Technical Appendix to

Entry and Competition in Takeover Auctions

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February 18, 2018

Appendix A. Entry Equilibrium

First, we formally obtain an expression for a particular bidder's expected profits conditional on entry. All variables are sale specific, so in this section we suppress both j subscripts and the target-level primitives θ_j to ease exposition. All derivations below should be interpreted as conditional on a particular realization of target-level primitives θ_j.

Let s_N^* denote the equilibrium threshold characterizing entry behavior among the N potential bidders for target j, $F^*(\cdot|N) \equiv F(\cdot|S_i \geq s_N^*)$ be the c.d.f. characterizing the selected distribution of valuations for each of the n bidders electing to enter, and $F_0(\cdot)$ be the distribution of the target's reservation value V_0. Let $Y_{k:n}$ denote the kth highest valuation among n entering bidders, let $y_{k:n}$ denote the realization of this random variable, and let v_0 denote the realization of the target's reservation value V_0. If $y_{1:n} \geq v_0$, the target is sold at $p = \max\{y_{2:n}, v_0\}$ so conditional on realizations of all random variables, the surplus of bidder with valuation v_i is thus

$$\begin{aligned} \mathbf{1}[v_i &\geq \max\{y_{1:n-1}, v_0\}]\,(v_i - p) \\ &= \mathbf{1}[v_i \geq \max\{y_{1:n-1}, v_0\}]\,(v_i - \max\{y_{1:n-1}, v_0\})\,. \end{aligned} \tag{A.1}$$

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Let $H^*_n(\cdot|N)$ be the equilibrium CDF of the random variable $\max\{Y_{1:n-1}, V_0\}$:

$$H^*_n(y|N) = F_0(y) \cdot F^*(y|N)^{n-1}. \tag{A.2}$$

By definition, $H^*_n(v|N)$ is the probability that a bidder with valuation v is the final standing bidder, with the associated density

$$h^*_n(v|N) = f_0(v) \cdot F^*(v|N)^{n-1} + (n-1)F_0(v)F^*(v|N)^{n-1}f^*(v|N), \tag{A.3}$$

describing the distribution of the bidder's outside option in this case, so the expected profit of an entrant with valuation v_i is thus

$$\begin{aligned} \pi^*(v_i; n, N) &= H^*_n(v_i|N) \int_0^{v_i} (v_i - y) \cdot \frac{h^*_n(y|N)}{H^*_n(y|N)} dy \\ &= \left[v_i H^*_n(v_i|N) - \int_0^{v_i} y \; h^*_n(y|N) \; dy \right] \\ &= \int_0^{v_i} \; F_0(y) \cdot F^*(y|N)^{n-1} \; dy, \end{aligned} \tag{A.4}$$

where the last equality follows from integration by parts.

Having obtained an expression for an entering bidder's expected profits, we now characterize the symmetric monotone pure strategy Bayesian Nash equilibrium. In any such equilibrium, entry decisions can be characterized by a signal threshold s^*_N such that bidder i chooses to enter if and only if $S_i \geq s^*_N$:

$$F^*(v; s^*_N\ ) = F(v|S_i \geq s^*_N\ ) = \frac{1}{1 - F_s(s^*_N\ )} \int_{s^*_N}^{\infty} F(v|t) \; f_s(t) \; dt, \tag{A.5}$$

The CDF of the distributions of valuations among entrants is then $F^*(v|N) = F^*(v; {s^*}_N)$. The following identity will be useful: for any (v, s^*),

$$(1 - F_s(s^*))\, F^*(v; s^*) = \int_{s^*}^{\infty} F(v|t)\ f_s(t)\ dt = F_v(v) - F_{vs}(v, s^*). \tag{A.6}$$

Independence of signals implies that the total number of entrants n follows a binomial distribution based on the entry probability $[1 - F_s(s_N^*)]$. Now consider the entry decision of potential acquirer i drawing signal realization $S_i = s_i$. Conditional on own signal s_i, the equilibrium threshold s_N^*, and total competition N, a potential bidder forecasts profits $\Pi(s_i; s_N^*, N)$. Expanding this term yields,

$$\begin{aligned}
&= E_V\left[E_n[\pi^*(v_i; n, N)|n \geq 1]|S_i = s_i\right] && \text{(A.7)}\\
&= \int_0^{\infty}\int_0^{v} F_0(y)\left[\sum_{n=1}^{N}\begin{pmatrix} N-1 \\ n-1 \end{pmatrix} F_s(s_N^*)^{N-n}\left([1 - F_s(s_N^*)]F^*(y; s_N^*)\right)^{n-1}\right] dy\ dF(v|s_i)\\
&= \int_0^{\infty}\int_0^{v} \left[F_0(y)\left[F_s(s_N^*) + (1 - F_s(s_N^*))F^*(y; s_N^*)\right]^{N-1}\right] dy\ dF(v|s_i)\\
&= \int_0^{\infty}\int_0^{v} \left[F_0(y)\left[F_s(s_N^*) + F_v(y) - F(y, s_N^*)\right]^{N-1}\right] dy\ dF(v|s_i),
\end{aligned}$$

where the third equality follows by properties of binomial series.

Reversing the order of integration yields our main expression for *ex ante* expected profit for potential acquirer with Stage 1 signal $S_i = s_i$:

$$\Pi(s_i; s_N^*, N) = \int_0^{\infty} [1 - F(v|s_i)] \cdot F_0(y) \cdot [F_s(s_N^*) + F_v(y) - F(y, s_N^*)]^{N-1}\, dy. \tag{A.8}$$

$F(v|s_i)$ is decreasing in s_i by stochastic dominance, and $F_s(s_N^*) + F_v(y) - F(y, s_N^*)$ is increasing in s_N^* by the identity

$$F_s(s_N^*) + F_v(y) - F(y, s_N^*) = F_s(s_N^*) + \int_{s^*}^{\bar{s}} F(v|t)\ f_s(t)\ dt \tag{A.9}$$

and it is easy to show that $F_s(s_N^*) + F_v(y) - F(y, s_N^*) \in [0, 1]$.

We now characterize equilibrium entry decisions. Bidder i enters into competitive bidding if expected profit from doing so is positive, so the equilibrium threshold s^*_N must thus satisfy the break even condition:

$$\Pi(s^*_N; s^*_N, N) - c = 0. \tag{A.10}$$

In other words, a marginal potential bidder with signal $S_i = s^*_N$ must be indifferent between entering and not entering. $\Pi(s_i; s^*_N, N)$ is increasing in its first argument and is strictly increasing in its second argument, so the break even condition (A.10) has a unique solution s^*_N. Further, since $\Pi(s_i; s^*_N, N)$ is decreasing in N, this solution s^*_N is increasing in N. Finally, by the form of the entry decision rule, the distribution of valuations among entering bidders is $F^*(v; s^*_N) = F(v|S_i \geq s^*)$. The signal threshold s^*_N is thus sufficient to characterize equilibrium entry behavior.

Appendix B. Estimation Algorithm

Recall the objective of our structural estimation procedure: to recover the deep structural parameters Γ_0 governing the distribution $g(\theta|X_j, \Gamma_0)$ of the target-level characteristics θ_j, accounting for the facts that (1) individual realizations of θ_j are unobserved to the econometrician, and (2) we observe only auctions resulting in sale. Toward this end, we consider maximum likelihood estimation based on events of the following form: for a given target j, n_j of bidders enter and the final sale price is $P_j = p_j$, conditional on target j inviting N_j potential bidders and holding an auction which results in sale. Integrating over unobserved target-level characteristics θ_j, we thereby obtain a target-level structural conditional likelihood function (in Γ) of the form:

$$\begin{aligned} L_j(p_j, n_j|sale_j = 1, N_j, X_j; \Gamma) = \\ \int f_p(p_j|sale_j, n_j, N_j; \theta_j) \cdot \Pr(sale_j|n_j, N_j; \theta_j) \cdot \Pr(n_j|N_j; \theta_j)\ g(\theta_j|X_j, \Gamma)\ d\theta_j \\ \Big/ \int \Pr(sale_j|n_j, N_j; \theta_j) \cdot \Pr(n_j|N_j; \theta_j)\ g(\theta_j|X_j, \Gamma)\ d\theta_j, \end{aligned} \tag{6}$$

In describing our procedure for estimating Γ based on equation (6), we proceed in four steps. First, we describe how we solve for the equilibrium entry threshold $s^*(N_j, \theta_j)$, which is the key prediction of the structural entry and bidding model above. Second, we discuss computation of the equilibrium objects $\Pr(n_t|N_t;\theta_j)$, $\Pr(sale_t|N_t;\theta_j)$, and $f_p(p_t|sale_t,n_t,N_t;\theta_j)$ appearing in (6). Third, we describe the importance sampling procedure by which we evaluate the integrals over θ_j appearing in the numerator and denominator of (6). Finally, we discuss the Markov Chain Monte Carlo algorithm by which we maximize (6) to recover point estimates and confidence intervals for $\hat{\Gamma}$.

B.1. Solving for the equilibrium entry threshold

Consider a takeover auction among N_j potential bidders competing for a target j with characteristics θ_j (observed to bidders, but unobserved to us). Let $s^*(N_j;\theta_j)$ denote the signal threshold characterizing equilibrium entry behavior in this takeover environment. From Equation (A.10), we know that we may compute this threshold $s^*(N_j;\theta_j)$ as the unique solution s^* to the breakeven condition:

$$c(\theta_j) = \int_0^\infty [1 - F_{v|s}(y|s^*;\theta_j)] \cdot F_0(y;\theta_j) \cdot H_{N_j}(y;\theta_j)dy, \tag{B.1}$$

where $H_{N_j}(\cdot;s^*,\theta_j)$ denotes the expected c.d.f. of the maximum valuation realized (through entry) among the N_j-1 potential rivals of each bidder i, accounting for the fact that some of these rivals may not enter in equilibrium:

$$H_{N_j}(y;s^*,\theta_j) \equiv [F_s(s^*;\theta_j) + F_v(y;\theta_j) - F_{vs}(y,s^*;\theta_j)]^{N_j-1}.$$

Taking $\theta_j \equiv (\mu_{vj},\sigma_{vj}^2,c_j,\alpha_j)$ as given, (V_j,S_j) are jointly log-normal with mean vector $[\mu_{vj},\mu_{vj}]$ and variance-covariance matrix

$$\text{Var}\left(\begin{bmatrix} V_j \\ S_j \end{bmatrix}\right) = \begin{bmatrix} \sigma_{vj}^2 & \sigma_{vj}^2 \\ \sigma_{vj}^2 & \frac{\sigma_{vj}^2}{1-\alpha_j} \end{bmatrix}.$$

Meanwhile, the conditional distribution of V_j given $S_j = s^*$ is normal with mean $\alpha_j\mu_{vj} + (1-\alpha_j)s^*$ and variance $\alpha_j\sigma_{vj}^2$. Given θ_j, computation of both $H_{N_j}(y;s^*;\theta_j)$ and $F_{v|s}(y|s^*)$ is therefore straightforward. For given θ_j and N_j, we may therefore solve (B.1) numerically to obtain the equilibrium entry threshold $s^*(N_j;\theta_j)$. In practice, we approximate this solution by interpolation over a fine grid in $\log s^*$, computing the right-hand side integral by the trapezoidal rule over a fine grid in $\log v$.[1]

B.2. Computing equilibrium objects in the likelihood function

With the equilibrium entry threshold $s^*(N_j;\theta_j)$ in hand, we turn to computation of the equilibrium objects $\Pr(n_t|N_t;\theta_j)$, $\Pr(sale_t|N_t;\theta_j)$, and $f_p(p_t|sale_t, n_t, N_t;\theta_j)$ appearing in (6).

Toward this end, first consider $\Pr(n_t|N_t;\theta_j)$. By construction, potential acquirers drawing signals $S_{ij} \geq s^*(N_j,\theta_j)$ elect to enter in equilibrium. The probability that any given potential acquirer elects to enter is therefore

$$q(N_j,\theta_j) = 1 - F_s(s^*(N_j,\theta_j);\theta_j). \tag{B.2}$$

Furthermore, conditional on θ_j, signal draws are independent across potential aquirers. Taking N_j and θ_j as given, the distribution of n_j therefore follows a binomial distribution with success probability $q(N_j,\theta_j)$:

$$\Pr(n_j|N_j,\theta_j) = \begin{pmatrix} N_j \\ n_j \end{pmatrix} q(N_j,\theta_j)^{n_j}(1-q(N_j,\theta_j))^{N_j-n_j}. \tag{B.3}$$

Next consider $\Pr(sale_j|N_j;\theta_j)$. By construction, the auction for target j ends in sale whenever at least one entering bidder draws a valuation above the seller's reservation value V_{0j}. It follows that:

$$\begin{aligned}\Pr(\text{sale}_j|N_j;\theta_j) &= \Pr(V_{0j} \leq Y_{1:N_j}|N_j,\theta_j) = 1 - \Pr(Y_{1:N_j} \leq V_{0j}|N_j,\theta_j) \\ &= 1 - \int_0^\infty \left[F_s(N_j;\theta_j) + F_v(v_0;\theta_j) - F_{vs}(v_0, s^*(N_j,\theta_j);\theta_j)\right]^{N_j}\ f_0(v_0,\theta_j)\ dv_0, \end{aligned} \tag{B.4}$$

[1] In practice, we consider grids of 200 points in both $\log s$ and $\log v$, with grid support between the 10^{-6}th and $(1-10^{-6})$th quantiles of $\log S$ and $\log V$. Numerical simulations confirm that the resulting solution is quite accurate in practice.

where (as above) the term in brackets represents the probability that potential acquirer i either does not enter or enters but draws a valuation less than v_0. As above, taking N_j and θ_j as given, the right-hand side integral is straightforward to compute, yielding a numeric solution for $\Pr(\text{sale}_j|N_j;\theta_j)$.[2]

Finally, consider the density $f_p(p_j|sale_j, n_j, N_j;\theta_j)$; i.e. the distribution of the sale premium p_j when target characteristics are θ_j, n_j of N_j potential bidders enter, and the auction results in sale. In characterizing this distribution, we adopt the following convention: through expressions such as

$$\Pr(sale_j \cap P_j = p|n_j, N_j;\theta_j) \tag{B.5}$$

we intend to indicate to the mixed joint density of the discrete random variable $sale_j$ and the continuous random variable P_j; i.e. more precisely,

$$\Pr(sale_j \cap P_j = p|n_j, N_j;\theta_j) := \lim_{h\downarrow 0} \frac{Pr(sale_j \cap P_j \in [p, p+h]|n_j, N_j;\theta_j)}{h}. \tag{B.6}$$

Applying this convention, we have by construction:

$$f_p(p_j|sale_j, n_j, N_j;\theta_j) = \frac{\Pr(sale_j \cap P_j = p_j|n_j, N_j;\theta_j)}{\Pr(sale_j|n_j, N_j;\theta_j)}. \tag{B.7}$$

$\Pr(sale_j|n_j, N_j;\theta_j)$ was characterized above, so having obtained an expression for $\Pr(sale_j \cap P_j = p_j|n_j, N_j;\theta_j)$ the argument will be complete.

By construction, a sale occurs when at least one entrant draws a valuation above the seller's reservation value v_{0j}. If only one entrant draws a valuation above v_{0j}, the transaction price p_j is the seller's reservation valuation v_{0j}. If at least two entrants draw valuations above v_{0j}, the transaction price p_j is the second highest entrant valuation $y_{2:n_j}$. Decomposing likelihoods of these events using

[2] In practice, as in computing $s^*(N_j;\theta_j)$ above, we approximate this integral via the trapezoidal rule on a grid of 200 points in $\log v_0$, with grid points spaced evenly (in $\log v_0$) between the 10^{-6}th and $(1-10^{-6})$th quantiles of $\log V_0$.

properties of order statistics yields the overall mixed density $\Pr(sale_j \cap P_j = p_j | n_j, N_j; \theta_j)$

$$
\begin{aligned}
&= \Pr(sale_j \cap Y_{2:n_j} = p_j | n_j, N_j, \theta_j) + \Pr(sale_j \cap V_{0j} = p_j | n_j, N_j, \theta_j) \\
&= \Pr(Y_{1:n_j} \geq p_j \cap Y_{2:n_j} = p_j \cap V_{0j} \leq p_j | n_j, N_j, \theta_j) \\
&\quad + \Pr(Y_{1:n_j} \geq p_j \cap Y_{2:n_j} \leq p_j \cap V_{0j} = p_j | n_j, N_j, \theta_j) \\
&= \left[n_j(n_j - 1) F^*(p_j; N_j, \theta_j)^{n_j - 2} [1 - F^*(p_j; N_j, \theta_j)] f^*(p_j; N_j, \theta_j) \right] \cdot F_0(p; \theta_j) \\
&\quad + \left[n_j F^*(p_j; N_j, \theta_j)^{n_j - 1} [1 - F^*(p_j; N_j, \theta_j)] \right] \cdot f_0(p_j; \theta_j).
\end{aligned}
$$

where as above $F^*(v|N_j, \theta_j)$ denotes the equilibrium distribution of valuations among entrants at (N_j, θ_j):

$$
F^*(v|N_j, \theta_j) = F(v|S_i \geq s^*(N_j, \theta_j)) = \frac{F_v(v; \theta_j) - F_{vs}(v, s^*(N_j, \theta_j); \theta_j)}{1 - F_s(s^*(N_j; \theta_j); \theta_j)} \tag{B.8}
$$

Again, taking θ_j and $s^*(N_j; \theta_j)$ as given, the distribution (B.8) can easily be computed as above. Having solved for the equilibrium entry threshold $s^*(N_j; \theta_j)$, computation of the equilibrium objects $\Pr(n_t|N_t; \theta_j)$, $\Pr(sale_t|N_t; \theta_j)$, and $f_p(p_t|sale_t, n_t, N_t; \theta_j)$ for given N_j, θ_j thus becomes a reasonably straightforward numerical exercise.

B.3. Importance sampling approach to integration over θ_j

Give numeric expressions for the equilibrium objects $\Pr(p_j, sale_j|n_j, N_j; \theta)$, $\Pr(n_j|N_j, \theta)$, and $\Pr(sale_j|N_j; \theta)$ obtained as above, we can in principle evaluate the likelihood (6) directly by computing the numerator and denominator integrals

$$
\int \Pr(p_j, sale_j|n_j, N_j; \theta) \Pr(n_j|N_j, \theta) \ g(\theta|X_j, \Gamma) \ d\theta \tag{B.9}
$$

and

$$
\int \Pr(sale_j|N_j; \theta) \ g(\theta|X_j, \Gamma) \ d\theta. \tag{B.10}
$$

Direct evaluation of the likelihood function is computationally prohibitive in practice since (B.9) and (B.10) depend on θ through the equilibrium condition (B.1), which itself requires solution of an equation involving integrals. We circumvent this challenge by implementing estimation via the simulated likelihood method of Ackerberg (2009)[3], which uses the principle of importance sampling to transform the complicated problem of repeated evaluation of the full likelihood into the much simpler problem of repeated evaluation of $g(\theta|X_j,\Gamma)$.

To illustrate the main idea of this method, let $\tilde{g}(\cdot)$ be any fixed *proposal distribution* over θ, and consider evaluation of the sale-level likelihood integral (B.9). By standard importance sampling arguments, we can rewrite this integral as follows:

$$\begin{aligned} & \int \Pr(p_j, sale_j|n_j, N_j;\theta)\Pr(n_j|N_j,\theta)\, g(\theta|X_j,\Gamma)\, d\theta \\ = & \int \left[\Pr(p_j, \mathrm{sale}_j|n_j, N_j;\theta)\Pr(n_j|N_j,\theta)\ \frac{g(\theta|X_j,\Gamma)}{\tilde{g}(\theta)}\right]\tilde{g}(\theta)\ d\theta \\ = & \ \tilde{E}\left[\Pr(p_j, \mathrm{sale}_j|n_j, N_j;\theta)\Pr(n_j|N_j,\theta)\ \frac{g(\theta|X_j,\Gamma)}{\tilde{g}(\theta)}\right], \end{aligned} \tag{B.11}$$

where the expectation in the last line is taken with respect to the proposal distribution $\tilde{g}(\cdot)$ rather than the true distribution $g(\cdot|X_j,\Gamma)$. If $\left\{\tilde{\theta}_r\right\}_{r=1}^{R}$ is a random sample drawn from $\tilde{g}(\cdot)$, it follows that for large enough R

$$\begin{aligned} & \int \Pr\left(p_j, sale_j|n_j, N_j;\theta\right)\Pr\left(n_j|N_j,\theta\right)\ g(\theta|X_j,\Gamma)\ d\theta \\ \approx & \sum_{r=1}^{R}\Pr\left(p_j, \mathrm{sale}_j|n_j, N_j;\theta_r\right)\Pr\left(n_j|N_j,\theta_r\right)\ \frac{g(\theta_r|X_j,\Gamma)}{\tilde{g}(\theta_r)}. \end{aligned} \tag{B.12}$$

If a new sample $\left\{\tilde{\theta}_r\right\}_{r=1}^{R}$ is drawn each time the integral (B.9) is evaluated, this importance sampling procedure will of course do nothing to simplify computation. Note, however, that the parameters Γ now appear *only* in the distribution $g(\theta_r|X_j,\Gamma)$, which itself only affects weights on elements in a sum. This in turn motivates Ackerberg (2009)'s reinterpretation of importance sampling.

Specifically, rather than drawing $\{\tilde{\theta}_r\}_{r=1}^{R}$ anew each time (B.9) is evaluated, Ackerberg (2009) propose to draw a single large sample $\{\tilde{\theta}_r\}_{r=1}^{R}$ from $\tilde{g}(\cdot)$ at the beginning of the algorithm. Holding

[3] Ackerberg, D., 2009. A new use of importance sampling to reduce computational burden in simulation estimation, *Quantitative Marketing and Economics* 7, 343-376.

this sample $\{\tilde{\theta}_r\}_{r=1}^R$ fixed, we may then calculate the integrand elements $\Pr(p_j, \text{sale}_j | n_j, N_j; \theta_r)$, $\Pr(n_j | N_j, \theta_r)$, and $\Pr(\text{sale}_j | N_j; \theta_r)$ for each θ_r *once for all* prior to estimation. Holding these pre-computed objects fixed, computation of the importance-sampling approximation (B.12) to the integral (C.1) at different values of Γ requires only recalculation of the importance sampling weights $g(\theta_j | X_j, \Gamma)$. As costs of computing $g(\theta_r | X_j, \Gamma)$ are trivial relative to costs of recomputing equilibrium, this allows for vastly accelerated estimation even net of higher setup costs, with the added advantage that the simulated likelihood function is automatically smooth in Γ. For our purposes, therefore, Ackerberg (2009) simulation is ideal; it mitigates the computational infeasibility that otherwise would be entailed by accommodating sample selection unobserved heterogeneity.

In practice, we implement this importance sampling procedure in two steps as follows. As a first pass, we draw a candidate importance sample $\{\theta_r\}_{r=1}^R$ of size $R = 10000$ for each target j from a multivariate uniform distribution over the following intervals: for $\mu_{vj}^r \sim U[-0.5192, 1.1825]$ (corresponding to 4 standard deviations of price above and below the mean), $\sigma_{vj}^r \sim 10^{-6} + U[0, 0.5]$, $c_j^r \sim U[0, 0.1]$, and $\alpha_j^r \sim U[0, 1]$. We then maximize the log-likelihood to obtain a first-step estimate $\hat{\Gamma}_0$ for Γ_0, and re-draw a new importance sample $\{\theta_r\}_{r=1}^R$ (also of size 10,000) for each auction j from the resulting predicted distribution $g(\cdot | X_j, \hat{\Gamma}_0)$. Finally, we maximize the simulated likelihood implied by this more accurate importance sample to obtain our final estimator $\hat{\Gamma}$ for Γ_0.

B.4. Inference: Pseudeo-Bayesian Markov Chain Monte Carlo Algorithm

In view of the importance sampling algorithm above, a variety of algorithms are feasible to maximize the (simulated analogue to) the log-likelihood (6). In practice, however, we focus on a Markov Chain Monte Carlo procedure in the spirit of Chernozhukov and Hong (2003).[4] Specifically, starting from a given initial point Γ^0, we use a Markov Chain Monte Carlo (MCMC) algorithm to obtain a sample $\{\Gamma^k\}_{k=1}^K$ of parameters from the conditional likelihood $\prod_j L(p_j, n_j | sale_j, N_j, X_j; \cdot)$, interpreted as (proportional to) the Bayesian posterior over Γ induced by the observed sample $\{(p_j, n_j | N_j)\}_{j=1}^J$ in conjunction with a flat (uninformative irregular) prior over Γ. We then consider the resulting posterior sample $\{\Gamma^k\}_{k=1}^K$ as a basis for inference on Γ_0.

[4] Chernozhukov, V. and H. Hong, 2003. "An MCMC approach to classical estimation," *Journal of Econometrics* 115:2, 293-346.

While not strictly necessary, this pseudo-Bayesian MCMC approach has several practical advantages. First, MCMC is a global maximization algorithm, which will eventually trace out the entire posterior of Γ (i.e. the entire likelihood function) from any initial point Γ^0. Second, under classical maximum likelihood regularity conditions, the Bernstein-Von Mises Theorem implies that (for any prior) any of the mean, median, and mode of $\{\Gamma^k\}_{k=1}^K$ will be asymptotically equivalent to the classical maximum likelihood estimator $\hat{\Gamma}$, with consistent classical standard errors provided by the standard deviation of $\{\Gamma^k\}_{k=1}^K$.[5] Third, even when classical maximum likelihood regularity conditions fail — for instance, in models which are only set, not point, identified — the posterior distribution traced out by the MCMC sample $\{\Gamma^k\}_{k=1}^K$ will still permit exact finite-sample Bayesian inference on Γ_0, leveraging only the information on Γ_0 revealed by the data. Finally, given the posterior sample $\{\Gamma^k\}_{k=1}^K$, conducting inference on any function $f(\Gamma)$ of Γ_0 is also straightforward — one need simply compute relevant quantiles of $f(\Gamma^k)$ over a (subsample of) $k = 1, ..., K$.

In practice, we implement MCMC at each iteration using a block Metropolis-Hastings algorithm, with four parameter blocks corresponding to the parameters governing the distributions of μ_{vj}, σ_{vj}, c_j, and α_j, and a multivariate normal proposal distribution for each block. We begin with a burnin phase of $K = 20,000$ iterations, using acceptance-rejection probabilities over this burnin phase to tune the proposal variances for each of our four parameter blocks. We then run another $K = 60,000$ MCMC iterations, taking the resulting posterior sample $\{\Gamma^k\}_{k=1}^{60,000}$ as a basis for inference on Γ_0 as described above. The resulting MCMC appears to have good mixing properties, converging quickly (within 5000 burnin iterations) to the neighborhood of the maximum likelihood even for relatively high-dimensional Γ. The high numbers of burnin and regular iterations above therefore primarily reflect an abundance of caution – in practice, much shorter chain lengths give virtually identical results.

Appendix C. The Negotiation Mechanisms

As described above, our counterfactual experiment of primary interest concerns an N-round sequential negotiation mechanism. This mechanism proceeds as follows. Round $n = 1, ..., N$ begins when the target approaches potential buyer $i = 1, ..., N$ (ordered at random) with an invitation to participate. The following events then take place in round n:

[5] See, e.g., Chernozhukov and Hong (2003) for one discussion of this property.

1. Potential buyer i observes its private signal S_{ij} for the target. Based on this signal S_{ij} and the entry and bidding history up to round $n-1$, potential bidder i determines whether to enter the negotiation at cost c.

2. Conditional on choosing to enter, potential buyer i learns its valuation V_{ij}. If another negotiating bidder has previously entered, potential buyer i and the current incumbent compete in an ascending button auction for the right to remain in the auction. The loser of this bidding round exits and the winner becomes the incumbent, with the current standing price being the level at which the loser drops out.

3. Conditional on outbidding the current incumbent, potential buyer i may submit a bid above the current standing price. If submitted, this jump bid is observed by all subsequent potential buyers, and becomes the standing price in round $n+1$.

Let b_{n-1} be the standing bid at the beginning of round n, and y_{n-1} be the valuation of the incumbent submitting bid b_{n-1}. In view of the sequence of events above, these objects evolve as follows. Upon being contacted by the acquirer in round n, potential acquirer i observes its signal s_{ij} and (based on this plus the history of the game to date) decides whether to enter. If i remains out in round n, the game proceeds to round $n+1$ with standing bid $b_n = b_{n-1}$. Alternatively, if i elects to enter and draws valuation v_{ij} upon entry, then three outcomes are possible. First, if v_{ij} is less than the current standing bid b_{n-1}, then i exits and the negotiation proceeds to round $n+1$ with standing bid $b_n = b_{n-1}$. Second, if v_{ij} is greater than b_{n-1} but less than y_{n-1}, then i bids up the price to v_{ij} before exiting, and the negotiation proceeds to round $n+1$ with standing bid $b_n = v_{ij}$. Finally, if v_{ij} is greater than y_{n-1}, then the current incumbent bids up the price to y_{n-1} before exiting, and i becomes the new incumbent. To signal strength and thereby deter future entry, incumbent i may then submit a *jump bid* $b_n \geq y_{n-1}$. This jump bid b_n then becomes the standing bid in round $n+1$.

In practice, we focus on the unique *separating perfect Bayesian equilibrium* within this sequential negotiation game. This equilibrium has two key components. First, entry decisions by bidder i in round $n = 1, ..., N$ are described by a signal threshold $s_n^*(y_{n-1})$ such that potential buyer i enters if and only if $S_{ij} \geq s_n^*(y_{n-1})$, where y_{n-1} denotes the valuation of the standing bidder and $s_n^*(y_{n-1})$ is strictly increasing in y_{n-1}. Second, conditional on outbidding an incumbent with valuation y_{n-1} in round $n < N$, new incumbent i with valuation $v_{ij} \geq y_{n-1}$ submits a jump bid $b_n \geq y_{n-1}$ described by a symmetric monotone jump bidding strategy $\beta_n(v_{ij}, y_{n-1})$. A separating perfect Bayesian equilibrium is therefore a collection of round-specific entry threshold functions $(s_1(y_0), ..., s_N(y_{N-1})$ and round-specific bidding strategies $(\beta_1(v, y_0), ..., \beta_{N-1}(v, y_{N-1}))$ such that all players are best-responding at each information node. We characterize these strategies by backward induction as follows.

First consider the deterrence bidding decision of a new incumbent in round N. As the game concludes at the end of round N, a new incumbent in round N has no incentive to submit a deterrence bid. Conditional on knocking out an incumbent with standing valuation y_{N-1}, a new entrant drawing valuation $v \geq y_{N-1}$ therefore trivially submits bid $b_N = y_{N-1}$ for all $v \geq y_{N-1}$. In this event the new incumbent earns ex post payoff $v - y_{N-1}$.

Next consider the entry decision of the potential bidder contacted in round N. By hypothesis, we are considering a separating equilibrium in which prior new incumbents have played bidding strategies strictly monotone in their valuations. Observing the history of the game to date, the potential entrant in round N will therefore infer the standing valuation y_{N-1} of the incumbent at the end of round $N-1$. Conditional on drawing signal realization $S_i = s_i$ against an incumbent with standing valuation y_{N-1}, potential entrant N therefore expects post-entry profit

$$\Pi_N(s_i, y_{N-1}) = \int (V_i - y_{N-1})\, dF_{v|s_N}(V_i|s_i). \tag{C.1}$$

Potential entrant N will enter when expected profits exceed costs, i.e. when $\Pi_N^*(s_9, y_{N-1}) \geq c_j$. This breakeven condition in turn determines the breakeven threshold $s_N(y_{N-1})$, which by arguments similar to those in Appendix A can be shown to be a strictly monotone function of the current standing valuation y_{N-1}.

Now consider the deterrence bidding decision of a new incumbent in round $N-1$. Conditional on knocking out a prior incumbent with valuation y_{N-2}, a new incumbent i with valuation $v_i \geq y_{N-2}$ faces the following tradeoff: by submitting a higher bid, i may pretend to be a higher type and thereby deter entry by a potential competitor in round N, but doing so will require i to pay a higher cost conditional on winning in this event. Specifically, if rivals expect i to bid according to the strategy $\beta_{N-1}(\cdot, y_{N-2})$, and entry decisions by i's potential round-N rival are taken according to the threshold $s_N(\cdot)$ above, then we may write i's deterrence bidding problem as

$$\max_{z \geq y_{N-2}} \pi_N(v_i, z; \beta_{N-1}(z, y_{N-2})) \tag{C.2}$$

where $\pi_N(v_i, z; b_{N-1})$ denotes the expected profit, at the start of round N with standing bid b_{N-1}, of an incumbent with true valuation v_i but whom potential rivals *believe* to have valuation z:

$$\begin{aligned} \pi_N(v_i, z; b_{N-1}) &= s_N(z) \cdot (v_i - b_{N-1}) \\ &\quad + (1 - s_N(z)) \int_0^{v_i} (v_i - \max\{Y_N, b_{N-1}\}) dF(Y_N | S_N \geq s_N(z)). \end{aligned} \tag{C.3}$$

Note that the first term of $\pi_N(v_i, b_{N-1}; z)$ reflects i's profit from events in which round N entry is successfully deterred, whereas the second term represents i's expected profit in events where i's round-N rival enters but draws a valuation below v_i.

Taking a first-order condition of (C.2) with respect to i's type report z, we obtain:

$$\frac{\partial}{\partial b_{N-1}} \pi_N(v_i, z; \beta_{N-1}(z; y_{N-2})) \cdot \beta'(z, y_{N-2}) + \frac{\partial}{\partial z} \pi_N(v_i, z; \beta_{N-1}(z; y_{N-2})) = 0.$$

Enforcing the restriction that in equilibrium the strategy $\beta_{N-1}(\cdot, y_{N-2})$ must be such that it is optimal for i to report truthfully, this in turn implies the following differential equation characterizing the unknown deterrence bidding strategy $\beta_{N-1}(\cdot; y_{n-2})$:

$$\beta'(v_i; y_{N-2}) = -\frac{\frac{\partial}{\partial z} \pi_N(v_i, v_i; \beta_{N-1}(v_i; y_{N-2}))}{\frac{\partial}{\partial b_{N-1}} \pi_N(v_i, v_i; \beta_{N-1}(v_i; y_{N-2}))}. \tag{C.4}$$

Combined with the boundary condition $\beta(y_{N-2}; y_{N-2}) = y_{N-2}$, this in turn determines the function $\beta(\cdot; y_{N-2})$ describing equilibrium deterrence bidding in round $N - 1$.

It is straightforward to show that, for given v_i, round-N profit $\pi_N(v_i, b_{N-1}; z)$ is strictly decreasing in b_{N-1} given z and strictly increasing in z given b_{N-1}. In other words, for given rival beliefs, i always prefers a strictly lower standing bid, and for a given standing bid, i always prefers rivals to believe she is a stronger type. Hence $\beta'(v_i; y_{N-2}) > 0$ above, which implies that the equilibrium deterrence bidding function $\beta(\cdot; y_{N-2})$ is strictly increasing as expected. This confirms that bidding in round $N-1$ is consistent with a strictly separating equilibrium, as desired.

Finally, consider the entry decision of potential bidder i with signal $S_i = s_i$ in round $N-1$, facing an incumbent with standing valuation y_{N-2}. Conditional on entry, the expected profit of this potential entrant will be equal to the optimal round N profit $\pi_N(v_i, v_i; \beta_{N-1}(v_i; y_{N-2}))$ described above, integrated over potential realizations V_i of v_i such that $V_i \geq y_{N-2}$:

$$\Pi_{N-1}(s_i; y_{N-2}) = \int_{y_{N-2}}^{\infty} \pi_N(V_i, V_i; \beta_{N-1}(V_i; y_{N-2}))\, dF_{v|s}(V_i|s_i). \tag{C.5}$$

It is again straightforward to show that the right-hand integrand must be increasing in V_i and decreasing in y_{N-2}. In view of the fact that V_i is stochastically increasing in s_i, this in turn implies that there will exist a unique threshold function $s_{N-2}(y_{N_2})$ such that i enters against an incumbent with standing valuation y_{N-2} only if $S_i \geq s_{N-2}(y_{N_2})$. We thereby obtain a complete characterization of entry and bidding behavior in round $N-2$.

Proceeding recursively in this fashion for rounds $N-3, N-4, ..., 1$, one ultimately obtains the desired series of strictly increasing entry functions $s_1(y_0), ..., s_N(y_{N-1})$ and strictly increasing bidding functions $\beta_1(v_i; y_0), ..., \beta_{N-1}(v_i; y_{N-2})$ characterizing the unique symmetric separating perfect Bayesian equilibrium of the sequential negotiation game. Roberts and Sweeting (2013) furthermore show that the resulting separating equilibrium is the *only* perfect Bayesian equilibrium to survive standard refinements (no weakly dominated strategies, sequential equilibrium, and the D1 refinements of Cho and Sobel (1990) and Ramey (1996)) on equilibria of the sequential negotiation game.

The go-shop mechanism is similar to the sequential negotiation but with only a single round of deterrence bidding. Bidding starts at a standing bid b_0 equal to the target's reservation valuation. The target approaches one potential bidder i at random, who observes their signal realization s_i and based on this and the standing bid b_0 decides whether to enter. If bidder i enters, i observes its valuation v_i, and may submit a jump bid above b_0, which becomes the new standing bid b_1. Otherwise, the standing bid remains $b_1 = b_0$. The game then proceeds to a go-shop round, in which the target contacts the other $N-1$ bidders, who based on the history of the game and their private signals decide whether or not to enter. If at least two bidders ultimately enter with values above the standing bid b_1, the game concludes with an ascending auction to determine the winning bidder. Otherwise the game concludes at the standing bid b_1.

Equilibrium in the go-shop mechanism is similar to that in the sequential mechanism, but simpler, since there is only one round of deterrence bidding. As above, we look for a symmetric separating perfect Bayesian equilibrium. The $N-1$ entrants in the final round observe the history of the game to date, including the initial bid b_0, whether a bidder entered in Round 1, and the entrant's jump bid b_1 if one was submitted. Based on this, all potential entrants infer the standing valuation upon entry, which we denote y_1: either $y_1 = v_0 = b_0$ if the standing bid is the seller's reservation price, or y_1 equal to the incumbent's valuation if the incumbent submitted a jump bid. Entry by all $N-1$ bidders contacted in the go-shop stage therefore proceeds according to a threshold $s^*(y_1)$, determined by the condition that a go-shop entrant drawing signal realization $S_i = s^*(y_1)$ must just break even from entry when the standing value is y_1 and the other go-shop entrants enter according to the threshold $s^*(y_1)$:

$$\int_{y_1}^{\infty} [F_s(s^*(y_1)) + F_v(v) - F_{vs}(v, s^*(y_1))]^{N-2} \, dF_{v|s}(v|S_i = s^*(y_1)). \tag{C.6}$$

Now consider Round 1 deterrence bidding by the incumbent. We seek a deterrence bidding strategy $B_I(v; b_0)$ which (for $v > b_0$) is strictly monotone in the incumbent's valuation v. As above, this strategy will be characterized by the condition that the incumbent's gains from pretending to be a higher type are just offset by the additional costs of a higher standing bid. Letting z denote the incumbent's pretended type, this yields the maximization problem

$$\max_z \Bigg\{ (v - \beta_I(z; b_0)) \cdot [F_s(s^*(z)) + F_v(\beta_I(z; b_0)) - F_{vs}(\beta_I(z; b_0), s^*(z))]^{N-1} \\ + \int_{\beta_I(z;b_0)}^{v} (v - y) \, \frac{d}{dy}[F_s(s^*(z)) + F_v(y) - F_{vs}(y, s^*(z))]^{N-1} \, dy \Bigg\}, \quad \text{(C.7)}$$

where the first term reflects incumbent profents in the event that the incumbent faces no rival entrant with a valuation above the go-shop standing bid $b_1 = \beta_I(z; b_0)$, and the second reflects expected profits in the event that at least one go-shop rival enters with a valuation above the chosen standing bid $\beta_I(z; b_0)$. Taking a first-order condition with respect to z and enforcing the equilibrium condition $v = z$, we ultimately obtain a differential equation characterizing the derivative $B_I'(v; b_0)$ of $B_I(v; b_0)$, which together with the boundary condition $B_I(b_0; b_0) = b_0$ uniquely determines the equilibrium deterrence bidding strategy $B_I(\cdot; b_0)$. As above, it is straightforward to show that this strategy $B_I(\cdot; b_0)$ must be strictly increasing, which confirms that $B_I(\cdot; b_0)$ describes a separating perfect Bayesian equilibrium as desired.

Appendix D. Equilibrium with an Alternate Bidding Model

This section provides details of the bidding and entry equilibrium in the case where rather than selling via a standard ascending auction, targets are sold via a final first-price sealed bidding round. The seller contacts N potential bidders, each of whom observe a private signal s_i and choose whether or not to incur the entry cost c. Then, in the second stage, K entering bidders with valuations above the target's reservation price v_0 compete in an auction according to the following bidding rules. Bidding starts from a seller-announced initial price v_0 and proceeds via an ascending auction until the auction reaches a price $\overline{p} \geq v_0$ at which at most $k = 2$ bidders remain. From this point, the remaining two bidders compete in a first-price sealed bid auction with reserve price $\overline{p}$ with the highest bidder in this final round winning the target and paying their bid. For now, we assume that the actual number of bidders with valuations above the target's reserve price is known to participants. Assuming that the these are unknown is unknown would slightly change the form of bidding strategies but would not alter seller expected revenue.

The bidding equilibrium can be characterized by a signal threshold s^* such that potential entrants with signals $s_i \geq s^*$ elect to enter. Let $F^*(\cdot)$ denote the corresponding (selected) distribution of valuations among entrants entering at s^*. Here, each bidder i infers that the maximum rival value among other bidders is drawn from the distribution

$$F^*_{1:k-1}(y|v_0) = \left[\frac{F^*(y) - F^*(v_0)}{1 - F^*(v_0)}\right]^{k-1}. \tag{D.1}$$

Conditional on reserve price v_0, the equilibrium final-round bidding strategy is thus

$$\beta_k(v|v_0) = v - \int_{v_0}^{v} \frac{F^*_{1:k-1}(y|v_0)}{F^*_{1:k-1}(v|v_0)}\, dy. \tag{D.2}$$

To understand the entry equilibrium, note that the bidding mechanism described above is a mechanism in which the entrant with the highest valuation wins the auction if and only if this valuation exceeds the seller's reservation price. This is the same allocation rule employed in the baseline ascending auction. Furthermore, bidders are ex ante symmetric with private signal-value pairs (s_i, v_i) distributed independently across bidders conditional on target type. Hence the revenue equivalence principle applies and conditional on any s^* and any realization of the seller's reserve value v_0 both expected revenue and expected bidder surplus in the auction will be identical to those prevailing in the simple ascending auction. This in turn implies that s^* describes an equilibrium of the first-price auction if and only if s^* describes an entry equilibrium in the simple ascending auction. Since we have already characterized this threshold in the simple ascending case, we are done.

We now describe computation of the likelihood function for a given target. As in our baseline model, we break this computation up into three steps: derivation of $\Pr(n, sale|N, \theta)$, derivation of $L(p, n, sale|N, \theta)$, and integration of these into an overall likelihood function (conditional on observing sale) via Ackerberg importance sampling.

First observe that sale occurs in the first-price auction whenever sale would occur in an ascending auction: i.e. whenever at least one bidder draws a value above v_0. Hence computation of $\Pr(n, sale|N, \theta)$ does not change.

Now consider construction of $L(p, n, sale|N, \theta)$. Due to a more complicated bidding equilibrium, this is now somewhat more complicated. However, we can derive a form for $L(p, n, sale|N, \theta)$ as follows. For ease of exposition we omit (N, θ) in the notation below; all derivations should be interpreted as conditional on (N, θ).

Toward this end, first consider likelihood of any given $(p, n, sale)$ combination conditional on realization of the seller's reserve value v_0. Noting that sale occurs if and only if $p \geq v_0$, we have $\Pr(p, n, sale|v_0) = 0$ for $p \leq v_0$. We may therefore decompose

$$L(p, n, sale) = \int_0^p \Pr(p, sale|n, v_0) \Pr(n) f_0(v_0) dv_0. \tag{D.3}$$

As in our main specification, $\Pr(n)$ is derived from a binomial PDF with parameters $(N, 1 - s^*_\theta)$. Hence consider $\Pr(p, sale|n, v_0)$. Let k be the number of entering bidders with valuations above the target's reservation price. Then the bidding equilibrium takes a different form for each realization of k. First, if $k = 1$ (i.e. only one entrant has a valuation above v_0), then the auction concludes at $p = v_0$. Second, if exactly $k \in \{2, ..., N\}$ entrants have valuations above v_0, then each bidder submits bids according to the corresponding k-specific bid strategy $\beta_k(\cdot|v_0)$.

Noting that these events are mutually exclusive, we may write:

$$\Pr(p, sale|n, v_0) = f_0(p) \Pr(k = 1|n, v_0) + \sum_{k=2}^{N} \Pr(p, sale, k|n, v_0) \tag{D.4}$$

First consider $\Pr(p, sale, k|n, v_0)$ for $k \in \{2, ..., N\}$.

$$\begin{aligned} \Pr(p, sale, k|n, v_0) &= \Pr(\beta_k(V_{1:n}|v_0) = p \cap V_{k:n} \geq v_0 \cap V_{k+1:n} \leq v_0) \\ &= \beta_k^{-1\prime}(p|v_0) \cdot \Pr(V_{1:n} = \beta_k^{-1}(p|v_0), V_{k:n} \geq v_0 \cap V_{k+1:n} \leq v_0) \end{aligned} \tag{D.5}$$

where the final line follows by change of variables.

But note that in this case we may write

$$\begin{aligned}\Pr(V_{1:n}=y, V_{k:n}\geq v_0, V_{k+1:n}\leq v_0) = \Pr\Big(&(\text{One of n } V_i = y),\\ &\cap (k \text{ of n } V_i \in [v_0, y] \cap (n-k-1 \text{ of n } V_i \leq v_0))\Big)\\ &= \frac{n!}{k!(n-k-1)!} F^*(v_0)^{n-k-1}[F^*(y)-F^*(v_0)]^k f^*(y). \quad \text{(D.6)}\end{aligned}$$

Substituting this expression into the relationship above, we ultimately obtain:

$$\begin{aligned}\Pr(p, sale, k|n, v_0) = \beta_k^{-1,\prime}(p|v_0)\cdot f^*(\beta_1^{-1}(p|v_0))&\\ \times \frac{n!}{(n-k-1)!k!} F^*(v_0)^{n-k-1}[F^*(\beta_k^{-1}(p|v_0)) - F^*(v_0)]^k.& \quad \text{(D.7)}\end{aligned}$$

Every term in this expression can be easily evaluated. We thus obtain a basis for numerical computation of $\Pr(p, sale, k|n, v_0)$ for each $k \in \{1, ..., K\}$.

It only remains to obtain an expression for $\Pr(k=1|n, v_0)$. Toward this end, observe that

$$\begin{aligned}\Pr(k &= 1|n, v_0) = \Pr(n-1 \text{ of } n\ V_i \leq v_0 \cap 1 \text{ of } n\ V_i \geq v_0) \quad \text{(D.8)}\\ &= nF^*(v_0)[1-F^*(v_0)]^{n-1}.\end{aligned}$$

Combining the resulting expressions in (2) yields an expression for $\Pr(p, sale|n, v_0)$ at any (n, v_0), and integrating (1) over v_0 yields a final form for $L(p, n, sale)$. Simulation of the overall likelihood can proceed as before.

Appendix E. Additional Empirical Results

(Continued on next page)

Table A.1
Additional Robustness Checks

This table reports estimates of the takeover market primitives recovered by the structural model described in Section 2, on the data described in Section 3. The table reports mean parameter estimates across all targets in our sample, and medians in brackets. The parameters of interest are the mean (μ) and variance (σ) of the potential entrant value distribution, average entry costs (c) and the average degree of pre-entry uncertainty (α). The estimates in columns (1) and (2) are obtained using an alternate bidding model. In the model, described in Section 6 of the main paper, along with Appendix D, the target first narrows the bidder pool via an ascending auction and then conducts first-price bidding competition among the two (column (1)) and three (column (2)) highest-valued bidders. The estimates in column (3) are conditioned on the predicted value from OLS regressions of the number of invited bidders on target characteristics, rather than on the actual number of invited bidders. The estimates in column (4) are based on a model that includes a separate reserve price distribution, described in Section 6 of the main paper. The estimates in column (5) include deals whose winning bid results in a price below the target's share price four weeks prior to announcement. All estimates are based on proposal distributions from by the baseline estimates in column (4) of table 3 in the main paper.

	(1)	(2)	(3)	(4)	(5)
Pre-entry uncertainty (α)	0.6491	0.6619	0.7429	0.4710	0.6600
	[0.6898]	[0.7082]	[0.7775]	[0.4713]	[0.7023]
Average entry cost (c)	0.0129	0.0130	0.0178	0.0207	0.0127
	[0.0056]	[0.0058]	[0.0085]	[0.0064]	[0.0055]
Average potential bidder valuation (μ)	0.2850	0.2845	0.2807	0.2010	0.2864
	[0.2846]	[0.2842]	[0.2804]	[0.1999]	[0.2860]
Spread of values across potential bidders (σ)	0.0601	0.0585	0.0651	0.1921	0.0560
	[0.0554]	[0.0538]	[0.0605]	[0.1853]	[0.0515]