.35)}$$

The concavity of the skill-to-luck ratio then implies that

$$f(n) > \tan^{-1}\left(\varphi_t \frac{n - \phi_t}{\sqrt{n}}\right), \quad n = \lfloor\phi_t\rfloor + 1, \ldots, 2\lfloor\phi_t\rfloor. \quad \text{(IA.36)}$$

Using this result I obtain a strict bound for Eq. (IA.31):

$$\Upsilon_t < \sum_{n \in [1, \lfloor\phi_t\rfloor] \cup (2\lfloor\phi_t\rfloor, \ldots, \infty)} \mu_t(n) \tan^{-1}\left(\frac{n - \phi_t}{\sqrt{n}} \varphi_t\right) + \sum_{n \in (\lfloor\phi_t\rfloor, 2\lfloor\phi_t\rfloor]} \mu_t(n) f(n), \quad \text{(IA.37)}$$

at all finite times. Symmetry further implies that

$$\sum_{n \in [1, \lfloor\phi_t\rfloor]} \mu_t(n) \tan^{-1}\left(\frac{n - \phi_t}{\sqrt{n}} \varphi_t\right) + \sum_{n \in (\lfloor\phi_t\rfloor, 2\lfloor\phi_t\rfloor]} \mu_t(n) f(n) = 0 \quad \text{(IA.38)}$$

and that μ_t has zero mass beyond twice its mean:

$$\mu_t(n) = 0, \quad n \geq 2\lfloor\phi_t\rfloor, \quad \text{(IA.39)}$$

since it is symmetric and its support is bounded from the left at 1. Hence, in the symmetric case Eq. (IA.37) directly leads to Eq. (IA.31).

IA.7. Details on computations in Section 6 (fund flows and fees)

In this appendix I discuss computational details related to the equilibrium solution in the presence of fund flows and fees. For the sake of brevity, I simply pinpoint results that differ from the baseline model.

Note first that flows and fees do not affect the results of Proposition 1. However, they modify portfolio strategies. To see how, I go over the main steps of Appendix B in the presence of fees and flows. Maximizing manager i's expected utility over compensation in Eq. (48) implies:

$$\max_{\theta^i} \mathbb{E}\left[-\exp\left(-\gamma f((\tau+1)W_T^i - \tau B_T)\right)\middle|\,\mathscr{F}_t^i\right]. \tag{IA.40}$$

The solution to this maximization problem is

$$\theta_{T-}^i \equiv \theta^i(\Psi, n, T-) = \frac{1}{f(\tau+1)\gamma}\left(o_{T-}(n)\right)^{-1}\lambda_{T-}^\top\Psi + \frac{\tau}{\tau+1}\left(\frac{\lambda_{1,T-}}{\lambda_{2,T-}}\Delta_{T-}^i + \widehat{\Theta}_{T-}^i\right). \tag{IA.41}$$

which substituted in the value function yields the boundary condition

$$J(W, B, \Psi, n, T-) = -\exp\left(-\gamma f((\tau+1)W - \tau B) - \frac{1}{2}\left(o_{T-}(n)\right)^{-1}\Psi^\top\Lambda_{T-}\Psi\right). \tag{IA.42}$$

The problem has an additional state variable, B, which applying Ito's lemma to Eq. (50) satisfies:

$$\mathrm{d}B_t = \widehat{\Theta}_t^c \mathrm{d}P_t = \begin{pmatrix} \frac{\lambda_{1,t}}{\lambda_{2,t}} & 1 \end{pmatrix}\Psi_t^i \mathrm{d}P_t \equiv \overline{\omega}_t^\top \Psi_t^i \mathrm{d}P_t. \tag{IA.43}$$

The associated HJB equation now satisfies

$$0 = \max_{\theta^i}\left\{J_W A_Q \Psi^i\theta^i + \frac{1}{2}J_{WW}B_Q^2(\theta^i)^2 + B_Q B_\Psi(n^i)^\top J_{W\Psi}\theta^i + J_{WB}B_Q^2\overline{\omega}^\top\Psi\theta^i\right\} + J_t + J_\Psi^\top A_\Psi \Psi^i \tag{IA.44}$$

$$+\frac{1}{2}\mathrm{tr}(J_{\Psi\Psi}B_\Psi(n^i)B_\Psi(n^i)^\top) + J_B(\Psi^i)^\top\overline{\omega}^\top A_Q\Psi^i + \frac{1}{2}J_{BB}B_Q^2(\Psi^i)^\top\overline{\omega}^\top\overline{\omega}\Psi^i + B_Q\overline{\omega}^\top\Psi^i B_\Psi(n^i)^\top J_{B\Psi} \tag{IA.45}$$

$$+\eta(n^i)\mathbb{E}^{\mathscr{L}_t(\widehat{Y}^i,\Delta n^i)}\left[J(W^i, B^i, \Psi^i + \sigma(n^i, \Delta n^i)\widehat{Y}^i, n^i + \Delta n^i, t) - J(W^i, B^i, \Psi^i, n^i, t)\right]. \tag{IA.46}$$

The first-order condition then yields the following portfolio policy:

$$\theta_t^i \equiv \theta_t(\Psi^i, n^i) = -\frac{J_W A_Q\Psi^i + B_Q B_\Psi(n^i)^\top J_{W\Psi} + J_{WB}B_Q^2\overline{\omega}^\top\Psi}{J_{WW}B_Q^2}. \tag{IA.47}$$

Substituting back in the HJB equation, tedious derivations show the ansatz of Theorem B.1 becomes:

$$J(W, B, \Psi, n, t) = -\exp\left(-\gamma f((\tau+1)W - \tau B) - u_t(n) - \frac{1}{2}\left(\Psi^\top R_t(n) + R_t(n)^\top\Psi + \Psi^\top M_t(n)\Psi\right)\right), \tag{IA.48}$$

where R and M satisfy the system of equations in Theorem B.1. The equation for u differs, but is irrelevant for portfolio strategies and is thus omitted. As a result, the solution for R and M is identical to Lemma B.3. Substituting these expressions in the optimal policy above yields Eq. (51).

To obtain price coefficients I now go over the main steps in Appendix C. Aggregating first portfolios at the horizon date:

$$\int_0^1 \theta_{T-}^i \mathrm{d}i = \sum_{n\in\mathbb{N}}\mu_T(n)\frac{1}{\gamma f}\left(o_{T-}(n)\right)^{-1}\lambda_{T-}^\top\Gamma_{T-}(n)\Psi_T + \tau\overline{\omega}_{T-}\Psi_T = (\tau+1)\mathbf{1}^\star\Psi_T, \tag{IA.49}$$

which yields the boundary conditions

$$\lambda_{1,T-} = \frac{o^c_{T-}\phi_T}{(\tau+1)(\phi_T o^c_{T-}+\sigma^2_S)} \quad \text{and} \quad \lambda_{2,T-} = -\gamma f \frac{o^c_{T-}\sigma^2_S}{(\phi_T o^c_{T-}+\sigma^2_S)} \quad . \tag{IA.50}$$

Similarly, aggregating portfolios at date t, Eq. (184) becomes:

$$\sum_{n\in\mathbb{N}} \mu_t(n) \left(A_{Q,t} - B_{Q,t}\left(o_t(n)\right)^{-1} B_{\Psi,t}(n)^\top \Lambda_t \right) \Gamma_t(n)\omega_t = -\frac{\lambda_{1,t}}{\lambda_{2,t}}\gamma f(\tau+1)B^2_{Q,t}, \tag{IA.51}$$

since $\overline{\omega}_t^\top \omega_t = 0$, which yields, $\frac{\lambda_{1,t}}{\lambda_{2,t}} = -\frac{\phi_t}{\gamma f(\tau+1)\sigma^2_S}$, and thus

$$o^c_t = \left(\frac{1}{\sigma^2_\Pi} + \left(\frac{\phi_0}{\gamma f(\tau+1)\sigma_\Theta \sigma^2_S} \right)^2 + \left(\frac{1}{\sigma_\Theta \sigma^2_S \gamma f(\tau+1)} \right)^2 \int_0^t \left(\frac{\mathrm{d}}{\mathrm{d}s}\phi_s \right)^2 \mathrm{d}s \right)^{-1} . \tag{IA.52}$$

Finally, spelling out the second equation of the system Eq. (191) becomes

$$\frac{\mathrm{d}}{\mathrm{d}t}\lambda_{2,t} = -k_t\lambda_{2,t}B_{Q,t} + B^2_{Q,t}\left(\lambda_{2,t}\frac{\phi_t o^c_t + \sigma^2_S}{o^c_t \sigma^2_S} + \gamma f \right), \tag{IA.53}$$

the solution of which is Eq. (52).

Furthermore, going through the steps of Appendix D and simplifying yields

$$\begin{aligned}
\widehat{\theta}^i_t &= \frac{n^i_t - \phi_t}{f\gamma(\tau+1)(\sigma^2_S + o^c_t\phi_t)} \left(\frac{\sigma^2_S(\tau+1) + \tau o^c_t \phi_t}{\sigma^2_S(\tau+1)} \Delta_t + f\gamma o^c_t \Theta_t \right) + \frac{\sqrt{n^i_t}}{f\gamma(\tau+1)\sigma_S}\epsilon^i_t & \text{(IA.54)} \\
&\quad + \tau \frac{\sigma^2_S k_t (f\gamma\sigma^2_S\sigma_\Theta(\tau+1) + k_t o^c_t \phi_t)}{(\sigma^2_S(\tau+1)(f\gamma\sigma_\Theta - k_t) - \tau k_t o^c_t \phi_t)^2} \left(\frac{\phi_t}{f\gamma\sigma^2_S(\tau+1)}\Delta_t - \Theta_t \right) & \text{(IA.55)} \\
&\equiv a_{\Delta,t}(n^i_t)\Delta_t + a_{\Theta,t}(n^i_t)\Theta_t + a_{\epsilon,t}(n^i_t)\epsilon^i_t, & \text{(IA.56)}
\end{aligned}$$

where the second line corresponds to the function $H(\cdot)$ in Eq. (53). To compute informational alphas, I substitute the equilibrium solution in Eq. (80) to obtain

$$\begin{aligned}
A_{Q,t} &= \tau \begin{pmatrix} \frac{k_t o^c_t (f\gamma\sigma^4_S\sigma_\Theta(\tau+1) + k_t o^c_t \phi_t (2\sigma^2_S + o^c_t\phi_t))}{(\tau+1)(\sigma^2_S + o^c_t\phi_t)^2} \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ \gamma f o^c_t \end{pmatrix} \frac{\sigma^4_S k_t o^c_t (k_t - (\tau+1)\sigma_\Theta\gamma f)}{(\sigma^2_S + o^c_t\phi_t)^2} & \text{(IA.57)} \\
&\equiv b_{\Delta,t}(n^i_t)\Delta_t + b_{\Theta,t}(n^i_t)\Theta_t, & \text{(IA.58)}
\end{aligned}$$

and

$$B_{Q,t} = \frac{o^c_t |\sigma^2_S(\tau+1)(k_t - f\gamma\sigma_\Theta) + \tau k_t \phi_t o^c_t|}{(\tau+1)(\sigma^2_S + \phi_t o^c_t)}. \tag{IA.59}$$

With these expressions can then compute $t-$statistics by simulations using Eq. (22).

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