Main Appendix of

IS THE ACTIVE FUND MANAGEMENT INDUSTRY CONCENTRATED ENOUGH?

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Latest Revision September 13, 2018

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Proof of Managers' Maximization Problems Equivalence: Profits and Expected Net Alpha

We will prove that when managers maximize fund expected net alphas they maximize profits and that they must do so in order to survive (that is, have wealth to manage and be solvent). [1] We also show that this maximization leads to a unique Nash equilibrium.

First, we establish that all managers offer the same level of fund expected net alpha. This is the case in PS, as well, and the rationale here is the same as there: managers who offer expected net alpha that is lower than the highest offered by some other manager attract no investments, as diversification benefits are irrelevant to risk-neutral investors and negligible to risk-averse ones, and are, thus, out of the AFMI.

Next, we will show that AFMI managers' competition drives the AFMI (unique) level of expected net alpha to be the highest possible one, where managers are still solvent; that is, where managers charge break-even fees.

Suppose that managers choose profit maximizing optimal effort and fees to set AFMI funds expected net alpha to be $\bar{\alpha}$. Without loss of generality, we assume that $\bar{\alpha}$ is between zero and the highest expected net alpha that allows solvency. We will show that, in equilibrium, $\bar{\alpha}$ is the maximum fund expected net alpha that mangers can produce (while staying solvent). Substituting $\bar{\alpha}$ into Equation (7) (our "state" equation that links effort, fees, and fund expected net alphas), yields

$$f_i^* = \hat{a} - \hat{b}\frac{S}{W} + A(e_i^*;H) - \bar{\alpha}\,. \tag{A1}$$

Denote the *profit rate* of manager i, as pro_i, $pro_i \triangleq f_i^* - C^i(e_i^*, s_i; H)$. Substituting Equation (A1) into pro_i, then substituting Equation (18) for the cost function, recalling that $c_{0,i} = c_0 \ \ \forall i$, and $c_{2,i} = c_2 \ \ \forall i$, and rearranging, we have

$$\bar{\alpha} = \hat{a} - \hat{b}\frac{S}{W} + A(e_i^*;H) - pro_i - c_0 - c_{1,i}s_i - c_2(e_i^*;H)\,.^{2} \tag{A2}$$

Now, consider manager i's total dollar profit function (size in dollars times the per dollar profit rate):

$$s_i\, pro_i = s_i[\, f_i^* - c_0 - c_{1,i}s_i - c_2(e_i^*;H)], \quad \forall i\,. \tag{A3}$$

---

[1] By our model assumptions, insolvent managers are out of the AFMI.

[2] We note that as all managers produce the same level of fund expected net alphas and effort levels [see Proposition RA1 (this is because the optimal effort levels are determined by alpha production functions and effort cost functions $A(e_i;H)$ and $c_2(e_i;H)$, respectively, which are the same across funds), Equation (A2) implies a relation, $pro_i + c_{1,i}s_i = pro_j + c_{1,j}s_j, \quad \forall i, j$.

For a given (non-zero) $s_i pro_i$, the first-order condition for optimal fund size is

$$s_i^* = \frac{f_i^* - c_0 - c_2(e_i^*; H)}{2c_{1,i}}. \tag{A4}$$

If we express the numerator of the last equation in terms of the current pro_i, we can rewrite the profit maximizing fund size, s_i^*, as

$$s_i^* = \frac{pro_i}{2c_{1,i}} + \frac{s_i}{2}. \tag{A5}$$

Equation (A5) relates the optimal fund size and its current size.[3] It shows that profit rates that are too high (with respect to profit maximizing profit rates) are associated with fund sizes that are too small. Thus, it might be possible that some manager j, $j \neq i$, increases her (dollar) profits by increasing (her) fund expected net alpha, reducing profit rates and increasing (her fund) size. As manager i does not observe other managers' cost functions,[4] she must consider the above possibility [to avoid losing (all) the wealth she manages].

We now demonstrate that the possible scenario described above indeed occurs. We will analyze a simple game between manager i and any other manager, denoted "$-i$". The actions of this game are to either maintain expected net alpha or improve it by an infinitesimal amount. Throughout, we assume that the diversification benefits of investing in both manager i and manager $-i$ are negligible. The payoffs are the profits of the two managers.

If manager i improves her fund expected net alpha infinitesimally and manager $-i$ does not follow, then manager i's profit change by an infinitesimal amount, say η_i, and manager $-i$ receives no investments and earns no profits. If, on the other hand, manager i does not follow manager $-i$ when she increases her fund's expected net alpha infinitesimally, then manager $-i$ profits change by η_{-i}, and manager i receives no investments and earns no profits. Suppose that manager i believes that manager $-i$'s strategy is to improve her fund expected net alpha, $\bar{\alpha}$ with (nontrivial) probability p and to maintain $\bar{\alpha}$ with probability $1 - p$. Suppose that manager i's strategy is to improve her fund expected net alpha with probability θ and maintain $\bar{\alpha}$ with probability $1 - \theta$.

The payoffs of such a game are illustrated in the following table, with the row (column) representing manager i's ($-i$'s) action, and with manager i's ($-i$'s) payoffs in the first (second)

[3]Note that Equation (A5) is defined only when profits are non-zero. Otherwise, the size derivative of Equation (A3) is not defined.

[4] If cost functions were common knowledge, each manager could have calculated the AFMI equilibrium independently.

figures in the brackets.[5] For simplicity and brevity, we do not introduce new notation to differentiate the infinitesimal profit changes when one or two players move. We use η_i and η_{-i} in both cases.

		Maintain $\bar{\alpha}$	Improve Infinitesimally
		$1-p$	p
Maintain $\bar{\alpha}$	$1-\theta$	$(pro_i s_i,\ \ pro_{-i} s_{-i})$	$(0,\ \ pro_{-i} s_{-i} + \eta_{-i})$
Improve Infinitesimally	θ	$(pro_i s_i + \eta_i,\ \ 0)$	$(pro_i s_i + \eta_i,\ pro_{-i} s_{-i} + \eta_{-i})$

We will show that in this game, manager i optimally chooses $\theta = 1$, until reaching the highest level of fund expected net alphas. (This is the break-even/zero-profits point, beyond which the manager becomes insolvent.) As manager i is a generic manager, this implies that all mangers do that. We will also show that once managers reach the point of producing the highest level of fund expected net alphas, they are in (a Nash) equilibrium.

The expected payoffs of manager i are[6]

$$\pi_i = (1-\theta)(1-p)pro_i s_i + \theta[(1-p)(pro_i s_i + \eta_i) + p(pro_i s_i + \eta_i)]\,. \tag{A6}$$

The first-order derivative with respect to θ is

$$\frac{d\pi_i}{d\theta} = \eta_i + p \times pro_i s_i\,. \tag{A7}$$

Equation (A7) shows that $\eta_i \to 0$, implies that $d\pi_i/d\theta > 0$. Thus, manager i's optimal choice to maximize π_i is $\theta = 1$. That is, increasing fund expected net alphas increases profits.

As managers keep increasing fund expected net alphas, they reach a level of fund expected net alpha where $\bar{\alpha}$ is the maximum fund expected net alpha. At this point, managers' profit rates must be zero (otherwise managers could use profits to increase fund expected net alphas). Moreover, further increases of fund expected net alphas (by increasing effort levels or decreasing fees) make managers insolvent. Thus, at this point, where $\bar{\alpha}$ is the optimal fund expected net alpha, η_i and η_{-i} are negative. Managers are, then, in a Nash equilibrium (Maintain $\bar{\alpha}$, Maintain $\bar{\alpha}$).

Therefore, we have shown that managers maximizing profits by choosing optimal fees

[5] For simplicity and brevity, we do not introduce new notation to differentiate the infinitesimal profit changes when one or two players move. We use η_i and η_{-i} in both cases. Please see the following footnote.

[6] We note that, generally, η_i and η_{-i} may be positive or negative, which does not affect our results, as they approach zero. If the infinitesimal profit change for manager i, when both players move, was denoted δ_i, Equation (A7) would have been $\frac{d\pi_i}{d\theta} = \eta_i + p(\delta_i - \eta_i) + p \times pro_i s_i$, yielding the same result as η_i and δ_i approach zero.

and effort levels is equivalent to their maximizing fund expected net alphas.

Next, we show that that managers' optimization leads to a unique AFMI equilibrium. Because at any fund expected net alpha level that is below the maximizing level, managers attract no investments and have incentives to increase fund expected net alphas; and because further increasing fund expected net alpha above the maximizing level drives managers to insolvency, this Nash equilibrium is unique.

Q.E.D.

Proof of Proposition RA0

$\left\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\right\}$ is a Nash equilibrium for the following reasons.

1. Given other managers' optimal choices, a manager has incentives to not deviate from $\mathbf{e}^*$ and/or $\mathbf{f}^*$. If a manager deviates from $\mathbf{e}^*$ or $\mathbf{f}^*$, this manager decreases the fund expected net alpha, either losing all investment or becoming insolvent. Managers also cannot deviate from both in offsetting ways and gain. This is because effort increases do not sufficiently improve performance to justify costs and fee increases, and effort reductions cause too great a loss of performance that cannot be returned to investors through fee reductions. The reason is that a manager's optimal effort and fee together determine his or her fund expected net alpha. If the manager deviates from the equilibrium and produces a higher fund expected net alpha, he or she incurs a loss; and if the manager deviates and produces a lower fund expected net alpha, he or she receives no investments. We proved these results in the previous proof of maximization problem equivalence.
2. Given managers' and other investors' optimal choices, an investor has no incentive to deviate from $\boldsymbol{\delta}_{\mathbf{j}}^*$. This is because, where there are infinitely many small mean-variance risk-averse investors, each investor's choice does not affect fund sizes and, thus, the AFMI size. Changing allocations across funds does not improve an investor's portfolio Sharpe ratio, whereas changing allocations between the AFMI and the passive benchmark decreases the portfolio Sharpe ratio.

$\left\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\right\}$ is unique.

$\mathbf{e}^*$ is unique because for each fund e_i^* is a unique solution to $B_{e_i}(e_i; H) = 0$;

$\mathbf{f}^*$ is unique because for each fund $f_i^* - C^i(e_i^*, s_i; H) = 0$, where $C^i(e_i^*, s_i; H)$ is a deterministic function of e_i^*, and e_i^* is unique;

$\boldsymbol{\delta}^*$ is unique because allocations to funds maximize investor portfolios' Sharpe ratios, driving fund expected net alphas to the same values. Deviating, thus, cannot help and to the extent that large deviation would affect fund sizes, they will decrease Sharpe ratios. Moreover, the uniqueness of $\mathbf{e}^*$ and $\mathbf{f}^*$ rules out the existence of additional equilibrium allocations. We show below (Proposition RA2) that each $\boldsymbol{\delta}_{\mathbf{j}}^*, \forall j$ is the weights vector of AFMI funds' "market portfolio."

Q.E.D.

Proof of Proposition RA1 and Lemma RA1

The proof of Proposition RA1.1 is in the Proof of RA0.1.

To maximize $\mathrm{E}(\alpha_i \mid D)$, manager i must choose the breakeven management fee. This is because choosing higher fee would decrease expected net alpha and choosing lower fee would induce insolvency. Moreover, changing both fees and effort levels would move managers away from optimal effort levels. Thus,

$$f_i^* - C^i(e_i^*, s_i; H) = 0. \tag{A8}$$

This proves Lemma RA1.1.

We defined the direct benefits of effort in Equation (21). If their partial derivative with respect to effort, at zero effort, is positive, i.e., $A_{e_i}(0;H) - c_{2e_i}(0;H) > 0$, then it pays to exert effort and the optimal effort level is strictly positive, i.e., $e_i^* > 0$. The first-order-condition, with respect to effort, to maximize $\mathrm{E}(\alpha_i \mid D)$ becomes

$$A_{e_i}(e_i^*;H) - c_{2e_i}(e_i^*;H) = B_{e_i}(e_i^*;H) = 0. \tag{A9}$$

The related second-order condition, $A_{e_i,e_i}(e_i^*;H) - c_{2e_i,e_i}(e_i^*;H) = B_{e_i,e_i}(e_i^*;H) < 0,$ is satisfied by assumptions. (This is because we assume that productivity effort decreases in scale, i.e., $A_{e_i,e_i}(e_i;H) < 0,\ \forall e_i$, and that the costs of effort increase in scale, i.e., $c_{2e_i,e_i}(e_i;H) > 0,\ \forall e_i$). Thus e_i^* is a maximum. (We assume that functional forms of effort productivities and effort costs induce a finite e_i^*.)

This proves Lemma RA1.2.

Next, as both f_i^* and e_i^* are functions of H, we can write

$$e_i^* = e_i^*(H), \tag{A10}$$

and

$$f_i^* = f_i^*(H). \qquad \text{(A11)}$$

Total differentiation of (A9) with respect to H, using the left-hand side, gives

$$e_i^{*}{}'(H) = -\frac{A_{e_i,H}(e_i^*;H) - c_{2e_i,H}(e_i^*;H)}{A_{e_i,e_i}(e_i^*;H) - c_{2e_i,e_i}(e_i^*;H)}. \qquad \text{(A12)}$$

Thus, if the numerator of Equation (A12), $A_{e_i,H}(e_i^*;H) - c_{2e_i,H}(e_i^*;H) \geq 0(<0)$, then $e_i^{*}{}'(H) \geq 0(<0)$. (We showed above that the denominator of Equation (A12) is negative.)

This proves Lemma RA1.3.

The optimal manager effort e_i^* is determined only by the functions $c_2(e_i;H)$ and $A(e_i;H)$, which are the same across funds. Thus, we have $e_i^* = e_j^*$ and $B(e_i^*;H) = B(e_j^*;H), \forall i, j$. Because, in equilibrium, managers produce the (same) level of fund expected net alphas (which we proved above in the Manager's Equivalence Problems theorem) and, as we just showed, exert the same optimal effort levels (i.e., $e_i^* = e_j^*, \forall i, j$), from Equation (A1) we have that $f_i^* = f_j^*, \forall i, j$.

These prove Proposition RA1.7.

In addition, by Equation (A8), we further have $C^i(e_i^*, s_i; H) = C^j(e_j^*, s_j; H), \forall i, j$. Recall that $C^i(e_i^*, s_i; H) = c_0 + c_{1,i}s_i + c_2(e_i^*;H)$. As c_0, e_i^*, and $C^i(e_i^*, s_i; H)$ are the same across funds, we have the following relationship between different funds' sizes and costs:

$$c_{1,i}s_i = c_{1,j}s_j, \ \forall i, j, \qquad \text{(A13)}$$

or $s_i / s_j = c_{1,j} / c_{1,i}, \forall i, j$.

This proves Lemma RA1.6.

Summing s_i / s_j with respect to i, $i = 1,2,...,M$, we have $\sum_{i=1}^{M} \frac{s_i}{s_j} = \frac{S}{s_j} = \sum_{i=1}^{M} \frac{c_{1,j}}{c_{1,i}}$, where we use Equation (A13) to write the second equality. Inversing the second equality and exchanging the subscripts j and i gives

$$\frac{s_i}{S} = \left(c_{1,i} \sum_{j=1}^{M} \left(c_{1,j}{}^{-1} \right) \right)^{-1}, \ \forall i. \qquad \text{(A14)}$$

This proves Lemma RA1.7.

Using the break-even fee condition and Equation (A14), we can write

$$
\begin{aligned}
f_i^* &= C^i(e_i^*, s_i; H) = c_0 + c_{1,i}s_i + c_2(e_i^*; H) \\
&= c_0 + c_{1,i}\frac{s_i}{S}\frac{S}{W}W + c_2(e_i^*; H) \\
&= c_0 + c_{1,i}W\left(c_{1,i}\sum_{j=1}^{M}\left(c_{1,j}^{-1}\right)\right)^{-1}\frac{S}{W} + c_2(e_i^*; H).
\end{aligned} \tag{A15}
$$

Differentiating f_i^*, as in the last equation, with respect to H, yields

$$
f_i^*{}'(H) = c_{1,i}W\left(c_{1,i}\sum_{j=1}^{M}\left(c_{1,j}^{-1}\right)\right)^{-1}\frac{d(S/W)}{dH} + c_{2e_i}(e_i^*; H)e_i^*{}'(H) + c_{2H}(e_i^*; H). \tag{A16}
$$

Thus, whether higher concentrations induce higher equilibrium optimal fees depends on whether they induce an increase in equilibrium AFMI sizes ($d(S/W)/dH$) and whether they induce an increase in equilibrium optimal effort levels ($e_i^*{}'(H)$).

This proves lemma RA1.4.

Differentiation of $B(e_i^*; H)$ with respect to H and the use of Equation (A9) give

$$
\frac{dB(e_i^*; H)}{dH} = B_{e_i}(e_i^*; H)e_i^*{}'(H) + A_H(e_i^*; H) - c_{2H}(e_i^*; H) = A_H(e_i^*; H) - c_{2H}(e_i^*; H). \tag{A17}
$$

Thus, if $A_H(e_i^*; H) - c_{2H}(e_i^*; H) \geq 0 (< 0)$, then $\frac{dB(e_i^*; H)}{dH} \geq 0 (< 0)$.

This proves Lemma RA1.5.

Taking expectations of both sides of Equation (1), for fund i, yields

$$
\mathrm{E}(r_{F,i} \mid D) = \mathrm{E}(\alpha_i \mid D) + \mu_p. \tag{A18}
$$

Because $\mathrm{E}(\alpha_i \mid D)$'s are the same across funds in equilibrium, $\mathrm{E}(r_{F,i} \mid D)$'s are the same across funds in equilibrium. Further, we have

$$
\mathrm{Var}(r_{F,i} \mid D) = \sigma_p^2 + \sigma_a^2 + \sigma_b^2\left(\frac{S}{W}\right)^2 + \sigma_x^2 + \sigma_\varepsilon^2, \tag{A19}
$$

implying that fund returns' variances, $\mathrm{Var}(r_{F,i} \mid D)$'s, are the same across funds. Combining (A18) and (A19) shows that all managers offer the same market competitive Sharpe ratio.

This proves Proposition RA1.4 and RA1.5.

We note that Proposition RA1.3 is a direct consequence of Lemma RA1.6 and RA1.7.

Finally, to prove Proposition RA1.2, recalling that aggregate skill is $\sum_{i=1}^{M}\left(c_{1,i}^{-1}\right)$, we differentiate S/W by parts, to get

$$\frac{d(S/W)}{d\left[\sum_{i=1}^{M} c_{1,i}{}^{-1}\right]} = \frac{d(S/W)}{d\left[\hat{b}+\left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right]} \frac{d\left[\hat{b}+\left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right]}{d\left[\sum_{i=1}^{M} c_{1,i}{}^{-1}\right]} > 0. \tag{A20}$$

The inequality is correct because, from Equation (A31), the first multiplicand of the middle expression is negative; because the variables in the second multiplicand of the middle expression are positive, the second multiplicand is also negative.

This proves Proposition RA1.2.

This proves Proposition RA1 except for Proposition RA1.6, which is proved in the next section.

Q.E.D.

Proof of Propositions RA1.6 and RA2, and Corollary to Proposition RA2

In this case, investors maximize their portfolio Sharpe ratios subject to the constraints described by Equations (13) and (14) in the paper (wealth constraints and no fund short selling constraints). Investors are small, so none affects fund sizes (i.e., s_i, $i = 1,\ldots, M$) and AFMI size (S/W),

We note that

$$\mathrm{E}(r_j \mid D) = \mu_p + \boldsymbol{\delta}_{\mathbf{j}}{}^{\mathbf{T}}\mathrm{E}(\boldsymbol{\alpha} \mid D) = \mu_p + \boldsymbol{\delta}_{\mathbf{j}}{}^{\mathbf{T}}\left[\hat{a} - \hat{b}\frac{S}{W} + A(e_i^*; H) - f_i^*\right]\boldsymbol{\iota}_{\mathbf{M}},\ \forall i, j. \tag{A21}$$

The first equality is Equation (10). The second equality holds because equilibrium fund expected net alphas are the same, as we show in Proposition RA1.4. Also, as we assume that there are no marginal diversification benefits across funds, implying that the term $\sigma_\varepsilon^2\left(\boldsymbol{\delta}_{\mathbf{j}}{}^{\mathbf{T}}\boldsymbol{\delta}_{\mathbf{j}}\right)$ is zero, we get

$$\mathrm{Var}(r_j \mid D) = \sigma_p^2 + \left[\sigma_a^2 + \sigma_x^2 + \sigma_b^2\left(\frac{S}{W}\right)^2\right]\left(\boldsymbol{\delta}_{\mathbf{j}}{}^{\mathbf{T}}\boldsymbol{\iota}_{\mathbf{M}}\right)^2,\ \forall j. \tag{A22}$$

Investors maximize $\frac{\mathrm{E}(r_j \mid D)}{\sqrt{\mathrm{Var}(r_j \mid D)}}$ by choosing $\boldsymbol{\delta}_{\mathbf{j}}$. The first-order condition (i.e., the derivative of $\frac{\mathrm{E}(r_j \mid D)}{\sqrt{\mathrm{Var}(r_j \mid D)}}$ with respect to $\boldsymbol{\delta}_{\mathbf{j}}$, set to be 0), when substituting the constraints $f_i^* - C^i(e_i^*, s_i; H) = 0,\ \forall i$, into it, is

$$
\begin{aligned}
&-\left(\mu_p / \sigma_p^2\right)\left[\sigma_a^2+\sigma_b^2\left(\frac{S}{W}\right)^2+\sigma_x^2\right]\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{T*}}\boldsymbol{\iota}_{\mathbf{M}}+\hat{a}-\hat{b}\frac{S}{W}+A(e_i^*;H)-c_0-c_{1,i}s_i-c_2(e_i^*;H)\\
&=-\left(\mu_p / \sigma_p^2\right)\left[\sigma_a^2+\sigma_b^2\left(\frac{S}{W}\right)^2+\sigma_x^2\right]\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{T*}}\boldsymbol{\iota}_{\mathbf{M}}-\left[\hat{b}+(c_{1,i}\frac{s_i}{S})W\right]\frac{S}{W}+\hat{a}+A(e_i^*;H)-c_0-c_2(e_i^*;H)\\
&=-\left(\mu_p / \sigma_p^2\right)\left[\sigma_a^2+\sigma_b^2\left(\frac{S}{W}\right)^2+\sigma_x^2\right]\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{T*}}\boldsymbol{\iota}_{\mathbf{M}}-\left[\hat{b}+\left(\sum_{i=1}^{M}c_{1,i}{}^{-1}\right)^{-1}W\right]\frac{S}{W}+X(e_i^*;H)\\
&=0.
\end{aligned}
\tag{A23}
$$

The first equality of Equation (A23) holds because

$$
c_{1,i}s_i=\left(c_{1,i}\frac{s_i}{S}\right)W\frac{S}{W}, \tag{A24}
$$

and we obtain the second equality by using the definition of $X(e_i^*;H)$ and Equation (A14).

Substituting $\gamma \triangleq \mu_p / \sigma_p^2$ and $S/W=\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{T*}}\boldsymbol{\iota}_{\mathbf{M}}$ (symmetric equilibrium) into Equation (A23), we have

$$
-\gamma\sigma_b^2\left(\frac{S}{W}\right)^3-\left[\gamma\sigma_a^2+\gamma\sigma_x^2+\hat{b}+\left(\sum_{i=1}^{M}c_{1,i}{}^{-1}\right)^{-1}W\right]\frac{S}{W}+X(e_i^*;H)=0. \tag{A25}
$$

If the constraint $\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\iota}_{\mathbf{M}}\le 1$ is not binding (i.e., $S/W<1$), the equilibrium optimal S/W is a real positive solution of this cubic equation. This is because the condition $X(e_i^*;H)>0, \forall H$ (positivity of the lowest order polynomial coefficient) and the negativity of the two higher order polynomial coefficients $-\gamma\sigma_b^2$, and $-\left[\gamma\sigma_a^2+\gamma\sigma_x^2+\hat{b}+\left(\sum_{i=1}^{M}c_{1,i}{}^{-1}\right)^{-1}W\right]$, (i.e., $-\gamma\sigma_b^2<0$, and $-\left[\gamma\sigma_a^2+\gamma\sigma_x^2+\hat{b}+\left(\sum_{i=1}^{M}c_{1,i}{}^{-1}\right)^{-1}W\right]<0$) guarantee the existence of exactly one positive real solution for S/W (and two imaginary ones). Also, as each investor cannot affect the value of S/W, Equation (A23) shows that the solution for $\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{T*}}\boldsymbol{\iota}_{\mathbf{M}}=S/W$ is unique given the parameter values and the market S/W.

If the constraint $\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{T}}\boldsymbol{\iota}_{\mathbf{M}}\le 1$ is binding, (i.e., $S/W=1$), there is an obviously unique solution where investors maximize their portfolio Sharpe ratios by allocating all their wealth to the AFMI (no passive index holdings).

We have, thus, demonstrated that $\delta_j=\delta_i, \forall i, j$, such that $S/W=\boldsymbol{\delta}_{\mathbf{j}}^{\mathbf{T*}}\boldsymbol{\iota}_{\mathbf{M}}$, induces a unique equilibrium.

This proves Proposition RA1.6 and Proposition RA2.2.

In addition, we have

$$\begin{aligned}\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} &= \hat{a} - \hat{b}\left(S/W\right) + A(e_i^*;H) - f_i^* \\ &= \hat{a} - \hat{b}\left(S/W\right) + A(e_i^*;H) - c_0 - c_{1,i}s_i - c_2(e_i^*;H) \\ &= -\left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right]\left(S/W\right) + X(e_i^*;H).\end{aligned} \tag{A26}$$

The first equality of Equality (A26) follows from the equilibrium break-even management fee condition, and the second equality of Equation (A26) follows from Equation (A24). Substituting Equation (A26) into Equation (A25) and rearranging, we offer two presentations of $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$, one as a function of $\frac{S}{W}$ and one as a function of $X(e_i^*;H)$, where the latter corresponds to Equation (33) in PS.

$$\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = \gamma\left(\sigma_\alpha^2 + \sigma_x^2\right)\frac{S}{W} = \frac{\gamma\left(\sigma_\alpha^2 + \sigma_x^2\right)}{\gamma\left(\sigma_\alpha^2 + \sigma_x^2\right) + \hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W} X(e_i^*;H), \tag{A27}$$

where

$$\sigma_\alpha^2 \triangleq \mathrm{Var}(\alpha_j \mid D) = \sigma_b^2\left(\frac{S}{W}\right)^2 + \sigma_a^2. \tag{A28}$$

Because all the components of Equation (A27) are positive, $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is positive. The intuition is as follows. From investors' portfolio variance formulas, Equation (A22), we can easily see that portfolios with allocations to the AFMI have higher variance that those holding the passive benchmark. If $\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = 0$, because of a sufficiently large amount of investment in funds, investors can always improve their portfolio Sharpe ratios (in particular, reduce their portfolios risk) by shifting wealth from AFMI to the passive benchmark.

This proves Proposition RA2.1.

This proves Proposition RA2.

Where the equilibrium optimal AFMI size is less than one (i.e., $S/W < 1$), differentiation of Equation (A25) with respect to $X(e_i^*;H)$ gives

$$\frac{d(S/W)}{dX} = \frac{1}{\gamma\left[3\sigma_b^2\left(\frac{S}{W}\right)^2 + \sigma_a^2 + \sigma_x^2\right] + \hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W}. \tag{A29}$$

As all the components of Equation (A29) are positive, we have that

$$\frac{d(S/W)}{dX} > 0. \tag{A30}$$

This proves Point 1 of the Corollary of Proposition RA2.

Differentiation of Equation (A25) with respect to $\left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right]$ gives

$$\begin{aligned}
\frac{d(S/W)}{d\left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right]} &= \frac{-(S/W)}{\gamma\left[3\sigma_b^2\left(\frac{S}{W}\right)^2 + \sigma_a^2 + \sigma_x^2\right] + \hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W} \\
&= -\frac{S}{W}\frac{d(S/W)}{dX} \\
&< 0.
\end{aligned} \tag{A31}$$

The last inequality holds because of Equation (A30) and the positivity of S/W.

This proves Point 2 of the Corollary of Proposition RA2.

Q.E.D.

Proof of Proposition RA3

By the chain rule,

$$\frac{d(S/W)}{dH} = \frac{d(S/W)}{dX(e_i^*;H)}\frac{dX(e_i^*;H)}{dH} = \frac{d(S/W)}{dX(e_i^*;H)}\frac{dB(e_i^*;H)}{dH}. \tag{A32}$$

The second equality of Equation (A32) follows from the definitions of $X(e_i^*;H)$ and $B(e_i^*;H)$ in equations (25) and (21), respectively. Recalling that $\frac{d(S/W)}{dX} > 0$, [see Equation (A30) above], we see that the sign of $d(S/W)/dH$ is determined by the sign of $dB(e_i^*;H)/dH$.

Also, differentiating Equation (A32), again with respect to H, we have

$$\begin{aligned}
\frac{d^2(S/W)}{dH^2} &= \frac{d(S/W)}{dX}\frac{d^2B(e_i^*;H)}{dH^2} + \left[\frac{dB(e_i^*;H)}{dH}\right]^2 \frac{d^2(S/W)}{dX^2} \\
&= \frac{d(S/W)}{dX}\frac{d^2B(e_i^*;H)}{dH^2} - \left[\frac{dB(e_i^*;H)}{dH}\right]^2 \left[\frac{d(S/W)}{dX}\right]^3 \gamma\left[6\sigma_b^2\frac{S}{W}\right].
\end{aligned} \tag{A33}$$

The first equality of Equation (A33) holds because of Equation (A32) and the 2nd derivative chain rule. To show that the second equality holds, we first differentiate Equation (A29) again, to get

$$\frac{d^2(S/W)}{dX^2} = -\left[\frac{d(S/W)}{dX}\right]^3 \gamma\left[6\sigma_b^2\frac{S}{W}\right], \tag{A34}$$

and then substitute the result.

We can write the second-order derivative of $B(e_i^*, H)$ with respect to H as

$$\frac{d^2 B(e_i^*;H)}{dH^2} = \frac{d\left[dB(e_i^*;H)/dH\right]}{dH} = \frac{d\left[A_H(e_i^*;H) - c_{2H}(e_i^*;H)\right]}{dH}, \tag{A35}$$

where the second equality of Equation (A35) holds because of (A17).

Noting that $d(S/W)/dX > 0$ from Equation (A30), $\gamma\left[6\sigma_b^2 \frac{S}{W}\right] > 0$, and $\left[A_H(e_i^*;H) - c_{2H}(e_i^*;H)\right]^2 \geq 0$, Equation (A33) implies that $d^2B(e_i^*;H)/dH^2 < 0$ implies $d^2(S/W)/dH^2 < 0$, whereas $d^2(S/W)/dH^2 > 0$ implies $d^2B(e_i^*;H)/dH^2 > 0$.

Differentiating Equation (A26) with respect to H and using Equations (A32), we have

$$\left.\frac{d\mathrm{E}(\alpha_i \mid D)}{dH}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = \left\{1 - \left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right] \frac{d(S/W)}{dX}\right\} \frac{dB(e_i^*;H)}{dH}. \tag{A36}$$

Substituting Equation (A29) into the right-hand side of Equation (A36) yields

$$1 - \left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right] \frac{d(S/W)}{dX} = \frac{\gamma\left[3\sigma_b^2\left(\frac{S}{W}\right)^2 + \sigma_a^2 + \sigma_x^2\right]}{\gamma\left[3\sigma_b^2\left(\frac{S}{W}\right)^2 + \sigma_a^2 + \sigma_x^2\right] + \hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W} > 0. \tag{A37}$$

The last inequality of Equation (A37) holds because the values of all the parameters and variables in the equation are positive. Then, from Equation (A37), Equation (A36) implies that the sign of $d\mathrm{E}(\alpha_i \mid D)/dH\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ is determined by the sign of $dB(e_i^*;H)/dH$.

Also, differentiating Equation (A36) again, with respect to H, and using Equations (A17), (A34), and (A35), we have

$$\left.\frac{d^2\mathrm{E}(\alpha_i \mid D)}{dH^2}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = \frac{d^2B(e_i^*;H)}{dH^2}\left\{1 - \left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right] \frac{d(S/W)}{dX}\right\} + \left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right] \times\left[A_H(e_i^*;H) - c_{2H}(e_i^*;H)\right]^2 \left[\frac{d(S/W)}{dX}\right]^3 \gamma\left[6\sigma_b^2 \frac{S}{W}\right]. \tag{A38}$$

As $\left\{1 - \left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right] \frac{d(S/W)}{dX}\right\} > 0$ by Equation (A37), $d(S/W)/dX > 0$ by

Equation (A30), $\left[A_H(e_i^*;H)-c_{2H}(e_i^*;H)\right]^2 \geq 0$, $\left[\hat{b}+\left(\sum_{i=1}^{M} c_{1,i}{}^{-1}\right)^{-1} W\right] > 0$, and $\gamma\left[6\sigma_b^2 \frac{S}{W}\right] > 0$, we have that $d^2\mathrm{E}(\alpha_i \mid D)/dH^2 |_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} < 0$ implies $d^2B(e_i^*;H)/dH^2 < 0$, whereas $d^2B(e_i^*;H)/dH^2 > 0$ implies $d^2\mathrm{E}(\alpha_i \mid D)/dH^2 |_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} > 0$.

Putting the results in this section together, we have

$$\frac{d(S/W)}{dH} \geq 0(<0) \Leftrightarrow \frac{dB(e_i^*;H)}{dH} \geq 0(<0) \Leftrightarrow \left.\frac{d\mathrm{E}(\alpha_i \mid D)}{dH}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} \geq 0(<0), \tag{A39}$$

$$\frac{d^2(S/W)}{dH^2} > 0 \Rightarrow \frac{d^2B(e_i^*;H)}{dH^2} > 0 \Rightarrow \left.\frac{d^2\mathrm{E}(\alpha_i \mid D)}{dH^2}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} > 0, \tag{A40}$$

and

$$\left.\frac{d^2\mathrm{E}(\alpha_i \mid D)}{dH^2}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} < 0 \Rightarrow \frac{d^2B(e_i^*;H)}{dH^2} < 0 \Rightarrow \frac{d^2(S/W)}{dH^2} > 0. \tag{A41}$$

This proves Proposition RA3.

Q.E.D.

Proof of Proposition RA4

Differentiating Equation (A25) with respect to $c_{1,i}$ gives

$$\frac{d(S/W)}{dc_{1,i}} = \frac{-(S/W)W}{\left\{\gamma\left[3\sigma_b^2(S/W)^2+\sigma_a^2+\sigma_x^2\right]+\hat{b}+\left(\sum_{l=1}^{M} c_{1,l}{}^{-1}\right)^{-1} W\right\}\left(\sum_{l=1}^{M} c_{1,l}{}^{-1}\right)^2 c_{1,i}^2} < 0, \tag{A42}$$

where the first equality holds because the derivative of $X(e_i^*,H)$ with respect to $c_{1,i}$ is 0, and the last inequality holds because all the parameter and variable values in Equation (A42) are positive, and we have a negative sign in the numerator.

Differentiating Equation (A26) with respect to $c_{1,i}$, we have

$$\begin{aligned}\left.\frac{d\mathrm{E}(\alpha_i \mid D)}{dc_{1,i}}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} &= -\left[\hat{b}+\left(\sum_{l=1}^{M} c_{1,l}{}^{-1}\right)^{-1} W\right]\frac{d(S/W)}{dc_{1,i}} - \frac{(S/W)W}{\left(\sum_{l=1}^{M} c_{1,l}{}^{-1}\right)^2 c_{1,i}^2} \\ &= \frac{-\gamma\left[3\sigma_b^2(S/W)^2+\sigma_a^2+\sigma_x^2\right](S/W)W}{\left\{\gamma\left[3\sigma_b^2(S/W)^2+\sigma_a^2+\sigma_x^2\right]+\hat{b}+\left(\sum_{l=1}^{M} c_{1,l}{}^{-1}\right)^{-1} W\right\}\left(\sum_{l=1}^{M} c_{1,l}{}^{-1}\right)^2 c_{1,i}^2} \\ &< 0,\end{aligned}$$

(A43)

where the second equality of Equation (A43) holds after substituting Equation (A42). The last inequality of Equation (A43) holds because all the parameters and variables in Equation (A43) are positive and we have a negative sign in the numerator.

Similarly, differentiating Equation (A26) with respect to $c_{1,j}$ yields

$$\left.\frac{d\mathrm{E}(\alpha_i \mid D)}{dc_{1,j}}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = \frac{-\gamma\left[3\sigma_b^2\left(S/W\right)^2+\sigma_a^2+\sigma_x^2\right]\left(S/W\right)W}{\left\{\gamma\left[3\sigma_b^2\left(S/W\right)^2+\sigma_a^2+\sigma_x^2\right]+\hat{b}+\left(\sum_{l=1}^{M}c_{1,l}{}^{-1}\right)^{-1}W\right\}\left(\sum_{l=1}^{M}c_{1,l}{}^{-1}\right)^2 c_{1,j}^2} < 0.$$

(A44)

For the case where $S/W=1$, from Equation (A26), we have

$\mathrm{E}(\alpha_i \mid D)\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = \left[\hat{b}+\left(\sum_{i=1}^{M}c_{1,i}{}^{-1}\right)^{-1}W\right] + X(e_i^*;H)$. Then,

$$\left.\frac{d\mathrm{E}(\alpha_i \mid D)}{dc_{1,i}}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = \frac{-W}{\left(\sum_{l=1}^{M}c_{1,l}{}^{-1}\right)^2 c_{1,i}^2} < 0, \quad \text{(A45)}$$

$$\left.\frac{d\mathrm{E}(\alpha_i \mid D)}{dc_{1,j}}\right|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = \frac{-W}{\left(\sum_{l=1}^{M}c_{1,l}{}^{-1}\right)^2 c_{1,j}^2} < 0\,, \quad \text{(A46)}$$

where the last inequalities of Equations (A45) and (A46) hold because all the parameter and variable values are positive and we have a negative sign in the numerators.

This proves Proposition RA4.2.

As s_i/S decreases in $c_{1,i}$ whereas s_j/S, $\forall j \neq i$ increases in $c_{1,i}$, from Equations (A14), (A43), (A44), (A45), and (A46), we find that $\mathrm{E}(\alpha_i \mid D)\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ and s_i/S are increasing/decreasing in the same direction due to changes in $c_{1,i}$, and that $\mathrm{E}(\alpha_j \mid D)\,|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}}$ and s_j/S, $\forall j \neq i$ are increasing/decreasing inversely due to changes in $c_{1,i}$, whether $S/W<1$ or $S/W=1$.

This proves Proposition RA4.1.

This proves Proposition RA4.

Q.E.D.

Proof of Proposition PS and Corollary

As we know from previous propositions, in equilibrium, managers are earning zero economic profit. Thus, we substitute $f_i = C^i(e_i, s_i; H)$ into $\mathrm{E}(\alpha_i \mid D)$ and then perform first-order differentiation with respect to e_i. We get $A_{e_i}(e_i^*; H) - c_{2e_i}(e_i^*; H) = 0$. The second-order condition, $A_{e_i,e_i}(e_i; H) - c_{2e_i,e_i}(e_i; H) < 0$, holds for all effort levels by our assumptions on the functional forms.

By construction, effort levels cannot be negative, i.e., $e_i^* \geq 0$. If $A_{e_i}(0; H) - c_{2e_i}(0; H) < 0$, managers (unable to exert negative effort levels) spend no effort, i.e., $e_i^* = 0$. In this case, managers charge an optimal proportional fee (break-even fee) equal to $f_i^* = c_0 + c_{1,i} s_i$.

This proves Proposition PS.

Then, our equilibrium is similar to that in Pastor and Stambaugh (2012), where managerial effort is not modeled [our $A(e_i; H)$ and $c_2(e_i; H)$ are both zero] and managers do not charge fees above opportunity costs.

Specifically, Equation 4 becomes

$$\alpha_i = a - b\frac{S}{W} - f_i, \tag{A47}$$

identical to Equation (8) in PS.

This proves the corollary to Proposition PS.

Q.E.D.

Proof of Proposition RN1

Infinitely Many Small Risk-Neutral Investors

Optimizing risk-neutral investors keep allocating wealth to the AFMI as long as fund expected net alphas are positive, maximizing their portfolio expected net returns.

In the case that investors have sufficient wealth, fund expected net alphas are driven down to zero. In this equilibrium, investors have additional wealth available to allocate to the AFMI even after funds have exhausted their abilities to produce positive fund expected net alphas.

Technically, $S/W < 1$, and

$$\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*, \mathbf{f}^*, \boldsymbol{\delta}^*\}} = \hat{a} - \hat{b}\left(S/W\right) + A(e_i^*; H) - f_i^* = 0, \ \forall i. \tag{A48}$$

Substituting into Equation (A48) the equilibrium conditions $f_i^* - C^i(e_i^*, s_i; H) = 0$,

$A_{e_i}(e_i^*;H) - c_{2e_i}(e_i^*;H) = 0$, and $\frac{s_i}{S} = \left(c_{1,i}\sum_{j=1}^{M}\left(c_{1,j}^{-1}\right)\right)^{-1}$, $\forall i$ [Equations (A8), (A9), (A14)] and rearranging, we have

$$\frac{S}{W} = \frac{\hat{a} + A(e_i^*;H) - c_0 - c_2(e_i^*;H)}{\hat{b} + (c_{1,i}\frac{s_i}{S})W} = \frac{X(e_i^*;H)}{\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}^{-1}\right)^{-1} W}. \tag{A49}$$

In the case that investors do not have sufficient wealth to allocate to the AFMI despite having positive fund expected net alphas, then $S/W = 1$. Now, substituting the equilibrium conditions $S/W = 1$, $f_i^* - C^i(e_i^*, s_i; H) = 0$, $A_{e_i}(e_i^*;H) - c_{2e_i}(e_i^*;H) = B_{e_i}(e_i^*;H) = 0$, and $\frac{s_i}{S} = \left(c_{1,i}\sum_{j=1}^{M}\left(c_{1,j}^{-1}\right)\right)^{-1}$, $\forall i$ into Equation (A26), we have

$$\mathrm{E}(\alpha_i \mid D)|_{\{\mathbf{e}^*,\mathbf{f}^*,\boldsymbol{\delta}^*\}} = X(e_i^*;H) - \left[\hat{b} + \left(\sum_{i=1}^{M} c_{1,i}^{-1}\right)^{-1} W\right]. \tag{A50}$$

This proves Proposition RN1.

Q.E.D.

Endogeneity in Measures of AFMI Concentration

Our model allows for an endogenous measure of AFMI concentration. Modeling an endogenous measure of concentration facilitates the use of available and prevalent empirical measures. If we define H to be the Herfindahl-Hirschman index (HHI), which is the sum of market shares squared, then H is endogenous to our model.[7] Using funds' equilibrium market share, as identified in Lemma RA1.7, we can write the equilibrium AFMI concentration H^* as

$$H^* \triangleq \sum_{i=1}^{M}\left(\frac{s_i}{S}\right)^2 = \sum_{i=1}^{M}\left(c_{1,i}\sum_{j=1}^{M}\left(c_{1,j}^{-1}\right)\right)^{-2} \tag{A51}$$

We can see that H^* is determined by $c_{1,i}$'s. Specifically, depending on the size of $c_{1,i}$ relative to that $c_{1,j}$, $\forall j \neq i$, an increase in $c_{1,i}$, holding $c_{1,j}$, $\forall j \neq i$ constant, increases or decreases H^*.

When the AFMI concentration is defined as H^*, propositions RA3 and RA4 imply that

[7] In an M-firm AFMI, for example, the HHI could have values between the highest concentration, 1, where one of the funds captures practically all the market share, and the lowest concentration, $1/M$, where market shares are evenly divided. That is, in an M-firms' market HHI $\in \left[\frac{1}{M}, 1\right)$.

the relation between the $c_{1,i}$'s and the equilibrium fund expected net alphas and AFMI size is complex. Where there are infinitely many risk-averse investors, an increase in $c_{1,i}$ affects the equilibrium fund expected net alphas in two ways: 1) its direct impact leads to lower equilibrium fund expected net alphas (Proposition RA4), and 2) depending on fund i 's size relative to rivals, it increases or decreases H^*, which consequently increases (decreases) the equilibrium fund expected net alphas if and only if $A_H(e_i^*;H^*)-c_{2H}(e_i^*;H^*)\geq(<)0$ (Proposition RA3). Similarly, an increase in $c_{1,i}$ affects the equilibrium AFMI size in two ways: 1) its direct impact leads to an (inverse direction) AFMI size change, and 2) it increases or decreases H^*, which consequently increases (decreases) the equilibrium AFMI size if and only if $A_H(e_i^*;H^*)-c_{2H}(e_i^*;H^*)\geq(<)0$. Thus, in the endogenous AFMI concentration measure case, the relation between the $c_{1,i}$'s and the equilibrium fund expected net alphas and AFMI size depend on fund i 's size relative to rivals.[8]

In general, we expect the theoretical concentration level in our framework to be influenced by industry characteristics such as regulations, transaction costs, tax rates, barriers to entry, and funds' idiosyncratic outcomes, in addition to funds' cost sensitivity to size (i.e., $c_{1,i}$'s). For example, Hong (2018) finds that a policy reform in Hong Kong (the Employee Choice Arrangement, 2012) substantially increased competition in the fund management industry by dramatically expanding the choices for pension plan participants from an average of eleven funds to more than four hundred funds. In these cases, the concentration level can change even when all the cost sensitivities (or fund manager skill) are constant. We do not model the various determinants of concentration levels and simply assume them to be exogenous. As long as real-world concentration is not exactly determined by the $c_{1,i}$'s (or any other exogenous parameter of our model), we are back to the case that when concentration is exogenous (that is, has an exogenous component), our predictions remain unaltered regarding the relation between

[8] We believe that our cost function, Equation (18), is a concise one that captures essential effects within our model. To assure that all our functional form restrictions of the non-specialized model (exogenous concentration), which we deem basic and simple, hold in the specialized one (endogenous measure of industry concentration); however, we need to impose additional, technical, "second order," parameter restrictions. For brevity and simplicity, we do not impose these restrictions. We call the parameter values that make the specialized model abide by these restrictions *plausible*. We later confirm that the said technical restrictions are not empirically binding. That is, imposing these restrictions would not change our empirical results. In other words, the empirically estimated parameters fall within the plausible parameters range.

changes in exogenous AFMI concentration level, the equilibrium fund expected net alphas, and AFMI size.