Online Appendix (Not for Publication)

A. The expression of $\pi(\tau_{x,i}, \tau_{y,i}; \tau_x, \tau_y)$ in Section 5.2.

In this appendix, we describe the expression of speculators’ ex-ante expected trading gains $\pi(\tau_{x,i}, \tau_{y,i}; \tau_x, \tau_y)$ in Section 5.2, where speculators acquire costly private information. Suppose that speculator $i$ considers to acquire private signals with precision levels $\tau_{x,i}$ and $\tau_{y,i}$, while the other speculators have acquired private signals with precision levels $\tau_x$ and $\tau_y$.

Following the same steps as in the main text, we can compute

$$E(\bar{P}|\bar{x}_i, \bar{y}_i, \bar{\omega}, \bar{\eta}) = \exp(b^p_0 + b^p_{\bar{x}}\bar{x}_i + b^p_{\bar{y}}\bar{y}_i + b^p_{\bar{\omega}}\bar{\omega} + b^p_{\bar{\eta}}\bar{\eta}),$$

$$E(\bar{V}|\bar{x}_i, \bar{y}_i, \bar{\omega}, \bar{\eta}) = \exp(b^p_0 + b^p_{\bar{x}}\bar{x}_i + b^p_{\bar{y}}\bar{y}_i + b^p_{\bar{\omega}}\bar{\omega} + b^p_{\bar{\eta}}\bar{\eta}),$$

where

$$b^p_0 = -\frac{g}{\lambda\sqrt{\tau_x^{-1} + \phi_y^2\tau_y^{-1}}} + \frac{1}{2\lambda^2\tau_x^{-1} + \phi_y^2\tau_y^{-1}} + \frac{1}{2\lambda^2(\tau_x^{-1} + \phi_y^2\tau_y^{-1})} + \frac{1}{\tau_a + \tau_{x,i} + \tau_\omega},$$

$$b^p_{\bar{x}} = \frac{1}{\lambda\sqrt{\tau_x^{-1} + \phi_y^2\tau_y^{-1}}},$$

$$b^p_{\bar{y}} = \frac{\phi_y}{\lambda\sqrt{\tau_x^{-1} + \phi_y^2\tau_y^{-1}}},$$

$$b^p_{\bar{\omega}} = \frac{1}{\lambda\sqrt{\tau_x^{-1} + \phi_y^2\tau_y^{-1}}} \left(\phi_\omega + \frac{\tau_\omega}{\tau_a + \tau_{x,i} + \tau_\omega}\right),$$

$$b^p_{\bar{\eta}} = \frac{1}{\lambda\sqrt{\tau_x^{-1} + \phi_y^2\tau_y^{-1}}} \left(\phi_\eta + \frac{\phi_\tau_\eta}{\tau_f + \tau_{y,i} + \tau_\eta}\right),$$

$$b^p_{0} = \log \left[\frac{\beta (1 - \beta)}{c}\right] + \frac{\tau_f + \tau_\eta + 2\tau_p}{2(\tau_f + \tau_\eta + \tau_p)^2} + \frac{2}{\tau_a + \tau_{x,i} + \tau_\omega} + \frac{1}{2} \left(1 + \frac{\tau_p}{\tau_f + \tau_\eta + \tau_p}\right)^2 \frac{1}{\tau_f + \tau_{y,i} + \tau_\eta},$$
\[ b^p_x = \frac{2\tau_{x,i}}{\tau_a + \tau_{x,i} + \tau_\omega}, \]
\[ b^p_y = \left(1 + \frac{\tau_p}{\tau_f + \tau_\eta + \tau_p}\right) \frac{\tau_{y,i}}{\tau_f + \tau_{y,i} + \tau_\eta}, \]
\[ b^p_\omega = \frac{2\tau_\omega}{\tau_a + \tau_{x,i} + \tau_\omega}, \]
\[ b^p_\eta = \frac{\tau_\eta}{\tau_f + \tau_\eta + \tau_p} + \left(1 + \frac{\tau_p}{\tau_f + \tau_\eta + \tau_p}\right) \frac{\tau_\eta}{\tau_f + \tau_{y,i} + \tau_\eta}. \]

Let us define
\[ \tilde{p}_i \equiv \log \left[ E(\tilde{P}|\tilde{x}_i, \tilde{y}_i, \tilde{\omega}, \tilde{\eta}) \right] = b^0_p + b^p_x \tilde{x}_i + b^p_y \tilde{y}_i + b^p_\omega \tilde{\omega} + b^p_\eta \tilde{\eta}, \]
\[ \tilde{v}_i \equiv \log \left[ E(\tilde{V}|\tilde{x}_i, \tilde{y}_i, \tilde{\omega}, \tilde{\eta}) \right] = b^0_v + b^v_x \tilde{x}_i + b^v_y \tilde{y}_i + b^v_\omega \tilde{\omega} + b^v_\eta \tilde{\eta}, \]
and
\[ \mu_p \equiv E(\tilde{p}_i) = b^0_p, \mu_v \equiv E(\tilde{v}_i) = b^0_v, \]
\[ \sigma^2_p \equiv \text{Var}(\tilde{p}_i) = \frac{(b^p_x + b^p_\omega)^2}{\tau_a} \frac{(b^p_y + b^p_\eta)^2}{\tau_f} + \left(\frac{(b^p_x)^2}{\tau_{x,i}} + \frac{(b^p_y)^2}{\tau_{y,i}} + \frac{(b^p_\omega)^2}{\tau_\omega} + \frac{(b^p_\eta)^2}{\tau_\eta}, \right. \]
\[ \sigma^2_v \equiv \text{Var}(\tilde{v}_i) = \frac{(b^v_x + b^v_\omega)^2}{\tau_a} \frac{(b^v_y + b^v_\eta)^2}{\tau_f} + \left(\frac{(b^v_x)^2}{\tau_{x,i}} + \frac{(b^v_y)^2}{\tau_{y,i}} + \frac{(b^v_\omega)^2}{\tau_\omega} + \frac{(b^v_\eta)^2}{\tau_\eta}, \right. \]
\[ \sigma_{vp} \equiv \text{Cov}(\tilde{v}_i, \tilde{p}_i) = \frac{(b^v_x + b^v_\omega)(b^v_y + b^v_\eta)}{\tau_a} \frac{(b^v_x)^2}{\tau_{x,i}} + \frac{(b^v_y)^2}{\tau_{y,i}} + \frac{(b^v_\omega)^2}{\tau_\omega} + \frac{(b^v_\eta)^2}{\tau_\eta}, \]
\[ + \frac{b^v_x b^v_\omega}{\tau_{x,i}} + \frac{b^v_x b^v_\eta}{\tau_{y,i}} + \frac{b^v_\omega b^v_\eta}{\tau_\omega} \frac{b^v_x b^v_\eta}{\tau_\eta}. \]

Then, using the law of iterated expectations, we have:
\[ \pi(\tau_{x,i}, \tau_{y,i}; \tau_x, \tau_y) \]
\[ = E\left[ d(\tilde{x}_i, \tilde{y}_i, \tilde{\omega}, \tilde{\eta}) E(\tilde{V} - \tilde{P}|\tilde{x}_i, \tilde{y}_i, \tilde{\omega}, \tilde{\eta}) - C(\tau_{x,i}, \tau_{y,i}) \right], \]
\[ = 2 \left\{ E\left[ e^{\tilde{v}_i} \Phi \left( \frac{\tilde{v}_i - \mu_{\tilde{v}_i}}{\sigma_{\tilde{v}_i}} \right) \right] + E\left[ e^{\tilde{p}_i} \Phi \left( \frac{\tilde{p}_i - \mu_{\tilde{p}_i}}{\sigma_{\tilde{p}_i}} \right) \right] \right\} - \left( e^{\mu_v + \frac{\sigma^2_v}{2}} + e^{\mu_p + \frac{\sigma^2_p}{2}} - 1 \right) - C(\tau_{x,i}, \tau_{y,i}), \]
where \( \Phi(\cdot) \) is the cumulative density function of a standard normal distribution, and
\[ \frac{\tilde{v}_i - \mu_{\tilde{v}_i}}{\sigma_{\tilde{v}_i}} = \frac{1 - \sigma_{vp}\sigma^{-2}_v}{\sqrt{\sigma^2_p - \sigma^2_{vp}\sigma^2_v}} \tilde{p}_i - \frac{\mu_p - \sigma_{vp}\sigma^{-2}_v \mu_v}{\sqrt{\sigma^2_p - \sigma^2_{vp}\sigma^2_v}} \tilde{v}_i, \]
\[ \frac{\tilde{p}_i - \mu_{\tilde{p}_i}}{\sigma_{\tilde{p}_i}} = \frac{1 - \sigma_{vp}\sigma^{-2}_p}{\sqrt{\sigma^2_v - \sigma^2_{vp}\sigma^2_p}} \tilde{v}_i - \frac{\mu_v - \sigma_{vp}\sigma^{-2}_p \mu_p}{\sqrt{\sigma^2_v - \sigma^2_{vp}\sigma^2_p}} \tilde{p}_i, \]
When we compute the derivative of $\pi(\tau_{x,i}, \tau_{y,i}; \tau_{x}, \tau_{y})$, we apply Leibniz’s rule. For instance,

$$\frac{\partial E \left[ e^{\tilde{y}_i} \Phi \left( \frac{\tilde{y}_i - \mu_{y_i|\tilde{v}_i}}{\sigma_{y_i|\tilde{v}_i}} \right) \right]}{\partial \tau_{x,i}} = \int_{-\infty}^{\infty} e^{\nu} \left[ \phi \left( \frac{1 - \sigma_{y_{x,i}y_{x,i}}^2}{\sqrt{\sigma_{y_{x,i}y_{x,i}}^2 - \sigma_{y_{x,i}y_{x,i}}^2 \sigma_{y_{x,i}y_{x,i}}^2}} - \nu \left( - \frac{\partial \mu_{y_{x,i}|\tilde{v}_i} v}{\partial \tau_{x,i}} - \frac{\partial \mu_{y_{x,i}|\tilde{v}_i} v}{\partial \sigma_{x,i}} \right) \right) dv, \right]$$

where $\phi(v; \mu_v, \sigma_v)$ is the probability density function of a normal distribution with mean $\mu_v$ and standard deviation $\sigma_v$. The above expression can be easily computed numerically since we know the expressions of $\mu$’s and $\sigma$’s.

**B. Speculators observe two private signals about one factor**

In this appendix, we consider a variation in which speculators receive two private signals about factor $\tilde{f}$ and where the other factor $\tilde{a}$ is common knowledge. Specifically, we shut down the uncertainty about $\tilde{a}$ by setting $\tau_{a} = \infty$. Each speculator $i$ observes two signals about factor $\tilde{f}$: (i) a speculator-specific signal, $\tilde{y}_i = \tilde{f} + \tilde{z}_{y,i}$, where $\tilde{z}_{y,i} \sim N(0, \tau_{y}^{-1})$ with $\tau_{y} > 0$, and (ii) a common private signal, $\tilde{s}_c = \tilde{f} + \tilde{z}_c$, where $\tilde{z}_c \sim N(0, \tau_{c}^{-1})$ with $\tau_{c} > 0$. Clearly, $\tilde{s}_c$ is not a private signal in the usual sense of the word, since it is observed by all speculators, but it is also not public since it is not observed by the real decision maker. Hence, we refer to it as a common private signal. We also allow the possibility that the public signal $\tilde{\eta}$ about factor $\tilde{f}$ might be correlated with $\tilde{s}_c$ by specifying $\tilde{\eta} = \tilde{f} + \gamma \tilde{z}_c + \tilde{\eta}$, where $\gamma$ is a constant and $\tilde{\eta} \sim N(0, \tau_{\eta}^{-1})$ with parameter $\tau_{\eta} \geq 0$ controlling the precision of public disclosure. The random variables $(\tilde{f}, \{\tilde{z}_{y,i}\}_i, \tilde{z}_c, \tilde{\eta})$ are mutually independent. All the other features of the model remain unchanged from Section 2..

We still consider linear monotone equilibria. In this setting, speculator $i$’s information set is $\{\tilde{y}_i, \tilde{s}_c, \tilde{\eta}\}$. We conjecture that speculators buy the asset whenever $\tilde{y}_i + \phi_c \tilde{s}_c + \phi_{\eta} \tilde{\eta} > g$, where $\phi$’s are endogenous parameters. Following the same steps as in the main text, we can compute the price as

$$\tilde{P} = \exp \left( \frac{\tilde{f} + \phi_c \tilde{s}_c + \phi_{\eta} \tilde{\eta} - g + \tilde{\xi}}{\lambda} \right).$$

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The real decision maker has information \{\tilde{\eta}, \tilde{p}\}. To him, the price is equivalent to the following signal in predicting the fundamental \(\tilde{f}\):

\[
\tilde{s}_p = \tilde{f} + \frac{\phi_c}{1 + \phi_c} \tilde{\xi} + \frac{\tilde{\xi}}{1 + \phi_c}.
\]

Thus, the optimal date-1 investment is

\[
K^* = \arg \max_K E \left( \beta \tilde{F} K - \frac{c}{2} K^2 \bigg| \tilde{\eta}, \tilde{s}_p \right) = \frac{\beta}{c} \exp \left[ \frac{1}{2} Var \left( \tilde{f} \bigg| \tilde{\eta}, \tilde{s}_p \right) \right].
\]

Using Bayes’ rule, we can compute

\[
E \left( \tilde{f} \bigg| \tilde{\eta}, \tilde{s}_p \right) = \beta_{\eta} \tilde{\eta} + \beta_{\tilde{s}_p} \tilde{s}_p,
\]

where

\[
\begin{align*}
\beta_{\eta} &= \frac{\tau_\eta (\tau_c + \phi_c^2 \tau_\xi - \gamma \phi_c^2 \tau_\xi - \gamma \phi_c \tau_\xi)}{\tau_c f + \tau_c \tau_\xi + \tau_c \tau_\eta + \gamma^2 \tau f \tau_\eta + \gamma^2 \tau_\xi \tau_\eta + \tau_c \phi_c^2 \tau_\xi + \phi_c^2 \tau f \tau_\eta + \phi_c^2 \tau_\xi \tau_\eta + 2 \tau_c \phi_c \tau_\xi - 2 \gamma \phi_c^2 \tau_\xi \tau_\eta + 2 \gamma \phi_c \tau_\xi \tau_\eta + \tau_c \phi_c + \gamma^2 \phi_c \tau_\eta - \gamma \phi_c \tau_\eta}, \\
\beta_{\tilde{s}_p} &= \frac{\tau_\xi (\phi_c + 1) (\tau_c + \tau_c \phi_c + \gamma^2 \phi_c \tau_\eta + \gamma \phi_c \tau_\eta)}{\tau_c f + \tau_c \tau_\xi + \tau_c \tau_\eta + \gamma^2 \tau f \tau_\eta + \gamma^2 \tau_\xi \tau_\eta + \tau_c \phi_c^2 \tau_\xi + \phi_c^2 \tau f \tau_\eta + \phi_c^2 \tau_\xi \tau_\eta + 2 \tau_c \phi_c \tau_\xi - 2 \gamma \phi_c^2 \tau_\xi \tau_\eta + 2 \gamma \phi_c \tau_\xi \tau_\eta + \gamma^2 \phi_c^2 \tau_\xi \tau_\eta - 2 \gamma \phi_c \tau_\xi \tau_\eta},
\end{align*}
\]

and

\[
Var \left( \tilde{f} \bigg| \tilde{\eta}, \tilde{s}_p \right) = \frac{\tau_c + \gamma^2 \tau_\eta + \phi_c^2 \tau_\xi}{\tau_c f + \tau_c \tau_\xi + \tau_c \tau_\eta + \gamma^2 \tau f \tau_\eta + \gamma^2 \tau_\xi \tau_\eta + \tau_c \phi_c^2 \tau_\xi + \phi_c^2 \tau f \tau_\eta + \phi_c^2 \tau_\xi \tau_\eta + 2 \tau_c \phi_c \tau_\xi - 2 \gamma \phi_c^2 \tau_\xi \tau_\eta + 2 \gamma \phi_c \tau_\xi \tau_\eta + \gamma^2 \phi_c^2 \tau_\xi \tau_\eta - 2 \gamma \phi_c \tau_\xi \tau_\eta}.
\]

We now go back to date 0 to compute the optimal trading strategy of speculators. Recall that speculator \(i\) has information set \{\tilde{y}_i, \tilde{s}_c, \tilde{\eta}\}. We can use the last two signals to obtain the following signal: \(\tilde{s}_i \equiv \tilde{\eta} - \gamma \tilde{s}_c = \tilde{f} + \frac{1}{1 - \gamma} \tilde{\xi} \tilde{\eta}\). Thus, in terms of predicting \(\tilde{f}\), speculator \(i\)'s information set is equivalent to \{\tilde{y}_i, \tilde{s}_c, \tilde{\eta}\}. We can compute

\[
E \left( \tilde{f} \bigg| \tilde{y}_i, \tilde{s}_c, \tilde{\eta} \right) = \frac{\tau_y \tilde{y}_i + \tau_c \tilde{s}_c + (1 - \gamma) \tau_\eta (\tilde{\eta} - \gamma \tilde{s}_c)}{\tau_f + \tau_y + \tau_c + (1 - \gamma)^2 \tau_\eta},
\]

\[
Var \left( \tilde{f} \bigg| \tilde{y}_i, \tilde{s}_c, \tilde{\eta} \right) = \frac{1}{\tau_f + \tau_y + \tau_c + (1 - \gamma)^2 \tau_\eta}.
\]

Using these expressions, we can compute

\[
E(\tilde{P} | \tilde{y}_i, \tilde{s}_c, \tilde{\eta}) = \exp \left( b^0 \tilde{y}_i + b^1 \tilde{s}_c + b^2 \tilde{\eta} \right),
\]

\[
E(\tilde{V} | \tilde{y}_i, \tilde{s}_c, \tilde{\eta}) = \exp \left( b^3 \tilde{y}_i + b^4 \tilde{s}_c + b^5 \tilde{\eta} \right),
\]
Thus, speculator \( i \) buys the risky asset if and only if

\[
E(\tilde{V} | \tilde{y}_i, \tilde{s}_c, \tilde{\eta}) > E(\tilde{P} | \tilde{y}_i, \tilde{s}_c, \tilde{\eta}) \iff
\]

\[
(b_v - b_y) \tilde{y}_i + (b_c - b_v) \tilde{c} + (b_v - b_y) \tilde{\eta} > b_0^v - b_0^y.
\]

Recall that we conjecture speculators’ trading strategy as buying the asset whenever \( \tilde{y}_i + \phi_c \tilde{s}_c + \phi_\eta \tilde{\eta} > g \). So, in a linear monotone equilibrium, we must have

\[
\phi_c = \frac{b_v^c - b_y^c}{b_y^v - b_y^c}, \quad \phi_\eta = \frac{b_y^v - b_y^\eta}{b_y^v - b_y^c}, \quad \text{and} \quad g = \frac{b_0^v - b_0^y}{b_y^v - b_y^c},
\]

provided that \( b_v^c - b_y^c > 0 \).

We use Figure OA1 to report the implications of disclosure for a variety of combinations of \( (\gamma, \tau_\xi) \). Other parameters are fixed at 1, i.e., \( \tau_f = \tau_y = \tau_c = \lambda = 1 \). To facilitate the comparison with Figure 1 in the main text, we also decompose the real efficiency effect into a direct and an indirect effect. Specifically, the real decision maker wishes to forecast \( \tilde{f} \) using the information set \( \{\tilde{\eta}, \tilde{s}_p\} \). His prior precision is \( \frac{1}{\text{Var}(\tilde{f})} \). After adding the public disclosure \( \tilde{\eta} \), his forecast precision increases to \( \frac{1}{\text{Var}(\tilde{f} | \tilde{\eta})} \), and so the difference of \( \frac{1}{\text{Var}(\tilde{f} | \tilde{\eta})} - \frac{1}{\text{Var}(\tilde{f})} \) captures
how much extra information the real decision maker obtains by directly observing the public signal $\tilde{\eta}$. Similarly, we will use the difference $\frac{1}{\text{Var}(\tilde{f}|\tilde{\eta}, s_p)} - \frac{1}{\text{Var}(\tilde{f}|\eta)}$ to capture how much extra information the real decision maker can learn from the price, which is the indirect effect of disclosure.

[INSERT FIG. OA1 HERE]

We observe that in this alternative setting, disclosing public information can also negatively affect price informativeness in many cases (such as the cases of $(\gamma = 0, \tau_\xi = 1)$ and $(\gamma = 0.5, \tau_\xi = 1)$). In addition, in some cases, for instance, when $\gamma = 0.5$ and $\tau_\xi = 10$, real efficiency can non-monotonically change with the precision $\tau_\eta$ of public information, so that greater disclosure can harm real efficiency, at least for some region of $\tau_\eta$. Thus, as in our main model, improving the precision of public information can backfire and reduce real efficiency, and this happens through causing the speculators to change weights across different signals they observe that are not observed by the real decision maker. While in our main setting this happens due to changing weights between private signals about different fundamentals, here it happens due to changing weights between a private signal that is specific to the speculator and one that is common across speculators.
Fig. OA1. Implications of disclosure when speculators observe two private signals. We have shut down the uncertainty of factor $\tilde{a}$. Speculators observe a speculator-specific signal and a common private signal about factor $\tilde{f}$. Parameter $\gamma$ controls the correlation between the common private signal and the public information. Parameter $\tau_\xi$ measures the precision of noise trading in the financial market. Parameter $\tau_\eta$ controls the precision of public information $\tilde{f}$ about factor $\tilde{f}$. Other parameters are fixed at 1, i.e., $\tau_f = \tau_y = \tau_c = \lambda = 1$. 