Internet Appendix

Volatility and the Cross-Section of Returns on FX Options

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1. Internet Appendix Tables

1.1. *Appendix – Panel fixed time effect regressions*

Table 1
Univariate Panel Time Fixed-Effects Regressions

This table reports the results of panel regressions with time fixed effects. The dependent variable in each case is the return on straddle positions in currencies. The single explanatory variable in each column is the implied volatility (IV) of the currency. Column (1) uses the full sample. In column (2) the currencies with the highest and lowest implied volatilities each week are excluded. In column (3) we exclude the two currencies with the highest volatility and the two currencies with the lowest volatility each week. Column (4) reports results using the first half of the sample while column (5) uses the second half of the data sample. Newey-West standard errors are used to compute the t-statistics reported in parentheses beneath the parameter estimates.

	(1) **Full**	(2) **P2-P8**	(3) **P3-P7**	(4) **H1**	(5) **H2**
IV	-6.600	-9.761	-8.679	-8.181	-5.606
	(-13.213)	(-12.807)	(-7.659)	(-10.206)	(-8.822)
Observations	7,875	6,055	4,236	3,780	4,095
R-squared	0.420	0.467	0.513	0.408	0.432

Table 2
Panel Time Fixed Effects Regressions – Spot Exchange Rate Changes

This table reports the results of panel regressions with time fixed effects. The dependent variable in each case is the one-month change in the spot exchange rate. Explanatory variables are denoted as follows: IV is the current implied volatility, and $RV - IV$ is the difference between realised and implied volatilities. Newey-West standard errors are used to compute the t-statistics reported in parentheses beneath the parameter estimates.

	(1)	(2)	(3)
IV	0.006		-0.015
	(0.351)		(-0.877)
$RV - IV$		-0.106	-0.111
		(-4.534)	(-5.007)
Observations	7,875	7,875	7,875
R-squared	0.596	0.598	0.598

Table 3
Panel Fixed Time Effects Regressions – Controls

This table reports the results of panel regressions with time fixed effects. The dependent variable in each case is the return on straddle positions in currencies. Explanatory variables are denoted as follows: IV is the current implied volatility, RV is the historical realised volatility, $RV - IV$ is the difference between realised and implied volatilities, dIV is the one month lagged change in implied volatility, $fdIV$ is the one month change in implied volatility measured over the life of the straddle position, and $TSIV$ is the term structure of implied volatility defined as the difference between the implied volatility of options with six months and one month to maturity. Newey-West standard errors are used to compute the t-statistics reported in parentheses beneath the parameter estimates.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
IV	-6.600		-7.008		-7.150	-4.332		-6.168
	(-13.213)		(-14.536)		(-14.614)	(-8.491)		(-11.464)
$RV - IV$		0.007	-2.123					
		(0.012)	(-3.612)					
dIV				-0.118	3.464			
				(-0.122)	(3.614)			
$fdIV$						14.629		
						(11.446)		
$TSIV$							11.421	2.069
							(6.792)	(1.134)
Observations	7,875	7,875	7,875	7,875	7,875	7,875	7,875	7,875
R-squared	0.420	0.402	0.422	0.402	0.422	0.446	0.410	0.421

Table 4
Panel Fixed Time Effects Regressions – Implied Volatility Decompositions

This table reports the results of panel regressions with time fixed effects. The dependent variable in each case is the return on straddle positions in currencies. Explanatory variables are denoted as follows: IV is the current implied volatility, βIV is the estimated coefficient from the regression of IV on the cross-sectional average level of IV, $IVavg$ is the time series average level of implied volatility, $IVidio$ is idiosyncratic volatility defined as the difference between total and systematic volatility as defined in section **??**, $BetaSkew$ and $BetaKurt$ denote estimated sensitivities to skew and kurtosis risks as described in the text. Newey-West standard errors are used to compute the t-statistics reported in parentheses beneath the parameter estimates.

	(1)	(2)	(3)	(4)	(5)
IV	-6.600	-7.169	-3.789	-6.760	-6.271
	(-13.213)	(-12.495)	(-6.009)	(-13.362)	(-12.628)
βIV		0.058			
		(2.157)			
$AvgIV$			-7.652		
			(-7.271)		
$IVidio$				-0.790	
				(-3.180)	
$BetaSkew$					0.003
					(0.260)
$BetaKurt$					0.070
					(5.852)
Observations	7,875	7,875	7,875	7,875	7,875
R-squared	0.420	0.421	0.426	0.421	0.425

Table 5
Iron Butterfly Fama-MacBeth Regressions – Controls

This table reports the results of Fama-MacBeth regressions. The dependent variable in each case is the return on 10-delta iron butterfly positions in currencies. Explanatory variables are denoted as follows: IV is the current implied volatility, RV is the historical realised volatility, $RV - IV$ is the difference between realised and implied volatilities, dIV is the one month lagged change in implied volatility, $fdIV$ is the one month change in implied volatility measured over the life of the straddle position, and $TSIV$ is the term structure of implied volatility defined as the difference between the implied volatility of six and one month options. Newey-West standard errors are used to compute the t-statistics reported in parentheses beneath the parameter estimates.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
IV	-9.574		-11.023		-10.870	-9.987		-10.258
	(-10.093)		(-9.728)		(-10.181)	(-9.663)		(-9.490)
$RV - IV$		-1.045	-4.260					
		(-0.702)	(-2.480)					
dIV				2.280	6.285			
				(1.236)	(3.000)			
$fdIV$						14.865		
						(6.070)		
$TSIV$							13.767	2.602
							(4.665)	(0.786)
Observations	7,875	7,875	7,875	7,875	7,875	7,875	7,875	7,875
R-squared	0.188	0.155	0.332	0.171	0.343	0.378	0.163	0.338
Number of groups	910	910	910	910	910	910	910	910

Table 6
Risk Adjusted Options Returns I

This table reports the results of OLS regressions of long-short options positions on explanatory variables. The dependent variables in columns (1) and (2) are single-currency and four-currency long-short straddle returns. The dependent variables in columns (3) and (4) are single-currency and four-currency long-short 10-delta iron butterly returns, while columns (5) and (6) use 25-delta iron butterfly returns. Column (7) reports results for $LMH1$ delta-hedged long currency positions, and column (8) repeats this for short currency positions. Explanatory variables are denoted as follows: $Agg.Vol_{straddle}$ is the equaly-weighted cross-sectional average return from the straddle positions, $Agg.Vol_{flyXX}$ is the equaly-weighted cross-sectional average return from the iron butterfly position with delta equal to XX (10 or 25), $Agg.Vol_{DHlong}$ is the equally-weighted cross-sectional average return from the delta-hedged long currency positions, $Agg.Vol_{DHshort}$ is the equally-weighted cross-sectional average return from the delta-hedged short currency positions, and $Agg.Forward$ is the equally-weighted cross-sectional average return on one-month forward positions. Newey-West standard errors are used to compute the t-statistics reported in parentheses beneath the parameter estimates.

	(1) LMH $strad$	(2) $LMH4$ $strad$	(3) LMH $iron10$	(4) $LMH4$ $iron10$	(5) LMH $iron25$	(6) $LMH4$ $iron25$	(7) LMH $DHlong$	(8) LMH $DHshort$
$Agg.Vol_{straddle}$	0.491	0.266						
	(4.750)	(6.695)						
$Agg.Forward$	2.059	0.547	0.626	0.209	0.292	0.286	11.097	-22.533
	(0.944)	(0.588)	(0.408)	(0.254)	(0.267)	(0.349)	(2.378)	(-4.147)
$Agg.Vol_{fly10}$			0.350	0.231				
			(4.529)	(5.561)				
$Agg.Vol_{fly25}$					0.319	0.345		
					(3.312)	(5.625)		
$Agg.Vol_{DHshort}$							0.522	
							(4.182)	
$Agg.Vol_{DHlong}$								0.891
								(5.010)
Constant	0.330	0.279	0.278	0.230	0.176	0.228	0.399	0.252
	(9.209)	(16.324)	(8.663)	(14.585)	(6.679)	(14.037)	(6.963)	(4.364)
Observations	910	910	910	910	910	910	910	910
R-squared	0.058	0.074	0.022	0.038	0.010	0.031	0.053	0.120

Table 7
Iron Butterfly Higher Moment Risks

This table reports the results of Fama-MacBeth regressions. The dependent variable in each case is the return on 10-delta iron butterfly positions in currencies. Explanatory variables are denoted as follows: IV is the current implied volatility, $BetaSkew$ and $BetaKurt$ denote estimated sensitivities to skew and kurtosis risks as described in the text. Newey-West standard errors are used to compute the t-statistics reported in parentheses beneath the parameter estimates.

	(1)	(2)
IV		-6.758
		(-7.357)
$BetaSkew$	0.203	0.273
	(3.715)	(5.115)
$BetaKurt$	0.732	0.788
	(5.314)	(6.210)
Observations	7,875	7,875
R-squared	0.405	0.548
Number of groups	910	910

1.2. Appendix – Implementation lag

In the text we form portfolios each Wednesday using mid market implied volatilities measured on the same day (i.e. with no implementation lag). Since there are no computations necessary to determine this implied volatility beyond simple averaging, we think this is sensible. However, in this subsection we form portfolios each Wednesday based on the implied volatilities observed one week earlier to avoid issues with implementation delays and the effects of bid-ask bounce. Table 8 reports mean returns on ranked portfolios using comtemporaneous and one-week lagged implied volatilities.

Table 8
Implementation lag

This table reports mean one-month holding period returns portfolios of curencies sorted on implied volatility. In the first row, portfolios are formed using contemporaneous implied volatility. In the second row they are formed using implied volatility from the previous week.

	P1	P2	P3	P4	P5	P6	P7	P8	P9
$IV(t)$	0.252	0.258	0.116	-0.001	0.091	-0.104	-0.083	-0.089	-0.138
$IV(t-1week)$	0.243	0.271	0.089	-0.002	0.095	-0.089	-0.066	-0.107	-0.132

1.3. Appendix – Performance charts

Figure 1. *LMH* Straddle Returns, Monthly Sampled Data

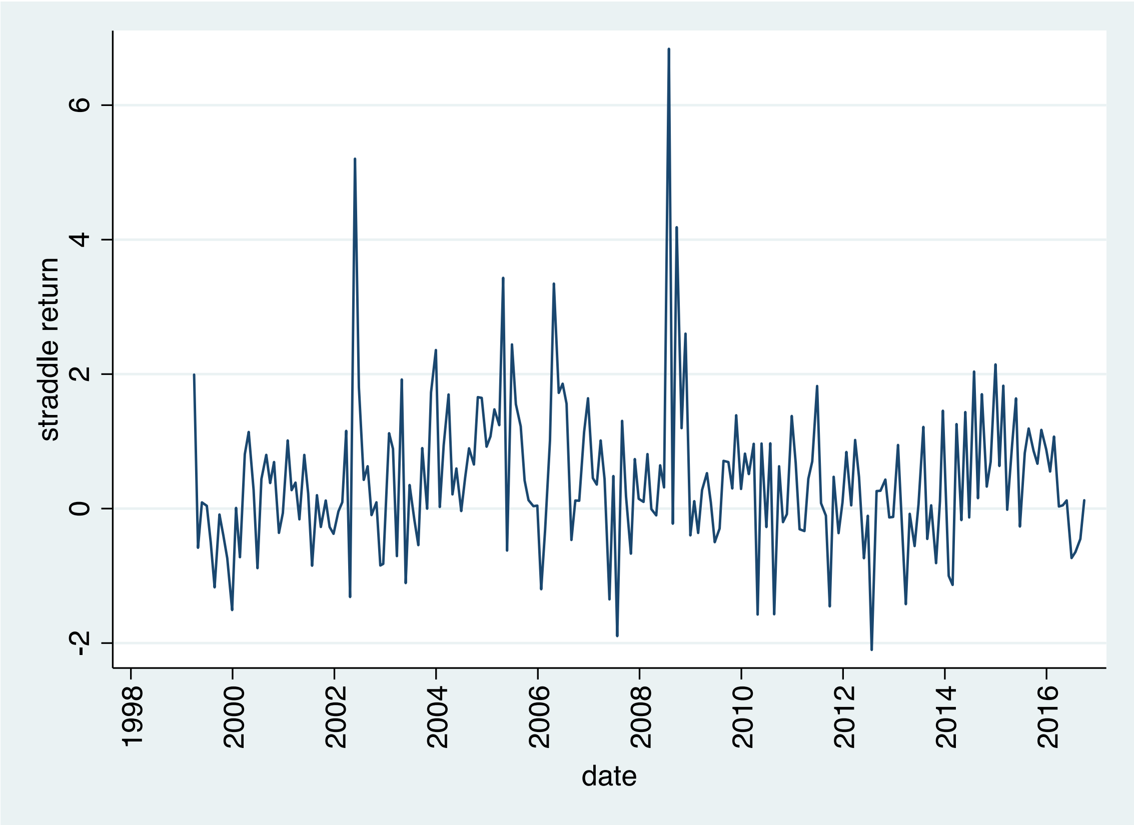


Figure 2. *LMH* Straddle Returns, Long Leg

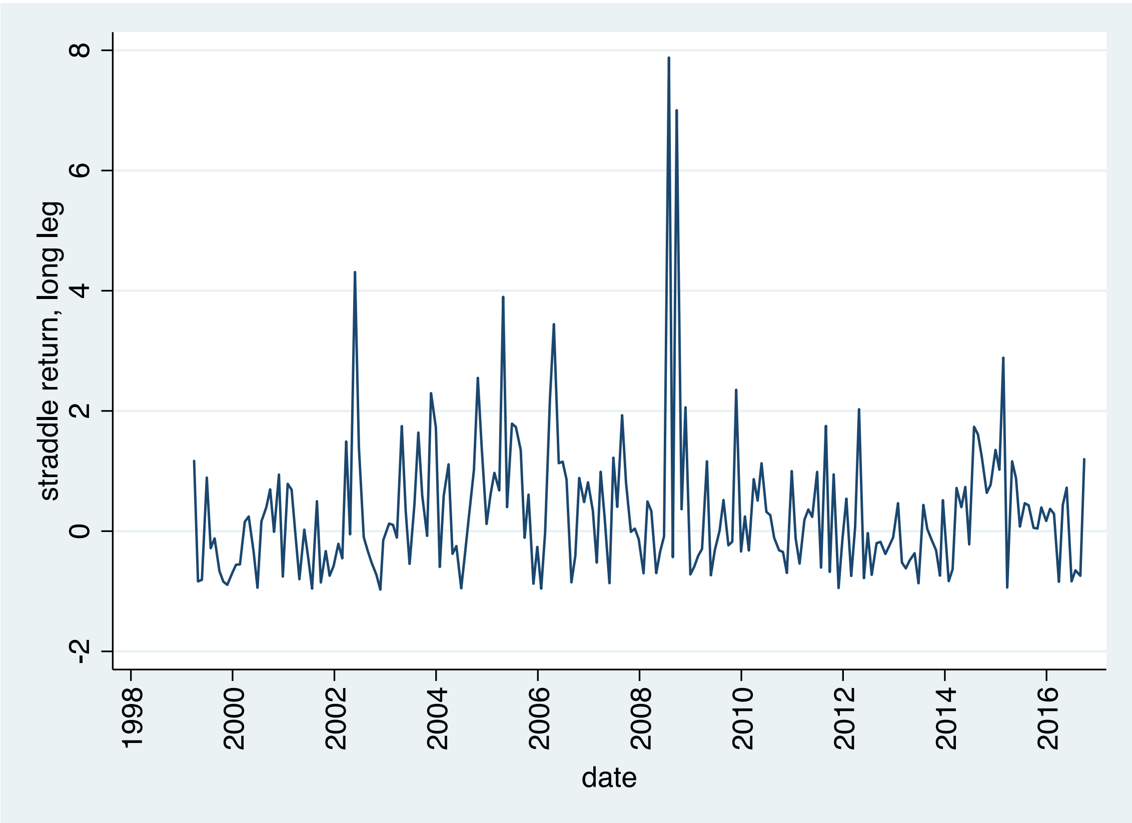


Figure 3. *LMH* Straddle Returns, Short Leg

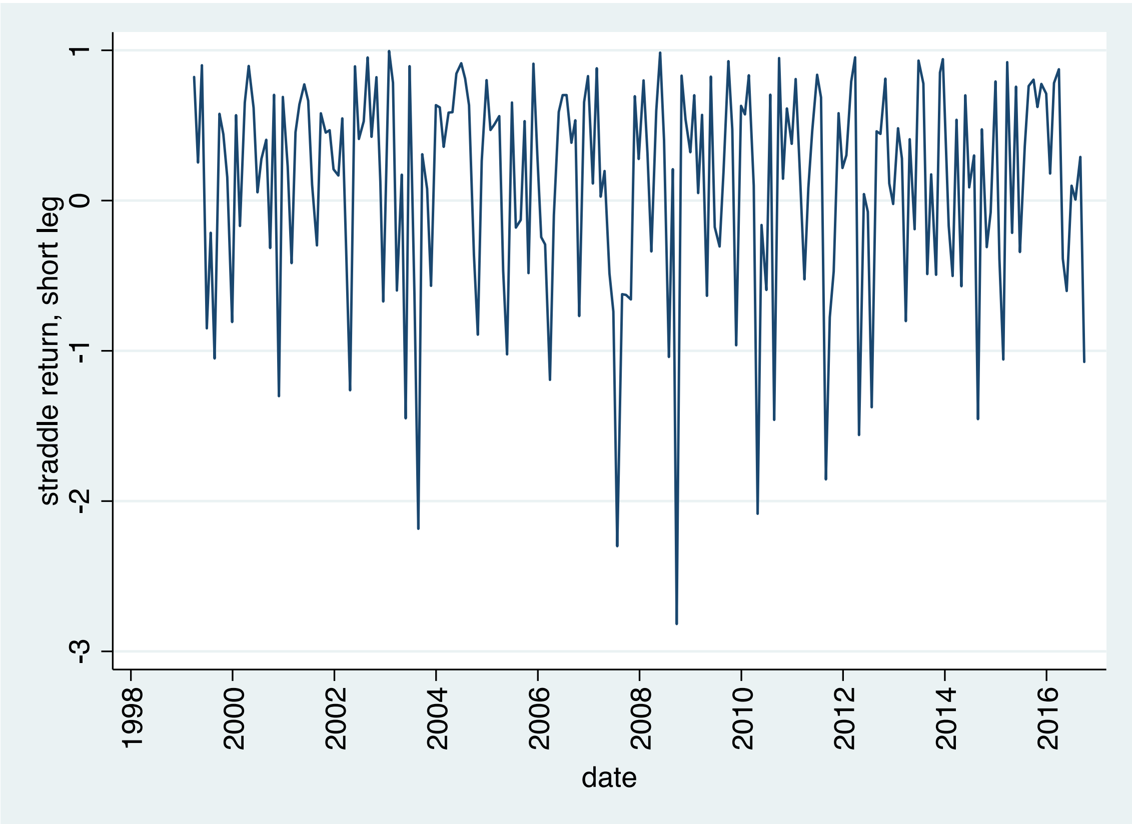
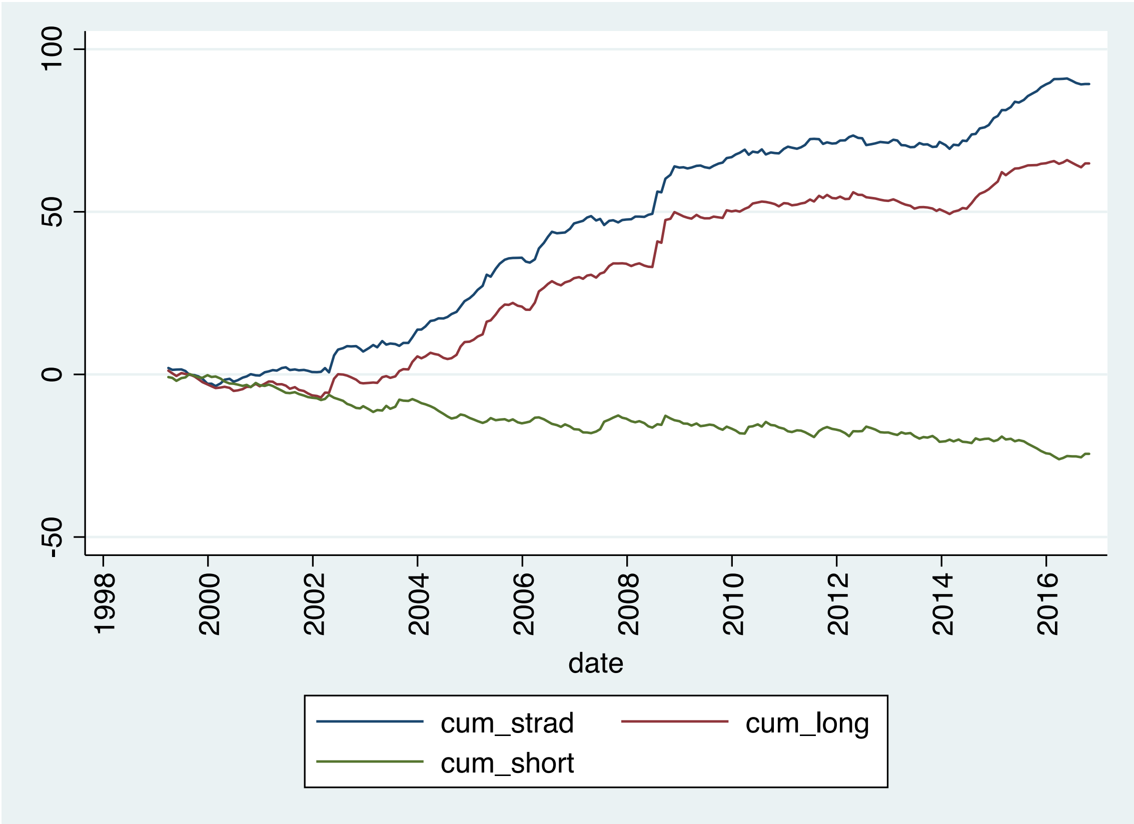


Figure 4. *LMH* Straddle Cumulative Returns, Long and Short Legs



Note: Short leg returns not inverted for clarity (ie these are returns to long position)

Figure 5. *LMH* 25 Delta Butterfly Returns, Monthly Sampled Data

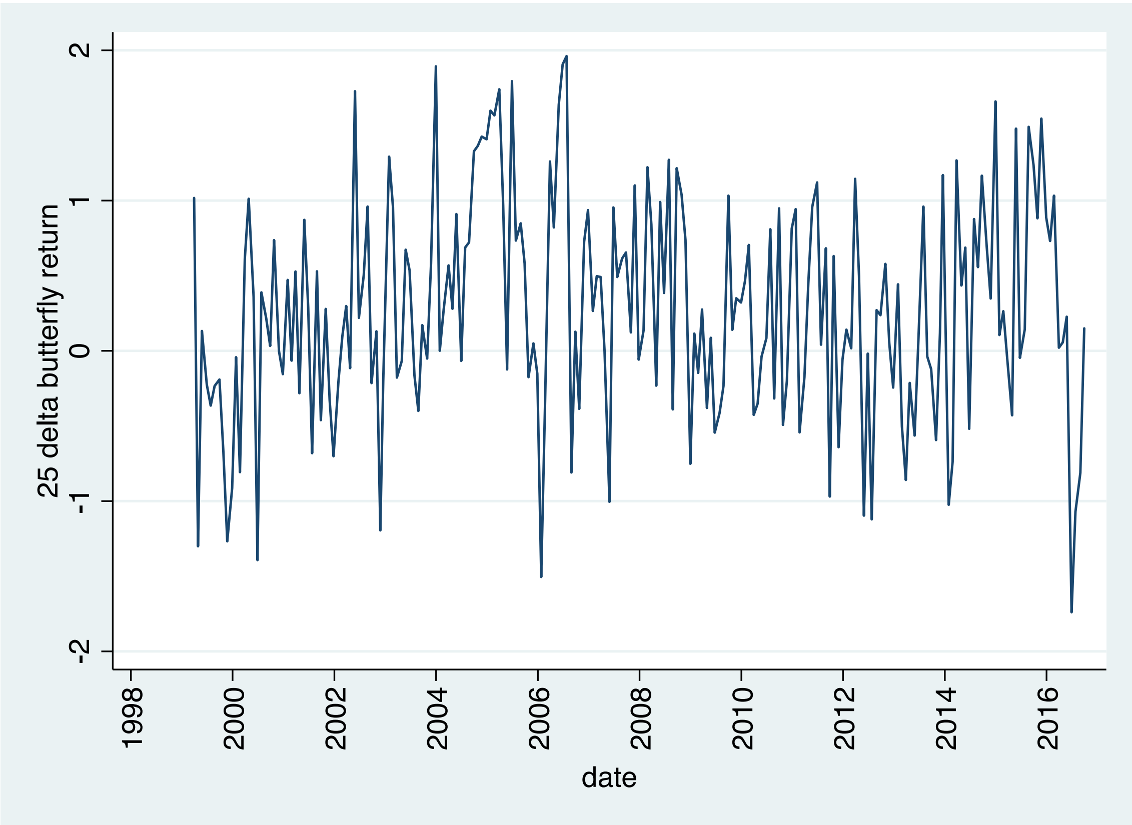
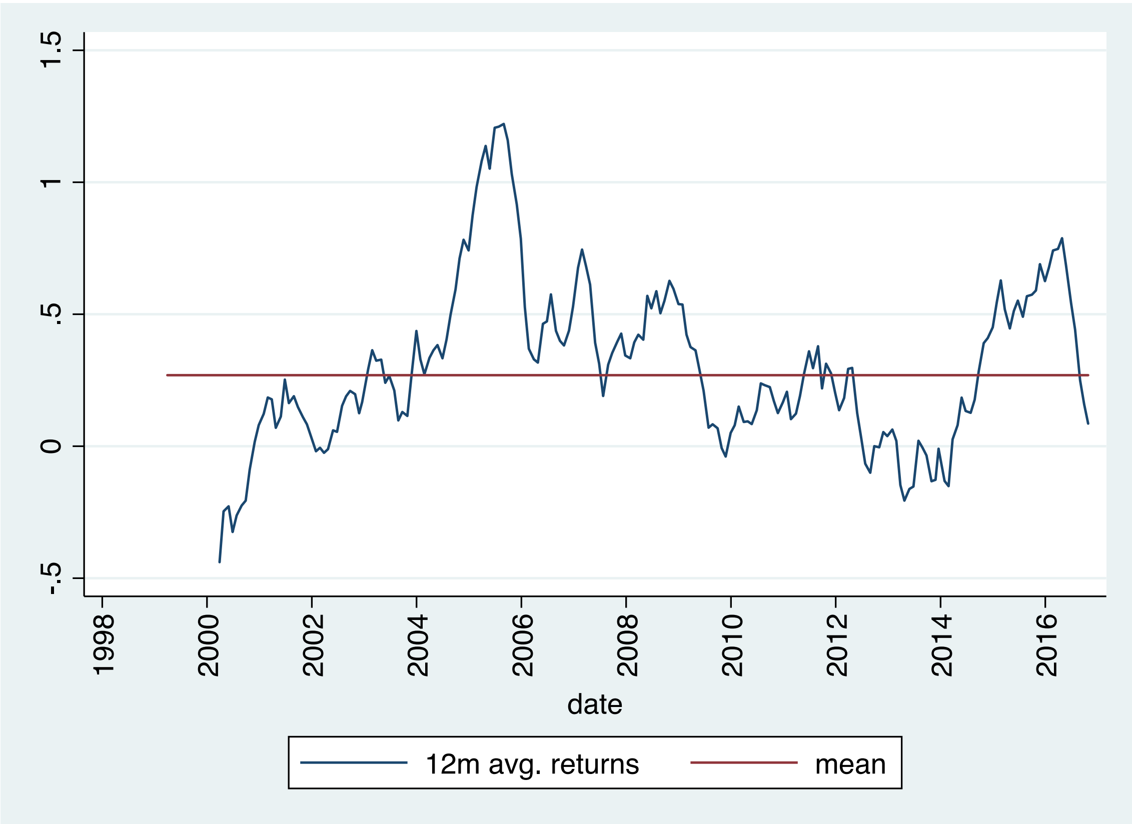


Figure 6. *LMH* 25 delta Butterfly Returns, 12 Month Moving Average



1.4. Appendix – Robsutness to jumps

We examine whether the results are primarily driven by periods when Chernov et al. (2018) identify jumps in exchange rates and/or volatility. CGZ identify jumps in the exchange rate and/or in volatility for four representative currencies (AUD, CHF, GBP and JPY) and for the 21-currency dollar index of Lustig, Roussanov and Verdelhan (2014). The AUD is quite frequently the highest IV currency while the GBP is very often the lowest IV currency and hence these two currencies often form part of the trading positions of our strategies. The CHF and JPY are less involved.

Table 9 gives mean returns of the LMH straddle and 25 delta butterfly using the full data set and excluding various permutations of CGZ's jump periods. We also exclude the whole of 2008 since this is the best performing calendar year for the strategy and is also a year full of CGZ jumps. We use the monthly sampled data and exclude returns for all months including an identified jump irrespective of whether that currency was involved in the trade that month.

Table 9
Straddle and Butterfly Mean Returns Excluding Identified Jump Months, Data Sampled Monthly

This table reports mean one-month holding period returns to LMH straddle and 25-delta iron butterfly positions on currencies sorted by implied volatilities. Data are sampled monthly to avoid ssues with overlapping observations. Months are excluded if the holding period includes one or more jumps as defined by CGZ (2008). CGZ identify jumps in the exchange rate ("rate jumps") and jumps in variance ("variance jumps"). They identify jumps in individual curencies ("currency jumps") and in the dollar index ("index jumps").

	Observations	LMH straddle	25 delta butterfly
Full sample	211	0.4253	0.2727
Excluding all jumps	151	0.4530	0.3234
Excluding jumps in index	190	0.3836	0.2690
Excluding only currency jumps	154	0.4452	0.3212
Excluding all rate jumps	188	0.4279	0.2818
Excluding all variance jumps	162	0.4296	0.3117
Excluding 2008	199	0.3707	0.2567

Removing months subject to identified jumps, if anything, increases the mean return on the strategies. Excluding just jumps in exchange rates or excluding just jumps in volatility barely affects mean returns. Excluding jumps in the 21-currency index (which are overwhelmingly jumps in volatility) does reduce mean straddle returns to 38% from 42.5%. Since many of the index jumps occur during 2008, we simply exclude this year in the final row of the table. This has the largest effect on returns, reducing straddles returns by 5% per month to 37%. Nevertheless, returns on the LMH straddle and 25 delta butterfly remain very large.

This result is also not inconsistent with our main explanation for the high returns. The height of the financial crisis would seem to be a period when the demand for hedging is high and the supply of arbitrage capital low. These are times when the mispricing we propose is most likely to occur. The mean return of the straddle strategy in 2008 was 133%, with both long and short legs contributing positively.

1.5. Appendix – Equilibrium expected returns: Calibration exercise

Period is $[0,T]$

S_t is the spot exchange rate (home currency per unit of foreign currency)

K is the strike price (the forward rate here)

r is the home interest rate

d is the foreign interest rate

μ is the expected rate of change in the spot rate

σ is the volatility of the spot price

C_t and P_t are the prices of calls and puts measured in home currency

Garman-Kohlhagen gives:

$C_0 = S_0 e^{-dT} N(d_1) - K e^{-rT} N(d_2)$

$P_0 = K e^{-rT} N(-d_2) - S_0 e^{-dT} N(-d_1)$

where

$d_1 = \dfrac{ln(s_0 e^{-dT}/K e^{-rT})}{\sigma\sqrt{T}} + \dfrac{1}{2}\sigma\sqrt{T}$

and

$d_2 = d_1 - \sigma\sqrt{T}$

If S is growing at rate μ rather than $r - d$ then:

$E_0[C_T] = S_0 e^{\mu T} N(d_1^*) - K N(d_2^*)$

$E_0[P_0] = K N(-d_2^*) - S_0 e^{\mu T} N(-d_1^*)$

where

$d_1^* = \dfrac{ln(S_0 e^{\mu T}/K)}{\sigma\sqrt{T}} + \dfrac{1}{2}\sigma\sqrt{T}$

and

$d_2^* = d_1^* - \sigma\sqrt{T}$

At the money straddles:

We are interested in $X_t = C_t + p_t$ with $K = S_0 e^{(r-d)T}$.

Substituting in from above

$X_0 = 2S_0 e{-}dT \left(2N\left(\frac{1}{2}\sigma\sqrt{T}\right) - 1\right)$

$E_0[X_T] = S_0 e^{\mu T}\left\{(2N(d_1^*) - 1 - e^{(-r+d-\mu)T}\,(2N(d_2^*) - 1)\right\}$

where $d_1^* = \frac{(\mu-r+d)\sqrt{T}}{\sigma} + \frac{1}{2}\sigma\sqrt{T}$

and

$d_2^* = d_1^* - \sigma\sqrt{T}$.

The expected return on the straddle, then, is:

$R^{straddle} = \frac{E_0[X_T]}{X_0} = e^{(\mu+d)T}\frac{\left(2N\left(a+\frac{1}{2}\sigma\sqrt{T}\right)-1\right)+e^{(-1+d-\mu)T}\left(2N\left(\frac{1}{2}\sigma\sqrt{T}-a\right)-1\right)}{2\left(2N\left(\frac{1}{2}\sigma\sqrt{T}\right)-1\right)}$

where

$a = \frac{(\mu-r+d)\sqrt{T}}{\sigma}$

Or expressing this in terms of a standardised risk premium λ:

$R^{straddle} = e^{rT}\frac{2e^{\lambda\sigma^2 T}N\left(\left(\frac{1}{2}+\lambda\right)\sigma\sqrt{T}\right)+2N\left(\left(\frac{1}{2}-\lambda\right)\sigma\sqrt{T}\right)-e^{\lambda\sigma^2 T}-1}{2\left(2N\left(\frac{1}{2}\sigma\sqrt{T}\right)-1\right)}$

where

$\lambda = \frac{\mu-r+d}{\sigma^2}$.

Simple calibration exercise:

In all our analysis T is one month (=1/12).

As noted in the paper, the interest rate differential between currencies in all portfolios is very small but to heighten the expected return we use the average AUD deposit rate of 5.13% (the highest average for any currency in the analysis) and the average USD deposit rate of 2.46%.

We assume a zero drift in the spot rate consistent with spot exchange rates following a random walk.

The average implied volatility for P1 is 7.9%. This gives an expected return on premium for ATMF straddles of 0.65%.

The average implied volatility for P9 is 13.1%, giving an expected return on premium for ATMF straddles of 0.45%.

Both of these expected returns are far from the actual returns we observe (25% for P1 and -13.9% for P9).

For an expected return on P1 equal to the observed mean, keeping interest rates and implied volatilities at the above values, the drift in the spot rate would need to be +17.25%.

For an expected return on P9 equal to the observed mean, keeping interest rates and implied volatilities at the above values, the minimum expected return is 0.23% when the drift in the spot rate is set to -3.1%. Negative expected returns can be generated with negative interest rates and a small negative drift in the spot rate for very high implied volatilities. Setting $r = -3\%$, $d = -1\%$, $\mu = -4\%$ and σ equal to the highest implied volatility value in the sample for any currency (39.5%) gives an expected return on the straddle of -0.32%.

It is clear that reasonable parameter values within the Garman-Kohlhagen framework are not capable of generating expected returns on straddles close to those observed in the data.

1.6. Appendix – Portfolio value

The cross-sectional ranking of currencies based on implied volatilities appears to provide robust patterns in the subsequent returns of straddles in those currencies. Here we consider how a long-short straddle strategy based on such rankings performs in a portfolio framework. The alternative assets we consider are long-short straddle strategies formed from ranking on (i) the difference between historical realised and curreny implied volatilities ($RV - IV$), (ii) the change in implied volatility over the previous month (dIV) and (iii) the term structure of implied volatility calculated as the difference between implied volatilities of options with six and one month to maturity ($TSIV$). Each strategy is motivated based on positive evidence both in the extant equity options literature and in our subsequent analysis using FX options, as shown in Table **??**.[1] We demonstrate the added value of the volatility-based LMH strategy in an in-sample portfolio allocation exercise. First, we estimate mean returns after transactions costs to each strategy and the associated variance-covariance matrix using the full sample of data. We then form mean-variance efficient portfolios using the strategies previously identified in the literature and subsequently consider the value of adding our new LMH strategy based on implied volatility ranking.

Table 10 reports the results. The top section of the first column reports the vector of optimal weights allocated to each of the three alternative strategies in forming the tangent portfolio with a risk-free rate of return of 3%p.a.[2] The optimal portfolio is formed with one-third allocated to the strategy based on changes in implied volatilities and two-thirds allocated to the strategy based on the term structure of implied volatilities. The lower section of column (1) gives the return and risk (standard deviation) of this tangent portfolio. Column (2) repeats this exercise but now adds the long-short strategy based on implied volatility. This strategy receives the greatest weighting across all four strategies and, in particular, reduces the weight allocated to the term structure-based strategy. Adding the IV-based strategy both increases the expected return of the tangent portfolio and reduces its risk. Accordingly, the annual Sharpe ratio increases by 0.34 with the addition of the LMH strategy. Columns (3) and (4) present analogous results for the global minimum variance portfolio. In this case, while the implied volatility-based strategy does not receive a particularly high portfolio weight it still contributes to raising the expected returns, reducing the risk, and increasing the Sharpe ratio of the optimised portfolio, in this case by 0.29.

[1] Each long-short strategy is constructed as high-minus-low or low-minus-high such that the expected return to the strategy is positive.

[2] The results are insensitive to the exact value of the risk-free rate used.

Table 10
Long-short Strategies – Portfolio Allocations

The top half of this table reports portfolio allocations to each of the long-short strategies lisetd in the first column. An in-sample mean-variance optimisation is performed using the full data sample. The first two columns report the portfolio allocations for the tangent portfolio, while columns (3) and (4) report the allocations for global minimum variance portfolios. Columns (1) and (3) exclude the IV-based strategy while columns (2) and (4) include it. The bottom half of the table report performance statistics of the portfolios.

	Tangent Port.		**GMV Port.**	
Asset	(1)	(2)	(3)	(4)
IV		0.392		0.177
$RV - IV$	0.040	0.015	0.330	0.281
dIV	0.307	0.256	0.445	0.404
$TSIV$	0.653	0.337	0.225	0.138
Ret_{port}	0.255	0.300	0.153	0.186
$Risk_{port}$	0.728	0.666	0.561	0.523
Sharpe ratio	1.201	1.547	0.929	1.215

Chernov, M., Graveline, J., Zviadadze, I., 2018. Crash risk in currency returns. Journal of Financial and Quantitative Analysis 53, 137–170.