Data Abundance and Asset Price Informativeness

On-Line Appendix

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This note is the on-line appendix for "Data Abundance and Asset Price Informativeness." It contains the proofs for findings mentioned in the paper but not proven there for the sake of brevity. It is not intended for publication. The on-line appendix is organized as follows.

- Section 1 derives the equilibrium demand for the raw signal (given in Lemma 1 in the paper).
- Section 2 analyzes the case in which the upper bound on the mass of speculators, i.e., $\bar{\alpha}$, is less than 2.
- Section 3 analyzes the case in which speculators can make their decision to buy the processed signal contingent on the realization of the price at date 1.
- Section 4 completes Proposition 3 (case 2).
- Section 5 completes Proposition 4 by considering the case in which $C_p > C_{min}(\theta, \alpha_1^e)$, derives sufficient and necessary conditions for $\partial \mathcal{E}_2(C_r, C_p)/\partial C_r > 0$, and shows that long run price informativeness jumps down when the market for the raw signal takes off, provided the processed signal is produced when the raw signal is not (i.e., $\mathcal{E}_2(\frac{\theta}{8}, C_p) < \mathcal{E}_2(\infty, C_p)$ if $C_p \leq C_{max}(\theta, 0)$).

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- Section 6 shows that $C_{max}(\theta, \alpha_1)$ decreases with α_1 as claimed in the proof of Proposition 3.
- Section 7 analyzes the case in which speculators who only buy one signal can trade at both dates if they wish.
- Section 8 shows that it is optimal for speculators to trade when they receive their signal if they cannot trade more than one share overall.

1 Equilibrium demand for the raw signal (Proof of Lemma 1).

Let $\pi_1^{gross,a}(\alpha_1) = \alpha_1 \bar{\pi}_1(\alpha_1) = \alpha_1 \max\left\{\frac{\theta}{2}(1-\alpha_1), 0\right\}$ be the aggregate gross expected profit for speculators who receive the raw signal. We represent by α_1^e the equilibrium value of α_1, the demand for the raw signal at date 1. Proceeding as in the market for the processed signal, we deduce that if $\alpha_1^e > 0$ then α_1^e solves:

$$\pi_1^{gross,a}(\alpha_1^e) = \alpha_1^e \max\left\{\frac{\theta}{2}(1-\alpha_1^e), 0\right\} = C_r. \tag{1.1}$$

As, $\pi_1^{gross,a}(\alpha_1)$ reaches its maximum for $\alpha_1 = 1/2$, we deduce that the previous equation has no solution if $C_r > \frac{\theta}{8}$. In this case, for all values of α_1, $\pi_1^{gross,a}(\alpha_1) < C_r$ and therefore there is no fee at which trades between the seller of the raw signal and buyers of this signal are mutually beneficial. Thus, in this case, $\alpha_1^e = 0$.

For $C_r < \frac{\theta}{8}$, eq.(1.1) has two solutions in $(0, 1)$. As explained in the text, we select the highest as the equilibrium demand since it yields the lowest fee charged by the information seller. This solution is:

$$\alpha_1^e = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{2C_r}{\theta}}, \tag{1.2}$$

and the corresponding equilibrium fee is therefore $F_1^e = C_r/\alpha_1^e$. Last, if $C_r = \frac{\theta}{8}$, the only solution to eq.(1.1) is $\alpha_1^e = 0$.

2 The case in which $\bar{\alpha} \leq 2$.

In equilibrium speculators' demands for each type of signal (i.e., α_1^e and α_2^e) must satisfy:

$$0 \leq \alpha_1^e \leq \bar{\alpha}, \text{and } \ 0 \leq \alpha_2^e \leq \bar{\alpha}.$$

When $\bar{\alpha} > 2$, the right hand sides of these constraints are never binding (as $\alpha_2^e \leq 2$ and $\alpha_1^e \leq 1$). This is the case on which we focused in the baseline version of the model. In contrast, when $\bar{\alpha} \leq 2$, these constraints can be binding, in which case one obtains a corner solution for equilibrium demands. We discuss each case in turn below. Note that a corner solution obtains for the demand for the raw signal iff $C_r > \frac{\theta}{8}$ (in which case $\alpha_1^e = 0$) or $C_r < \hat{C}_r$ (in which case $\alpha_1^e = 1$), where $\hat{C}_r = \frac{\theta}{2}(\frac{1}{4} - (\min\{(\bar{\alpha} - 0.5), 0\})^2)$.

1. **Case 1.** The zero profit conditions eq.(19) in the text and eq.(1.1) in this appendix have solutions α_1^e and α_2^e such that $0 < \alpha_1^e < \bar{\alpha}$ and $0 < \alpha_2^e < \bar{\alpha}$. In this case, the equilibria of the markets for the raw signal and the processed signal are interior. This case is identical to that analyzed in the paper and all results are identical to those obtained when $\bar{\alpha} > 2$.

2. **Case 2.** The equilibrium demand for (i) the raw signal is a corner solution, that is $\alpha_1^e = 0$ (if $C_r > \frac{\theta}{8}$) or $\alpha_1^e = \bar{\alpha}$ (if $C_r < \hat{C}_r$) and (ii) the equilibrium demand for the processed signal is interior ($0 < \alpha_2^e < \bar{\alpha}$). In this case, the cost of producing the raw signal C_r has (locally) no impact on α_1^e and thus has no local effect on α_2^e. Therefore, if $C_r < \hat{C}_r$ or $C_r > \frac{\theta}{8}$ then a small change in the cost of the raw signal has no effect on asset price informativeness or trade and price patterns. For other values of C_r, results are identical to that in the text.

3. **Case 3.** The equilibrium demand for the processed signal is a corner solution, that is $\alpha_2^e = 0$ or $\alpha_2^e = \bar{\alpha}$. Thus, α_2^e does not depend on α_1^e since its value is either zero or $\bar{\alpha}$. Therefore, a change in cost of the raw signal C_r has (locally) no impact on α_2^e. Thus, it does not affect price informativeness in the long run.

3 Speculators can buy the processed signal at date 2.

In this section, we consider the case in which speculators can buy the processed signal *after* observing the price of the asset at date 1, i.e., just before trading at date 2. The equilibrium of the market for the raw signal is unchanged in this case and is still given by the second part of Lemma 1. The main difference with the case considered in the text is that (i) the demand for the processed signal at date 2, α_2, depends on p_1, the price of the asset at the end of the first period and (ii) the seller the processed signal can charge a different fee for this signal depending on the price realized at date 1.

We consider two subcases. In the first case (Section 3.1), we assume that the seller of the processed signal pays the fixed cost of information processing *after* observing the realization of the price at date 1. In the second case (Section 3.2), we assume that this cost is paid *before* observing the realization of the price at date 1. The break-even conditions for the information seller differ in each case. In the second case, the information seller must set its fees for the processed signal so that it covers its fixed cost of producing the signal *on average across the possible realizations for the price at date* 1. In the first case, the information seller sets its fee so that it breaks even conditional on *each* realization of the price at date 1. The two cases yield an identical equilibrium outcome when there is a demand for the processed signal for each realization of the price at date 1. Equilibrium outcomes (e.g., the fees for the processed signal) differ when the demand for the processed signal is nil for some realization of the price at date 1. However, we show that the implications obtained in the baseline model (about price informativeness and price and trade patterns) are preserved in each case.

3.1 Investment in information processing takes place after the realization of the price at date 1

At the end of the first period, there are three possible outcomes (see Panel A of Figure 3 in the text) : (i) the asset price has not changed, $p_1 = p_0$; (ii) the asset price has increased $p_1 = \frac{1+\theta}{2}$; (iii) the asset price has decreased $p_1 = \frac{1-\theta}{2}$. Due to the symmetry of the model, the expected profit from trading on the processed signal is the same in the last two cases. Thus, the decision to buy the processed signal is identical whether the price has increased or decreased during the first period. This means that only two states are relevant for the analysis of the market for information at date 2: either (i) the price has not changed in the first period ($p_1 = p_0$) or (ii) the price has changed ($p_1 \neq p_0$). We denote by $(\alpha_2^{nc}, F_2^{nc})$ the equilibrium of the market for information in the first case and by (α_2^c, F_2^c) the equilibrium of this market in the second case (superscript "c" stands for "change" while superscript "nc" stands for no change).

Case 1: $p_1 \neq p_0$. We first derive the equilibrium of the market for the processed signal when $p_1 \neq p_0$, i.e., (α_2^c, F_2^c). As in the baseline model, (α_2^c, F_2^c) must satisfy the following zero profit conditions when $\alpha_2^c > 0$:

$$\text{Zero profit for speculators:} \quad \bar{\pi}_2^{net,c}(\alpha_2^c, F_2^c, \theta) = \pi_2^c(\alpha_2^c, \theta) - F_2^c = 0. \tag{3.1}$$

$$\text{Zero profit for the information seller:} \quad \bar{\Pi}_2^{seller,c}(\alpha_2^c, F_2^c) = \alpha_2^c \times F_2^c - C_p = 0, \tag{3.2}$$

where superscript c for expected profits indicates that these profits are computed conditional on a price *change* in the first period. If there is no solution (α_2^c, F_2^c) to this system of equations for which $\alpha_2^c > 0$ then the market for the processed signal is inactive when $p_1 \neq p_0$ and we set $\alpha_2^c = 0$.

As $p_1 \neq p_0$, the *gross* expected profit (per speculator) from trading on the processed signal is (see Step 3 in the proof of Proposition 2 in the text for a derivation):

$$\pi_2^c(\alpha_2^c) = \theta(1-\theta)(1-\alpha_2^c).$$

Thus, in *aggregate*, speculators' net expected profit is:

$$\pi_2^{net,c,a}(\alpha_2^c, C_p) = \alpha_2^c \times (\pi_2^c(\alpha_2^c) - F_2^c) = \alpha_2^c\theta(1-\theta)(1-\alpha_2^c) - C_p,$$

where the second equality follows from eq.(3.2). In equilibrium, if $\alpha_2^c > 0$, speculators' net expected profit must be equal to zero (see eq.(3.1)). Thus, we deduce from the previous equation that α_2^c solves:

$$\pi_2^{net,a,c}(\alpha_2^c, C_p) = \alpha_2^c\theta(1-\theta)(1-\alpha_2^c) - C_p = 0. \tag{3.3}$$

As in the baseline model, if eq.(3.3) has multiple positive solutions then we retain the highest because it yields the smallest (hence most competitive) fee for the information seller. If eq.(3.3) has no positive solution then the market for information for the processed signal is inactive (i.e., $\alpha_2^c = 0$) when $p_1 \neq 1/2$. Solving for eq.(3.3), we obtain:

$$\alpha_2^c(\theta, C_p) = \begin{cases} \frac{1}{2} + \left(\frac{1}{4} - \frac{C_p}{\theta(1-\theta)}\right)^{\frac{1}{2}} & \text{if } 0 \leq C_p \leq \frac{\theta(1-\theta)}{4}, \\ 0 & \text{if } C_p > \frac{\theta(1-\theta)}{4}, \end{cases} \tag{3.4}$$

It follows from eq.(3.2) that the equilibrium fee for the processed signal is $F_2^c = \frac{C_p}{\alpha_2^c}$ when $\alpha_2^c > 0$.

Case 2: $p_1 = p_0 = 1/2$**.** In this case, there is no change in the price at date 1. The net expected profit (per speculator) from trading on the processed signal is then (see Step 3 in the proof of Proposition 2 in the text for a derivation):

$$\pi_2^{nc}(\alpha_2) = \begin{cases} \frac{\theta}{2(2-\theta)}\left(2-\theta-\alpha_2\right) & \text{if} \quad \alpha_2 \leq 1, \\ \frac{\theta}{2}\frac{1-\theta}{2-\theta}\left(2-\alpha_2\right) & \text{if} \quad \alpha_2 > 1, \end{cases} \tag{3.5}$$

where superscript *nc* for the expected profit indicates that it is computed conditional on no price change in the first period. We can then solve for the equilibrium demand for the processed signal as in the case in which $p_1 \neq 1/2$. After some algebra, we obtain:

$$\alpha_2^{nc}(\theta, C_p) = \begin{cases} 1 + \left(1 - \frac{2(2-\theta)}{\theta(1-\theta)}C_p\right)^{\frac{1}{2}} & \text{if } C_p \leq \frac{\theta}{2}\frac{1-\theta}{2-\theta}, \\ \frac{2-\theta}{2} + \left(\frac{(2-\theta)^2}{4} - \frac{2(2-\theta)}{\theta}C_p\right)^{\frac{1}{2}} & \text{if } \frac{\theta}{2}\frac{1-\theta}{2-\theta} < C_p \leq \frac{\theta(2-\theta)}{8} \\ 0 & \text{if } C_p > \frac{\theta(2-\theta)}{8}, \end{cases} \tag{3.6}$$

Thus, when $p_1 = 1/2$, it follows from eq.(3.2) that the equilibrium fee for the processed signal is $F_2^{nc} = \frac{C_p}{\alpha_2^{nc}}$ when $\alpha_2^{nc} > 0$. .

Using the expressions for α_2^c and α_2^{nc} (eq.(3.4) and eq.(3.6)), it is easily shown that $\alpha_2^c < \alpha_2^{nc}$ when $C_p > 0$. Thus, the demand for processed information is smaller when prices have changed at date 1 than when they have not. In equilibrium, the expected demand for the processed signal in equilibrium is:

$$E(\alpha_2^e) = Pr(p_1 \neq p_0)\alpha_2^c + (1 - Pr(p_1 \neq p_0))\alpha_2^{nc} = \alpha_1\alpha_2^c + (1 - \alpha_1)\alpha_2^{nc},$$

where α_2^e denotes the realization of the demand for the processed signal at date 2 (i.e., α_2^c or α_2^{nc}). An increase in the demand for the raw signal increases the probability that the price changes at date 1 since $Pr(p_1 \neq p_0) = \alpha_1$. As the demand for the processed signal is smaller when the asset price changes at date 1 than when it does not ($\alpha_2^c < \alpha_2^{nc}$), we obtain the following result.

Proposition 3.1. *The expected demand for the processed signal in equilibrium decreases with the demand for the raw signal, that is,* $\frac{\partial E(\alpha_2^e)}{\partial \alpha_1} < 0$*. Thus, a decrease in the cost of the raw signal reduces the expected demand for the processed signal in equilibrium.*

This result is the analog of Proposition 3 in the baseline model. It is stronger in the sense that it holds for all parameter values. Figure 3.1 illustrates the previous proposition for specific parameter values.

Asset price informativeness at date t=2. We now study the effect of a reduction in the cost of producing the raw signal on the informativeness of the price at date 2. As in the baseline model, we define asset price informativeness at date 2 as:

$$\mathcal{E}_2(C_r, C_p) = \frac{1}{4} - E[(\tilde{V} - p_2^*)^2], \tag{3.7}$$

where p_2^* denotes the realization of the equilibrium price at date 2. We obtain the following result.

Proposition 3.2. *If* $C_p \leq \frac{\theta(2-\theta)}{8}$ *then*

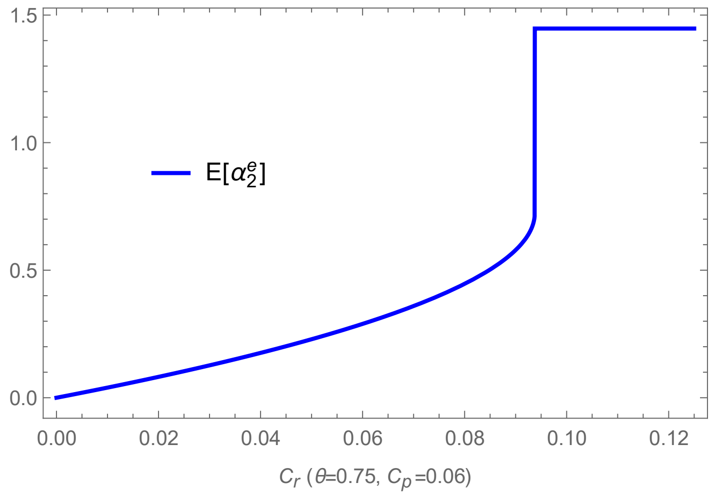

Figure 3.1: Expected demand for the processed signal ($\mathbf{E}(\alpha_2^e)$)

1. *When $\theta \leq 1/2$, a decrease in the cost of producing the raw signal reduces asset price informativeness at date 2.*

2. *When $\theta > 1/2$, a decrease in the cost of producing the raw signal reduces asset price informativeness at date 2 iff $C_p \leq \bar{C}_p$ where $\bar{C}_p$ is a threshold that belongs to $\left[\frac{\theta(1-\theta)}{2(2-\theta)}, \frac{\theta(2-\theta)}{8}\right]$*

Thus, when $C_p < \frac{\theta(2-\theta)}{8}$, the informativeness of the price at date 2 can decline when C_r declines as emphasized by Proposition 4 in the paper. Figure 3.2 illustrates this claim for specific parameter values.

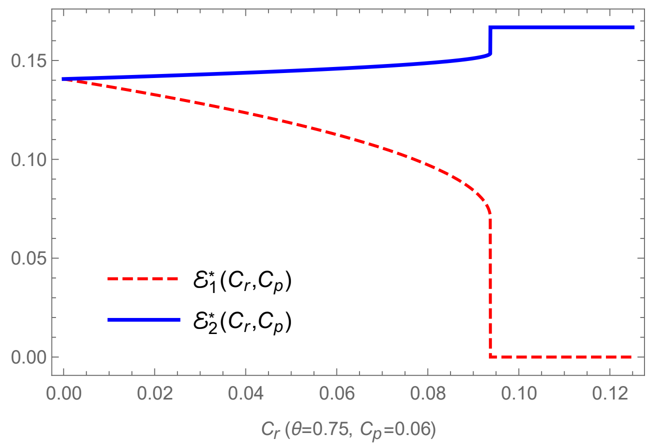

Figure 3.2: The informativeness of the price at date 2

When $C_p > \frac{\theta(2-\theta)}{8}$, the cost of producing the processed signal is so high that there is no fee at which it can be profitably produced (i.e., a which buyers and sellers of the processed signal can mutually trade), for all possible realizations of the price at date 1. Thus, the processed signal is not produced and the informativeness of the price at date 2 is equal to that at date 1. The latter increases when α_1 increases and therefore, in this case, a reduction in the cost of the raw signal raises price informativeness at date 2 as well.

Price and trade patterns. If $C_r \leq \theta/8$, some speculators buy the raw signal (see Lemma 1 in the text of the paper) . Using Proposition 1 in the paper, we can write the equilibrium strategy of a speculator receiving the raw signal, s, as

$$x_1^*(s, C_r) = \mathbb{I}_{s=1} - \mathbb{I}_{s=0} = u \times [\mathbb{I}_{V=1} - \mathbb{I}_{V=0}] + (1-u) \times [\mathbb{I}_{\epsilon=1} - \mathbb{I}_{\epsilon=0}]. \tag{3.8}$$

If $C_r > \frac{\theta}{8}$, no speculator buys the raw signal in equilibrium and therefore $x_1^*(s, C_r) = 0$. In sum:

$$x_1^*(s, C_r) = \begin{cases} \mathbb{I}_{s=1} - \mathbb{I}_{s=0} = u \times [\mathbb{I}_{V=1} - \mathbb{I}_{V=0}] + (1-u) \times [\mathbb{I}_{\epsilon=1} - \mathbb{I}_{\epsilon=0}] & \text{if } 0 \leq C_r \leq \theta/8, \\ 0 & \text{if } C_r > \theta/8. \end{cases} \tag{3.9}$$

Similarly, using Proposition 2 in the paper, we can write the optimal trading strategy of speculators at date 2 as:

$$x_2^*(p_1, u, C_p) = \begin{cases} u \times [\mathbb{I}_{V=1} - \mathbb{I}_{V=0}] + (1-u) \times [\mathbb{I}_{p_1=(1-\theta)/2} - \mathbb{I}_{p_1=(1+\theta)/2}] & \text{if } 0 \leq C_p \leq \frac{\theta(1-\theta)}{4}, \\ u \times [\mathbb{I}_{V=1} - \mathbb{I}_{V=0}] \times \mathbb{I}_{p_1=1/2} & \text{if } \frac{\theta(1-\theta)}{4} < C_p \leq \frac{\theta(2-\theta)}{8}, \\ 0 & \text{if } C_p > \frac{\theta(2-\theta)}{8}, \end{cases} \tag{3.10}$$

where the last equality follows from the fact that there is no demand for the processed signal if $C_p > \frac{\theta(2-\theta)}{8}$.

Now, assume $C_r \leq \frac{\theta}{8}$ and $C_p \leq \frac{\theta(2-\theta)}{8}$. These assumptions guarantee that some speculators trade at date 1 ($x_1^* \neq 0$) and that some speculators trade at date 2 at least if

$p_1 = 1/2$. Using eq.(3.9) and eq.(3.10), we obtain:

$$Cov(x_1^*, x_2^*) = \begin{cases} \theta(1-\alpha_1^e), \text{ if } C_p > \frac{\theta(1-\theta)}{4}, \\ \theta - (1-\theta)\alpha_1^e, \text{ if } C_p \leq \frac{\theta(1-\theta)}{4}. \end{cases}$$

Thus, as implied by Corollary 4 in the baseline model, we obtain that $Cov(x_1^*, x_2^*)$ declines when α_1^e increases, i.e., when the cost of the raw signal decreases. Moreover, when $C_p \leq \frac{\theta(1-\theta)}{4}$, one obtains exactly the same expression for the covariance as that in Corollary 4 in the baseline model. Thus, as in this case, $Cov(x_1^*, x_2^*)$ becomes negative if $\theta < 1/2$ and C_r is small enough.

Using the expression for the equilibrium price at date 1 (Proposition 1 in the paper) and eq.(3.10), we also obtain

$$Cov(r_1, x_2^*) = E\left[\left(p_1 - \frac{1}{2}\right)x_2\right] = \begin{cases} 0, \text{ if } C_p > \frac{\theta(1-\theta)}{4}, \\ \theta(2\theta-1)\alpha_1^e, \text{ if } C_p \leq \frac{\theta(1-\theta)}{4} \text{ (as in the paper)}, \end{cases}$$

where $r_1 = p_1^* - p_0$ is the return from date 0 to date 1. Thus, when $C_p \leq \frac{\theta(1-\theta)}{4}$ (so that some speculators buy the processed signal whether the price at date 1 has changed or not), we obtain the same expression for $Cov(r_1, x_2^*)$ as that in Corollary 5 in the paper. Otherwise $Cov(r_1, x_2^*)$ is zero. Hence, Corollary 5 holds true even if speculators can make their decision to buy the processed signal contingent on the price realized at date 1.

Last consider $Cov(x_1^*, r_2)$ where $r_2 = (p_2^* - p_1^*)$ is the return from date 1 to date 2. Calculations yield:

$$Cov(x_1^*, r_2) = \begin{cases} \frac{\theta(1-\alpha_1^e)\alpha_2^{nc}}{2(2-\theta)}, \text{ when } C_p > \frac{\theta(1-\theta)}{2(2-\theta)}, \\ \frac{\theta(1-\alpha_1^e)(1+(1-\theta)(\alpha_2^{nc}-1))}{2(2-\theta)}, \text{ when } C_p \leq \frac{\theta(1-\theta)}{2(2-\theta)}. \end{cases}$$

Observe that this is the same expression as that obtained in the baseline model (see eq.(27)), with α_2^{nc} replacing α_2^e. The intuition is the following. In equilibrium, the innovation in $r_2 = (p_2^* - p_1^*)$ is orthogonal to dealers' information set set at $t = 1$ since equilibrium prices follow a martingale. If the order flow at date 1 reveals the raw signal then x_1^* is known to dealers at date 1 and therefore $r_2 = p_2^* - p_1^*$ is orthogonal to x_1^*. This means that if $p_1^* \neq 1/2$ then r_2 is orthogonal to x_1^*. Thus, the covariance between x_1^* and r_2 is only driven by the cases in which $p_1^* = 1/2$, in which case, the demand for the processed signal is α_2^{nc}. This explains why only α_2^{nc} affects $Cov(x_1^*, r_2)$.

As α_2^{nc} does not depend on α_1^e, we obtain that $Cov(x_1, r_2)$ decreases when α_1^e increases, i.e., when C_r decreases, for the same reason as in the baseline model.

Proofs for Section 3.1

Proof of Proposition 3.1. When $C_p > \frac{\theta(2-\theta)}{8}$, the demand for the processed signal is nil whether the price changes at date 1 or not. Thus, $E[\alpha_2^e] = 0$ in this case. When $\frac{\theta(2-\theta)}{8} \geq C_p > \frac{\theta(1-\theta)}{4}$, the demand for the processed signal is nil if the price changes at date 1 and strictly positive otherwise. Thus, in this case, $E[\alpha_2^e] = (1-\alpha_1)\alpha_2^{nc}$, which is clearly decreasing with α_1. Finally, when $C_p \leq \frac{\theta(1-\theta)}{4}$, we have, using the expressions for the equilibrium demand for the processed signal:

$$E[\alpha_2^e] = (1-\alpha_1)\left[1 + \left(1 - \frac{2(2-\theta)}{\theta(1-\theta)}C_p\right)^{\frac{1}{2}}\right] + \alpha_1\left[\frac{1}{2} + \left(\frac{1}{4} - \frac{C_p}{\theta(1-\theta)}\right)^{\frac{1}{2}}\right],$$

which decreases with α_1 since $\left(\frac{1}{4} - \frac{C_p}{\theta(1-\theta)}\right)^{\frac{1}{2}} \leq 1/2$.

Proof of Proposition 3.2. First, observe that $\mathcal{E}_2(C_r, C_p)$ is inversely and linearly related to $E[(\tilde{V} - p_2^e)^2]$. It is easily shown that $E[(\tilde{V} - p_2^*)^2] = E[p_2^*(1-p_2^*)]$. To obtain the proposition, we can therefore analyze how $E[p_2^*(1-p_2^*)]$ varies with C_r.

We first compute $E[p_2^*(1-p_2^*)]$. If $\alpha_2^{nc} \leq 1$, we obtain (e.g., using Figure 3, in the paper):

$$
\begin{aligned}
E[p_2^*(1-p_2^*)] &= \alpha_1 \times \left[(1-\theta)\alpha_2^c \times \frac{1}{4} + (1-\alpha_2^c) \times \frac{(1-\theta)(1+\theta)}{4}\right] \\
&+ (1-\alpha_1) \times \left[(1-\alpha_2^{nc}) \times \frac{1}{4} + \left(\frac{\alpha_2^{nc}}{2} + \frac{(1-\theta)\alpha_2^{nc}}{2}\right) \times \frac{1-\theta}{(2-\theta)^2}\right] \\
&= \frac{1}{4}\alpha_1[1-\theta+(1-\theta)\theta(1-\alpha_2^c)] + \frac{1}{4}(1-\alpha_1)\left[1-\left(1-2(2-\theta)\frac{1-\theta}{(2-\theta)^2}\right)\alpha_2^{nc}\right] \\
&= \frac{1}{4}\alpha_1[1-\theta+(1-\theta)\theta(1-\alpha_2^c)] + \frac{1}{4}(1-\alpha_1)\left(1-\frac{\theta}{2-\theta}\alpha_2^{nc}\right) \\
&= \frac{1}{4}\alpha_1(1-\theta) + \frac{1}{4}\alpha_1(1-\theta)\theta(1-\alpha_2^c) + \frac{1}{4}(1-\alpha_1)\theta\left(1-\frac{1}{2-\theta}\alpha_2^{nc}\right) + \frac{1}{4}(1-\alpha_1)(1-\theta) \\
&= \frac{1}{4}(1-\theta) + \frac{1}{4}\alpha_1\pi_2^c(\alpha_2^c) + \frac{1}{2}(1-\alpha_1)\pi_2^{nc}(\alpha_2^{nc}).
\end{aligned}
$$

Similarly, if $1 \leq \alpha_2^c \leq 2$, we obtain:

$$
\begin{aligned}
E[p_2^*(1-p_2^*)] &= \alpha_1 \times \left[(1-\theta)\alpha_2^c \times \frac{1}{4} + (1-\alpha_2^c) \times \frac{(1-\theta)(1+\theta)}{4}\right] \\
&+ (1-\alpha_1) \times \left[(1-\theta)(\alpha_2^{nc}-1) \times \frac{1}{4} + \left(\frac{2-\alpha_2^{nc}}{2} + \frac{(1-\theta)(2-\alpha_2^{nc})}{2}\right) \times \frac{1-\theta}{(2-\theta)^2}\right] \\
&= \frac{1}{4}\alpha_1(1-\theta) + \frac{1}{4}\alpha_1\pi_2^c(\alpha_2^c) \\
&+ \frac{1}{4}(1-\alpha_1)(1-\theta) + (1-\alpha_1) \times \left[(1-\theta)(\alpha_2^{nc}-2) \times \frac{1}{4} + \frac{2-\alpha_2^{nc}}{2}(2-\theta) \times \frac{1-\theta}{(2-\theta)^2}\right] \\
&= \frac{1}{4}\alpha_1(1-\theta) + \frac{1}{4}\alpha_1\pi_2^c(\alpha_2^c) \\
&+ \frac{1}{4}(1-\alpha_1)(1-\theta) + \frac{1}{2}(1-\alpha_1) \times \left[\frac{1-\theta}{2-\theta} - \frac{1-\theta}{2}\right](2-\alpha_2^{nc}) \\
&= \frac{1}{4}\alpha_1(1-\theta) + \frac{1}{4}\alpha_1\pi_2^c(\alpha_2^c) \\
&+ \frac{1}{4}(1-\alpha_1)(1-\theta) + \frac{1}{2}(1-\alpha_1)\frac{\theta(1-\theta)}{2(2-\theta)}(2-\alpha_2^{nc}) \\
&= \frac{1}{4}(1-\theta) + \frac{1}{4}\alpha_1\pi_2^c(\alpha_2^c) + \frac{1}{2}(1-\alpha_1)\pi_2^{nc}(\alpha_2^{nc})
\end{aligned}
$$

Thus, in all cases:

$$\frac{\partial E[p_2^*(1-p_2^*)]}{\partial \alpha_1} = \frac{\pi_2^c(\alpha_2^c) - 2\pi_2^{nc}(\alpha_2^{nc})}{4}. \tag{3.11}$$

Thus, a reduction in C_r (i.e., an increase in α_1) increases the informativeness of the price at date 2 (i.e., reduces $E[p_2^*(1-p_2^*)]$) if and only if $\pi_2^c(\alpha_2^c) < 2\pi_2^{nc}(\alpha_2^{nc})$. Now we analyze when this is the case and when it is not. For this, note that: $\pi_2^c(\alpha_2^c) = C_p/\alpha_2^c$ if $\alpha_2^c > 0$ and $\pi_2^c(0) = \theta(1-\theta)$ if $\alpha_2^c = 0$. Similarly, $\pi_2^{nc}(\alpha_2^{nc}) = C_p/\alpha_2^{nc}$ if $\alpha_2^{nc} > 0$ and $\pi_2^c(0) = \theta/2$ if $\alpha_2^{nc} = 0$ (see eq.(3.5)).

- If $C_p > \frac{\theta(2-\theta)}{8}$, $\alpha_2^{nc} = \alpha_2^c = 0$. In this case, using eq.(3.11), we deduce from the previous remarks that:

$$\frac{\partial E[p_2^*(1-p_2^*)]}{\partial \alpha_1} = -\frac{\theta^2}{4} < 0,$$

  which is decreasing with α_1.

- If $\frac{\theta(1-\theta)}{4} < C_p \leq \frac{\theta(2-\theta)}{8}$ we have $\alpha_2^c = 0$ and $\alpha_2^{nc} > 0$. In this case, In this case, using eq.(3.11), we obtain that:

$$\frac{\partial E[p_2^*(1-p_2^*)]}{\partial \alpha_1} = \theta(1-\theta) - C_p/\alpha_2^{nc}. \tag{3.12}$$

  Thus, the pricing error increases with α_1 if and only if

$$\alpha_2^{nc}(\theta, C_p) \geq \frac{2C_p}{\theta(1-\theta)}. \tag{3.13}$$

  Using eq.(3.6), we deduce that $\alpha_2^{nc}(\theta, C_p)$ increases when C_p decreases. And using eq.(3.11), we deduce that the derivative of the pricing error with respect to α_1 increases when C_p decreases. We also observe that when $C_p \leq \frac{\theta(1-\theta)}{2(2-\theta)}$, $\alpha_2^{nc} \geq 1$ and therefore eq.(3.13) is satisfied. Moreover, when $C_p = \frac{\theta(2-\theta)}{8}$, $\alpha_2^{nc} = \frac{2-\theta}{2}$. Thus, in this case, eq.(3.13) is satisfied if and only if $\theta < 1/2$. Combining these two observations, we deduce that when $\frac{\theta(1-\theta)}{4} < C_p \leq \frac{\theta(2-\theta)}{8}$, there is a $\bar{C}_p \in \left[\frac{\theta(1-\theta)}{2(2-\theta)}, \frac{\theta(2-\theta)}{8}\right]$ such that asset price informativeness decreases when C_r decreases if (i) $\theta < 1/2$ or if (ii) $\theta > 1/2$ and $C_p < \bar{C}_p$.

- If $C_p \leq \frac{\theta(1-\theta)}{4}$, using eq.(3.11), we obtain that:

$$\frac{\partial E[p_2^*(1-p_2^*)]}{\partial \alpha_1} = \frac{C_p}{2}\left(\frac{1}{2\alpha_2^c} - \frac{1}{\alpha_2^{nc}}\right) > 0, \tag{3.14}$$

  where the inequality follows from the fact that $2\alpha_2^c \leq \alpha_2^{nc}$ when $C_p \leq \frac{\theta(1-\theta)}{4}$.

3.2 Investment in information processing takes place before the realization of the price at date 1

Now we consider the case in which the seller and the buyers of the processed signal contract *before* observing the realization of the price at date 1 but the buyers can make their decision to eventually buy the signal contingent on the realization of the price at date 1. This corresponds to the case in which (i) information processing must start early at date 1 for the processed signal to be delivered in time at date 2 and (ii) the seller offers a schedule (F_2^c,F_2^{nc}), i.e., a fee for the processed signal that is contingent on the realization of the price. A speculator can then decide to buy the processed signal only when (i) the price has changed at date 1, (ii) only when the price has not changed or (iii) in both cases.

Denote $R_2^{nc} = \alpha_2^{nc} F_2^{nc}$ and $R_2^c = \alpha_2^c F_2^c$. An equilibrium of the market for the processed signal is a vector $(\alpha_2^{nc}, \alpha_2^c, F_2^{nc}, F_2^{nc})$ such that:

$$\text{Zero profit for speculators if } p_1 \neq p_0\text{:} \quad \bar{\pi}_2^{net,c}(\alpha_2^c, F_2^c, \theta) = \pi_2^c(\alpha_2^c, \theta) - F_2^c = 0. \tag{3.15}$$

$$\text{Zero profit for speculators if } p_1 = p_0\text{:} \quad \bar{\pi}_2^{net,c}(\alpha_2^c, F_2^c, \theta) = \pi_2^c(\alpha_2^c, \theta) - F_2^c = 0. \tag{3.16}$$

Zero profit for the information seller:

$$\bar{\Pi}_2^{seller}(\alpha_2^{nc}, \alpha_2^c, F_2^c, F_2^{nc}) = Pr(p_1 \neq p_0)R_2^c + (1 - Pr(p_1 \neq p_0))R_2^{nc} - C_p = \alpha_1 R_2^c + (1-\alpha_1)R_2^{nc} - C_p = 0. \tag{3.17}$$

The only difference with the case analyzed in Section 3.1 is the zero profit condition for the seller of the processed signal. This condition accounts for the fact that the information seller must pay the fixed cost of information processing before knowing whether $p_1 \neq p_0$ (which happens with probability $Pr(p_1 \neq p_0) = \alpha_1$) or $p_1 = p_0$ (which happens with probability $(1 - \alpha_1)$).

Note that $R_2^{nc} = \alpha_2^{nc} F_2^{nc}$ and $R_2^c = \alpha_2^c F_2^c$ are the revenues of the seller of the processed signal conditional on the two possible outcomes at the end of period 1: (i) no change (nc) in the price of the asset or (i) a change (c) in the price of the asset. These revenues are bounded by the largest possible value for the aggregate expected profits from trading on the processed signal conditional on each possible outcome, that is:

$$R_2^{nc} \leq \frac{\theta(2-\theta)}{8}, \text{ and } R_2^c \leq \frac{\theta(1-\theta)}{4}$$

Thus, the largest possible expected profit for the seller of the processed signal is $\bar{\Pi}(\alpha_1) \equiv (1-\alpha_1)\frac{\theta(2-\theta)}{8}+\alpha_1\frac{\theta(1-\theta)}{4}$ We deduce that a necessary condition for the seller of the processed signal to pay the fixed cost of producing this signal is that:

$$C_p \leq \bar{\Pi}(\alpha_1). \tag{3.18}$$

Henceforth we assume that this condition is satisfied. Otherwise the processed signal is not produced and $\alpha_2^c = \alpha_2^{nc} = 0$.

Observe that there might be multiple pairs $(R_2^c,\ R_2^{nc})$ for the zero expected profit condition for the seller of the processed signal (in eq.(3.2)), which leads to the possibility of multiple equilibria in the market for the processed signal. When this happens, we we select the equilibrium that minimize the variance of revenues for the information seller, i.e., $(R_2^{nc} - R_2^c)^2$ since the seller's revenue is binomial random variable. Indeed, this would be the strictly preferred outcome for the information seller if it is risk averse.

We can rewrite the zero profit condition (3.17):

$$R_2^{nc} - R_2^c = \frac{C_p - R_2^c}{1-\alpha_1} \tag{3.19}$$

We then consider two cases. The first case arises when $C_p \leq \frac{\theta(1-\theta)}{4}$. In this case, the variance of revenues is minimized for $R_2^c = C_p$ and is equal to zero, so that $R_2^{nc} = C_p$ as well. This outcome is possible because $C_p < \frac{\theta(1-\theta)}{4}$, the largest possible value for R_2^c. The zero profit condition (3.2) can thus be replaced by two conditions:

$$\alpha_2^c F_2^c = C_p,$$

and

$$\alpha_2^{nc} F_2^{nc} = C_p.$$

The analysis is then identical to the case analyzed in Section 3.1. In particular, as in Section 3.1, we obtain that the equilibrium demand for the processed signal when $p_1 \neq p_0$ and when $p_1 = p_0$ are respectively:

$$\alpha_2^c(\theta, C_p) = \frac{1}{2} + \left(\frac{1}{4} - \frac{C_p}{\theta(1-\theta)}\right)^{\frac{1}{2}} \tag{3.20}$$

and

$$\alpha_2^{nc}(\theta, C_p) = 1 + \left(1 - \frac{2(2-\theta)}{\theta(1-\theta)} C_p\right)^{\frac{1}{2}} \tag{3.21}$$

The second case arises when $\frac{\theta(2-\theta)}{8} \geq C_p > \frac{\theta(1-\theta)}{4}$. In this case, the constraint that $R_2^c \leq \theta(1-\theta)/4$ is binding. Inspection of eq.(3.19) shows that the equilibrium such that the variance of revenues for the information seller is minimal is such that R_2^c is set at its maximal possible value, i.e., $\frac{\theta(1-\theta)}{4}$, which is therefore the aggregate expected trading profits obtained by speculators buying the processed signal when the price changes at date 1. This implies that in this case the demand for the processed signal must be:

$$\alpha_2^c = \frac{1}{2}, \tag{3.22}$$

and therefore $F_2^c = \theta(1-\theta)/2$. Moreover, using eq.(3.19), we deduce that R_2^{nc} must be such that

$$R_2^{nc} = \alpha_2^{nc} F_2^{nc} = \frac{\theta(1-\theta)}{4} + \frac{C_p - \frac{\theta(1-\theta)}{4}}{1-\alpha_1}.$$

Let denote the R.H.S of this equation by $R(\alpha_1, C_p)$ (i.e., $R(\alpha_1, C_p) = \frac{\theta(1-\theta)}{4} + \frac{C_p - \frac{\theta(1-\theta)}{4}}{1-\alpha_1}$). We can then derive the equilibrium demand for the processed signal when $p_1 = p_0$ as we did in Section 3.1, except that the role of C_p is now played by $R(\alpha_1, C_p)$. We deduce that the demand the processed signal conditional on no change in the price of the asset at date 1 is:

$$\alpha_2^{nc}(\alpha_1, \theta, C_p) = \begin{cases} 1 + \left(1 - \frac{2(2-\theta)}{\theta(1-\theta)} R(\alpha_1, C_p)\right)^{\frac{1}{2}} & \text{if } \frac{\theta(1-\theta)}{4} < R(\alpha_1, C_p) \leq \frac{\theta}{2}\frac{1-\theta}{2-\theta}, \\ \frac{2-\theta}{2} + \left(\frac{(2-\theta)^2}{4} - \frac{2(2-\theta)}{\theta} R(\alpha_1, C_p)\right)^{\frac{1}{2}} & \text{if } \frac{\theta}{2}\frac{1-\theta}{2-\theta} < R(\alpha_1, C_p) \leq \frac{\theta(2-\theta)}{8} \end{cases} \tag{3.23}$$

In sum, the only difference with the case studied in Section 3, is the case where $\frac{\theta(2-\theta)}{8} \geq C_p > \frac{\theta(1-\theta)}{4}$. There are two differences relative to what we obtain in 3.

First, even if the price changes at date 1, there is a demand for the processed signal when $C_p > \frac{\theta(1-\theta)}{4}$ while this demand was nil in the case considered in Section 3. The reason is that conditional on the realization of this state (a change in price), the information seller sells the processed signal at a price that is too low to recover its fixed cost of producing the signal. As this cost has been sunk, selling the signal is optimal anyway but, in this state, the fee charged for the signal is less than the cost of producing it. This implies that when $p_1 = p_0$, the seller of the processed signal sells it at a price that exceeds his fixed cost of producing the processed signal. This price cannot be profitably undercut because the decision to produce the processed signal (and pay the associated fixed cost) must be made before observing the realization of the price of the asset at date 1.

Second, α_2^{nc} is negatively related to α_1. Yet, as in Section 3.1, we still have $\alpha_2^{nc} > \alpha_2^{c}$. Moreover, the expected demand for information is equal to $E(\alpha_2^e) = \alpha_1 \frac{1}{2} + (1-\alpha_1)\alpha_2^{nc}(\alpha_1)$. As in Section 3.1, the expected demand for the processed signal decreases with the demand for the raw signal since $E(\alpha_2^e) = \alpha_1 \frac{1}{2} + (1-\alpha_1)\alpha_2^{nc}(\alpha_1)$ and $\alpha_2^{nc}(\alpha_1)$ decreases with α_1.

Asset price informativeness at date t=2. As in Section 3, we analyze the effect of a reduction of C_r (i.e., an increase in α_1) on the informativeness of prices at $t = 2$.

Proposition 3.3. *If $C_p > \bar{\Pi}(\alpha_1)$, a decrease in the cost of producing the raw signal improves asset price informativeness at date 2. If $C_p \leq \bar{\Pi}(\alpha_1)$ then there is a $\bar{C}_p(\alpha_1) \in \left[\frac{\theta(1-\theta)}{4}, \bar{\Pi}(\alpha_1)\right]$ such that a decrease in the cost of producing the raw signal reduces asset* *price informativeness at date 2 if and only if* $C_p < \bar{C}_p(\alpha_1)$.

Qualitatively, we find the same result as in Section 3.1: when the cost of producing processed information is "high", a decrease in the cost of producing the raw signal improves asset price informativeness at date 2, and conversely when the cost of producing processed information is "low".

Price and trade patterns. As speculators are either active in both cases ("c" or "nc") or not at all, a speculator's trade at $t = 2$ is as follows,

$$x_2^*(p_1, u, C_p) = \begin{cases} 0 & \text{if } C_p > \bar{\Pi}(\alpha_1), \\ u \times [\mathbb{I}_{V=1} - \mathbb{I}_{V=0}] + (1-u) \times [\mathbb{I}_{p1=(1-\theta)/2} - \mathbb{I}_{p1=(1+\theta)/2}] & \text{if } 0 \leq C_p \leq \bar{\Pi}(\alpha_1). \end{cases} \tag{3.24}$$

Therefore, proceeding as in Section 3.1, we obtain:

$$Cov(x_1^*, x_2^*) = \begin{cases} 0, \text{ if } C_p > \bar{\Pi}(\alpha_1), \\ \theta - (1-\theta)\alpha_1^e, \text{ if } C_p \leq \bar{\Pi}(\alpha_1). \end{cases}$$

$$Cov(r_1, x_2^*) = E\left[\left(p_1 - \frac{1}{2}\right) x_2\right] = \begin{cases} 0, \text{ if } C_p > \bar{\Pi}(\alpha_1), \\ \theta(2\theta - 1)\alpha_1^e, \text{ if } C_p \leq \bar{\Pi}(\alpha_1). \end{cases}$$

$$Cov(x_1^*, r_2) = \begin{cases} 0, \text{ if } C_p > \bar{\Pi}(\alpha_1), \\ \frac{\theta(1-\alpha_1^e)\alpha_2^{nc}(\alpha_1)}{2(2-\theta)}, \text{ if } \frac{\theta(1-\theta)}{4} < C_p \leq \bar{\Pi}(\alpha_1) \text{ and } R(\alpha_1, C_p) > \frac{\theta(1-\theta)}{2(2-\theta)}, \\ \frac{\theta(1-\alpha_1^e)(1+(1-\theta)(\alpha_2^{nc}(\alpha_1)-1))}{2(2-\theta)}, \text{ if } \frac{\theta(1-\theta)}{4} < C_p \leq \bar{\Pi}(\alpha_1) \text{ and } R(\alpha_1, C_p) \leq \frac{\theta(1-\theta)}{2(2-\theta)}, \\ \frac{\theta(1-\alpha_1^e)(1+(1-\theta)(\alpha_2^{nc}-1))}{2(2-\theta)}, \text{ if } C_p \leq \frac{\theta(1-\theta)}{4}. \end{cases}$$

The expression of covariances $Cov(x_1^*, x_2^*)$ and $Cov(r_1, x_2^*)$ are as in the paper. The only difference is the threshold $\bar{\Pi}(\alpha_1)$ that replaces C_{max}. Compared to Section 3.1, we lose the "intermediate" cases which correspond to the situation where $\alpha_2^c = 0$ and $\alpha_2^{nc} > 0$, when $\frac{\theta(2-\theta)}{8} \geq C_p > \frac{\theta(1-\theta)}{4}$.

$Cov(x_1^*, r_2)$ admits more cases than in the paper or in Section 3.1. Notice that in the second and third cases, α_1 also enters the expression via $\alpha_2^{nc}(\alpha_1)$, which is not the case in the fourth case, and in Section 3. Yet, in all cases, $Cov(x_1^*, r_2)$ decreases with α_1, as in Section 3.1.

Proofs for Section 3.2

Proof of Proposition 3.3. When $C_p > \bar{\Pi}(\alpha_1)$, we have $\alpha_2^{nc} = \alpha_2^c = 0$ then informativeness increases with α_1 as in Proposition 3.2, when $C_p > \frac{\theta(2-\theta)}{8}$.

When $C_p > \frac{\theta(1-\theta)}{4}$ Using equation (3.11), we can calculate the derivative of the pricing error with respect to α_1 as follows

$$\frac{\partial E[p_2^*(1-p_2^*)]}{\partial \alpha_1} = \frac{\pi_2^c(1/2) - 2\pi_2^{nc}(\alpha_2^{nc})}{4} = \frac{1}{4}\left[\frac{\theta(1-\theta)}{2} - \frac{2}{\alpha_2^{nc}}\left(\frac{\theta(1-\theta)}{4} + \frac{C_p - \frac{\theta(1-\theta)}{4}}{1-\alpha_1}\right)\right],$$

We can first notice that this derivative is decreasing function of C_p since α_2^{nc} declines with C_p and $\pi_2^{nc}(\alpha_2^{nc})$ declines with α_2^{nc}.

Second, the derivative can be rewritten as

$$\frac{1}{4}\left[\frac{\theta(1-\theta)}{2}\left(1 - \frac{1}{\alpha_2^{nc}}\right) - \frac{2}{\alpha_2^{nc}}\frac{C_p - \frac{\theta(1-\theta)}{4}}{1-\alpha_1}\right].$$

If $\frac{\theta}{2}\frac{1-\theta}{2-\theta} \leq R(\alpha_1, C_p) \leq \frac{\theta(2-\theta)}{8}$, we have $\alpha_2^{nc} \leq 1$, then the derivative of the pricing error is negative. It follows that price informativeness increases with α_1.

When $R(\alpha_1, C_p) < \frac{\theta}{2}\frac{1-\theta}{2-\theta}$, $\alpha_2^{nc} > 1$. Consider the case where $R(\alpha_1, C_p)$ converges to $\frac{\theta(1-\theta)}{4}$, or equivalently C_p converges to $\frac{\theta(1-\theta)}{4}$ from above. The derivative converges towards

$$\frac{\theta(1-\theta)}{8}\left(1 - \frac{1}{\alpha_2^{nc}}\right) > 0.$$

Since $\partial E[p_2^*(1-p_2^*)]/\partial \alpha_1$ is a decreasing function of C_p, it follows that there is a threshold $\bar{C}_p(\alpha_1)$ such that price informativeness decreases with α_1, i.e., $\partial E[p_2^*(1-p_2^*)]/\partial \alpha_1 > 0$, if and only if $C_p < \bar{C}_p(\alpha_1)$.

When $C_p \leq \frac{\theta(1-\theta)}{4}$, we are in the same case as in Proposition 3.2. Asset price informativeness decreases with α_1.

4 Complement to the proof of Proposition 3.

Let $\bar{\alpha}_1(\theta) = \frac{(1-\theta)^2+\theta^2}{(1-2\theta)[2(1-\theta)(2-\theta)-1]}$ and $\bar{C}_p(\theta) = \frac{(\theta(1-\theta)^2)(1-2\theta)}{(2-\theta)\left(2(1-\theta)-\frac{1}{2-\theta}\right)^2}$. We first prove the following result.

Lemma 4.1. *For $\theta < 1/2$ and $C_{min}(\theta, \alpha_1) \leq C_p < C_{max}(\theta, \alpha_1)$, $\frac{\partial \alpha_2^e}{\partial \alpha_1} > 0$ if and only if (i) $\theta < \frac{\sqrt{2}-1}{\sqrt{2}}$, (ii) $\alpha_1 > \bar{\alpha}_1(\theta)$, and (iii) $C_p > \bar{C}_p(\theta)$.*

Proof. When $\theta < 1/2$, we know from Corollary 2 that $\frac{\partial \alpha_2^e}{\partial \alpha_1} > 0$ if and only if

$$\alpha_2^e(\alpha_1) < \hat{\alpha}_2(\theta) = \frac{(2-\theta)(1-2\theta)}{2(2-\theta)(1-\theta)-1}$$

Using the expression of α_2^e in Lemma 1 in the text, we obtain that $\alpha_2^e(\alpha_1) < \hat{\alpha}_2(\theta)$ if and only if

$$\alpha_2^{max}(\theta, \alpha_1)\left(1 + \sqrt{1 - \frac{C_p}{C_{max}(\theta, \alpha_1)}}\right) < \hat{\alpha}_2(\theta). \tag{4.1}$$

That is, if and only if

$$\sqrt{1 - \frac{C_p}{C_{max}(\theta, \alpha_1)}} < \frac{\hat{\alpha}_2(\theta)}{\alpha_2^{max}(\theta, \alpha_1)} - 1. \tag{4.2}$$

For this inequality to be verified, a necessary condition is that the right hand side is positive. That is, we must have:

$$\hat{\alpha}_2(\theta) > \alpha_2^{max}(\theta, \alpha_1) = \frac{1}{2}\frac{1-(2\theta-1)\alpha_1}{\frac{1}{2-\theta} + \left(2(1-\theta) - \frac{1}{2-\theta}\right)\alpha_1}\left(= \frac{(2-\theta)(1-(2\theta-1)\alpha_1)}{2(1+(2(2-\theta)(1-\theta)-1)\,\alpha_1)}\right). \tag{4.3}$$

Observe that $\alpha_2^{max}(\theta, \alpha_1)$ decreases with α_1. Moreover, $\alpha_2^{max}(\theta, 0) = (2-\theta)/2$ and $\alpha_2^{max}(\theta, 1) = 1/2$. We have:

$$\begin{aligned}
\alpha_2^{max}(\theta, 0) - \hat{\alpha}_2(\theta) &= \frac{2-\theta}{2} - \frac{1-2\theta}{2(1-\theta) - \frac{1}{2-\theta}} = \frac{2(1-\theta)(2-\theta) - 1 - 2(1-2\theta)}{2\left(2(1-\theta) - \frac{1}{2-\theta}\right)} \\
&= \frac{2(1-\theta)(2-\theta) - 4(1-\theta) + 1}{2\left(2(1-\theta) - \frac{1}{2-\theta}\right)} = \frac{-2\theta(1-\theta)+1}{2\left(2(1-\theta) - \frac{1}{2-\theta}\right)} > 0,
\end{aligned}$$

for $\theta \leq 1/2$. Moreover,

$$\begin{aligned}
\alpha_2^{max}(\theta,1) - \hat{\alpha}_2(\theta) &= \frac{1}{2} - \frac{1-2\theta}{2(1-\theta) - \frac{1}{2-\theta}} = \frac{1-\theta-\frac{1}{2(2-\theta)} - 1 + 2\theta}{2(1-\theta) - \frac{1}{2-\theta}} \\
&= \frac{\theta - \frac{1}{2(2-\theta)}}{2(1-\theta) - \frac{1}{2-\theta}} = \frac{\theta(2-\theta) - \frac{1}{2}}{2(1-\theta)(2-\theta) - 1} \\
&= \frac{-(1-\theta)^2 + \frac{1}{2}}{2(1-\theta)(2-\theta) - 1}.
\end{aligned}$$

Therefore $\alpha_2^{max}(\theta,1) - \hat{\alpha}_2(\theta) > 0$ iff $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$. Consequently if $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$ then $\alpha_2^{max}(\theta,\alpha_1) - \hat{\alpha}_2(\theta) > 0$ for all α_1 and Condition (4.3) cannot hold true. Hence, $\frac{\partial \alpha_2^e}{\partial \alpha_1} > 0$ cannot hold true if $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$.

If $\theta < \frac{\sqrt{2}-1}{\sqrt{2}}$, there exists $\bar{\alpha}_1(\theta) \in [0,1]$ such that $\alpha_2^{max}(\theta, \bar{\alpha}_1(\theta)) = \hat{\alpha}_2(\theta)$ and for all $\alpha_1 > \bar{\alpha}_1(\theta)$, $\alpha_2^{max}(\theta,\alpha_1) < \hat{\alpha}_2(\theta)$, since $\alpha_2^{max}(\theta,\alpha_1)$ is decreasing with α_1. Solving the equation $\alpha_2^{max}(\theta,\bar{\alpha}_1(\theta)) = \hat{\alpha}_2(\theta)$ for $\bar{\alpha}_1(\theta)$, we obtain:

$$\bar{\alpha}_1(\theta) = \frac{1}{\left(1 - \frac{\theta}{1-\theta}\right)\left[2(2-\theta) - \frac{1}{1-\theta}\right]} + \frac{\theta^2}{(1-2\theta)[2(1-\theta)(2-\theta) - 1]}. \tag{4.4}$$

We deduce that for $\theta < \frac{\sqrt{2}-1}{\sqrt{2}}$, $\bar{\alpha}_1(\theta)$ increases with θ. Moreover,

$$\bar{\alpha}_1(0) = \frac{1}{3}$$

and

$$\bar{\alpha}_1(\frac{\sqrt{2}-1}{\sqrt{2}}) = \frac{\frac{1}{2} + \left(1 - \frac{1}{\sqrt{2}}\right)^2}{(\sqrt{2}-1)\left[2\frac{1}{\sqrt{2}}\left(1 + \frac{1}{\sqrt{2}}\right) - 1\right]} = 1.$$

Thus, if $\theta < \frac{\sqrt{2}-1}{\sqrt{2}}$ and $\alpha_1 > \bar{\alpha}_1(\theta)$, Condition (4.2) can be satisfied. This condition is equivalent to:

$$C_p > \bar{C}_p(\theta), \tag{4.5}$$

where

$$\bar{C}_p(\theta) \equiv \frac{C_{max}(\theta,\alpha_1)}{\alpha_2^{max}(\theta,\alpha_1)^2}(2\alpha_2^{max}(\theta,\alpha_1) - \hat{\alpha}_2(\theta))\hat{\alpha}_2(\theta)$$

We have

$$\frac{C_{max}(\theta,\alpha_1)}{\alpha_2^{max}(\theta,\alpha_1)^2} = \frac{\theta}{2}\left[\frac{1}{2-\theta} + \left(2(1-\theta) - \frac{1}{2-\theta}\right)\alpha_1\right].$$

Moreover:

$$\begin{aligned} 2\alpha_2^{max}(\theta,\alpha_1) - \hat{\alpha}_2(\theta) &= \frac{1-(2\theta-1)\alpha_1}{\frac{1}{2-\theta} + \left(2(1-\theta) - \frac{1}{2-\theta}\right)\alpha_1} - \frac{1-2\theta}{2(1-\theta) - \frac{1}{2-\theta}} \\ &= \frac{\frac{2(1-\theta)^2}{2-\theta}}{\left(2(1-\theta) - \frac{1}{2-\theta}\right)\left[\frac{1}{2-\theta} + \left(2(1-\theta) - \frac{1}{2-\theta}\right)\alpha_1\right]} \end{aligned}$$

Thus:

$$\bar{C}_p(\theta) = \frac{\theta(1-\theta)^2}{2-\theta}\frac{\hat{\alpha}_2(\theta)}{2(1-\theta) - \frac{1}{2-\theta}} = \frac{\theta(1-\theta)^2}{2-\theta}\frac{1-2\theta}{\left(2(1-\theta) - \frac{1}{2-\theta}\right)^2} = \frac{\theta(1-\theta)^2(2-\theta)(1-2\theta)}{(2(1-\theta)(2-\theta)-1)^2}.$$

This achieves the proof of Lemma 4.1.**Q.E.D**

Finally, we observe in equilibrium, the condition $\alpha_1^e(C_r) > \bar{\alpha}_1(\theta)$ is equivalent to:

$$C_r < \frac{\theta}{2}\left(\frac{1}{4} - \max\left(\bar{\alpha}_1(\theta) - \frac{1}{2}, 0\right)^2\right) = \bar{C}_r(\theta).$$

5 Sufficient and necessary conditions for $\partial\mathcal{E}_2(C_r, C_p)/\partial C_r > 0$.

In Proposition 4, we have shown that if $C_p < C_{min}(\theta, \alpha_1^e)$ then a decrease in the cost of producing the raw signal, C_r, reduce the informativeness of the price at date 2 (i.e., $\partial\mathcal{E}_2(C_r, C_p)/\partial C_r > 0$). We now derive sufficient and necessary conditions for this result to hold when $C_{min}(\theta, \alpha_1^e) < C_p < C_{max}(\theta, \alpha_1^e)$ (Case A). We also analyze the case $C_p > C_{max}(\theta, \alpha_1^e)$ at the end of this section (Case B). Overall the conditions derived in Cases A and B complete the proof of Proposition 4 and yields Figure 9 in the paper.

In case C, we show that long run price informativeness jumps down when the market for the raw signal takes off provided the processed signal is produced when the raw signal is not, i.e., $\mathcal{E}_2(\frac{\theta}{8}, C_p) < \mathcal{E}_2(\infty, C_p)$ if $C_p \leq C_max(\theta, 0)$. This supports the claim in the two first paragraphs on page 32 in Section 6.1.

Case A. Consider the case in which $C_{min}(\theta, \alpha_1^e) < C_p < C_{max}(\theta, \alpha_1^e)$. In this case, $0 < \alpha_2^e \leq 1$ (Proposition 1). Using eq.(13) and eq.(54), we have:

$$\mathcal{E}_2(C_r, C_p) = \frac{\theta}{4} - \frac{1}{2}\left(\bar{\pi}_2(\alpha_1^e, \alpha_2^e) - \frac{\theta(1-\theta)}{2}\alpha_1^e(1-\alpha_2^e)\right), \tag{5.1}$$

where we omit the arguments of functions α_1^e and α_2^e to simplify notations. As $\alpha_2^e > 0$, in equilibrium, $\alpha_2^e \bar{\pi}_2^e = C_p$ (see eq.(18)). Thus, we can rewrite eq.(5.1) as:

$$\mathcal{E}_2(C_r, C_p) = \frac{\theta}{4} - \frac{1}{2}\left(\frac{C_p}{\alpha_2^e} - \frac{\theta(1-\theta)}{2}\alpha_1^e(1-\alpha_2^e)\right), \tag{5.2}$$

Using the fact that C_r affects α_2^e only through its effect on α_1^e, we deduce from eq.(5.1) that:

$$\frac{\partial \mathcal{E}_2(C_r, C_p)}{\partial C_r} = \left(\frac{1}{2}\frac{\partial \alpha_1^e}{\partial C_r}\right)\left(\frac{\partial \alpha_2^e}{\partial \alpha_1^e}\left(\frac{C_p}{(\alpha_2^e)^2} - \frac{\theta(1-\theta)}{2}\alpha_1^e\right) + \frac{\theta(1-\theta)}{2}(1-\alpha_2^e)\right), \tag{5.3}$$

As $\frac{\partial \alpha_1^e}{\partial C_r} \leq 0$, we deduce that the sign of $\frac{\partial \mathcal{E}_2(C_r, C_p)}{\partial C_r}$ is opposite to the sign of the following function:

$$G(\alpha_1^e, \alpha_2^e) \equiv \frac{\partial \alpha_2^e}{\partial \alpha_1^e}\left(\frac{C_p}{(\alpha_2^e)^2} - \frac{\theta(1-\theta)}{2}\alpha_1^e\right) + \frac{\theta(1-\theta)}{2}(1-\alpha_2^e). \tag{5.4}$$

To determine the sign of $G(\alpha_1^e, \alpha_2^e)$, we first compute $\frac{\partial \alpha_2^e}{\partial \alpha_1^e}$. Using eq.(51), we obtain:

$$-\frac{\partial \alpha_2^e}{\partial \alpha_1^e} = \frac{\frac{\partial[\alpha_2^e \bar{\pi}_2(\alpha_1^e, \alpha_2^e)]}{\partial \alpha_1^e}}{\frac{\partial[\alpha_2^e \bar{\pi}_2(\alpha_1^e, \alpha_2^e)]}{\partial \alpha_2^e}} = \frac{\alpha_2^e \frac{\partial \bar{\pi}_2(\alpha_1^e, \alpha_2^e)}{\partial \alpha_1^e}}{\alpha_2^e \frac{\partial \bar{\pi}_2(\alpha_1^e, \alpha_2^e)}{\partial \alpha_2^e} + \bar{\pi}_2(\alpha_1^e, \alpha_2^e)}.$$

Moreover, as $0 < \alpha_2^e \leq 1$, we deduce from Proposition 2 that:

$$\bar{\pi}_2(\alpha_1^e, \alpha_2^e) = \frac{\theta}{2}\left\{1 - (2\theta - 1)\alpha_1^e - \left[\frac{1}{2-\theta} + \left(2(1-\theta) - \frac{1}{2-\theta}\right)\alpha_1^e\right]\alpha_2^e\right\}.$$

This implies that

$$\frac{\partial \bar{\pi}_2(\alpha_1^e, \alpha_2^e)}{\partial \alpha_1^e} = -\frac{\theta}{2}\left[2\theta - 1 + \left(2(1-\theta) - \frac{1}{2-\theta}\right)\alpha_2^e\right],$$

$$\frac{\partial \bar{\pi}_2(\alpha_1^e, \alpha_2^e)}{\partial \alpha_1^e} = -\frac{\theta}{2}\left[\frac{1}{2-\theta} + \left(2(1-\theta) - \frac{1}{2-\theta}\right)\alpha_1^e\right].$$

Therefore,

$$-\frac{\partial \alpha_2^e}{\partial \alpha_1^e} = \frac{\alpha_2^e\left[2\theta - 1 + \left(2(1-\theta) - \frac{1}{2-\theta}\right)\alpha_2^e\right]}{\alpha_2^e\left[\frac{1}{2-\theta} + \left(2(1-\theta) - \frac{1}{2-\theta}\right)\alpha_1^e\right] - \frac{2C_p}{\theta}\frac{1}{\alpha_2^e}}.$$

The denominator of this expression is equal to $-\frac{\partial[\alpha_2^e \bar{\pi}_2(\alpha_1^e, \alpha_2^e)]}{\partial \alpha_2^e}$, which is strictly positive (see the discussion that precedes Lemma 1 in the paper or Figure 5). Hence, we deduce that $G(\alpha_1^e, \alpha_2^e) < 0$ iff:

$$\begin{aligned}&\alpha_2^e\left[2\theta - 1 + \left(2(1-\theta) - \frac{1}{2-\theta}\right)\alpha_2^e\right]\left(\frac{2C_p}{\theta}\frac{1}{(\alpha_2^e)^2} - (1-\theta)\alpha_1^e\right),\\ &-(1-\theta)(1-\alpha_2^e)\left[\alpha_2^e\left(\frac{1}{2-\theta} + \left(2(1-\theta) - \frac{1}{2-\theta}\right)\alpha_1^e\right) - \frac{2C_p}{\theta}\frac{1}{\alpha_2^e}\right] > 0\end{aligned}$$

After some algebra, one can show that this condition is equivalent to:

$$\Upsilon(C_r, C_p, \theta) > 0,$$

where

$$\Upsilon(C_r, C_p, \theta) \equiv \frac{1-\theta}{2-\theta}\left(\frac{2C_p}{\theta} - \alpha_2^e(1-\alpha_2^e)\right) + 2C_p\left(\frac{1}{\alpha_2^e} - 1\right) - \alpha_1^e\alpha_2^e\frac{(1-\theta)^2}{2-\theta}. \quad (5.5)$$

In sum, $G(\alpha_1^e, \alpha_2^e) < 0$ iff $\Upsilon(C_r, C_p, \theta) > 0$. Thus, when $C_{min}(\theta, \alpha_1^e) < C_p < C_{max}(\theta, \alpha_1^e)$, $\frac{\partial \mathcal{E}_2(C_r, C_p)}{\partial C_r} > 0$ iff $\Upsilon(C_r, C_p, \theta) > 0$. In other words, when $C_{min}(\theta, \alpha_1^e) < C_p < C_{max}(\theta, \alpha_1^e)$, a decrease in the cost of the raw signal lowers the informativeness of the price at date 2 if and only if $\Upsilon(C_r, C_p, \theta) > 0$.

Case B. Last consider the case in which $C_{max}(\theta, \alpha_1^e) < C_p$. In this case, no speculator buys the processed signal ($\alpha_2^e = 0$). Thus, $\mathcal{E}_2(C_r, C_p) = \mathcal{E}_1(C_r, C_p)$ (see eq.(54) in the proof of Corollary 3). That is, long run price informativeness is equal to short run price informativeness because there is no information production after date 1. In this case, a decrease in C_r raises price informativeness simply because it raises the demand for the raw signal and thereby price informativeness at date 1.

Case C. Figure 5.1 plots for each pair (C_p, θ) the sign of $\mathcal{E}_2(\frac{\theta}{8}, C_p) - \mathcal{E}_2(C_r > \frac{\theta}{8}, C_p)$. The grey (blue) area are the pairs for which this difference is negative (resp., positive). The red curve plots $C_{max}(\theta, 0) = \frac{\theta(2-\theta)}{8}$. For a given θ, if C_p is below this curve then the demand for the process signal is strictly positive when there is no market for the raw signal ($\alpha_1 = 0$). The graphic shows that for all pairs (C_p, θ) such this condition is satisfied ($C_p < C_{max}(\theta, 0)$) then $\mathcal{E}_2(\frac{\theta}{8}, C_p) < \mathcal{E}_2(C_r > \frac{\theta}{8}, C_p)$.[1] This proves that long run price informativeness jumps down when the market for the raw signal takes off provided the processed signal is produced when the raw signal is not (i.e., $\mathcal{E}_2(\frac{\theta}{8}, C_p) < \mathcal{E}_2(\infty, C_p)$ if $C_p \leq C_{max}(\theta, 0)$).

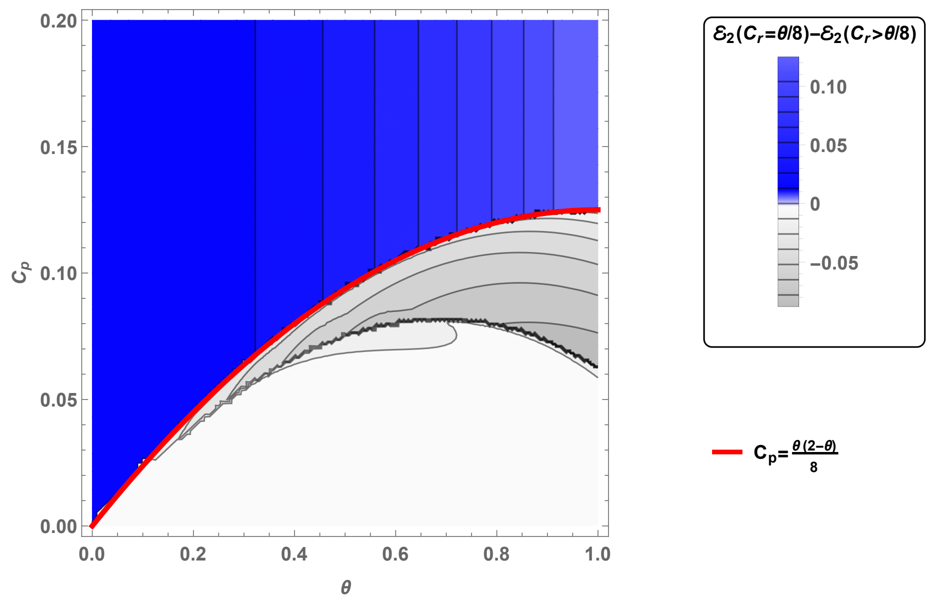

Figure 5.1

6 Proof that C_{max} decreases with α_1 when $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$.

Case 1. For $\theta > 1/2$, we can write C_{max} as

$$C_{max}(\theta, \alpha_1) = \frac{\theta}{4}\bar{\alpha}_2(\theta, \alpha_1)(1 - (2\theta - 1)\alpha_1),$$

[1] Note that Figure 5.1 covers all possible cases for the parameter values since C_r is fixed here. Thus, a graphical proof is sufficient.

which is the product of two decreasing and positive functions of α_1. Thus, in this case, C_{max} decreases with α_1.

Case 2. For $\theta < 1/2$, we use the fact that for a given α_2, the aggregate speculators' profit at $t = 2$, that is $\alpha_2\bar{\pi}_2(\alpha_1, \alpha_2)$ can be written as

$$\begin{aligned}\alpha_2\bar{\pi}_2(\alpha_1, \alpha_2) &= \frac{C_{max}(\theta, \alpha_1)}{\alpha_2^{max}(\theta, \alpha_1)^2}(2\alpha_2^{max}(\theta, \alpha_1) - \alpha_2)\alpha_2 \\ &= C_{max}(\theta, \alpha_1)\left(2 - \frac{\alpha_2}{\alpha_2^{max}(\theta, \alpha_1)}\right)\frac{\alpha_2}{\alpha_2^{max}(\theta, \alpha_1)}\end{aligned}$$

We know that for $\alpha_2 = \hat{\alpha}_2(\theta)$, the expected profit $\bar{\pi}_2(\alpha_1, \hat{\alpha}_2(\theta))$ does not depend on α_1. Therefore the aggregate profit taken in $\hat{\alpha}_2(\theta)$ is also independent of α_1 and happen to be equal to $\bar{C}_p(\theta)$ calculated previously. Indeed,

$$\hat{\alpha}_2(\theta)\bar{\pi}_2(\alpha_1, \hat{\alpha}_2(\theta)) = C_{max}(\theta, \alpha_1)\left(2 - \frac{\hat{\alpha}_2(\theta)}{\alpha_2^{max}(\theta, \alpha_1)}\right)\frac{\hat{\alpha}_2(\theta)}{\alpha_2^{max}(\theta, \alpha_1)} = \bar{C}_p(\theta) > 0. \quad (6.1)$$

Notice that the the function $h(x) = x(2 - x)$ reaches a maximum for $x = 1$, increases for $x < 1$ and decreases for $x > 1$. In previous section 4, we have shown that when $\theta < 1/2$, $\bar{C}_p(\theta) > 0$, $\alpha_2^{max}(\theta, \alpha_1)$ decreases with α_1, and $\alpha_2^{max}(\theta, \alpha_1) > \hat{\alpha}_2(\theta)$ if $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$. Therefore, the ratio $r(\theta, \alpha_1) = \frac{\hat{\alpha}_2(\theta)}{\alpha_2^{max}(\theta,\alpha_1)}$ increases with α_1, and is always less than 1. Hence $r(\theta, \alpha_1)(2 - r(\theta, \alpha_1))$ increases with α_1. We deduce that $C_{max}(\theta, \alpha_1)$ must decrease with α_1 since $C_{max}(\theta, \alpha_1)r(\theta, \alpha_1)(2 - r(\theta, \alpha_1))$ is strictly positive and does not depend on α_1 (see eq.(6.1)).

7 Speculators can trade at both dates even if they buy only one signal.

In this section we consider the case in which speculators who buy only one signal can trade at both dates if they wish. We follow the same steps as in the baseline model. In Section 7.1, we derive speculators' equilibrium trading strategies (Proposition 7.1) and their expected profits (Proposition 7.2), holding (α_1, α_2) constant. We then show in Section 7.2 that an increase in the demand for the raw signal can exert a positive or negative externality on investors trading on the processed signal, as in the baseline model, and we write conditions characterizing equilibrium demands for each type of signal. Using these conditions, in Section 7.3, we show numerically that a decrease in the cost of the raw signal can lead to a decrease in long run price informativeness, as in the baseline model. In Section, 7.4, we show that the implications of the model for price and trade patterns are preserved in this extension.

7.1 Equilibrium strategies and profits

It is straightforward to show that, at date $t = 1$, the equilibrium is the same as in the baseline model. In particular, speculators who only buy the raw signal behaves as described in Proposition 1 in the paper while speculators who only buy the processed signal do not trade. Indeed, the latter expects to trade at $\mathsf{E}(p_1^*) = p_0$, which is just their valuation for the asset given that they have no information at date 1.

Thus, as in the baseline model, there are two possible outcomes at date 1: (i) either the price has changed relative to date 0 (with probability α_1) or (ii) it has not changed (with probability $(1 - \alpha_1)$). If the price has changed at date 1, it fully reflects the information in the raw signal. In this case, speculators who only purchase the raw signal cannot profitably trade at date 2 since they have no informational advantage relative to the dealers. Thus, the equilibrium at date 2 is identical to that in the baseline model if the price changes at date 1. Hence, in this case, speculators trading on the raw signal gets a zero expected profit at date 2 while speculators who trade on the processed signal obtain the same expected profit as in the baseline model, i.e., $\pi_2^c(\alpha_2) = \theta(1-\theta)\max[1-\alpha_2, 0]$.

If the price does not change at date 1, speculators who only purchase the raw signal might find optimal to trade again at date 2 (see below). We denote by $\pi_2^{1,nc}(\alpha_1, \alpha_2)$, the expected profit of a raw signal speculator in this case. Hence, the ex-ante expected trading profit of trading on the raw signal is:

$$\bar{\pi}_1(\alpha_1, \alpha_2) = \pi_1(\alpha_1) + (1-\alpha_1)\pi_2^{1,nc}(\alpha_1, \alpha_2) \tag{7.1}$$

with $\pi_1(\alpha_1) = \frac{\theta}{2}\max[1-\alpha_1, 0]$.

Similarly, we denote by $\pi_2^{2,nc}(\alpha_1, \alpha_2)$, the expected profit of trading on the processed signal when the price has not changed at $t = 1$. The total expected profit of trading on the processed signal is therefore:

$$\bar{\pi}_2(\alpha_1, \alpha_2) = \alpha_1 \pi_2^c(\alpha_2) + (1-\alpha_1)\pi_2^{2,nc}(\alpha_1, \alpha_2). \tag{7.2}$$

Lemma 7.1. *If $\pi_2^{1,nc}(\alpha_1, \alpha_2) > 0$ then no speculator buys both signals.*

Proof. The expected profit of a speculator who buys the raw and the processed signals net of the fees charged by information sellers is:

$$\pi_1(\alpha_1) + \bar{\pi}_2(\alpha_1, \alpha_2) - F_r - F_p = \pi_1(\alpha_1) - F_r < \bar{\pi}_1(\alpha_1, \alpha_2) - F_r,$$

where the second equality follows from the fact that, in equilibrium, the fee charged for the processed signal is such that $\bar{\pi}_2(\alpha_1, \alpha_2) = F_p$ and the last inequality from eq.(7.1) and the condition $\pi_2^{1,nc}(\alpha_1, \alpha_2) > 0$. This inequality implies that buying only the raw signal dominates buying both signals when $\pi_2^{1,nc}(\alpha_1, \alpha_2) > 0$. □

Thus, in the rest of this section, we call speculators who only trade on the raw signal as "raw information speculators" and those who only trade on the processed signal as "processed information speculators". The next proposition describes the equilibrium trading strategies of both types of speculators at date 2 when the price has no changed at date 1. An equilibrium of the market at date 2 when the price has not changed at date 1 is a triplet $(x_{2r}^*(s), x_2^*(s,u), p_2^*(f_2))$ such that (i) $p_2^*(f_2)$ is equal to the expected payoff of the asset given f_2, (ii) each speculator receiving the raw signal maximizes his expected profit by trading $x_{2r}^*(s)$ shares given other speculators' trading strategies and the market maker's pricing policy $(p_2^*(f_2))$ and (iii) each speculator receiving the processed signal maximizes his expected profit by trading $x_{2p}^*(s,u)$ shares given other speculators' trading strategies and and the market maker's pricing policy $(p_2^*(f_2))$. Let denote $k = \min[\alpha_1/\alpha_2, 1]$, $\alpha = \alpha_1 + \alpha_2$, and $\beta = \alpha_1 - k \times \alpha_2$.

Proposition 7.1. *At $t = 2$, when the price has not changed at $t = 1$,*

1. *The speculators who receive the raw signal buy one share when $s = 1$ and sell one share when $s = 0$, i.e., $x^*_{2r}(1) = 1$ and $x^*_{2r}(0) = -1$.*

2. *The speculators who receive the processed signal buy one share when $(s, u) = (1, 1)$, sell one share when $(s, u) = (0, 1)$, sell k shares when $(s, u) = (1, 0)$, and buy k shares when $(s, u) = (0, 0)$. That is, $x^*_{2p}(1, 1) = 1$, $x^*_{2p}(0, 1) = -1$,$x^*_{2p}(1, 0) = -k$, and $x^*_{2p}(-1, 0) = k$.*

3. *The equilibrium price is:*

$$p^*_2(f_2) = \frac{\theta\phi(f_2 - \alpha) + \frac{1-\theta}{2}\phi(f_2 + \beta) + \frac{1-\theta}{2}\phi(f_2 - \beta)}{\frac{\theta}{2}\phi(f_2 - \alpha) + \frac{\theta}{2}\phi(f_2 + \alpha) + \frac{1-\theta}{2}\phi(f_2 + \beta) + \frac{1-\theta}{2}\phi(f_2 - \beta)} \times \frac{1}{2}. \tag{7.3}$$

In line with intuition, when the price has not changed at date 1, speculators who receive the raw signal trade in the same way at dates 1 and 2 since their informational advantage relative to dealers is identical at both dates. When $u = 1$, speculators who receive the processed signal trade as in the baseline case. In contrast, when $u = 0$, they act differently. Indeed, they trade even when $u = 0$ and $p_1 = p_0$ while they do not in this situation in the baseline case. For instance, when $s = 1$ and $u = 0$, the process information speculators expect the raw information speculators to buy the asset at date 2. These buys tend to push the price above processed information speculators' estimate of the payoff of the asset given that $u = 0$ (i.e., 1/2). Thus, when $s = 1$ and $u = 0$, they sell the asset. Given the aggregate size of their trade, k, the order flow from all speculators (those trading on the processed signal and those trading on the raw signal) is $\beta = \alpha_1 - k\alpha_2$. The trade size of processed information speculators in equilibrium (k) is such that $\beta \geq 0$ so that their sells never more than offset raw information speculators' buys when $s = 1$ and $u = 0$. Intuitively doing so cannot be optimal in equilibrium because the expected price for the asset at date 2 would then be less than the estimate of the asset by the processed information speculators. Intuitions are symmetric when when $s = 0$ and $u = 0$

From Propositions 7.1, we deduce the expected profit of the speculators (on their trade at date $t = 2$) when the price has not changed at date $t = 1$.

Proposition 7.2. *Speculators' expected profits on their trade at date* 2*, conditional on no change in the price at date* $t = 1$*, are as followed*

1. $\Gamma_1 : \alpha_1 + \alpha_2 \leq 1,$ *and* $\alpha_2 \geq \alpha_1$

$$\pi_2^{1,nc}(\alpha_1, \alpha_2) = \pi_2^{2,nc}(\alpha_1, \alpha_2) = \frac{\theta}{2}\left(1 - \frac{1}{2-\theta}(\alpha_1 + \alpha_2)\right) \tag{7.4}$$

2. $\Gamma_2 : \alpha_1 + \alpha_2 \leq 1,$ *and* $\alpha_2 < \alpha_1$

$$\pi_2^{1,nc}(\alpha_1, \alpha_2) = \frac{\theta}{2}\left[1 - \alpha_1 - \frac{\theta}{2-\theta}\alpha_2\right] \tag{7.5}$$

$$\pi_2^{2,nc}(\alpha_1, \alpha_2) = \frac{\theta}{2}\left[1 - (2\theta - 1)\alpha_1 - \underbrace{\left(\frac{2}{2-\theta} - (2\theta - 1)\right)}_{\geq 0}\alpha_2\right] \tag{7.6}$$

3. $\Gamma_3 : 1 < \alpha_1 + \alpha_2 \leq 2,$ *and* $\alpha_2 \geq \alpha_1$

$$\pi_2^{1,nc}(\alpha_1, \alpha_2) = \pi_2^{2,nc}(\alpha_1, \alpha_2) = \frac{\theta(1-\theta)}{2(2-\theta)}(2 - \alpha_1 - \alpha_2) \tag{7.7}$$

4. $\Gamma_4 : 1 < \alpha_1 + \alpha_2 \leq 2,\ \alpha_2 < \alpha_1,$ *and* $\alpha_1 < 1$

$$\pi_2^{1,nc}(\alpha_1, \alpha_2) = \frac{\theta(1-\theta)}{2-\theta}(1 - \alpha_1) \tag{7.8}$$

$$\pi_2^{2,nc}(\alpha_1, \alpha_2) = \frac{\theta(1-\theta)}{2(2-\theta)}(2 + 2(1-\theta)\alpha_1 - 2(2-\theta)\alpha_2) \tag{7.9}$$

5. $\Gamma_5 : \alpha_2 \leq 1,$ *and* $\alpha_1 \geq 1$

$$\pi_2^{1,nc}(\alpha_1, \alpha_2) = 0 \tag{7.10}$$

$$\pi_2^{2,nc}(\alpha_1, \alpha_2) = \theta(1-\theta)(1 - \alpha_2) \tag{7.11}$$

6. $\Gamma_6 : \alpha_1 + \alpha_2 > 2,$ *and* $\alpha_2 > 1$

$$\pi_2^{1,nc}(\alpha_1, \alpha_2) = \pi_2^{2,nc}(\alpha_1, \alpha_2) = 0 \tag{7.12}$$

Figure 7.1 shows the case that obtains (denoted Γ_j for $j \in \{1, ..., 6\}$) for each possible pair $((\alpha_1, \alpha_2))$. Proposition 7.2 shows that in case Γ_6, all speculators obtain zero expected profits. This case cannot arise when the costs of producing the signals are strictly positive. Thus we ignore it in the rest of the analysis. Similarly, in case Γ_5, speculators trading on the raw signal obtain a zero expected profit. Again this case cannot arise when the raw signal is costly to produce. Thus, we ignore it as well.

When $\alpha_2 \geq \alpha_1$ (cases Γ_1 and Γ_3), the expected profits of speculators on their trades at date 2 are the same whether they buy the processed or the raw signal, when the price has not changed at date 1 (i.e., $\pi_2^{1,nc}(\alpha_1, \alpha_2) = \pi_2^{2,nc}(\alpha_1, \alpha_2)$). The reason is that in this case, $\beta = 0$. Thus, the aggregate demand from speculators trading on the raw signal is perfectly offset by that of speculators trading on the processed signal when $u = 0$. Thus, when $u = 0$, all speculators trade at price $1/2$ and make zero expected profits. When $u = 1$, they trade as the speculators who receive the processed signal and therefore obtain the same expected profit.

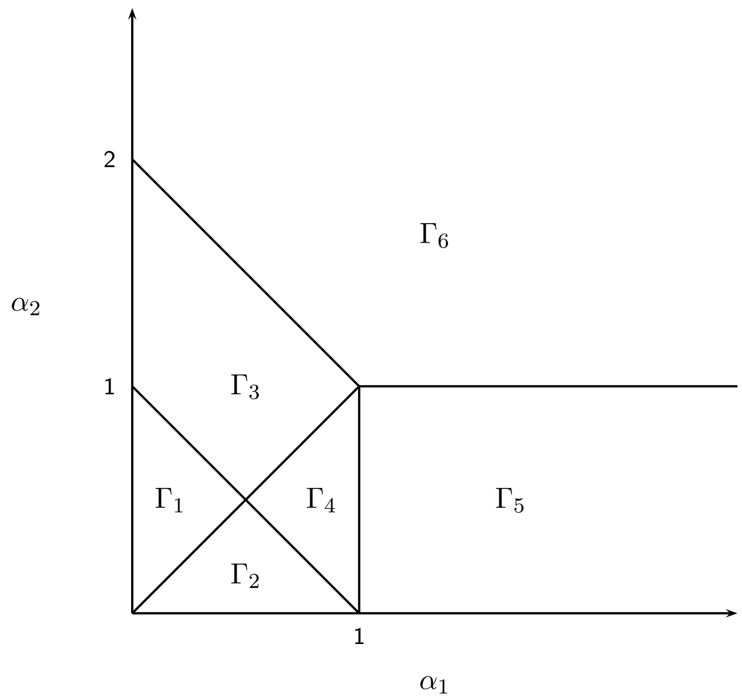

Figure 7.1: Cases in Proposition 7.2 for each pair (α_1, α_2)**.**

7.2 Equilibrium in the market for information

Let $\bar{\pi}_1^{agg}(\alpha_1, \alpha_2) = \alpha_1 \bar{\pi}_1(\alpha_1, \alpha_2)$ denote the aggregate profits of speculators who buy the raw signal and $\bar{\pi}_2^{agg}(\alpha_1, \alpha_2) = \alpha_2 \bar{\pi}_2(\alpha_1, \alpha_2)$ denote the aggregate profits of those who buy the processed signal. An interior equilibrium in the market for information is a pair $(\alpha_1^e(C_r, C_p), \alpha_2^e(C_r, C_p)$ such that,

$$\bar{\pi}_1^{agg}(\alpha_1^e(C_r, C_p), \alpha_2^e(C_r, C_p)) = C_r, \tag{7.13}$$

$$\bar{\pi}_2^{agg}(\alpha_1^e(C_r, C_p), \alpha_2^e(C_r, C_p)) = C_p, \tag{7.14}$$

and,

$$\frac{\partial \bar{\pi}_1^{agg}}{\partial \alpha_1}(\alpha_1^e(C_r, C_p), \alpha_2^e(C_r, C_p)) < 0, \ \frac{\partial \bar{\pi}_2^{agg}}{\partial \alpha_2}(\alpha_1^e(C_r, C_p), \alpha_2^e(C_r, C_p)) < 0. \tag{7.15}$$

Condition (7.15) guarantees that we select as an equilibrium the pair $(\alpha_1^e(C_r, C_p), \alpha_2^e(C_r, C_p)$ with the largest demand for each signal when there are multiple solutions to the system of equation eq.(7.13) and eq.(7.14).

Demand externalities. As in the paper, we can assess how an exogenous increase in the demand for the raw signal, α_1, affects the demand for the processed signal α_2, around an equilibrium, by computing the "best response function", $\alpha_2(\alpha_1)$, implicitly given by $\bar{\pi}_2^{agg}(\alpha_1, \alpha_2) = C_p$, around $(\alpha_1^e(C_r, C_p), \alpha_2^e(C_r, C_p))$. This function is (locally) increasing if and only if

$$\frac{\partial \bar{\pi}_2}{\partial \alpha_1}(\alpha_1^e, \alpha_2^e) < 0.$$

Using eq.(7.2) and the definition of $\bar{\pi}_2^{agg}(\alpha_1, \alpha_2)$, we obtain:

$$\frac{\partial \bar{\pi}_2}{\partial \alpha_1} = \pi_2^c(\alpha_2) - \pi_2^{2,nc}(\alpha_1, \alpha_2) + (1 - \alpha_1)\frac{\partial \pi_2^{2,nc}}{\partial \alpha_1}. \tag{7.16}$$

We obtain a similar the same expression for $\frac{\partial \bar{\pi}_2}{\partial \alpha_1}$ in the baseline model (see eq.(15)), except for the last term $((1-\alpha_1)\frac{\partial \pi_2^{2,nc}}{\partial \alpha_1}$. As explained in the baseline model, the first term $(\pi_2^c(\alpha_2) - \pi_2^{2,nc}(\alpha_1, \alpha_2))$ can be positive or negative, which implies that an increase in the demand for the raw signal can increase or reduce the value of trading on the processed signal. Eq.(7.16) shows that this key mechanism for our results still operates in the case in which the speculators can trade on the raw signal at date 2.

In the baseline model, however, an increase in the demand for the raw signal affects the expected profit of speculators receiving the processed signal only through its effect on the likelihood α_1 that the price at date 1 reveals the raw signal s. When speculators receiving the raw signal can also trade at date 2, an increase in their mass also directly affects the expected profit of trading on the processed signal when there is no change in the price at date 1. Indeed, as shown in the previous section (Proposition 7.1), in this case, speculators with the raw signal trade at date 2 and this affects the expected profit of speculators who receive the processed signal. This effect is captured by the last term in eq.(7.16) $(\frac{\partial \pi_2^{2,nc}}{\partial \alpha_1})$.

The sign of this effect can be positive or negative. The reason is identical to the reason for which $\pi_2^c(\alpha_2) - \pi_2^{2,nc}(\alpha_1, \alpha_2)$ can be positive or negative. Indeed, when the raw information speculators' signal is correct ($u = 1$), trading by these speculators increases competition for the speculators trading on the processed signal and thereby reduces their profits. In contrast, when their signal is incorrect ($u = 0$), trading by the raw speculators at date 2 open new profit opportunities for the speculators trading on the processed signal since they can trade against the noise introduced by raw signal speculators in the price at date 2. This is exactly the same mechanism as that present in the baseline model, which is already captured by the first term in eq.(7.16) $((\pi_2^c(\alpha_2) - \pi_2^{2,nc}(\alpha_1, \alpha_2))$.

Thus, as in the baseline model, an increase in the demand for the raw signal can either be (locally) (i) a positive externality for those buying the processed signal (it increases their expected profit) or (ii) a negative externality. To see this, consider two cases. First, consider the subcase of case Γ_3, in Proposition 7.2, where $\alpha_2 \geq 1$. In this case, $\pi_2^c(\alpha_2) = 0$ and therefore, using eq.(7.2) and eq.(7.7), we have:

$$\bar{\pi}_2^{agg}(\alpha_1, \alpha_2) = \frac{\theta(1-\theta)}{2(2-\theta)}\alpha_2(1-\alpha_1)\left(2-\alpha_1-\alpha_2\right).$$

It follows that $\partial\bar{\pi}_2^{agg}/\partial\alpha_1 < 0$ and thus $\partial\alpha_2/\partial\alpha_1 < 0$. Alternatively, consider case Γ_4. We deduce from eq.(7.2) and (7.9) that in this case,

$$\bar{\pi}_2^{agg}(\alpha_1, \alpha_2) = \alpha_1\theta(1-\theta)\alpha_2(1-\alpha_2) + (1-\alpha_1)\frac{\theta(1-\theta)}{2(2-\theta)}\alpha_2\left(2+2(1-\theta)\alpha_1 - 2(2-\theta)\alpha_2\right).$$

The partial derivative with respect to α_1 can be positive for some values of α_2.

Another difference with the baseline case is that the demand for the processed signals (α_2) affects the expected profit of speculators who receive the raw signal. Indeed, when they trade at date 2, their expected profit is affected by the demand of speculators who receive the processed signal (see Proposition 7.2) and therefore their total expected profit depends on this demand as well. In all cases, considered in Proposition 7.2, the expected profit for speculators trading on the raw signal at date 2 decreases with the demand for the processed signal. We deduce that:

$$\frac{\partial\bar{\pi}_1^{agg}}{\partial\alpha_2} < 0. \tag{7.17}$$

Solving for the equilibrium. Equations (7.13)and (7.15) show that when speculators can trade on the raw signal at date 2, the demand for the raw signal and the demand for the processed signal are *simultaneously* determined in equilibrium because (i) the expected profit from trading on the processed signal depends on the demand for the raw signal, α_1 (as in the baseline model) and (ii) the expected profit from trading on the raw signal depends on the demand for the processed signal, α_2. In contrast, in the baseline model, the expected profit from trading on the raw signal does not depend on the demand for the processed signal. As a result, in the baseline model, one can solve for equilibrium demands in closed-form by first solving for the demand for the raw signal and then the demand for the processed signal at the equilibrium point obtained for the demand for the raw signal. The simultaneous determination of the demands for both signals in the extension considered here precludes the use of this approach and, for this reason, no analytical solutions can be obtained for equilibrium demands. However one can solve numerically for the system of equations (7.13) and (7.15) to find the equilibrium. Panel A in Figure 7.2 provides an example. In this example, when C_r declines, the demand for the raw signal (blue line) increases and the demand for the processed signal (red line) increases. Thus, as in the baseline model, this is a case in which an increase in the demand for the raw signal exerts a negative externality on the demand for the processed signal.

7.3 Asset price informativeness.

Remember that as asset price informativeness at date t is defined as,

$$\mathcal{E}_t = \frac{1}{4} - E[(V - p_t)^2] = \frac{1}{4} - E[p_t(1 - p_t)].$$

At date 1, the equilibrium is as in the baseline model and we therefore obtain the same expression for $\mathcal{E}_1$ (eq.(53) in the baseline model). For date 2, we obtain the following result. We obtain the following result.

Proposition 7.3. *Asset price informativeness at date 2 is:*

$$\mathcal{E}_2(C_r, C_p) = \frac{\theta}{4} - \frac{1}{4}\left[(1-\alpha_1^e)\left(\pi_2^{1,nc}(\alpha_1^e, \alpha_2^e) + \pi_2^{2,nc}(\alpha_1^e, \alpha_2^e)\right) + \alpha_1^e \pi_2^c(\alpha_2^e)\right], \tag{7.18}$$

where α_1^e and α_2^e solve (7.13)and (7.15) and expressions for expected profits are given in Proposition 7.2.

We just consider the cases in which $(\alpha_1, \alpha_2) \in \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_4$ because other cases cannot arise in an interior equilibrium in which the demand for both signals is strictly positive.

Solving numerically for α_1^e and α_2^e, we can analyze the effect of a change in the cost of the raw signal on price informativeness at date 2. Panels B and D of Figure 7.2 show that the key finding of the baseline model still obtains in this extension. Indeed, a decrease in the cost of the raw signal can trigger a decrease in price informativeness at date 2 because it generates a drop in the value of the processed signal and therefore the demand for this signal (Panels A and C of Figure 7.2). In contrast to the baseline model, we cannot delineate in a clear way for which values of the parameters this happens because we cannot solve analytically for equilibrium demands.

7.4 Price and trade patterns

We only consider cases in which in equilibrium, the demand for both signals is strictly positive ($\alpha_1^e(C_r, C_p) > 0$ and $\alpha_2^e(C_r, C_p) > 0$). Indeed, in other cases, the covariances derived below are all zero, as in the baseline model, since $x_1 = 0$ or $x_2 = 0$.

Corollary 7.1. *If $\alpha_1^e(C_r, C_p) > 0$ and $\alpha_2^e(C_r, C_p) > 0$, then the covariance between a raw signal speculator's trade, x_1 (at date 1 or 2), and a processed signal speculator's trade, x_2, is*

$$Cov(x_1, x_2) = \theta - (1-\theta)\alpha_1^e - (1-\theta)(1-\alpha_1^e)\min\left[1, \frac{\alpha_1^e}{\alpha_2^e}\right]. \tag{7.19}$$

The first two terms in $Cov(x_1, x_2)$ $(\theta - (1-\theta)\alpha_1^e)$ are identical to that in the baseline model (see Corollary 4 in the baseline model). As explained in the paper, the second

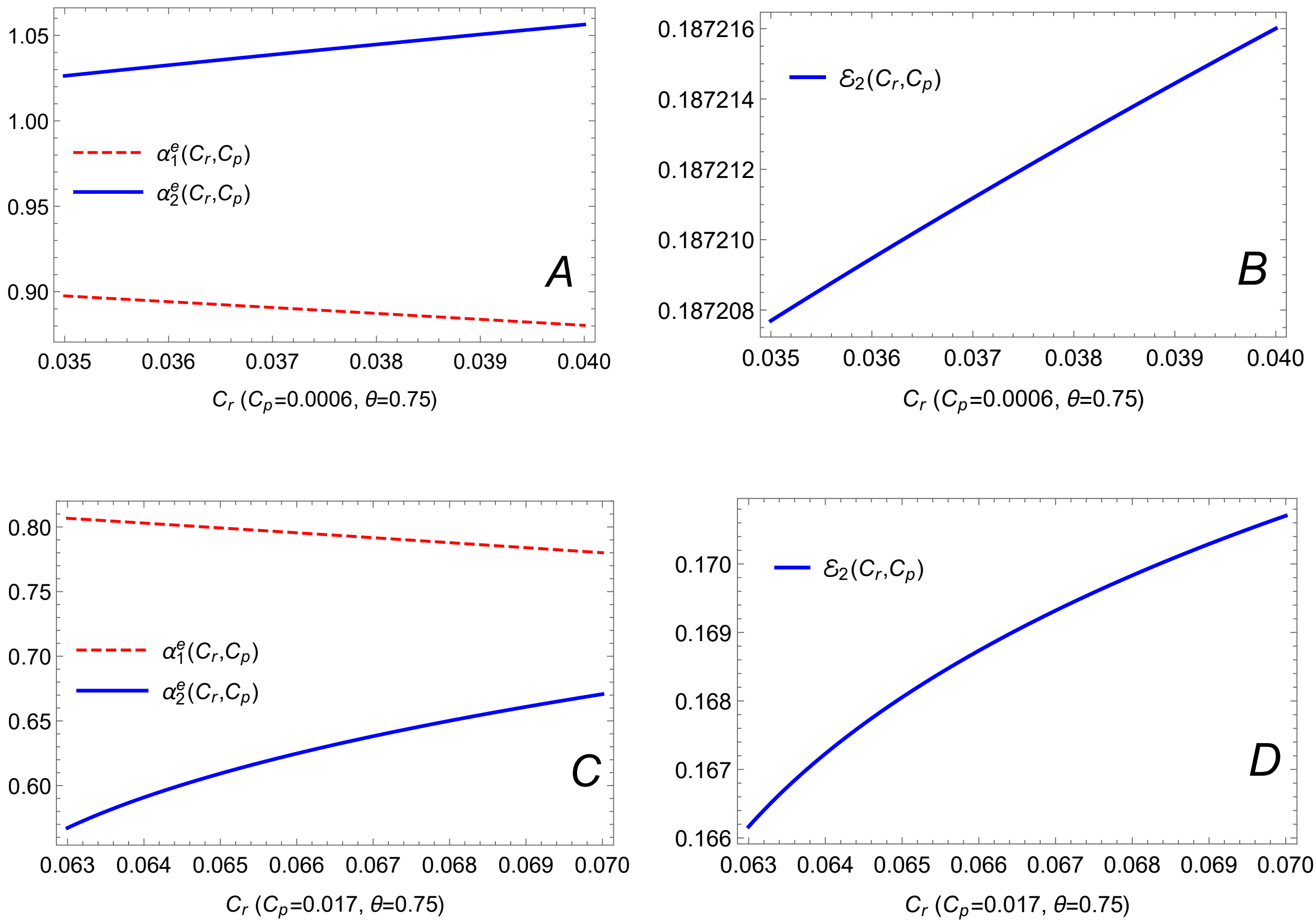

Figure 7.2: Panels A and C depicts demands for information, $\alpha_1^e(C_r, C_p)$ and $\alpha_2^e(C_r, C_p)$, and Panels B and D depicts long run price informativeness, $\mathcal{E}_2(C_r, C_p)$, as functions of C_r, respectively for $C_p = 0.0006$ and $C_p = 0.017$, and for $\theta = 0.75$.

term captures the fact that when raw speculators' signal is noise, processed information speculators trade in a direction opposite to raw information speculators, which tends to create a negative correlation between the trades of raw and processed information speculators. In the baseline model, processed information speculators trade in a direction opposite to that of raw information speculators only the raw signal is noise ($u = 0$) and the price has changed at date 1. In contrast, in the extension considered here, they also do so when the price has not changed at date 1, which reinforces the tendency for the correlation between the trades of raw and processed information speculators to be negatively correlated. This effect is captured by the last term in the expression for $Cov(x_1, x_2)$. Its presence does not change our main conclusions for $Cov(x_1, x_2)$ in the baseline model. That is, $Cov(x_1, x_2)$ can be positive or negative and is more likely to be negative when C_r and θ are small, as shown in Figure 7.3.

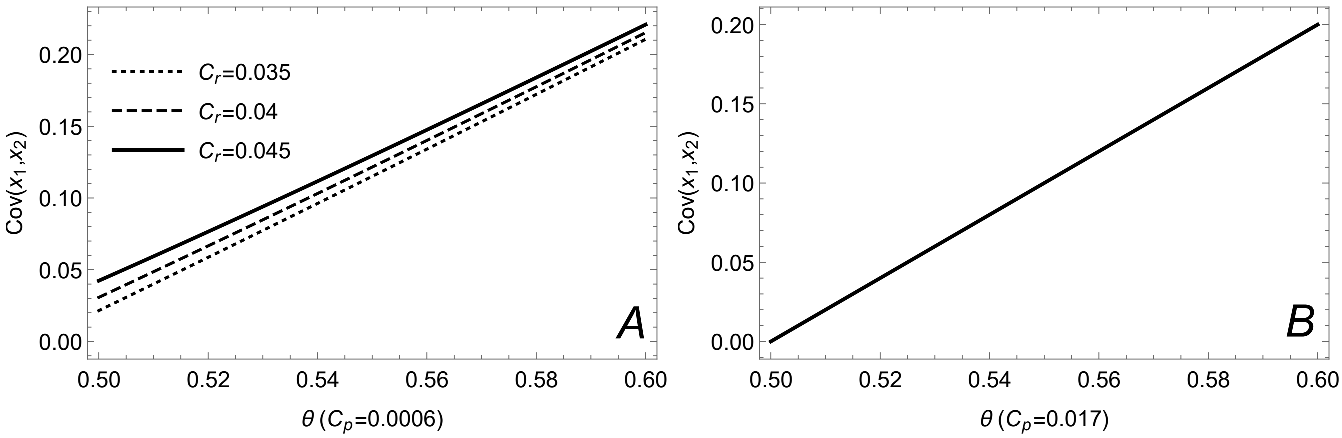

Figure 7.3: Panel A depicts the covariance between speculators' trades, as functions of θ, for $C_r = 0.035$, $C_r = 0.04$ and $C_r = 0.045$, and for $C_p = 0.0006$. Panel B depicts the covariance between speculators' trades, as functions of θ, for $C_p = 0.017$. In the last case, $\alpha_2^e < \alpha_1^e$, and thus $Cov(x_1, x_2) = 2\theta - 1$ and does not depend on C_r.

Corollary 7.2. *The covariance between date 1 return, r_1 (at date 1 or 2), and a processed signal speculator's trade, x_2, is*

$$Cov(r_1, x_2) = \theta(2\theta - 1)\alpha_1^e. \tag{7.20}$$

We obtain exactly the same expression for $Cov(r_1, x_2)$ in the baseline model (see Corollary 5). Thus, the results obtained in this case still apply. In a particular, (i) $Cov(r_1, x_2)$ can be positive or negative and (ii) a decrease in the cost of the raw signal raises the demand for this signal (α_1^e and therefore increases $Cov(r_1, x_2)$ in absolute value.

Corollary 7.3. *If the covariance between a raw signal speculator's trade, x_1, and date 2 return, $r_2 = p_2 - p_1$, is*

$$Cov(x_1, r_2) = \frac{1}{2}(1 - \alpha_1^e)\left(\theta - 2\pi_2^{1,nc}(\alpha_1^e, \alpha_2^e)\right). \tag{7.21}$$

Thus, as in the baseline model, $Cov(x_1, r_2)$ is always positive because $\pi_2^{1,nc}(\alpha_1^e, \alpha_2^e) < \frac{\theta}{2}$. Figure 7.4 shows that $Cov(x_1, r_2)$ can increase with C_r. But again, in contrast with the baseline model, we cannot clearly determine under which parameters' values this result would or would not hold.

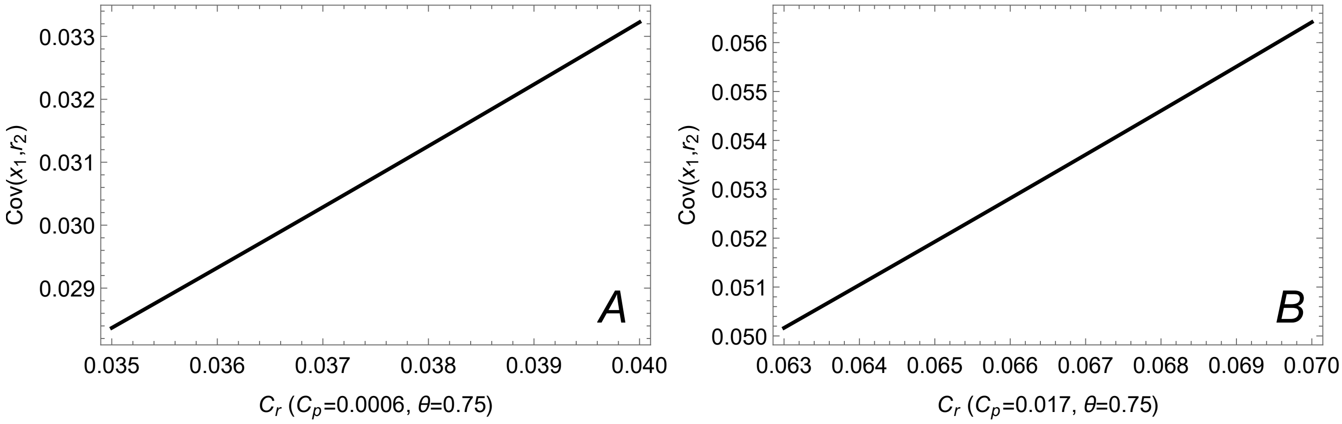

Figure 7.4: Panel A and B depict the covariance between raw signal speculators' trades and date 2 return, as functions of C_r, respectively for $C_p = 0.0006$ and $C_p = 0.017$.

Proofs

Proof of Propositions 7.1 and 7.2. We first show that speculators' trading strategies and the equilibrium price given in Proposition 7.1 form an equilibrium. Given these trading strategies, the aggregate order flow at date 2 when there is no change in price at date 1 is

$$\tilde{f}_2 = \tilde{l} + \begin{cases} \alpha & \text{if } s = 1, u = 1, \\ \beta & \text{if } s = 1, u = 0, \\ -\alpha & \text{if } s = 0, u = 1, \\ -\beta & \text{if } s = 0, u = 0, \end{cases} \tag{7.22}$$

where $\alpha = \alpha_1 + \alpha_2$ and $\beta = \alpha_1 - k\alpha_2$. Observe that α and β are the components of the order flow that comes from speculators when $u = 1$ and $u = 0$, respectively. equilibrium. It follows from eq.(7.22) that:

$$Pr[\tilde{f} = f] = \frac{\theta}{2}\phi(f - \alpha) + \frac{\theta}{2}\phi(f + \alpha) + \frac{1-\theta}{2}\phi(f + \beta) + \frac{1-\theta}{2}\phi(f - \beta),$$

$$Pr[\tilde{f} = f | V = 1] = \theta\phi(f - \alpha) + \frac{1-\theta}{2}\phi(f + \beta) + \frac{1-\theta}{2}\phi(f - \beta),$$

$$Pr[\tilde{f} = f|V = 0] = \theta\phi(f + \alpha) + \frac{1-\theta}{2}\phi(f + \beta) + \frac{1-\theta}{2}\phi(f - \beta).$$

This yields the expression for the equilibrium price at date 2 when the price has not changed at date 1 since:

$$\begin{aligned} p_2^*(f_2) = Pr[V = 1|\tilde{f}_2 = f_2] &= \frac{Pr[\tilde{f}_2 = f_2|V = 1]}{2Pr[\tilde{f}_2 = f_2]} \\ &= \frac{\theta\phi(f_2 - \alpha) + \frac{1-\theta}{2}\phi(f_2 + \beta) + \frac{1-\theta}{2}\phi(f_2 - \beta)}{\frac{\theta}{2}\phi(f_2 - \alpha) + \frac{\theta}{2}\phi(f_2 + \alpha) + \frac{1-\theta}{2}\phi(f_2 + \beta) + \frac{1-\theta}{2}\phi(f_2 - \beta)} \times \frac{1}{2} \end{aligned}$$

We now show that the trading strategies of each type of speculator given in Parts 1 and 2 of Proposition 7.1 form an equilibrium. We consider 4 different cases.

Case 1: Γ_1 and Γ_2: $\alpha = \alpha_1 + \alpha_2 \leq 1$. In this case, the last column of the next table gives the possible realizations of the equilibrium price at date 2 for each realization of the order flow at date 2, $\tilde{f}_2$. The equilibrium price is identical for all realizations of the order flow that belongs to the interval I indicated in the first column. The probability distribution of possible realizations for the equilibrium price is given in the second column. The third column gives this distribution conditional on $V = 1$ and is useful for some calculations.

$f_2 \in I$	$Pr[\tilde{f}_2 \in I]$	$Pr[\tilde{f}_2 \in \|V = 1]$	p_2
$[-1-\alpha, -1-\beta)$	$\frac{\theta}{2}\frac{\alpha-\beta}{2}$	0	0
$[-1-\beta, -1+\beta)$	$\frac{1}{2}\beta$	$\frac{1-\theta}{2}\beta$	$\frac{1-\theta}{2}$
$[-1+\beta, -1+\alpha)$	$\frac{2-\theta}{2}\frac{\alpha-\beta}{2}$	$(1-\theta)\frac{\alpha-\beta}{2}$	$\frac{1-\theta}{2-\theta}$
$[-1+\alpha, 1-\alpha)$	$1-\alpha$	$1-\alpha$	$\frac{1}{2}$
$[1-\alpha, 1-\beta)$	$\frac{2-\theta}{2}\frac{\alpha-\beta}{2}$	$\frac{\alpha-\beta}{2}$	$\frac{1}{2-\theta}$
$[1-\|\beta\|, 1+\beta)$	$\frac{1}{2}\beta$	$\frac{1+\theta}{2}\beta$	$\frac{1+\theta}{2}$
$[1+\beta, 1+\alpha)]$	$\frac{\theta}{2}\frac{\alpha-\beta}{2}$	$\theta\frac{\alpha-\beta}{2}$	1

To derive speculators' optimal trading strategy, it is also useful to compute the probability distributions of the equilibrium price at date 2 conditional on (i) $s = 1$ and $u = 1$ and

(ii) $s = 1$ and $u = 0$. We obtain:

$f_2 \in I$	$Pr[\tilde{f}_2 \in I \mid s = 1, u = 1]$	$Pr[\tilde{f}_2 \in I \mid s = 1, u = 0]$	p_2
$[-1-\alpha, -1-\beta)$	0	0	0
$[-1-\beta, -1+\beta)$	0	0	$\frac{1-\theta}{2}$
$[-1+\beta, -1+\alpha)$	0	$\frac{\alpha-\beta}{2}$	$\frac{1-\theta}{2-\theta}$
$[-1+\alpha, 1-\alpha)$	$1-\alpha$	$1-\alpha$	$\frac{1}{2}$
$[1-\alpha, 1-\beta)$	$\frac{\alpha-\beta}{2}$	$\frac{\alpha-\beta}{2}$	$\frac{1}{2-\theta}$
$[1-\beta, 1+\beta)$	β	β	$\frac{1+\theta}{2}$
$[1+\beta, 1+\alpha)]$	$\frac{\alpha-\beta}{2}$	0	1

We deduce that as $\beta \geq 0$,

$$\begin{aligned} E[\tilde{p}_2 \mid s = 1, u = 1] &= \frac{\alpha-\beta}{2} \times 1 + \beta \times \frac{1+\theta}{2} + \frac{\alpha-\beta}{2} \times \frac{1}{2-\theta} + (1-\alpha) \times \frac{1}{2}, \\ &= \frac{1}{2} + \frac{1}{2(2-\theta)}\alpha + \left(\frac{\theta}{2} - \frac{1}{2(2-\theta)}\right)\beta. \end{aligned} \tag{7.23}$$

$$\begin{aligned} E[\tilde{p}_2 \mid s = 1, u = 0] &= \beta \times \frac{1+\theta}{2} + \frac{\alpha-\beta}{2} \times \frac{1}{2-\theta} + (1-\alpha) \times \frac{1}{2} + \frac{\alpha-\beta}{2} \times \frac{1-\theta}{2-\theta}, \\ &= \frac{1}{2} + \frac{\theta}{2}\beta. \end{aligned} \tag{7.24}$$

Thus, $E[\tilde{p}_2 \mid s = 1, u = 0] \geq \frac{1}{2}$ with a strict inequality if $\beta > 0$. Suppose first that $\alpha_1 > \alpha_2$. In this case, $k = 1$ and $\beta > 0$. It is therefore strictly optimal for speculators receiving the processed signal to sell the asset when they receive the signal $s = 1$ and $u = 0$. Thus, all speculators with the processed signal sell the asset when $s = 1$ and $u = 0$. If $\alpha_1 < \alpha_2$ then $k = \frac{\alpha_1}{\alpha_2}$ and $\beta = 0$. Each speculator with the processed signal is therefore indifferent between selling the asset or doing nothing in the state $s = 1$ and $u = 0$. Thus, selling k shares (or equivalently selling one share with probability k) is a best response when $\alpha_1 < \alpha_2$ $(k < 1)$ and $s = 1$ and $u = 0$.

Moreover, using eq.(7.23) and (7.24), we deduce that:

$$E[\tilde{p}_2|s=1] = \frac{1+\beta\theta}{2} + \frac{\theta(\alpha-\beta)}{2(2-\theta)}.$$

Thus, as $k \leq 1$ and $\alpha_1 + \alpha_2 \leq 1$, we deduce that:

$$E[\tilde{p}_2|s=1] < E[V|s=1] = \frac{1+\theta}{2}.$$

It follows that a speculator who receives the raw signal optimally buys the asset when $s = 1$ since he expects to trade at a price less than his estimate of the value of the asset.

The analysis is symmetric when $s = 0$ and is skipped for brevity. This proves that when $\alpha = \alpha_1 + \alpha_2 \leq 1$, speculators' strategies in Proposition 7.1 form an equilibrium.

Profits. Given his equilibrium trading strategy, the expected profit of a processed signal speculator when there is no change in the price at date 1 is:

$$\begin{aligned}
\pi_2^{2,nc}(\alpha_1,\alpha_2) &= \theta E[(V-p_2)|s=1,u=1] + (1-\theta)E[(p_2-V)|s=1,u=0]k \\
&= \theta\left(\frac{1}{2} - \frac{1}{2(2-\theta)}\alpha - \left(\frac{\theta}{2} - \frac{1}{2(2-\theta)}\right)\beta\right) + (1-\theta)\frac{\theta}{2}k\beta \\
&= \theta\left(\frac{1}{2} - \frac{1}{2(2-\theta)}(\alpha_1+\alpha_2) - \left(\frac{\theta}{2} - \frac{1}{2(2-\theta)}\right)\max\left[0,\alpha_1-\alpha_2\right]\right) \\
&\quad + (1-\theta)\frac{\theta}{2}\max\left[0,\alpha_1-\alpha_2\right]\min\left[\frac{\alpha_1}{\alpha_2},1\right]
\end{aligned}$$

Given his equilibrium trading strategy, the expected profit of speculator trading on the raw signal is:

$$\begin{aligned}
\pi_2^{1,nc}(\alpha_1,\alpha_2) &= \theta E[(V-p_2)|s=1,u=1] + (1-\theta)E[(V-p_2)|s=1,u=0] \\
&= \theta\left(\frac{1}{2} - \frac{1}{2(2-\theta)}\alpha - \left(\frac{\theta}{2} - \frac{1}{2(2-\theta)}\right)\beta\right) - (1-\theta)\frac{\theta}{2}\beta \\
&= \theta\left(\frac{1}{2} - \frac{1}{2(2-\theta)}(\alpha_1+\alpha_2) - \left(\frac{\theta}{2} - \frac{1}{2(2-\theta)}\right)\max\left[0,\alpha_1-\alpha_2\right]\right) \\
&\quad - (1-\theta)\frac{\theta}{2}\max\left[0,\alpha_1-\alpha_2\right]
\end{aligned}$$

If $\alpha_2 > \alpha_1$, then

$$\pi_2^{1,nc}(\alpha_1, \alpha_2) = \pi_2^{2,nc}(\alpha_1, \alpha_2) = \frac{\theta}{2}\left(1 - \frac{1}{2-\theta}(\alpha_1 + \alpha_2)\right)$$

If $\alpha_2 < \alpha_1$, then

$$\pi_2^{1,nc}(\alpha_1, \alpha_2) = \frac{\theta}{2}\left[1 - \alpha_1 - \frac{\theta}{2-\theta}\alpha_2\right] \ (> 0)$$

$$\pi_2^{2,nc}(\alpha_1, \alpha_2) = \frac{\theta}{2}\left[1 - (2\theta - 1)\alpha_1 - \underbrace{\left(\frac{2}{2-\theta} - (2\theta - 1)\right)}_{\geq 0}\alpha_2\right]$$

This proves Proposition 7.2 in cases Γ_1 and Γ_2.

Other cases. The analysis of other cases follow similar steps. We use the following shorthands for some notations:

1. P1 for $Pr[\tilde{f} \in I]$
2. P2 for $Pr[\tilde{f} \in I|V = 1]$
3. P3 for $Pr[\tilde{f} \in I|s = 1, u = 1]$
4. P4 for $Pr[\tilde{f} \in I|s = 1, u = 0]$ if $\beta \geq 0$

Case 2: Γ_3 and Γ_4: Assume $1 \leq \alpha \leq 2$ and (i) $\alpha_2 \geq \alpha_1$ or (ii) $\alpha_2 < \alpha_1$ and $\alpha_1 < 1$. As in the previous case, the next table gives the probability distribution of the equilibrium price by considering various intervals (I) for the realization of the order flow at date 2, $\tilde{f}_2$. It also provides the distribution of the equilibrium price at date 2 conditional on various events in Columns 2 to 5. For instance, if $f_2 \in [-1 - \alpha, -1 - \beta)$, the equilibrium price at date 2 is zero. This happens with probability $\frac{\theta}{2}\frac{\alpha - |\beta|}{2}$ unconditionally and with probability zero conditional on $V = 1$ ($P2$), or $s = u = 1$ ($P3$) or $s = 1$ and $u = 0$ ($P4$).

$f_2 \in I$	$P1$	$P2$	$P3$	$P4$	$\tilde{p}_2$
$[-1-\alpha, -1-\beta)$	$\frac{\theta}{2}\frac{\alpha-\beta}{2}$	0	0	0	0
$[-1-\beta, -1+\beta)$	$\frac{1}{2}\beta$	$\frac{1-\theta}{2}\beta$	0	0	$\frac{1-\theta}{2}$
$[-1+\beta, 1-\alpha)$	$\frac{2-\theta}{2}\frac{2-(\alpha+\beta)}{2}$	$(1-\theta)\frac{2-(\alpha+\beta)}{2}$	0	$\frac{2-(\alpha+\beta)}{2}$	$\frac{1-\theta}{2-\theta}$
$[1-\alpha, -1+\alpha)$	$(1-\theta)(\alpha-1)$	$(1-\theta)(\alpha-1)$	0	$\alpha-1$	$\frac{1}{2}$
$[-1+\alpha, 1-\beta)$	$\frac{2-\theta}{2}\frac{2-(\alpha+\beta)}{2}$	$\frac{2-(\alpha+\beta)}{2}$	$\frac{2-(\alpha+\beta)}{2}$	$\frac{2-(\alpha+\beta)}{2}$	$\frac{1}{2-\theta}$
$[1-\beta, 1+\beta)$	$\frac{1}{2}\beta$	$\frac{1+\theta}{2}\beta$	β	β	$\frac{1+\theta}{2}$
$[1+\beta, 1+\alpha]$	$\frac{\theta}{2}\frac{\alpha-\beta}{2}$	$\theta\frac{\alpha-\beta}{2}$	$\frac{\alpha-\beta}{2}$	0	1

As $\beta \geq 0$,

$$\begin{aligned} E[\tilde{p}_2 | s=1, u=1] &= \frac{\alpha-\beta}{2} \times 1 + \beta \times \frac{1+\theta}{2} + \frac{2-(\alpha+\beta)}{2} \times \frac{1}{2-\theta} \\ &= \frac{1}{2-\theta} + \frac{1-\theta}{2(2-\theta)}\alpha - \frac{(1-\theta)^2}{2(2-\theta)}\beta \end{aligned}$$

$$\begin{aligned} E[\tilde{p}_2 | s=1, u=0] &= \beta \times \frac{1+\theta}{2} + \frac{2-(\alpha+\beta)}{2} \times \frac{1}{2-\theta} + (\alpha-1) \times \frac{1}{2} + \frac{2-(\alpha+\beta)}{2} \times \frac{1-\theta}{2-\theta} \\ &= \frac{1}{2} + \frac{\theta}{2}\beta \end{aligned}$$

Then, proceeding exactly as in Case 1, one can show that speculators' strategies in Proposition 7.1 form an equilibrium in the case Γ_3 and Γ_4.

Speculators' expected profits. Given his equilibrium trading strategy, the expected profit of a processed information speculator is:

$$\pi_2^{2,nc}(\alpha_1, \alpha_2) = \theta E[(V - p_2) | s=1, u=1] + (1-\theta) E[(p_2 - V) | s=1, u=0] k$$

$$= \theta \left(\frac{1-\theta}{2-\theta} - \frac{1-\theta}{2(2-\theta)}\alpha + \frac{(1-\theta)^2}{2(2-\theta)}\beta \right) + (1-\theta)\frac{\theta}{2}k\beta$$

$$= \theta \left(\frac{1-\theta}{2-\theta} - \frac{1-\theta}{2(2-\theta)}\alpha + \frac{(1-\theta)^2}{2(2-\theta)} \max[0, \alpha_1 - \alpha_2] \right) + (1-\theta)\frac{\theta}{2} \max[0, \alpha_1 - \alpha_2] \min\left[\frac{\alpha_1}{\alpha_2}, 1\right]$$

Given his equilibrium trading strategy, the expected profit of a raw information speculator is:

$$\begin{aligned}
\pi_2^{1,nc}(\alpha_1,\alpha_2) &= \theta E[(V-p_2)|s=1,u=1] + (1-\theta)E[(V-p_2)|s=1,u=0] \\
&= \theta\left(\frac{1-\theta}{2-\theta} - \frac{1-\theta}{2(2-\theta)}\alpha + \frac{(1-\theta)^2}{2(2-\theta)}\beta\right) - (1-\theta)\frac{\theta}{2}\beta \\
&= \theta\left(\frac{1-\theta}{2-\theta} - \frac{1-\theta}{2(2-\theta)}\alpha + \frac{(1-\theta)^2}{2(2-\theta)}\max\left[0,\alpha_1-\alpha_2\right]\right) - (1-\theta)\frac{\theta}{2}\max\left[0,\alpha_1-\alpha_2\right]
\end{aligned}$$

When $\alpha_2 \geq \alpha_1$, $\beta = 0$, which implies that $\alpha = \alpha_1 + \alpha_2 \leq 2$. In this case, we deduce from the previous expressions for the expected profits that:

$$\pi_2^{1,nc}(\alpha_1,\alpha_2) = \pi_2^{2,nc}(\alpha_1,\alpha_2) = \frac{\theta(1-\theta)}{2(2-\theta)}\left(2-\alpha_1-\alpha_2\right)$$

When $\alpha_2 < \alpha_1$, $\beta = \alpha_2 - \alpha_1$, which implies that $\alpha + \beta \leq 2$ and therefeore $\alpha_1 \leq 1$. In this case, we deduce from the previous expressions for the expected profits that:

$$\pi_2^{1,nc}(\alpha_1,\alpha_2) = \frac{\theta(1-\theta)}{2-\theta}\left(1-\alpha_1\right)\ \ ,(\geq 0)$$

$$\pi_2^{2,nc}(\alpha_1,\alpha_2) = \frac{\theta(1-\theta)}{2(2-\theta)}\left(2+2(1-\theta)\alpha_1-2(2-\theta)\alpha_2\right)$$

This proves Proposition 7.2 in cases Γ_3 and Γ_4.

Case 3: Γ_5**:** $\alpha_2 \leq 1$ **and** $\alpha_1 \geq 1$**.** We distinguish two subcases in this case.

Case 3.1. In the first subcase, we have $\alpha_2 \geq \alpha_1 - 1$. The probability distribution of the order flow at date 2, $\tilde{f}_2$, and the equilibrium price p_2 for each possible realization of

$\tilde{f}_2$ are then as follows.

I	$P1$	$P2$	$P3$	$P4$	$\tilde{p}_2$
$[-1-\alpha, -1-\beta)$	$\frac{\theta}{2}\frac{\alpha-\beta}{2}$	0	0	0	0
$[-1-\beta, 1-\alpha)$	$\frac{1}{2}\frac{2-(\alpha-\beta)}{2}$	$\frac{1-\theta}{2}\frac{2-(\alpha-\beta)}{2}$	0	0	$\frac{1-\theta}{2}$
$[1-\alpha, -1+\beta)$	$\frac{1-\theta}{2}\frac{\alpha+\beta-2}{2}$	$\frac{1-\theta}{2}\frac{\alpha+\beta-2}{2}$	0	0	$\frac{1}{2}$
$[-1+\beta, 1-\beta)$	$(1-\theta)(1-\beta)$	$(1-\theta)(1-\beta)$	0	$1-\beta$	$\frac{1}{2}$
$[1-\beta, -1+\alpha)$	$\frac{1-\theta}{2}\frac{\alpha+\beta-2}{2}$	$\frac{1-\theta}{2}\frac{\alpha+\beta-2}{2}$	0	$\frac{\alpha+\beta-2}{2}$	$\frac{1}{2}$
$[-1+\alpha, 1+\beta)$	$\frac{1}{2}\frac{2-(\alpha-\beta)}{2}$	$\frac{1+\theta}{2}\frac{2-(\alpha-\beta)}{2}$	$\frac{2-(\alpha-\beta)}{2}$	$\frac{2-(\alpha-\beta)}{2}$	$\frac{1+\theta}{2}$
$[1+\beta, 1+\alpha]$	$\frac{\theta}{2}\frac{\alpha-\beta}{2}$	$\theta\frac{\alpha-\beta}{2}$	$\frac{\alpha-\beta}{2}$	0	1

We deduce that:

$$\begin{aligned} E[\tilde{p}_2|s=1, u=1] &= \frac{\alpha-\beta}{2} \times 1 + \frac{2-(\alpha-\beta)}{2} \times \frac{1+\theta}{2} \\ &= \frac{1+\theta}{2} + \frac{1-\theta}{4}\alpha - \frac{1-\theta}{4}\beta \end{aligned}$$

$$\begin{aligned} E[\tilde{p}_2|s=1, u=0] &= \frac{2-(\alpha-\beta)}{2} \times \frac{1+\theta}{2} + \frac{\alpha+\beta-2}{2} \times \frac{1}{2} + (1-\beta) \times \frac{1}{2} \\ &= \frac{1+\theta}{2} - \frac{\theta}{4}(\alpha-\beta) \end{aligned}$$

Then, proceeding exactly as in Case 1, one can show that speculators' strategies in Proposition 7.1 form an equilibrium in the case Γ_5 when $\alpha_2 \geq \alpha_1 - 1$.

Speculators' expected profits. As $\alpha_2 \geq \alpha_1 - 1$, given his equilibrium trading strategy, the expected profit of a processed information speculator is then:

$$
\begin{aligned}
\pi_2^{2,nc}(\alpha_1,\alpha_2) &= \theta\left(\frac{1-\theta}{2} - \frac{1-\theta}{4}\alpha + \frac{1-\theta}{4}(\alpha_1-\alpha_2)\right) \\
&+ (1-\theta)\left(\frac{\theta}{2} - \frac{\theta}{4}\alpha + \frac{\theta}{4}(\alpha_1-\alpha_2)\right) \\
&= \theta(1-\theta)(1-\alpha_2)
\end{aligned}
$$

The expected profit of a raw information speculator is

$$
\begin{aligned}
\pi_2^{1,nc}(\alpha_1,\alpha_2) &= \theta\left(\frac{1-\theta}{2} - \frac{1-\theta}{4}\alpha + \frac{1-\theta}{4}(\alpha_1-\alpha_2)\right) \\
&- (1-\theta)\left(\frac{\theta}{2} - \frac{\theta}{4}\alpha + \frac{\theta}{4}(\alpha_1-\alpha_2)\right) \\
&= 0
\end{aligned}
$$

This proves Proposition 7.2 in in the case Γ_5 when $\alpha_2 \geq \alpha_1 - 1$.

Case 3.2. In the second subcase, we have $\alpha_2 < \alpha_1 - 1$. The probability distribution of the order flow at date 2, $\tilde{f}_2$, and the equilibrium price p_2 for each possible realization of $\tilde{f}_2$ are then as follows.

I	$P1$	$P2$	$P3$	$P4$	$\tilde{p}_2$
$[-1-\alpha, -1-\beta)$	$\frac{\theta}{2}\frac{\alpha-\beta}{2}$	0	0	0	0
$[-1-\beta, 1-\alpha)$	$\frac{1}{2}\frac{2-(\alpha-\beta)}{2}$	$\frac{1-\theta}{2}\frac{2-(\alpha-\beta)}{2}$	0	0	$\frac{1-\theta}{2}$
$[1-\alpha, 1-\beta)$	$\frac{1-\theta}{2}\frac{\alpha-\beta}{2}$	$\frac{1-\theta}{2}\frac{2-(\alpha-\beta)}{2}$	0	0	$\frac{1}{2}$
$[1-\beta, -1+\beta)$	0	0	0	0	$\frac{1}{2}$
$[-1+\beta, -1+\alpha)$	$\frac{1-\theta}{2}\frac{\alpha-\beta}{2}$	$\frac{1-\theta}{2}\frac{\alpha-\beta}{2}$	0	$\frac{\alpha-\beta}{2}$	$\frac{1}{2}$
$[-1+\alpha, 1+\beta)$	$\frac{1}{2}\frac{2-(\alpha-\beta)}{2}$	$\frac{1+\theta}{2}\frac{2-(\alpha-\beta)}{2}$	$\frac{2-(\alpha-\beta)}{2}$	$\frac{2-(\alpha-\beta)}{2}$	$\frac{1+\theta}{2}$
$[1+\beta, 1+\alpha]$	$\frac{\theta}{2}\frac{\alpha-\beta}{2}$	$\theta\frac{\alpha-\beta}{2}$	$\frac{\alpha-\beta}{2}$	0	1

We deduce that:

$$
\begin{aligned}
E[\tilde{p}_2|s=1,u=1] &= \frac{\alpha-\beta}{2}\times 1+\frac{2-(\alpha-\beta)}{2}\times\frac{1+\theta}{2}\\
&= \frac{1+\theta}{2}+\frac{1-\theta}{4}\alpha-\frac{1-\theta}{4}\beta
\end{aligned}
$$

$$
\begin{aligned}
E[\tilde{p}_2|s=1,u=0] &= \frac{2-(\alpha-\beta)}{2}\times\frac{1+\theta}{2}+\frac{\alpha-\beta}{2}\times\frac{1}{2}\\
&= \frac{1+\theta}{2}-\frac{\theta}{4}(\alpha-\beta).
\end{aligned}
$$

These are exactly the same expressions as in case 3.1 and so the rest of the proof is identical. We skip it brevity.

As explained in the text, this case cannot arise when the market for information is in equilibrium since some speculators make zero expected profits. Thus, for brevity, we do not compte price informativeness in this case.

Case 4: Γ_6: $\alpha=\alpha_1+\alpha_2\geq 2$ and $\alpha_2\geq 1$. Remember that β is the order flow from all speculators in equilibrium. When $\alpha_2\geq 1$ $\beta=0$. In this case, the market maker can perfectly infer the realization of (s,u) from the order flow at date 2. Indeed, when $u=0$, the support of the order flow is $[-1-\beta,1-\beta]\cup[-1+\beta,1+\beta]$, i.e., $[-1,1]$. In contrast, when $u=s=1$, this support is $[-1+\alpha,1+\alpha]$ while when $u=s=0$ the support for the order flow is $[-1-\alpha,1-\alpha]$. These three supports never overlap when $\alpha\geq 2$. Thus, the order flow is fully revealing in this case and speculators make zero expected profits.[2]

Proof of Proposition 7.3. First, the pricing error date 2 conditional on $p_1=1/2$ can be rewritten as

$$
\begin{aligned}
E[(V-p_2)^2|p_1=1/2] &= E[V(V-p_2)|p_1=1/2]+E\,\underbrace{E[p_2(V-p_2)|p_2,p_1=1/2]}_{=0}|p_1=1/2]\\
&= \frac{1}{2}E[(2V-1)(V-p_2)|p_1=1/2]+\frac{1}{2}E\,\underbrace{[V-p_2|p_1=1/2]}_{=0}
\end{aligned}
$$

[2]In this case, speculators are indifferent between trading or not and therefore there are other equilibria. However, all equilibria are such that speculators make zero expected profits. As they never arise when information production is costly, we ignore them. In all other cases, the equilibrium is unique.

Now consider the trading strategy of speculators at date 2 conditional on $p_1 = 1/2$:

$$x_1 = 2s - 1 = u(2V - 1) + (1 - u)(2\varepsilon - 1) = 2V - 1 + 2(1 - u)(\varepsilon - V)$$

$$x_2 = u(2V - 1) - k(1 - u)(2\varepsilon - 1) = 2V - 1 + 2(1 - u)(-k\varepsilon - V)$$

Then one can write,

$$\begin{aligned}
&\pi_2^{1,nc}(\alpha_1, \alpha_2) + \pi_2^{2,nc}(\alpha_1, \alpha_2) \\
=&E[(x_1 + x_2)(V - p_2)|p_1 = 1/2] \\
=&2E[(2V - 1)(V - p_2)|p_1 = 1/2] + 2E[(1 - u)((1 - k)\varepsilon - 2V)(V - p_2)|p_1 = 1/2] \\
=&2E[(2V - 1)(V - p_2)|p_1 = 1/2] + 2(1 - \theta)E[(1 - k)\varepsilon(V - p_2)|p_1 = 1/2, u = 0] \\
&- 4(1 - \theta)E[V(V - p_2)|p_1 = 1/2, u = 0]
\end{aligned}$$

Conditional on $u = 0$, V and p_2 are independent. Therefore

$$\begin{aligned}
E[V(V - p_2)|p_1 = 1/2, u = 0] &= E[V^2|p_1 = 1/2, u = 0] - E[V|p_1 = 1/2, u = 0]E[p_2|p_1 = 1/2, u = 0] \\
&= \frac{1}{2} - \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}
\end{aligned}$$

In equilibrium, we have $k = \min[\alpha_1/\alpha_2, 1$. When $k = 1$, obviously we have,

$$E[(1 - k)\varepsilon(V - p_2)|p_1 = 1/2, u = 0] = 0$$

When $k = \alpha_1/\alpha_2$, we also have $\beta = 0$. Then, in this case, conditional on $u = 0$, the aggregate is the liquidity trade order flow is $f_2 = l_2$, and then p_2 is independent of ε. Therefore

$$\begin{aligned}
&E[(1 - k)\varepsilon(V - p_2)|p_1 = 1/2, u = 0] \\
=&(1 - k)E[\varepsilon|p_1 = 1/2, u = 0] \left(E[V|p_1 = 1/2, u = 0] - E[p_2|p_1 = 1/2, u = 0]\right) \\
=&(1 - k)\frac{1}{2}\left(\frac{1}{2} - \frac{1}{2}\right) = 0
\end{aligned}$$

Overall, we obtain,

$$\pi_2^{1,nc}(\alpha_1, \alpha_2) + \pi_2^{2,nc}(\alpha_1, \alpha_2) = 4E[(V - p_2)^2 | p_1 = 1/2] - (1 - \theta)$$

and thus

$$E[(V - p_2)^2 | p_1 = 1/2] = \frac{1-\theta}{4} + \frac{1}{4}\left(\pi_2^{1,nc}(\alpha_1, \alpha_2) + \pi_2^{2,nc}(\alpha_1, \alpha_2)\right)$$

When the equilibrium price at date 1 reveals the raw signal s, the equilibrium at date 2 is as in the baseline model (since speculators who receive the raw signal do not trade in this case). Hence, using the results of the baseline model, we obtain that:

$$E[(V - p_2) | p_1 \neq 1/2] = (1 - \alpha_2)\frac{(1+\theta)(1-\theta)}{4} + \alpha_2\frac{1-\theta}{4} = \frac{1-\theta}{4} + \frac{1}{4}\pi_2^c(\alpha_2)*$$

Consequently,

$$E[(V - p_2)^2] = \frac{1-\theta}{4} + \frac{\alpha_1}{4}\pi_2^c(\alpha_2) + \frac{1-\alpha_1}{4}\left(\pi_2^{1,nc}(\alpha_1, \alpha_2) + \pi_2^{2,nc}(\alpha_1, \alpha_2)\right)$$

And the results for informativeness follows.

Proof of Corollary 7.1. As in the baseline version of the paper (see proof of Corollary 4 in the paper), we have:

$$Cov(x_1, x_2) = \mathsf{E}[x_1 x_2] = \frac{1}{2}\mathsf{E}[x_2 | s = 1] - \frac{1}{2}\mathsf{E}[x_2 | s = 0].$$

Thus, denoting the event $\{s = 1, u = 0\}$ by I_1 and the event $\{s = 0, u = 0\}$ by I_0, we obtain:

$$\begin{aligned} Cov(x_1, x_2) = &\frac{\theta}{2}\mathsf{E}\left[x_2 | V = 1, u = 1\right] + \frac{(1-\theta)}{2}(\alpha_1^e \mathsf{E}\left[x_2 | I_1, p_1 = \frac{1+\theta}{2}\right] + (1-\alpha_1^e)\mathsf{E}\left[x_2 | I_1, p_1 = \frac{1}{2}\right]) \\ &- \frac{\theta}{2}\mathsf{E}\left[x_2 | V = 0, u = 1\right] - \frac{(1-\theta)}{2}(\alpha_1^e \mathsf{E}\left[x_2 | I_0, p_1 = \frac{1-\theta}{2}\right] + (1-\alpha_1^e)\mathsf{E}\left[x_2 | I_0, p_1 = \frac{1}{2}\right] \\ &= \theta - (1-\theta)[\alpha_1^e + (1-\alpha_1^e)k] \end{aligned}$$

where $k = \min\{1, \frac{\alpha_1^e}{\alpha_2^e}\}$.

Proof of Corollary 7.2. As $E(x_2) = 0$, we have:

$$Cov(r_1 x_2) = E(r_1 x_2) = \alpha_1^e E(r_1 x_2 \mid r_1 \neq 0).$$

When $r_1 \neq 0$, the equilibrium at date 2 is as in the baseline model. Thus, $E(r_1 x_2 \mid r_1 \neq 0)$, and therefore $Cov(r_1 x_2)$ has the same expression as in the baseline model.

Proof of Corollary 7.3.

We first compute the expression for $Cov(x_1, r_2)$ given in eq.(27). As $E[x_1] = 0$,

$$Cov(x_1, r_2) = E[(p_2^* - p_1^*)x_1] - E[p_2^* - p_1^*]E[x_1] = E[(p_2^* - p_1^*)x_1].$$

Now:

$$\begin{aligned} E[p_1^* x_1] &= \frac{1}{2}(E[p_1^* x_1 | s = 1] + E[p_1^* x_1 | s = 0]) = \frac{1}{2}(E[p_1^* | s = 1] - E[p_1 | s = 0]) \\ &= \frac{1}{2}\left((1 - \alpha_1^e)\frac{1}{2} + \alpha_1^e \frac{1+\theta}{2}\right) - \frac{1}{2}\left((1 - \alpha_1^e)\frac{1}{2} + \alpha_1^e \frac{1-\theta}{2}\right) \\ &= \frac{\theta \alpha_1^e}{2}. \end{aligned}$$

Similarly , we have that:

$$E[p_2^* x_1] = \frac{1}{2}(E[p_2^* | s = 1] - E[p_2^* | s = 0]).$$

We first compute $E[p_2^* | s = 1]$. We have:

$$E[p_2^* | s = 1] = \alpha_1^e E\left[p_2 \middle| s = 1, p_1^* = \frac{1+\theta}{2}\right] + (1 - \alpha_1^e) E\left[p_2^* \middle| s = 1, p_1^* = \frac{1}{2}\right].$$

The event $p_1 = \frac{1+\theta}{2}$ implies that $s = 1$. Thus,

$$E\left[p_2^* \middle| s = 1, p_1^* = \frac{1+\theta}{2}\right] = E\left[p_2 \middle| p_1^* = \frac{1+\theta}{2}\right] = p_1^* = \frac{1+\theta}{2}.$$

Similarly,

$$E[p_2^*|s=0] = \alpha_1^e E\left[p_2 \middle| s=0, p_1^* = \frac{1-\theta}{2}\right] + (1-\alpha_1^e) E\left[p_2^* \middle| s=0, p_1^* = \frac{1}{2}\right].$$

The event $p_1 = \frac{1-\theta}{2}$ implies that $s = 0$. Thus,

$$E\left[p_2^* \middle| s=0, p_1^* = \frac{1-\theta}{2}\right] = E\left[p_2 \middle| p_1^* = \frac{1-\theta}{2}\right] = p_1^* = \frac{1-\theta}{2},$$

Hence,

$$\begin{aligned} E[(p_2^* - p_1^*)x_1] &= \frac{1}{2}(1-\alpha_1^e)\left(E\left[p_2^* \middle| s=1, p_1^* = \frac{1}{2}\right] - E\left[p_2^* \middle| s=0, p_1^* = \frac{1}{2}\right]\right) \\ &= \frac{1}{2}(1-\alpha_1^e)\left(2E\left[p_2^* \middle| s=1, p_1^* = \frac{1}{2}\right] - 1\right) \\ &= \frac{1}{2}(1-\alpha_1^e)\left(2E\left[V \middle| s=1, p_1^* = \frac{1}{2}\right] + 2E\left[p_2^* - V \middle| s=1, p_1^* = \frac{1}{2}\right] - 1\right) \\ &= \frac{1}{2}(1-\alpha_1^e)\left(\theta - 2\pi_2^{1,nc}(\alpha_1^e, \alpha_2^e)\right). \end{aligned}$$

8 Optimal timing of their trades by raw information speculators

In the baseline model, we assume (i) that speculators who receive the raw signal trade at date 1 and (ii) that they have a maximum trading capacity of one share at this date. In this section, we relax the first assumption while maintaining the assumption that speculators's total trade size cannot exceed more than one share. We assume that speculators buy either the raw or the processed signal. This is without loss of generality since speculators make zero expected profit net of the fees paid for these signals. Thus, they are indifferent between buying none, one, or two signals. As a result, one can assume that the population of speculators buying the raw signal is distinct from that buying the processed signal without affecting the results.

Consider a speculator i receiving the raw signal first. Let $x_{1i}(s)$ be his signed order at date 1 and let $x_{2i}(s, p_1)$ be his optimal signed order at date 2, when he receives the raw signal s and the price realized at the end of date 1 is p_1. Speculator i's trade at date 2, can depend on the realization of the price at date 1 since this price is observed by all market participants between dates 1 and 2. As the speculator's trade size cannot exceed one share overall, we have:

$$|x_{1i}(s)| + |x_{2i}(s, p_1)| \leq 1. \tag{8.1}$$

At date 1, the aggregate demand from speculators receiving the raw signal is:

$$q_1 = \int_0^{\alpha_1} x_{1i}(s)di.$$

For given trading strategies of speculators receiving the raw signal, the equilibrium at date 1 is identical to that described in the model, except that q_1 plays the role of α_1[3]. Thus, with probability q_1, the price at date 1 fully reflects the raw signal ($p_1 = E(v \mid s)$) and with probability $(1 - q_1)$, the price at date 1 does not change. In the first case, it cannot be optimal for a speculator receiving the raw signal to trade at date 2 since his estimate of the payoff of the asset after the first trading round is identical to that of dealers at date 2. Thus, in any equilibrium:

$$x_2^*(s, p_1) = 0 \quad if \quad p_1 \neq 1/2.$$

We now show that it is optimal for a raw speculator to trade only at date 1, i.e., $x_2^*(s, 1/2) = 0$ as well. The next lemma is useful to establish this result. In this lemma, we denote by p_2^{nc}, the equilibrium price at date 2 *conditional on no change* in the price at date 1 (i.e., if $p_1 = 1/2$).

Lemma 1. *In any equilibria at date 2 (i.e., whether or not some speculators trading on the raw signal choose to trade at date 2), we have:*

$$E[p_2^{nc}|s = 1] > 1/2 > E[p_2^{nc}|s = 0].$$

This result is intuitive. It says that, in equilibrium, traders who observe the raw

[3] When $x_{1i}(s) = 1$ for all i then $q_1 = \alpha_1$, which is the case considered in the baseline model.

signal at date 1 must expect the price at date 2 to be on average strictly greater (resp., smaller) than the price at date 1 if (1) they have received a positive (resp., negative) signal, $s = 1$, (resp., $s = 0$) and (2) the price at date 1 has not changed. The reason is that, conditional on no change in the price at date 1, the optimal trading strategy of the speculators receiving the processed signal is such that they trade in the same direction as the traders receiving the raw signal. Thus, the latter expects the former to submit buy (sell) orders when they submit buy (sell) orders and therefore the price at date 2 to be on average higher than at date 1 when $s = 1$.

Intuitively, this means that the expected return on the trade at date 2 is strictly smaller than the expected return on the trade at date 1 for a speculator who receives the raw signal. Thus, he should optimally allocate all his trading capacity to date 1. To establish this point formally, consider the expected profit of a speculator who receives the raw signal, s. This expected profit is:

$$(1 - q_1)[x_1(s) \times (E(v \mid s) - \frac{1}{2}) + x_2(s, 1/2) \times (E(v \mid s) - E[p_2^{nc}|s = 1])],$$

since if $p_1 \neq 1/2$, the speculator optimally does not trade. Lemma 1 implies that

$$(E(v \mid s) - E[p_2^{nc}|s = 1])] < (E(v \mid s) - \frac{1}{2}).$$

. It follows that under the constraint (8.1), a speculator who receives the raw signal maximizes his expected profit with $x_2^*(s, 1/2) = 0$ and $x_1^*(s) = 1$.[4] Thus, under constraint (8.1), there cannot be an equilibrium in which $x_2^*(s, 1/2) \neq 0$. Thus, in equilibrium $x_2^*(s, 1/2) = 0$.

Now consider a trader who receives the processed signal. At date 1, this trader has no information. hence, he cannot profitably trade with the market maker. Thus, trading only at date 2 is a dominant strategy for speculator who buys the processed signal.

Proofs.

[4]As there is a continuum of speculators, the order of each speculator trading on the raw signal is too small to affect prices at date 1 or 2. Thus, in choosing his optimal trading strategy, the speculator takes prices as given (as in the baseline model).

Proof of Lemma 1. Let $q_2(s)^* = \int_0^{\alpha_1} x^*_{2i}(s, 1/2)di$ be the demand of speculators receiving the raw signal at date 2 in a given equilibrium. Proceeding as in the proof of Proposition 7.1, it is possible to show that if $x^*_{2i}(s, 1/2) \neq 0$ then it is strictly positive when $s = 1$ and strictly negative when $s = 0$ (raw information speculators buy when they receive good news and sell when they receive bad news. Thus, $q^*_2(1) \geq 0$ and $q^*_2(0) \leq 0$ (with equalities if no speculator receiving the raw signal trade at date 2). Then, we can proceed exactly as in the proof of Proposition 7.1 in Section 7 to derive the equilibrium trading strategy of the processed information speculators with q_2 playing the role of α_1. The lemma then follows from the fact in all cases considered in this proof, the expected price of the asset at date 2 is strictly larger than 1/2 when $s = 1$ and strictly smaller than 1/2 when $s = 0$ (see, for instance, eq.(7.23) and eq.(7.24) in case 1 of the proof of Proposition 7.1).