

**Internet Appendix for  
“Common pricing across asset classes: Empirical  
evidence revisited”**

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This Internet Appendix presents propositions for the LMW model in Section A. The bootstrap details are in Section B. Additional results for HKM at quarterly and monthly frequencies can be found in Sections C and D, respectively. Results on individual asset pricing errors are provided in Section E. The (approximate) finite-sample rank test reported in the paper is described in Section F. Finally, in Section G, we derive the asymptotic distributions of the GLS lambda and gamma estimators (and of their misspecification-robust  $t$ -statistics) in misspecified and unidentified beta-pricing models. We use the same notation and table format as in the paper.

## Appendix A: Propositions for the LMW model

Let  $f_t^a = [f_t, f_t^-]'$ , where  $f_t^- = f_t$  if  $f_t \leq \delta$  and NaN otherwise. In addition, denote the mean vector and covariance matrix of  $f_t^a$  by  $\mu_f^a$  and  $V_f^a$ , respectively. Finally, let  $D = \text{Diag}(V_f^a)$  be a diagonal matrix of the diagonal elements of  $V_f^a$ ,  $D_t = \text{Diag}\left((f_t^a - \mu_f^a)(f_t^a - \mu_f^a)'\right)$ , and  $G_t = (r_t - \mu_r)(f_t^a - \mu_f^a)' - BD_t$ . The following proposition provides the asymptotic distribution of  $\hat{B}_1$ .

**Proposition 1.** *Assume that  $[f_t^a, r_t']'$  is jointly stationary and ergodic with a finite fourth moment. With  $T^- < T$  observations available for estimation, we have*

$$\sqrt{T^-}(\hat{B}_1 - B_1) \overset{A}{\rightsquigarrow} \mathcal{N}\left(0_N, V(\hat{B}_1)\right), \quad (\text{A.1})$$

where  $V(\hat{B}_1) \equiv \sum_{j=-\infty}^{\infty} E[h_t h_{t+j}']$  and

$$h_t = G_t D^{-1} \iota. \quad (\text{A.2})$$

**Proof:** Define  $Y_t = [f_t^a, r_t']'$  and its mean and covariance matrix as  $\mu$  and  $V$ , respectively. The corresponding sample moments of  $Y_t$  are given by  $\hat{\mu}$  and  $\hat{V}$ . This and the following proofs rely on the fact that  $\hat{B}_1$  and  $\hat{\gamma}^-$  are smooth functions of  $\hat{\mu}$  and  $\hat{V}$ . Therefore, once we have the asymptotic distribution of  $\hat{\mu}$  and  $\hat{V}$ , we can use the delta method to obtain the asymptotic distribution of  $\hat{B}_1$  in this proposition and  $\hat{\gamma}^-$  in Propositions 2 and 3. Let

$$\varphi = \begin{bmatrix} \mu \\ \text{vec}(V) \end{bmatrix}, \quad \hat{\varphi} = \begin{bmatrix} \hat{\mu} \\ \text{vec}(\hat{V}) \end{bmatrix}. \quad (\text{A.3})$$

We first note that  $\hat{\mu}$  and  $\hat{V}$  can be written as the GMM estimator that uses the moment conditions  $E[g_t] = 0_{(N+K)(N+K+1)}$ , where

$$g_t = \begin{bmatrix} Y_t - \mu \\ \text{vec}((Y_t - \mu)(Y_t - \mu)' - V) \end{bmatrix}. \quad (\text{A.4})$$

Since this is an exactly identified system of moment conditions, it is straightforward to verify that under the assumption that  $Y_t$  is stationary and ergodic with a finite fourth moment, we have that  $\hat{\varphi}$  is normally distributed with mean  $\varphi$  and covariance matrix  $S_0$ ,<sup>1</sup> where

$$S_0 = \sum_{j=-\infty}^{\infty} E[g_t g_{t+j}']. \quad (\text{A.5})$$

Using the delta method, the asymptotic distribution of  $\hat{B}_1$  is

$$\sqrt{T}(\hat{B}_1 - B_1) \stackrel{A}{\approx} \mathcal{N}\left(0_N, \left[\frac{\partial B_1}{\partial \varphi'}\right] S_0 \left[\frac{\partial B_1}{\partial \varphi'}\right]'\right). \quad (\text{A.6})$$

Note that

$$\frac{\partial B_1}{\partial \text{vec}(V)'} = (\iota' \otimes I_N) \frac{\partial \text{vec}(B)}{\partial \text{vec}(V)'}. \quad (\text{A.7})$$

It can be shown that

$$\frac{\partial \text{vec}(B)}{\partial \text{vec}(V)'} = [D^{-1}, 0_{2 \times N}] \otimes [0_{N \times 2}, I_N] - (D^{-1} \otimes B) \Theta_1 ([I_2, 0_{2 \times N}] \otimes [I_2, 0_{2 \times N}]), \quad (\text{A.8})$$

where  $\Theta_1$  is a  $4 \times 4$  matrix such that  $\text{vec}(D) = \Theta_1 \text{vec}(V_f^a)$ .<sup>2</sup> By substituting the latter expression into Eq. (A.7), we obtain

$$\frac{\partial B_1}{\partial \text{vec}(V)'} = [\iota' D^{-1}, 0_N'] \otimes [0_{N \times 2}, I_N] - (\iota' D^{-1} \otimes B) \Theta_1 ([I_2, 0_{2 \times N}] \otimes [I_2, 0_{2 \times N}]). \quad (\text{A.9})$$

Since  $B_1$  does not depend on  $\mu$ , it follows that

$$\begin{aligned} \frac{\partial B_1}{\partial \varphi'} g_t &\equiv h_t = [(r_t - \mu_r)(f_t^a - \mu_f^a)' - B D_t] D^{-1} \iota \\ &= G_t D^{-1} \iota. \end{aligned} \quad (\text{A.10})$$

This concludes the proof.

<sup>1</sup>Note that  $S_0$  is a singular matrix as  $\hat{V}$  is symmetric, so there are redundant elements in  $\hat{\varphi}$ . We could have written  $\hat{\varphi}$  as  $[\hat{\mu}', \text{vech}(\hat{V})']'$ , but the results are the same under both specifications.

<sup>2</sup>Specifically,  $\Theta_1$  is a matrix with its (1, 1)-th and (4, 4)-th elements equal to one, and zero elsewhere.

Let  $\hat{V}_f^a$ ,  $\hat{\mu}_f^a$ , and  $\hat{\mu}_r$  be the sample estimates of  $V_f^a$ ,  $\mu_f^a$ , and  $\mu_r$ , respectively. To conduct statistical tests, we need a consistent estimator of  $V(\hat{B}_1)$ . This can be obtained by replacing  $h_t$  with

$$\hat{h}_t = \hat{G}_t \hat{D}^{-1} \iota, \quad (\text{A.11})$$

where  $\hat{D} = \text{Diag}(\hat{V}_f^a)$ ,  $\hat{D}_t = \text{Diag}((f_t^a - \hat{\mu}_f^a)(f_t^a - \hat{\mu}_f^a)')$ , and  $\hat{G}_t = (r_t - \hat{\mu}_r)(f_t^a - \hat{\mu}_f^a)' - \hat{B} \hat{D}_t$ . In particular, if  $h_t$  is uncorrelated over time, then we have  $V(\hat{B}_1) = E[h_t h_t']$ , and its consistent estimator is given by

$$\hat{V}(\hat{B}_1) = \frac{1}{T^-} \sum_{t=1}^{T^-} \hat{h}_t \hat{h}_t'. \quad (\text{A.12})$$

When  $h_t$  is autocorrelated, one can use Newey and West's (1987) method to obtain a consistent estimator of  $V(\hat{B}_1)$ . Then, a test of  $H_0 : B_{1,i} = 0$  (for  $i = 1, \dots, N$ ) can be performed based on the  $t$ -statistic of  $\hat{B}_{1,i}$ . Similarly, we can test  $H_0 : B_1 = 0_N$  by noting that  $T^- \hat{B}_1' \hat{V}(\hat{B}_1)^{-1} \hat{B}_1 \sim \chi_N^2$ .

Next, we derive a standard error for the  $DR$  premium OLS estimate that accounts for the EIV bias induced by the estimation of the betas and that is robust to model misspecification.

**Proposition 2.** *Assume that  $[f_t^{a'}, r_t']'$  is jointly stationary and ergodic with a finite fourth moment and that  $B_1 \neq 0_N$ . Suppose we have  $T^- < T$  observations available for estimation. Let  $C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$  and  $\gamma_t^- = A(r_t - BCf_t^a)$ . Then,*

$$\sqrt{T^-}(\hat{\gamma}^- - \gamma^-) \overset{A}{\approx} \mathcal{N}(0, V(\hat{\gamma}^-)), \quad (\text{A.13})$$

where  $V(\hat{\gamma}^-) \equiv \sum_{j=-\infty}^{\infty} E[s_t s_{t+j}]$ .

(a) When  $e = 0_N$ ,

$$s_t = (\gamma_t^- - \gamma^-) - AG_t D^{-1}(\iota \gamma^- + C \mu_f^a). \quad (\text{A.14})$$

(b) When  $e \neq 0_N$ ,

$$s_t = (\gamma_t^- - \gamma^-) - AG_t D^{-1}(\iota \gamma^- + C \mu_f^a) - H e' G_t D^{-1} \iota. \quad (\text{A.15})$$

**Proof:** It is straightforward to show that

$$\frac{\partial \gamma^-}{\partial \mu_f^{a'}} = -ABC, \quad (\text{A.16})$$

$$\frac{\partial \gamma^-}{\partial \mu_r'} = A. \quad (\text{A.17})$$

After some tedious algebra, we obtain

$$\begin{aligned} \frac{\partial \gamma^-}{\partial \text{vec}(V)'} &= [H\iota'D^{-1}, 0'_N] \otimes [0'_2, e'] - [(\gamma^- \iota' + \mu_f^{a'} C') D^{-1}, 0'_N] \otimes [0'_2, A] \\ &\quad + [(\gamma^- \iota' + \mu_f^{a'} C') D^{-1} \otimes AB] \Theta_1 ([I_2, 0_{2 \times N}] \otimes [I_2, 0_{2 \times N}]) \\ &\quad - [H\iota'D^{-1} \otimes e'B] \Theta_1 ([I_2, 0_{2 \times N}] \otimes [I_2, 0_{2 \times N}]). \end{aligned} \quad (\text{A.18})$$

Combining terms, it follows that

$$\frac{\partial \gamma^-}{\partial \varphi'} g_t \equiv s_t = (\gamma_t^- - \gamma^-) - AG_t D^{-1} (\iota \gamma^- + C \mu_f^a) - H e' G_t D^{-1} \iota. \quad (\text{A.19})$$

When  $e = 0_N$ , the last term in the previous equation drops out and we obtain

$$s_t = (\gamma_t^- - \gamma^-) - AG_t D^{-1} (\iota \gamma^- + C \mu_f^a). \quad (\text{A.20})$$

This concludes the proof.

The first term of  $s_t$  in Eq. (A.15),  $(\gamma_t^- - \gamma^-)$ , represents the Fama and MacBeth (1973) contribution to the total asymptotic variance of  $\hat{\gamma}^-$ . This is the only part of the asymptotic variance considered by LMW in their empirical analysis. The second term,  $-AG_t D^{-1} (\iota \gamma^- + C \mu_f^a)$ , is the EIV adjustment due to the estimation of the betas. Finally, the third component,  $-H e' G_t D^{-1} \iota$ , is the additional variability due to model misspecification.

To obtain a consistent misspecification-robust estimator of  $V(\hat{\gamma}^-)$ , we need to replace  $s_t$  in Eq.(A.15) with  $\hat{s}_t$ , where

$$\hat{s}_t = (\hat{\gamma}_t^- - \hat{\gamma}^-) - \hat{A} \hat{G}_t \hat{D}^{-1} (\iota \hat{\gamma}^- + C \hat{\mu}_f^a) - \hat{H} \hat{e}' \hat{G}_t \hat{D}^{-1} \iota, \quad (\text{A.21})$$

with  $\hat{\gamma}_t^- = \hat{A}(r_t - \hat{B} C f_t^a)$  and  $\hat{e} = \hat{\mu}_{rc} - \hat{B}_1 \hat{\gamma}^-$ .

Turning to the GLS case, let  $H = (B_1' V_r^{-1} B_1)^{-1}$ ,  $A = H B_1' V_r^{-1}$ ,  $\gamma^- = A \mu_{rc}$ , and  $e = \mu_{rc} - B_1 \gamma^-$ . Denote the GLS estimate of  $\gamma^-$  by  $\hat{\gamma}^- = \hat{A} \hat{\mu}_{rc}$ , where  $\hat{A} = \hat{H} \hat{B}_1' \hat{V}_r^{-1}$ , and  $\hat{H} = (\hat{B}_1' \hat{V}_r^{-1} \hat{B}_1)^{-1}$ , with  $\hat{V}_r$  being the sample equivalent of  $V_r$ .

**Proposition 3.** *Assume that  $[f_t^{a'}, r_t']'$  is jointly stationary and ergodic with a finite fourth moment and that  $B_1 \neq 0_N$ . Suppose we have  $T^- < T$  observations available for estimation. Let  $C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$  and  $\gamma_t^- = A(r_t - B C f_t^a)$ . Then,*

$$\sqrt{T^-} (\hat{\gamma}^- - \gamma^-) \overset{A}{\rightsquigarrow} \mathcal{N}(0, V(\hat{\gamma}^-)), \quad (\text{A.22})$$

where  $V(\hat{\gamma}^-) \equiv \sum_{j=-\infty}^{\infty} E[s_t s_{t+j}]$ .

(a) When  $e = 0_N$ ,

$$s_t = (\gamma_t^- - \gamma^-) - AG_t D^{-1}(\iota \gamma^- + C \mu_f^a). \quad (\text{A.23})$$

(b) When  $e \neq 0_N$ ,

$$s_t = (\gamma_t^- - \gamma^-) - AG_t D^{-1}(\iota \gamma^- + C \mu_f^a) - H e' V_r^{-1} G_t D^{-1} \iota - e' V_r^{-1} (r_t - \mu_r)(r_t - \mu_r)' A'. \quad (\text{A.24})$$

**Proof:** It is straightforward to show that

$$\frac{\partial \gamma^-}{\partial \mu_f^{a'}} = -ABC, \quad (\text{A.25})$$

$$\frac{\partial \gamma^-}{\partial \mu_r'} = A. \quad (\text{A.26})$$

After some tedious algebra, we obtain

$$\begin{aligned} \frac{\partial \gamma^-}{\partial \text{vec}(V)'} &= [H \iota' D^{-1}, 0'_N] \otimes [0'_2, e' V_r^{-1}] - [(\gamma^- \iota' + \mu_f^{a'} C') D^{-1}, \mu_{rc}' V_r^{-1}] \otimes [0'_2, A] \\ &\quad + [0'_2, \gamma^- A] \otimes [0'_2, B_1' V_r^{-1}] + [(\gamma^- \iota' + \mu_f^{a'} C') D^{-1} \otimes AB] \Theta_1 ([I_2, 0_{2 \times N}] \otimes [I_2, 0_{2 \times N}]) \\ &\quad - [H \iota' D^{-1} \otimes e' V_r^{-1} B] \Theta_1 ([I_2, 0_{2 \times N}] \otimes [I_2, 0_{2 \times N}]). \end{aligned} \quad (\text{A.27})$$

Combining terms, it follows that

$$\frac{\partial \gamma^-}{\partial \rho'} g_t \equiv s_t = (\gamma_t^- - \gamma^-) - AG_t D^{-1}(\iota \gamma^- + C \mu_f^a) - H e' V_r^{-1} G_t D^{-1} \iota - e' V_r^{-1} (r_t - \mu_r)(r_t - \mu_r)' A'. \quad (\text{A.28})$$

When  $e = 0_N$ , the last two terms in the previous equation drop out and we obtain

$$s_t = (\gamma_t^- - \gamma^-) - AG_t D^{-1}(\iota \gamma^- + C \mu_f^a). \quad (\text{A.29})$$

This concludes the proof.

The last two terms in Eq. (A.24) vanish when  $e$  is a zero vector, that is, when the model is correctly specified. In particular, the last term in Eq. (A.24) is due to the fact that in the GLS case the weighting matrix needs to be estimated. A consistent misspecification-robust estimator of  $V(\hat{\gamma}^-)$  can then be obtained by replacing  $s_t$  in Eq. (A.24) with  $\hat{s}_t$ , where

$$\hat{s}_t = (\hat{\gamma}_t^- - \hat{\gamma}^-) - \hat{A} \hat{G}_t \hat{D}^{-1}(\iota \hat{\gamma}^- + C \hat{\mu}_f^a) - \hat{H} \hat{e}' \hat{V}_r^{-1} \hat{G}_t \hat{D}^{-1} \iota - \hat{e}' \hat{V}_r^{-1} (r_t - \hat{\mu}_r)(r_t - \hat{\mu}_r)' \hat{A}', \quad (\text{A.30})$$

with  $\hat{\gamma}_t^- = \hat{A}(r_t - \hat{B} C f_t^a)$  and  $\hat{e} = \hat{\mu}_{rc} - \hat{B}_1 \hat{\gamma}^-$ .

## Appendix B: Bootstrap procedure

Let  $r_t^{(i)}$  denote the excess return for asset  $i$  ( $i = 1, \dots, N$ ) at time  $t$  ( $t = 1, \dots, T$ ), and  $f_t^{(j)}$  be the  $t$ -th observation on the  $j$ -th factor ( $j = 1, \dots, K$ ). We then stack the  $K$  factors and  $N$  test asset returns in a  $T \times (N + K)$  matrix  $Z$  with rows  $z_t = [f_t', r_t']$  for  $t = 1, \dots, T$ , where  $f_t = [f_t^{(1)}, \dots, f_t^{(K)}]'$  and  $r_t = [r_t^{(1)}, \dots, r_t^{(N)}]'$ . The regularity conditions (stationarity and ergodicity) are imposed on the multivariate process  $z_t = [f_t', r_t']$ . The bootstrap samples  $\{z_t^*\}_{t=1}^T$  are constructed by drawing with replacement blocks of  $m$  ( $1 \leq m < T$ ) observations from matrix  $Z$ . To induce stationarity in the bootstrap world, we use the circular block bootstrap that “wraps” the data. (See Politis and Romano 1994.) The bootstrap sample is denoted by  $Z^* = \{(z_1^*, z_2^*, \dots, z_m^*), (z_{m+1}^*, z_{m+2}^*, \dots, z_{2m}^*), \dots, (z_{T-m}^*, z_{T-m+1}^*, \dots, z_T^*)\}$  with  $z_t^* = [f_t^{*'}, r_t^{*'}]$  denoting the resampled analog of the original data  $z_t = [f_t', r_t']$ . By resampling jointly all cross-sectional observations, our bootstrap allows for unknown forms of cross-sectional heterogeneity and remains agnostic about the precise sources of factor structure and model misspecification. At the same time, the blocking scheme along the time series dimension allows for mimicking general but unknown forms of serial correlation and conditional heteroskedasticity.

The resampled data  $\{f_t^{*'}, r_t^{*'}\}_{t=1}^T$  is used to obtain the relevant bootstrap estimates that are denoted by  $*$ . The bootstrap  $p$ -values for the tests of statistical significance are constructed by bootstrapping standardized (asymptotically pivotal) statistics.

### B.1. Bootstrap inference in the HKM model

In the HKM model, to obtain the bootstrap  $p$ -values of the tests for statistical significance, we resample the data matrix with rows  $z_t = [f_t', r_t']$  ( $t = 1, \dots, T$ ), where  $f_t = [MKT_t, CPTLT_t]'$  or  $f_t = [MKT_t, CPTLT_t]'$ . For the specification tests, we need to ensure that the null of correct specification holds in the resampled data. For this reason, in bootstrapping the specification test, we impose the following restriction on the means of returns:

$$\tilde{\mu}_r = \hat{X}(\hat{X}'W\hat{X})^{-1}\hat{X}'W\hat{\mu}_r, \quad (\text{B.1})$$

where  $W = I_N$  or  $W = \hat{V}_r^{-1}$ , and use  $z_t = [f_t', (r_t - \hat{\mu}_r + \tilde{\mu}_r)']$  to generate bootstrap samples. The bootstrap analog for the  $j$ -th bootstrap sample of the CSR test of Shanken (1985) is given by

$$\hat{S}_j^* = (\hat{\mu}_r^* - \hat{X}^*\hat{\gamma}^*)'W^*(\hat{\mu}_r^* - \hat{X}^*\hat{\gamma}^*), \quad (\text{B.2})$$

where  $\hat{\mu}_r^*$ ,  $\hat{X}^* = [1_N, \hat{\beta}^*]$ ,  $W^*$ , and  $\hat{\gamma}^*$  denote the bootstrap estimates of  $\mu_r$ ,  $X$ ,  $W$ , and  $\gamma$ , respectively. With  $\hat{S}$  denoting the sample estimate of the specification test statistic, the bootstrap  $p$ -value is computed as  $\frac{1}{B} \sum_{j=1}^B \mathbb{I} \left\{ \hat{S}_j^* > \hat{S} \right\}$ , where  $\mathbb{I}\{\cdot\}$  denotes the indicator function and  $B$  is the number of bootstrap replications.

Similarly, for the constrained zero-beta rate case, we impose the following restriction on the means of the returns:

$$\tilde{\mu}_r = \hat{\beta}(\hat{\beta}'W\hat{\beta})^{-1}\hat{\beta}W\hat{\mu}_r. \quad (\text{B.3})$$

The bootstrap test statistic for the  $j$ -th bootstrap sample takes the form

$$\hat{S}_j^* = (\hat{\mu}_r^* - \hat{\beta}^* \hat{\gamma}^*)' W^* (\hat{\mu}_r^* - \hat{\beta}^* \hat{\gamma}^*), \quad (\text{B.4})$$

with a bootstrap  $p$ -value of  $\frac{1}{B} \sum_{j=1}^B \mathbb{I} \left\{ \hat{S}_j^* > \hat{S} \right\}$ .

Finally, let  $\hat{t}_i = \sqrt{T} \hat{\gamma}_i / \sqrt{\hat{V}_{ii}(\hat{\gamma})}$  denote the sample misspecification-robust  $t$ -test of statistical significance for parameter  $\gamma_i$  ( $t$ -stat <sub>$m$</sub>  in the tables), where  $\hat{V}_{ii}(\hat{\gamma})$  is the  $(i, i)$ -th element of the estimated misspecification-robust covariance matrix (see KRS 2013), and  $\hat{t}_{i,j}^* = \sqrt{T}(\hat{\gamma}_i^* - \hat{\gamma}_i) / \sqrt{\hat{V}_{ii}^*(\hat{\gamma}^*)}$  be its bootstrap analog for the  $j$ -th bootstrap sample. The symmetric bootstrap  $p$ -value is computed as  $\frac{1}{B} \sum_{j=1}^B \mathbb{I} \left\{ \hat{t}_{i,j}^* > \hat{t}_i \right\}$ .

## B.2. Bootstrap inference in the LMW model

For the LMW model, we resample the data matrix with rows  $z_t = [f_t, r_t']$  ( $t = 1, \dots, T$ ), where  $f_t$  is the market log excess return. Let  $\{f_t^*\}_{t=1}^T$  denote the bootstrap data for the market log excess return with mean  $\hat{\mu}_f^*$  and variance  $\hat{V}_f^*$ , and  $r_t^*$  be the  $N$ -vector of bootstrap log excess returns at time  $t$ , with mean vector  $\hat{\mu}_r^*$  and covariance matrix  $\hat{V}_r^*$ . We then obtain the bootstrap estimate  $\hat{\beta}^*$  by projecting the  $N$ -vector of bootstrap returns,  $r_t^*$ , on a constant and  $f_t^*$ . Similarly,  $\hat{\beta}^{-*}$  is computed based on the projection of  $r_t^*$  on a constant and  $f_t^{-*}$  (where  $f_t^{-*} = f_t^*$  if  $f_t^* \leq \hat{\mu}_f^* - \sqrt{\hat{V}_f^*}$  and NaN otherwise). Thus,  $\hat{B}_1^* = [\hat{\beta}^*, \hat{\beta}^{-*}] \iota$ .

Consider the individual  $t$ -tests of  $H_0 : B_{1,i} = 0$  for  $i = 1, \dots, N$  and the joint test of  $H_0 : B_1 = 0_N$ , where  $N = \dim(r_t)$ . Imposing the null hypothesis, the sample test statistics take the form  $\hat{t}_i = \sqrt{T}^{-1} \hat{B}_{1,i} / \sqrt{\hat{V}_{ii}(\hat{B}_1)}$  for  $i = 1, \dots, N$  and  $JT = T^{-1} \hat{B}_1' \hat{V}(\hat{B}_1)^{-1} \hat{B}_1$ . With  $\hat{B}_1^* = [\hat{B}_{1,1}^*, \dots, \hat{B}_{1,N}^*]'$  and  $\hat{V}^*(\hat{B}_1^*)$ , the bootstrap versions of these test statistics are  $\hat{t}_i^* = \sqrt{T}^{-1} (\hat{B}_{1,i}^* - \hat{B}_{1,i}) / \sqrt{\hat{V}_{ii}^*(\hat{B}_1^*)}$  for  $i = 1, \dots, N$  and  $JT^* = T^{-1} (\hat{B}_1^* - \hat{B}_1)' \hat{V}^*(\hat{B}_1^*)^{-1} (\hat{B}_1^* - \hat{B}_1)$ . The bootstrap  $p$ -values are then



computed as  $\frac{1}{B} \sum_{j=1}^B \mathbb{I} \left\{ \hat{t}_{i,j}^* > \hat{t}_i \right\}$  for  $i = 1, \dots, N$  and  $\frac{1}{B} \sum_{j=1}^B \mathbb{I} \left\{ JT_j^* > JT \right\}$ , where  $B$  is the number of bootstrap replications.

The bootstrap estimate of the downside beta risk premium is given by

$$\hat{\gamma}^{-*} = (\hat{B}_1^{*'} W^* \hat{B}_1^*)^{-1} \hat{B}_1^* W^* \hat{\mu}_{rc}^*, \quad (\text{B.5})$$

where  $\hat{\mu}_{rc}^* = \hat{\mu}_r^* - \hat{\beta}^* \hat{\mu}_f^*$ , and  $W^* = I_N$  for OLS or  $W^* = \hat{V}_r^{*-1}$  for GLS. With the sample misspecification-robust  $t$ -test for statistical significance of  $\gamma^-$  defined as  $\hat{t} = \sqrt{T^-} \hat{\gamma}^- / \sqrt{\hat{V}(\hat{\gamma}^-)}$ , where  $\hat{V}(\hat{\gamma}^-) = \sum_{l=1}^k \hat{s}_t \hat{s}_{t+l}$  for some  $k < T^-$  and  $\hat{s}_t$  is the sample analog of  $s_t$  in part (b) of Proposition 2 or 3. The bootstrap  $t$ -test is constructed as  $\hat{t}^* = \sqrt{T^-} (\hat{\gamma}^{-*} - \hat{\gamma}^-) / \sqrt{\hat{V}^*(\hat{\gamma}^{-*})}$ , where  $\hat{V}^*(\hat{\gamma}^{-*}) = \sum_{l=1}^k \hat{s}_t^* \hat{s}_{t+l}^*$  and  $\hat{s}_t^*$  is the bootstrap analog of  $s_t$  in part (b) of Proposition 2 or 3. The bootstrap  $p$ -value is then computed as  $\frac{1}{B} \sum_{j=1}^B \mathbb{I} \left\{ \hat{t}_j^* > \hat{t} \right\}$ .

## C. Quarterly frequency: Traded capital factor

Tables C.1 to C.3 report results for the traded version of the capital risk factor in HKM’s two-factor and single-factor models. To be consistent with HKM, in all the two-pass CSR analyses (at quarterly as well as monthly frequencies) we do not subtract the risk-free rate from traded capital, that is,  $CPTLT$  is a return, and not an excess return.

Tables C.1 and C.2 correspond to Tables 2 and 3 in the paper (but with the traded instead of the nontraded factor), and Table C.3 corresponds to Panel B in Tables 6 and 7 in the paper, respectively, for the traded capital factor. Overall, the results with traded capital are similar to the ones for nontraded capital in the paper.

## D. Monthly frequency

Tables D.1 to D.11 replicate Tables 1 to 9 in the paper but at a monthly frequency with Tables D.3 and D.5 reporting additional results for the traded capital factor. Looking at the OLS gammas in HKM’s two-factor model, the results based on monthly data are substantially weaker than the ones for quarterly data. As in the paper, the OLS and GLS-based inference for the price of covariance risk with misspecification-robust standard errors (asymptotic and bootstrap) suggests little or no incremental pricing ability for the capital risk factor at the 5% significance level. Finally, in line with the main findings in the paper, the results for the single-factor models do not reveal substantial differences between the pricing ability of the market and capital ratio factors.

## E. Individual pricing errors

To prepare for our analysis, we define  $Y = [f', r']'$  and its mean and covariance matrix as

$$\mu = E[Y] \equiv \begin{bmatrix} \mu_f \\ \mu_r \end{bmatrix}, \quad (\text{E.1})$$

$$V = \text{Var}[Y] \equiv \begin{bmatrix} V_f & V_{fr} \\ V_{rf} & V_r \end{bmatrix}. \quad (\text{E.2})$$

We assume that  $X = [1_N, \beta]$  is of full column rank. To simplify the notation, we suppress the subscript  $W$  from  $\gamma_W$ ,  $\lambda_W$ ,  $e_W$ , and  $\rho_W^2$  when the choice of  $W$  is clear from the context.

Let  $Y_t = [f_t', r_t']'$  and assume that the time series  $Y_t$  is jointly stationary and ergodic with a finite fourth moment. Denote the sample moments of  $Y_t$  by

$$\hat{\mu} = \begin{bmatrix} \hat{\mu}_f \\ \hat{\mu}_r \end{bmatrix} = \frac{1}{T} \sum_{t=1}^T Y_t, \quad (\text{E.3})$$

$$\hat{V} = \begin{bmatrix} \hat{V}_f & \hat{V}_{fr} \\ \hat{V}_{rf} & \hat{V}_r \end{bmatrix} = \frac{1}{T} \sum_{t=1}^T (Y_t - \hat{\mu})(Y_t - \hat{\mu})'. \quad (\text{E.4})$$

Starting from the fixed  $W$  case, the following proposition provides the asymptotic distribution of  $\hat{e}$  under correctly specified and misspecified models.

**Proposition 4.** *Let  $\epsilon_t = r_t - \mu_r - \beta(f_t - \mu_f)$ ,  $H = (X'WX)^{-1}$ ,  $A = HX'W$ ,  $y_t = 1 - \gamma_1' V_f^{-1}(f_t - \mu_f)$ ,  $z_t = [0, (f_t - \mu_f)' V_f^{-1}]'$ , and  $u_t = e'W(r_t - \mu_r)$ .*

(a) *When  $e = 0_N$ ,*

$$\sqrt{T}\hat{e} \overset{A}{\rightsquigarrow} \mathcal{N} \left( 0_N, \sum_{j=-\infty}^{\infty} E[l_t l_{t+j}'] \right), \quad (\text{E.5})$$

where

$$l_t = (I_N - XA)\epsilon_t y_t. \quad (\text{E.6})$$

(b) *When  $e \neq 0_N$ ,*

$$\sqrt{T}(\hat{e} - e) \overset{A}{\rightsquigarrow} \mathcal{N} \left( 0_N, \sum_{j=-\infty}^{\infty} E[l_t l_{t+j}'] \right), \quad (\text{E.7})$$

where

$$l_t = (I_N - XA)\epsilon_t y_t - XH z_t u_t. \quad (\text{E.8})$$

**Proof:** Using the delta method and the same notation as in the proof of Proposition 1, it is straightforward to show that

$$\frac{\partial e}{\partial \varphi'} \equiv l_t = \frac{\partial e}{\partial \mu_r'} (r_t - \mu_r) + \frac{\partial e}{\partial \text{vec}(V)} \text{vec}((Y_t - \mu)(Y_t - \mu)' - V). \quad (\text{E.9})$$

For the first term, we have

$$\frac{\partial e}{\partial \mu_r'} = [I_N - X(X'WX)^{-1}X'W], \quad (\text{E.10})$$

and for the second term, we use the chain rule to obtain

$$\begin{aligned}
\frac{\partial e}{\partial \text{vec}(V)'} &= -(\gamma' \otimes I_N) \frac{\partial \text{vec}(X)}{\partial \text{vec}(V)'} - X \frac{\partial \gamma}{\partial \text{vec}(V)'} \\
&= -(\gamma' \otimes I_N) \left( ([0_K, V_f^{-1}]', 0_{(K+1) \times N}) \otimes [-\beta, I_N] \right) \\
&\quad - X \left( [H[0_K, V_f^{-1}]', 0_{(K+1) \times N}] \otimes [0'_K, e'W] - [\gamma'_1 V_f^{-1}, 0'_N] \otimes [-A\beta, A] \right) \\
&= -[\gamma'_1 V_f^{-1}, 0'_N] \otimes [-\beta, I_N] \\
&\quad - X \left( [H[0_K, V_f^{-1}]', 0_{(K+1) \times N}] \otimes [0'_K, e'W] - [\gamma'_1 V_f^{-1}, 0'_N] \otimes [-A\beta, A] \right).
\end{aligned} \tag{E.11}$$

It follows that

$$\begin{aligned}
l_t &= [I_N - X(X'WX)^{-1}X'W](r_t - \mu_r) - \text{vec}([- \beta, I_N](Y_t - \mu)(Y_t - \mu)'[\gamma'_1 V_f^{-1}, 0'_N]') \\
&\quad - X[H z_t u_t - (\phi_t - \phi)w_t] \\
&= e_t - e - w_t \epsilon_t - X[H z_t u_t - (\phi_t - \phi)w_t],
\end{aligned} \tag{E.12}$$

where  $\gamma_t = [\gamma_{0t}, \gamma_{1t}] = Ar_t$ ,  $\phi_t = [\gamma_{0t}, (\gamma_{1t} - f_t)']'$ ,  $\phi = [\gamma_0, (\gamma_1 - \mu_f)']'$ ,  $w_t = 1 - y_t$ , and  $e_t = (I_N - XA)r_t$ . Using the fact that

$$\phi_t - \phi = A\epsilon_t, \tag{E.13}$$

and

$$e_t - e = (I_N - XA)(r_t - \mu_r) = (I_N - XA)[\beta(f_t - \mu_f) + \epsilon_t] = (I_N - XA)\epsilon_t, \tag{E.14}$$

we can write

$$l_t = (I_N - XA)\epsilon_t y_t - XH z_t u_t. \tag{E.15}$$

When  $e = 0_N$ , we have  $u_t = 0$ . It follows that

$$l_t = (I_N - XA)\epsilon_t y_t. \tag{E.16}$$

This concludes the proof.

Under a correctly specified model, it is easy to see that the covariance matrix of  $\hat{e}$  has rank  $N - K - 1$  because  $X'Wl_t = 0_{K+1}$ . For the case of a misspecified model, we use the fact that  $X'W(I_N - XA) = 0_{(K+1) \times N}$  to obtain

$$X'Wl_t = -z_t u_t. \tag{E.17}$$

In particular, the first element of  $X'Wl_t$  (that is,  $1'_N Wl_t$ ) is zero. It follows that under the misspecified model, the asymptotic covariance matrix of  $\hat{e}$  has rank  $N - 1$  because  $1'_N W\hat{e} = 0$ .

Turning our attention to the GLS case, the next proposition provides the asymptotic distribution of  $\hat{e}$  under correctly specified and misspecified models.

**Proposition 5.** *Let  $H = (X'V_r^{-1}X)^{-1}$ ,  $A = HX'V_r^{-1}$ ,  $y_t = 1 - \gamma'_1 V_f^{-1}(f_t - \mu_f)$ , and  $u_t = e'V_r^{-1}(r_t - \mu_r)$ .*

(a) *When  $e = 0_N$ ,*

$$\sqrt{T}\hat{e} \overset{A}{\underset{\sim}{\mathcal{N}}}(0_N, V(\hat{e})), \quad (\text{E.18})$$

where  $V(\hat{e}) \equiv \sum_{j=-\infty}^{\infty} E[l_t l'_{t+j}]$  and

$$l_t = (I_N - XA)\epsilon_t y_t. \quad (\text{E.19})$$

(b) *When  $e \neq 0_N$ ,*

$$\sqrt{T}(\hat{e} - e) \overset{A}{\underset{\sim}{\mathcal{N}}}(0_N, V(\hat{e})), \quad (\text{E.20})$$

where  $V(\hat{e}) \equiv \sum_{j=-\infty}^{\infty} E[l_t l'_{t+j}]$  and

$$l_t = (I_N - XA)\epsilon_t y_t - X[H z_t - A(r_t - \mu_r)]u_t. \quad (\text{E.21})$$

**Proof:** It is straightforward to show that

$$l_t = \frac{\partial e}{\partial \mu'_r} (r_t - \mu_r) + \frac{\partial e}{\partial \text{vec}(V)'} \text{vec}((Y_t - \mu)(Y_t - \mu)' - V). \quad (\text{E.22})$$

For the first term, we have

$$\frac{\partial e}{\partial \mu'_r} = [I_N - X(X'V_r^{-1}X)^{-1}X'V_r^{-1}], \quad (\text{E.23})$$

and for the second term, we use the chain rule to obtain

$$\frac{\partial e}{\partial \text{vec}(V)'} = -(\gamma' \otimes I_N) \frac{\partial \text{vec}(X)}{\partial \text{vec}(V)'} - X \frac{\partial \gamma}{\partial \text{vec}(V)'}. \quad (\text{E.24})$$

As in the fixed  $W$  case,

$$\frac{\partial \text{vec}(X)}{\partial \text{vec}(V)'} = \left( [0_K, V_f^{-1}]', 0_{(K+1) \times N} \right) \otimes [-\beta, I_N]. \quad (\text{E.25})$$

In addition, using the first order condition  $\beta'V_r^{-1}e = 0_K$ , we have

$$\begin{aligned} \frac{\partial \gamma}{\partial \text{vec}(V)'} &= \left[ H[0_K, V_f^{-1}]', 0_{(K+1) \times N} \right] \otimes [0'_K, e'V_r^{-1}] \\ &\quad - \left[ \gamma'_1 V_f^{-1}, 0'_N \right] \otimes [-A\beta, A] - [0'_K, e'V_r^{-1}] \otimes [0_{(K+1) \times K}, A]. \end{aligned} \quad (\text{E.26})$$

It follows that

$$\begin{aligned} \frac{\partial e}{\partial \text{vec}(V)'} &= -(\gamma' \otimes I_N) \left( ([0_K, V_f^{-1}]', 0_{(K+1) \times N}) \otimes [-\beta, I_N] \right) \\ &\quad - X \left( \left[ H[0_K, V_f^{-1}]', 0_{(K+1) \times N} \right] \otimes [0'_K, e'V_r^{-1}] \right) \\ &\quad + X \left( \left[ \gamma'_1 V_f^{-1}, 0'_N \right] \otimes [-A\beta, A] + [0'_K, e'V_r^{-1}] \otimes [0_{(K+1) \times K}, A] \right). \end{aligned} \quad (\text{E.27})$$

Therefore, we have

$$\begin{aligned} l_t &= [I_N - X(X'V_r^{-1}X)^{-1}X'V_r^{-1}](r_t - \mu_r) - \text{vec}([- \beta, I_N](Y_t - \mu)(Y_t - \mu)'[\gamma'_1 V_f^{-1}, 0'_N]') \\ &\quad - X[H z_t u_t - (\phi_t - \phi)w_t - (\gamma_t - \gamma)u_t], \\ &= e_t - e - w_t \epsilon_t - X[H z_t u_t - (\phi_t - \phi)w_t - (\gamma_t - \gamma)u_t], \end{aligned} \quad (\text{E.28})$$

where  $\gamma_t = Ar_t$ ,  $\phi_t = [\gamma_{0t}, (\gamma_{1t} - f_t)']'$ ,  $\phi = [\gamma_0, (\gamma_1 - \mu_f)']'$ ,  $w_t = 1 - y_t$ , and  $e_t = (I_N - XA)r_t$ . Using the fact that  $e_t - e = (I_N - XA)\epsilon_t$ ,  $\phi_t - \phi = A\epsilon_t$ , and  $\gamma_t - \gamma = A(r_t - \mu_r)$  we also can write

$$l_t = (I_N - XA)\epsilon_t y_t - X[H z_t - A(r_t - \mu_r)]u_t. \quad (\text{E.29})$$

When  $e = 0_N$ , we have  $u_t = 0$ . It follows that

$$l_t = (I_N - XA)\epsilon_t y_t. \quad (\text{E.30})$$

This concludes the proof.

Under a correctly specified model, it is easy to see that the covariance matrix of  $\hat{e}$  has rank  $N - K - 1$  because  $X'V_r^{-1}l_t = 0_{K+1}$ . For the case of a misspecified model, the asymptotic covariance of  $\hat{e}$  has rank  $N$ . Using the fact that  $X'V_r^{-1}(I_N - XA) = 0_{(K+1) \times N}$ , we obtain

$$X'V_r^{-1}l_t = -z_t u_t + X'V_r^{-1}(r_t - \mu_r)u_t, \quad (\text{E.31})$$

which does not have any zero element (the first element of  $X'V_r^{-1}l_t$  is given by  $1'_N V_r^{-1}(r_t - \mu_r)u_t$  and is nonzero).

To conduct statistical tests, we need a consistent estimator of  $V(\hat{e})$ . This can be obtained in the fixed weighting matrix case by replacing  $l_t$  with

$$\hat{l}_t = (I_N - \hat{X}\hat{A})\hat{e}_t\hat{y}_t - \hat{X}\hat{H}\hat{z}_t\hat{u}_t, \quad (\text{E.32})$$

where  $\hat{X} = [1_N, \hat{\beta}]$ ,  $\hat{e}_t = r_t - \hat{\mu}_r - \hat{\beta}(f_t - \hat{\mu}_f)$ ,  $\hat{H} = (\hat{X}'W\hat{X})^{-1}$ ,  $\hat{A} = \hat{H}\hat{X}'W$ ,  $\hat{y}_t = 1 - \hat{\gamma}'_1\hat{V}_f^{-1}(f_t - \hat{\mu}_f)$ ,  $\hat{z}_t = [0, (f_t - \hat{\mu}_f)'\hat{V}_f^{-1}]'$ , and  $\hat{u}_t = \hat{e}'W(r_t - \hat{\mu}_r)$ . In particular, if  $l_t$  is uncorrelated over time, then we have  $V(\hat{e}) = E[l_t l_t']$ , and its consistent estimator is given by

$$\hat{V}(\hat{e}) = \frac{1}{T} \sum_{t=1}^T \hat{l}_t \hat{l}_t'. \quad (\text{E.33})$$

When  $l_t$  is autocorrelated, one can use Newey and West's (1987) method to obtain a consistent estimator of  $V(\hat{e})$ . Similarly, in the GLS case, we need to replace  $l_t$  with

$$\hat{l}_t = (I_N - \hat{X}\hat{A})\hat{e}_t\hat{y}_t - \hat{X}[\hat{H}\hat{z}_t - \hat{A}(r_t - \hat{\mu}_r)]\hat{u}_t, \quad (\text{E.34})$$

where  $\hat{H} = (\hat{X}'\hat{V}_r^{-1}\hat{X})^{-1}$ ,  $\hat{A} = \hat{H}\hat{X}'\hat{V}_r^{-1}$ ,  $\hat{y}_t = 1 - \hat{\gamma}'_1\hat{V}_f^{-1}(f_t - \hat{\mu}_f)$ ,  $\hat{u}_t = \hat{e}'\hat{V}_r^{-1}(r_t - \hat{\mu}_r)$ , and  $\hat{\gamma}_1$  is the GLS risk premium estimator of  $\gamma_1$ . Finally, it should be emphasized that although the asymptotic covariance matrix of  $\hat{e}$  can have a deficient column rank, the tests of individual significance of the pricing errors for both OLS and GLS can be implemented based on the usual  $t$ -statistics.<sup>3</sup>

## F. Rank test

When the factors have very low correlations with the returns, it is sensible to test whether the  $N \times (K + 1)$  matrix  $X = [1_N, \beta]$  has a full column rank before running the two-pass CSR. Let  $P$  be an  $N \times (N - 1)$  orthonormal matrix whose columns are orthogonal to  $1_N$ , such that  $P'P = I_{N-1}$  and  $PP' = I_N - 1_N(1_N'1_N)^{-1}1_N'$ . Note that premultiplying by  $P$  removes the column of 1s from the matrix  $X$ . Thus, performing a rank test on the  $(N - 1) \times K$  matrix  $\Pi = P'\beta$  provides a convenient way of testing for rank deficiency of  $X$ . Let  $\hat{\Pi} = P'\hat{\beta}$ , we have

$$\sqrt{T}\text{vec}(\hat{\Pi} - \Pi) \xrightarrow{d} \mathcal{N}(0_{(N-1)K}, S_{\hat{\Pi}}), \quad (\text{F.1})$$

where

$$S_{\hat{\Pi}} = \sum_{j=-\infty}^{+\infty} E[\tilde{x}_t \tilde{x}'_{t+j}], \quad (\text{F.2})$$

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<sup>3</sup>Analytical derivations for the WLS case are available from the authors upon request.

and

$$\tilde{x}_t = V_f^{-1}(f_t - \mu_f) \otimes P' \epsilon_t. \quad (\text{F.3})$$

Denoting by  $\hat{S}_{\hat{\Pi}}$  a consistent estimator of  $S_{\hat{\Pi}}$ , Cragg and Donald (1997) suggest that we can test  $H_0 : \text{rank}(\Pi) = K - 1$  using

$$\mathcal{J}_1 = T \min_{\Pi \in \Gamma(K-1)} \text{vec}(\hat{\Pi} - \Pi)' \hat{S}_{\hat{\Pi}}^{-1} \text{vec}(\hat{\Pi} - \Pi) \stackrel{A}{\sim} \chi_{N-K}^2, \quad (\text{F.4})$$

where  $\Gamma(K - 1)$  is the space of an  $(N - 1) \times K$  matrix with rank  $K - 1$ . In general, this test is not computationally attractive since we need to optimize over  $N(K - 1)$  parameters. Gospodinov, Kan, and Robotti (2017) propose an alternative expression for  $\mathcal{J}_1$  that greatly reduces the computational burden. Their test is given by

$$\mathcal{J}_2 = T \min_c [-1, c'] \hat{\Pi}' (C \hat{S}_{\hat{\Pi}} C')^{-1} \hat{\Pi} [-1, c'] \stackrel{A}{\sim} \chi_{N-K}^2, \quad (\text{F.5})$$

where  $c$  is a  $(K - 1)$ -vector and  $C = [-1, c'] \otimes I_{N-1}$ . With this new expression, one can easily test whether  $X$  has a full column rank even when  $K$  is large.

If we further assume that  $\tilde{x}_t$  is serially uncorrelated and  $\text{Var}[r_t | f_t] = \Sigma = V_r - \beta V_f \beta'$  (conditional homoskedasticity case),

$$S_{\hat{\Pi}} = V_f^{-1} \otimes P' \Sigma P, \quad (\text{F.6})$$

and we have the following simple test of  $H_0 : \text{rank}(\Pi) = K - 1$ :

$$\mathcal{J}_3 = T \xi_K \stackrel{A}{\sim} \chi_{N-K}^2, \quad (\text{F.7})$$

where  $\xi_K$  is the smallest eigenvalue of  $\hat{V}_f \hat{\beta}' P (P' \hat{\Sigma} P)^{-1} P' \hat{\beta}$  and  $\hat{\Sigma} = \hat{V}_r - \hat{\beta} \hat{V}_f \hat{\beta}'$ . Then, based on Theorem 10.4.2 of Muirhead (2005), the (approximate)  $F$ -test of  $H_0 : \text{rank}(X) = K$  that we employ in the paper (RANK TEST) is given by

$$\mathcal{J}_4 = \xi_K (T - N) / (N - K) \sim F_{N-K, T-N}. \quad (\text{F.8})$$

For the normality case, the  $p$ -value for  $\mathcal{J}_4$  is exact when  $K = 1$ . For  $K > 1$ , using Theorem 6 of Schott (1984), an upper bound for the  $p$ -value  $P[\xi_K > c]$  is obtained as

$$P[\xi_K > c] \leq P[F_{N-K, T-N} > (T - N)c / (N - K)]. \quad (\text{F.9})$$

In the constrained zero-beta rate case,

$$\mathcal{J}_4 = \xi_K (T - N) / (N - K + 1) \sim F_{N-K+1, T-N}, \quad (\text{F.10})$$

where  $\xi_K$  is the smallest eigenvalue of  $\hat{V}_f \hat{\beta}' \hat{\Sigma}^{-1} \hat{\beta}$ .



## G. Asymptotic Distributions of the GLS Lambda and Gamma Estimators in Misspecified and Unidentified Beta-Pricing Models

In this section, we consider a beta-pricing model with a constant,  $K - 1$  useful factors, and one spurious factor.<sup>4</sup> The beta-pricing model is assumed to be globally misspecified, that is, the beta-pricing relation does not hold in population. In addition, the model is unidentified due to the presence of a spurious factor, that is, a factor that is uncorrelated with the test asset returns and the useful factors for all time periods. The analysis follows closely Gospodinov, Kan, and Robotti (2014), and the main results can be summarized as follows. All of the asymptotic distributions for the GLS parameter estimates are non-normal. The GLS estimators of the parameters for the constant and useful factors converge to bounded random variables and, hence, they are inconsistent. The GLS estimators of the parameters associated with the spurious factor diverge at rate  $\sqrt{T}$ . In addition, for the spurious factor, the GLS misspecification-robust  $t$ -statistics proposed by Kan, Robotti, and Shanken (KRS, 2013) are asymptotically distributed as  $\mathcal{N}(0, 1)$ . The squared  $t$ -statistics for the intercepts and the useful factors are bounded by  $\chi_1^2$  random variables. This implies that when the model is misspecified, the standard inference based on the misspecification-robust  $t$ -statistics will deliver asymptotically valid, although somewhat conservative, inference even when the model is potentially unidentified due to the presence of a spurious factor.

Below, in the interest of brevity, we only report the statistical details for the GLS lambda estimator because the results and conclusions for the GLS gamma estimator and associated  $t$ -statistics are qualitatively the same.<sup>5</sup> Let  $f_t$  be a  $(K - 1)$ -vector of useful factors, with mean  $\mu_f$  and covariance matrix  $V_f$ , and let  $g_t$  be a spurious factor, with mean 0 and variance 1.<sup>6</sup> We assume that  $N > K + 1$  and that  $[r_t', f_t', g_t']'$  are jointly stationary and ergodic processes with finite fourth moments. Denote by  $V_{rf}$  the matrix of asset covariances w.r.t. the first  $K - 1$  useful factors and by  $d = 0_N$  the vector of asset covariances w.r.t. the spurious factor, ordered last.

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<sup>4</sup>The limiting results in the case where a linear combination of useful factors is spurious are the same as the ones for the baseline spurious factor scenario. Therefore, we do not discuss them separately in the following analysis.

<sup>5</sup>Specifically, all the results for the constant and spurious factor are unaltered. For the useful factors, we prove that the limiting distribution of the misspecification-robust  $t$ -statistic is different from the one for the lambda case. However, it is still true that the squared misspecification-robust  $t$ -statistic is stochastically bounded by a  $\chi_1^2$  random variable. The results for the gamma analysis are available from the authors upon request.

<sup>6</sup>This assumption does not affect our asymptotic results on statistical inference for the risk premia parameters of the linear beta-pricing model. It does, however, affect the limiting distribution of the estimated intercept and the statistical inference on it. The limiting results derived under a generic mean and variance of the spurious factor are available from the authors upon request.

The sample counterparts of  $V_{rf}$  and  $d$  are  $\hat{V}_{rf}$  and  $\hat{d}$ . Let  $D = [C, d]$  and  $C = [1_N, V_{rf}]$ , and denote by  $\hat{D} = [\hat{C}, \hat{d}]$  and  $\hat{C} = [1_N, \hat{V}_{rf}]$  their sample counterparts.<sup>7</sup> In addition, let  $\lambda^* = [\lambda_0^*, \lambda_{1,1}^*, \dots, \lambda_{1,K}^*]' = [\lambda_0^*, \lambda_1^*, \lambda_2^*]' \equiv [\lambda_{01}^{*'}, \lambda_2^{*'}]'$ , where  $\lambda_{01}^* = [\lambda_0^*, \lambda_{1,1}^*, \dots, \lambda_{1,K-1}^*]'$  and  $\lambda_2^* = \lambda_{1,K}^*$  denotes the pseudo-true value of the risk premium on the spurious factor.<sup>8</sup> Define the GLS estimator of  $\lambda^*$  as

$$\hat{\lambda}^{GLS} = \begin{bmatrix} \hat{\lambda}_0^{GLS} \\ \hat{\lambda}_1^{GLS} \\ \hat{\lambda}_2^{GLS} \end{bmatrix} = \begin{bmatrix} \hat{\lambda}_{01}^{GLS} \\ \hat{\lambda}_2^{GLS} \end{bmatrix} = (\hat{D}'\hat{V}_r^{-1}\hat{D})^{-1}\hat{D}'\hat{V}_r^{-1}\hat{\mu}_r. \quad (\text{G.1})$$

Note that this is equivalent to running an ordinary least squares regression of  $\hat{V}_r^{-\frac{1}{2}}\hat{\mu}_r$  on  $\hat{V}_r^{-\frac{1}{2}}\hat{C}$  and  $\hat{V}_r^{-\frac{1}{2}}\hat{d}$ . In order to obtain  $\hat{\lambda}_2^{GLS}$ , we can project  $\hat{V}_r^{-\frac{1}{2}}\hat{\mu}_r$  and  $\hat{V}_r^{-\frac{1}{2}}\hat{d}$  on  $\hat{V}_r^{-\frac{1}{2}}\hat{C}$ , and then regress the residuals from the first projection onto the residuals from the second projection. It follows that

$$\hat{\lambda}_2^{GLS} = \frac{\hat{d}'\hat{V}_r^{-\frac{1}{2}}[I_N - \hat{V}_r^{-\frac{1}{2}}\hat{C}(\hat{C}'\hat{V}_r^{-1}\hat{C})^{-1}\hat{C}'\hat{V}_r^{-\frac{1}{2}}]\hat{V}_r^{-\frac{1}{2}}\hat{\mu}_r}{\hat{d}'\hat{V}_r^{-\frac{1}{2}}[I_N - \hat{V}_r^{-\frac{1}{2}}\hat{C}(\hat{C}'\hat{V}_r^{-1}\hat{C})^{-1}\hat{C}'\hat{V}_r^{-\frac{1}{2}}]\hat{V}_r^{-\frac{1}{2}}\hat{d}}. \quad (\text{G.2})$$

Similarly, the parameter vector  $\hat{\lambda}_{01}^{GLS}$  is obtained by projecting  $\hat{V}_r^{-\frac{1}{2}}\hat{\mu}_r$  and  $\hat{V}_r^{-\frac{1}{2}}\hat{C}$  on  $\hat{V}_r^{-\frac{1}{2}}\hat{d}$ , and then regressing the residuals from the first projection onto the residuals from the second projection, which yields

$$\begin{aligned} \hat{\lambda}_{01}^{GLS} &= (\hat{C}'\hat{V}_r^{-\frac{1}{2}}[I_N - \hat{V}_r^{-\frac{1}{2}}\hat{d}(\hat{d}'\hat{V}_r^{-1}\hat{d})^{-1}\hat{d}'\hat{V}_r^{-\frac{1}{2}}]\hat{V}_r^{-\frac{1}{2}}\hat{C})^{-1} \\ &\quad \times \hat{C}'\hat{V}_r^{-\frac{1}{2}}[I_N - \hat{V}_r^{-\frac{1}{2}}\hat{d}(\hat{d}'\hat{V}_r^{-1}\hat{d})^{-1}\hat{d}'\hat{V}_r^{-\frac{1}{2}}]\hat{V}_r^{-\frac{1}{2}}\hat{\mu}_r. \end{aligned} \quad (\text{G.3})$$

We adopt the following simplifying notation. Let  $\tilde{C} = V_r^{-\frac{1}{2}}C$ ,  $\tilde{\mu}_r = V_r^{-\frac{1}{2}}\mu_r$ ,  $M = I_N - \tilde{C}(\tilde{C}'\tilde{C})^{-1}\tilde{C}'$ , and  $z_N \sim \mathcal{N}(0_N, I_N)$ . Let  $P$  be an  $N \times (N - K)$  orthonormal matrix whose columns are orthogonal to  $\tilde{C}$  so that  $PP' = I_N - \tilde{C}(\tilde{C}'\tilde{C})^{-1}\tilde{C}'$ . Then, we have

$$\sqrt{T}\hat{V}_r^{-\frac{1}{2}}\hat{d} \xrightarrow{d} z_N, \quad (\text{G.4})$$

$$\hat{M} = I_N - \hat{V}_r^{-\frac{1}{2}}\hat{C}(\hat{C}'\hat{V}_r^{-1}\hat{C})^{-1}\hat{C}'\hat{V}_r^{-\frac{1}{2}} \xrightarrow{p} M = PP', \quad (\text{G.5})$$

$$\hat{V}_r^{-\frac{1}{2}}\hat{\mu}_r \xrightarrow{p} \tilde{\mu}_r, \quad (\text{G.6})$$

and

$$T^{-\frac{1}{2}}\hat{\lambda}_2^{GLS} \xrightarrow{d} \frac{z_N' M \tilde{\mu}_r}{z_N' M z_N} = \frac{\sqrt{Q}z_1}{z_{N-K}' z_{N-K}}, \quad (\text{G.7})$$

<sup>7</sup>The matrices  $C$  ( $N \times K$ ) and  $D$  ( $N \times (K + 1)$ ) have a column rank  $K$ .

<sup>8</sup>Since  $\lambda_2^*$  is not defined under identification failure, we set it equal to zero.

where  $\tilde{Q} = \tilde{\mu}'_r P P' \tilde{\mu}_r$ ,  $z_1 = \tilde{\mu}'_r P P' z_N / \sqrt{\tilde{Q}} \sim \mathcal{N}(0, 1)$ , and  $z_{N-K} = P' z_N \sim \mathcal{N}(0_{N-K}, I_{N-K})$ . Similarly, using that  $\hat{V}_r^{-\frac{1}{2}} \hat{C} \xrightarrow{p} \tilde{C}$ , we have

$$\hat{\lambda}_{01}^{GLS} \xrightarrow{d} (\tilde{C}' [I_N - z_N (z'_N z_N)^{-1} z'_N] \tilde{C})^{-1} \tilde{C}' [I_N - z_N (z'_N z_N)^{-1} z'_N] \tilde{\mu}_r. \quad (\text{G.8})$$

Furthermore, using  $\lambda_{01}^* = (\tilde{C}' \tilde{C})^{-1} \tilde{C}' \tilde{\mu}_r$  and the identity

$$H = (\tilde{C}' [I_N - z_N (z'_N z_N)^{-1} z'_N] \tilde{C})^{-1} = (\tilde{C}' \tilde{C})^{-1} + \frac{(\tilde{C}' \tilde{C})^{-1} \tilde{C}' z_N z'_N \tilde{C} (\tilde{C}' \tilde{C})^{-1}}{z'_{N-K} z_{N-K}}, \quad (\text{G.9})$$

we obtain

$$\hat{\lambda}_{01}^{GLS} - \lambda_{01}^* \xrightarrow{d} -\frac{\sqrt{\tilde{Q}} z_1}{z'_{N-K} z_{N-K}} (\tilde{C}' \tilde{C})^{-1} \tilde{C}' z_N. \quad (\text{G.10})$$

In order to compute the asymptotic covariance of the parameter estimates, we consider the  $\hat{h}_t$  derived in the Online Appendix of KRS for the GLS case:

$$\begin{aligned} \hat{h}_t &\equiv \begin{bmatrix} \hat{h}_{01t} \\ \hat{h}_{2t} \end{bmatrix} = (\hat{D}' \hat{V}_r^{-1} \hat{D})^{-1} \hat{D}' \hat{V}_r^{-1} \hat{e}_t + (\hat{D}' \hat{V}_r^{-1} \hat{D})^{-1} \begin{pmatrix} 0 \\ f_t - \hat{\mu}_f \\ g_t \end{pmatrix} - \hat{D}' \hat{V}_r^{-1} (r_t - \hat{\mu}_r) \hat{u}_t \\ &\quad + (\hat{D}' \hat{V}_r^{-1} \hat{D})^{-1} \hat{D}' \hat{V}_r^{-1} \left( \hat{V}_{rf} \hat{\lambda}_1^{GLS} + \hat{d} \hat{\lambda}_2^{GLS} \right), \end{aligned} \quad (\text{G.11})$$

where  $\hat{u}_t = \hat{e}' \hat{V}_r^{-1} (r_t - \hat{\mu}_r)$ ,  $\hat{e} = \hat{\mu}_r - \hat{D} \hat{\lambda}^{GLS}$ , and  $\hat{e}_t = (r_t - \hat{\mu}_r) \left[ 1 - \hat{\lambda}_1^{GLS} (f_t - \hat{\mu}_f) - \hat{\lambda}_2^{GLS} g_t \right]$ .

Note that

$$\hat{e}_t = -\frac{\sqrt{T} \sqrt{\tilde{Q}} z_1}{z'_{N-K} z_{N-K}} (r_t - \mu_r) g_t + O_p(1), \quad (\text{G.12})$$

and it can be shown that

$$\hat{u}_t \xrightarrow{d} \tilde{\mu}'_r P [I_{N-K} - z_{N-K} (z'_{N-K} z_{N-K})^{-1} z'_{N-K}] P' V_r^{-\frac{1}{2}} (r_t - \mu_r), \quad (\text{G.13})$$

$$(\hat{D}' \hat{V}_r^{-1} \hat{D})^{-1} = \begin{bmatrix} H + O_p(T^{-\frac{1}{2}}) & -\sqrt{T} \frac{(\tilde{C}' \tilde{C})^{-1} \tilde{C}' z_N}{z'_{N-K} z_{N-K}} + O_p(1) \\ -\sqrt{T} \frac{z'_N \tilde{C} (\tilde{C}' \tilde{C})^{-1}}{z'_{N-K} z_{N-K}} + O_p(1) & \frac{T}{z'_{N-K} z_{N-K}} + O_p(T^{\frac{1}{2}}) \end{bmatrix}, \quad (\text{G.14})$$

and

$$(\hat{D}' \hat{V}_r^{-1} \hat{D})^{-1} \hat{D}' \hat{V}_r^{-1} = \begin{bmatrix} (\tilde{C}' \tilde{C})^{-1} \tilde{C}' V_r^{-\frac{1}{2}} - \frac{(\tilde{C}' \tilde{C})^{-1} \tilde{C}' z_N z'_{N-K} P' V_r^{-\frac{1}{2}}}{z'_{N-K} z_{N-K}} + O_p(T^{-\frac{1}{2}}) \\ \frac{\sqrt{T} z'_{N-K} P' V_r^{-\frac{1}{2}}}{z'_{N-K} z_{N-K}} + O_p(1) \end{bmatrix}. \quad (\text{G.15})$$

Putting all the pieces together and eliminating the dominated terms, we obtain:

$$\frac{\hat{h}_{2t}}{T} \xrightarrow{d} -\frac{\tilde{\mu}'_r P z_{N-K}}{(z'_{N-K} z_{N-K})^2} z'_{N-K} P' V_r^{-\frac{1}{2}} (r_t - \mu_r) g_t + \frac{\tilde{\mu}'_r P \left( I_{N-K} - \frac{z_{N-K} z'_{N-K}}{z'_{N-K} z_{N-K}} \right) P' V_r^{-\frac{1}{2}} (r_t - \mu_r) g_t}{z'_{N-K} z_{N-K}}. \quad (\text{G.16})$$

Note that the two terms in the previous equation are uncorrelated because

$$\left( I_{N-K} - \frac{z_{N-K} z'_{N-K}}{z'_{N-K} z_{N-K}} \right) z_{N-K} = 0_{N-K}. \quad (\text{G.17})$$

It follows that<sup>9</sup>

$$\begin{aligned} \frac{s_m^2 \left( \hat{\lambda}_2^{GLS} \right)}{T} &= \frac{1}{T^3} \sum_{t=1}^T \hat{h}_{2t} \xrightarrow{d} \frac{(\tilde{\mu}'_r P z_{N-K})^2}{(z'_{N-K} z_{N-K})^4} z'_{N-K} z_{N-K} \\ &\quad + \frac{1}{(z'_{N-K} z_{N-K})^2} \tilde{\mu}'_r P \left( I_{N-K} - \frac{z_{N-K} z'_{N-K}}{z'_{N-K} z_{N-K}} \right) P' \tilde{\mu}_r \\ &= \frac{\tilde{Q}}{(z'_{N-K} z_{N-K})^2} \end{aligned} \quad (\text{G.18})$$

and

$$t\text{-stat}_m \left( \hat{\lambda}_2^{GLS} \right) = \frac{\hat{\lambda}_2^{GLS}}{s_m \left( \hat{\lambda}_2^{GLS} \right)} \xrightarrow{d} z_1 \sim \mathcal{N}(0, 1). \quad (\text{G.19})$$

Turning to the constant and useful factors, it is possible to show, after eliminating the dominated terms, that

$$\begin{aligned} \frac{\hat{h}_{01t}}{\sqrt{T}} &\xrightarrow{d} -\frac{\tilde{\mu}'_r P z_{N-K}}{z'_{N-K} z_{N-K}} (\tilde{C}' \tilde{C})^{-1} \tilde{C}' \left( I_N - \frac{z_N z'_{N-K}}{z'_{N-K} z_{N-K}} P' \right) V_r^{-\frac{1}{2}} (r_t - \mu_r) g_t \\ &\quad - \frac{(\tilde{C}' \tilde{C})^{-1} \tilde{C}' z_N}{z'_{N-K} z_{N-K}} \tilde{\mu}'_r P \left( I_{N-K} - \frac{z_{N-K} z'_{N-K}}{z'_{N-K} z_{N-K}} \right) P' V_r^{-\frac{1}{2}} (r_t - \mu_r) g_t. \end{aligned} \quad (\text{G.20})$$

Using that  $P' \tilde{C} = 0_{(N-K) \times K}$  and  $(I_{N-K} - z_{N-K} (z'_{N-K} z_{N-K})^{-1} z'_{N-K}) z_{N-K} = 0_{N-K}$ , it can be shown that the two terms in the previous equation are uncorrelated. Therefore, we have

$$\frac{\hat{V}_m \left( \hat{\lambda}_{01}^{GLS} \right)}{T} = \frac{1}{T^2} \sum_{t=1}^T \hat{h}_{01t} \hat{h}'_{01t} \xrightarrow{d} \frac{\tilde{Q}}{(z'_{N-K} z_{N-K})^2} \left[ z_1^2 (\tilde{C}' \tilde{C})^{-1} + (\tilde{C}' \tilde{C})^{-1} \tilde{C}' z_N z'_N \tilde{C} (\tilde{C}' \tilde{C})^{-1} \right]. \quad (\text{G.21})$$

Let  $c_i$  be the  $i$ -th diagonal element of  $(\tilde{C}' \tilde{C})^{-1}$  and  $z_{1,i} = -\frac{\iota'_i (\tilde{C}' \tilde{C})^{-1} \tilde{C}' z_N}{\sqrt{c_i}} \sim \mathcal{N}(0, 1)$ , for  $i = 1, \dots, K$ , where  $\iota_i$  is a selector vector with one for its  $i$ -th element and zero otherwise (the length of  $\iota_i$  is implied by the matrix that it is multiplied to). Then,

$$\frac{s_m^2 \left( \hat{\lambda}_{01,i}^{GLS} \right)}{T} = \frac{\iota'_i \hat{V}_m \left( \hat{\lambda}_{01}^{GLS} \right) \iota_i}{T} \xrightarrow{d} \frac{\tilde{Q} c_i}{(z'_{N-K} z_{N-K})^2} (z_1^2 + z_{1,i}^2). \quad (\text{G.22})$$

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<sup>9</sup>Serial correlation could be accommodated in our analysis at the cost of more tedious notation.

We can also express the  $i$ -th element of Eq. (G.10) as

$$\hat{\lambda}_{01,i}^{GLS} - \lambda_{01,i}^* \xrightarrow{d} \frac{\sqrt{\tilde{Q}}z_1}{z'_{N-K}z_{N-K}} \sqrt{c_i}z_{1,i}. \quad (\text{G.23})$$

Therefore, we obtain

$$t\text{-stat}_m^2(\hat{\lambda}_{01,i}^{GLS}) = \frac{\left(\hat{\lambda}_{01,i}^{GLS} - \lambda_{01,i}^*\right)^2}{s_m^2(\hat{\lambda}_{01,i}^{GLS})} \xrightarrow{d} \frac{z_1^2 z_{1,i}^2}{z_1^2 + z_{1,i}^2}, \quad (\text{G.24})$$

which is stochastically bounded by  $\chi_1^2$ . In summary, the squared misspecification-robust  $t$ -statistic for the spurious factor is  $\chi_1^2$  distributed when the sample size is large. For the constant term and the useful factors, the squared misspecification-robust  $t$ -statistic is stochastically bounded by a  $\chi_1^2$  random variable when  $T \rightarrow \infty$ .

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**Table C.1**

OLS cross-sectional asset-pricing tests by asset class.

The table presents the OLS estimates of the prices of beta and covariance risks for HKM's two-factor model. *MKT* and *CPTLT* denote the market and traded capital factors, respectively. *INT* is the cross-sectional intercept estimate. For each parameter estimate, we report the Jagannathan and Wang (1998) *t*-ratio (*t*-stat<sub>*c*</sub>), the KRS model misspecification-robust *t*-ratio (*t*-stat<sub>*m*</sub>), and the bootstrap *p*-value (boot *p*-val). In addition, we present the sample OLS cross-sectional  $R^2$  ( $R^2$ ) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) *p*-values. Finally, RANK TEST and fs *p*-val are the (approximate) finite-sample rank test discussed in Appendix F and its finite-sample *p*-value. *N* and *T* represent the number of assets and time series observations, respectively. *p*-values are in square brackets.

Panel A: Price of beta risk

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
<i>CPTLT</i>	5.96	6.62	6.95	28.53	12.96	6.95	19.26	8.69
<i>t</i> -stat <sub><i>c</i></sub>	(1.89)	(2.55)	(1.52)	(1.71)	(3.00)	(1.77)	(3.40)	(2.39)
<i>t</i> -stat <sub><i>m</i></sub>	(1.81)	(2.34)	(1.75)	(1.82)	(2.01)	(0.86)	(3.50)	(1.31)
boot <i>p</i> -val	[0.075]	[0.009]	[0.157]	[0.199]	[0.123]	[0.582]	[0.006]	
<i>MKT</i>	1.38	2.17	2.38	2.91	1.62	0.06	8.62	1.74
<i>t</i> -stat <sub><i>c</i></sub>	(0.89)	(1.15)	(0.60)	(0.53)	(0.56)	(0.03)	(1.80)	(0.97)
<i>t</i> -stat <sub><i>m</i></sub>	(0.86)	(1.04)	(0.67)	(0.52)	(0.30)	(0.02)	(1.26)	(1.01)
boot <i>p</i> -val	[0.389]	[0.275]	[0.478]	[0.569]	[0.784]	[0.988]	[0.184]	
<i>INT</i>	0.33	0.29	0.26	-1.08	-0.40	0.65	-0.75	-0.22
<i>t</i> -stat <sub><i>c</i></sub>	(0.24)	(2.22)	(0.22)	(-0.24)	(-2.60)	(0.57)	(-0.68)	(-0.26)
<i>t</i> -stat <sub><i>m</i></sub>	(0.21)	(2.09)	(0.26)	(-0.23)	(-5.12)	(0.51)	(-0.58)	(-0.23)
boot <i>p</i> -val	[0.829]	[0.025]	[0.805]	[0.795]	[0.008]	[0.605]	[0.509]	
$R^2$	0.45	0.85	0.74	0.99	0.68	0.26	0.59	0.68
SPEC TEST	80.81	70.69	8.36	10.10	46.32	19.73	15.08	
asy <i>p</i> -val	[0.000]	[0.000]	[0.039]	[0.814]	[0.000]	[0.475]	[0.089]	
boot <i>p</i> -val	[0.005]	[0.030]	[0.134]	[0.886]	[0.912]	[0.883]	[0.423]	
RANK TEST	1.96	2.17	1.10	0.41	1.62	0.88	1.24	
fs <i>p</i> -val	[0.009]	[0.007]	[0.363]	[0.977]	[0.125]	[0.614]	[0.273]	
<i>N</i>	25	20	6	18	20	23	12	124
<i>T</i>	172	148	65	103	47	105	135	172

Panel B: Price of covariance risk

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
<i>CPTLT</i>	7.97	7.45	6.40	44.13	18.13	12.07	16.07	12.23
<i>t</i> -stat <sub><i>c</i></sub>	(2.37)	(0.98)	(0.56)	(1.50)	(1.64)	(1.74)	(1.54)	(1.98)
<i>t</i> -stat <sub><i>m</i></sub>	(2.22)	(0.86)	(0.72)	(1.62)	(1.05)	(1.37)	(1.12)	(1.73)
boot <i>p</i> -val	[0.031]	[0.320]	[0.531]	[0.244]	[0.347]	[0.446]	[0.263]	
<i>MKT</i>	-8.05	-6.67	-5.27	-52.31	-22.91	-15.51	-8.63	-12.82
<i>t</i> -stat <sub><i>c</i></sub>	(-2.38)	(-0.56)	(-0.28)	(-1.41)	(-1.41)	(-1.57)	(-0.48)	(-2.12)
<i>t</i> -stat <sub><i>m</i></sub>	(-2.30)	(-0.49)	(-0.36)	(-1.53)	(-0.76)	(-1.36)	(-0.32)	(-1.37)
boot <i>p</i> -val	[0.017]	[0.565]	[0.749]	[0.264]	[0.535]	[0.407]	[0.738]	



**Table C.2**

GLS cross-sectional asset-pricing tests by asset class.

The table presents the GLS estimates of the prices of beta and covariance risks for HKM's two-factor model. *MKT* and *CPTLT* denote the market and traded capital factors, respectively. *INT* is the cross-sectional intercept estimate. For each parameter estimate, we report the Jagannathan and Wang (1998) *t*-ratio ( $t\text{-stat}_c$ ), the KRS model misspecification-robust *t*-ratio ( $t\text{-stat}_m$ ), and the bootstrap *p*-value (boot *p*-val). In addition, we present the sample GLS cross-sectional  $R^2$  ( $R^2$ ) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) *p*-values. *N* and *T* represent the number of assets and time series observations, respectively. *p*-values are in square brackets.

Panel A: Price of beta risk							
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
<i>CPTLT</i>	-3.98	4.09	2.64	12.25	6.19	3.51	12.21
$t\text{-stat}_c$	(-1.93)	(2.64)	(0.76)	(2.67)	(2.52)	(1.32)	(3.49)
$t\text{-stat}_m$	(-1.63)	(2.01)	(0.56)	(1.76)	(1.92)	(0.93)	(2.47)
boot <i>p</i> -val	[0.140]	[0.049]	[0.570]	[0.493]	[0.196]	[0.486]	[0.069]
<i>MKT</i>	-1.88	2.02	4.78	1.66	2.67	-0.27	9.25
$t\text{-stat}_c$	(-1.73)	(1.80)	(1.90)	(0.77)	(1.64)	(-0.20)	(3.02)
$t\text{-stat}_m$	(-1.59)	(1.47)	(1.48)	(0.56)	(1.43)	(-0.17)	(2.24)
boot <i>p</i> -val	[0.107]	[0.160]	[0.140]	[0.581]	[0.290]	[0.847]	[0.137]
<i>INT</i>	3.56	0.08	1.37	-0.47	-0.09	0.30	-1.78
$t\text{-stat}_c$	(4.31)	(6.31)	(2.01)	(-0.30)	(-3.53)	(0.67)	(-2.18)
$t\text{-stat}_m$	(3.62)	(5.97)	(1.36)	(-0.19)	(-2.31)	(0.58)	(-1.72)
boot <i>p</i> -val	[0.005]	[0.000]	[0.218]	[0.887]	[0.210]	[0.634]	[0.137]
$R^2$	0.06	0.09	0.28	0.27	0.08	0.17	0.36
SPEC TEST	67.93	72.90	8.59	38.89	70.47	26.67	24.50
asy <i>p</i> -val	[0.000]	[0.000]	[0.035]	[0.001]	[0.000]	[0.145]	[0.004]
boot <i>p</i> -val	[0.005]	[0.009]	[0.083]	[0.369]	[0.699]	[0.549]	[0.150]
<i>N</i>	25	20	6	18	20	23	12
<i>T</i>	172	148	65	103	47	105	135

  

Panel B: Price of covariance risk							
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
<i>CPTLT</i>	-3.12	2.92	-5.54	18.02	4.29	6.78	0.44
$t\text{-stat}_c$	(-1.37)	(1.24)	(-0.86)	(1.78)	(1.66)	(1.99)	(0.08)
$t\text{-stat}_m$	(-1.03)	(0.80)	(-0.52)	(1.12)	(1.27)	(1.47)	(0.05)
boot <i>p</i> -val	[0.360]	[0.502]	[0.619]	[0.736]	[0.344]	[0.307]	[0.960]
<i>MKT</i>	1.53	-1.10	12.68	-20.76	-2.76	-9.11	12.53
$t\text{-stat}_c$	(0.60)	(-0.29)	(1.26)	(-1.48)	(-0.63)	(-1.95)	(1.16)
$t\text{-stat}_m$	(0.43)	(-0.20)	(0.78)	(-0.90)	(-0.51)	(-1.50)	(0.80)
boot <i>p</i> -val	[0.711]	[0.875]	[0.422]	[0.769]	[0.697]	[0.275]	[0.436]

**Table C.3**

OLS and GLS cross-sectional asset-pricing tests by asset class (price of beta risk in HKM's single-factor model).

The table presents the OLS and GLS estimates of the price of beta risk for HKM's single-factor model (HKMSF). *CPTLT* denotes HKM's traded capital factor. *INT* is the cross-sectional intercept estimate. For each parameter estimate, we report the Jagannathan and Wang (1998) *t*-ratio ( $t\text{-stat}_c$ ), the KRS model misspecification-robust *t*-ratio ( $t\text{-stat}_m$ ), and the bootstrap *p*-value (boot *p*-val). In addition, we present the sample OLS and GLS cross-sectional  $R^2$ s ( $R^2$ ) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) *p*-values. Finally, RANK TEST and fs *p*-val are the (approximate) finite-sample rank test discussed in Appendix F and its finite-sample *p*-value.  $N$  and  $T$  represent the number of assets and time series observations, respectively. *p*-values are in square brackets.

Panel A: OLS								
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
<i>CPTLT</i>	-0.38	5.31	6.22	14.16	9.33	0.94	18.97	3.41
$t\text{-stat}_c$	(-0.14)	(3.10)	(1.77)	(2.99)	(2.92)	(0.31)	(3.44)	(1.07)
$t\text{-stat}_m$	(-0.13)	(3.06)	(1.79)	(3.03)	(2.39)	(0.11)	(3.25)	(1.03)
boot <i>p</i> -val	[0.898]	[0.002]	[0.119]	[0.006]	[0.023]	[0.911]	[0.012]	
<i>INT</i>	2.43	0.35	0.39	-5.19	-0.37	0.29	-1.08	-0.03
$t\text{-stat}_c$	(1.79)	(1.67)	(0.48)	(-2.67)	(-3.73)	(0.46)	(-1.38)	(-0.06)
$t\text{-stat}_m$	(1.55)	(1.67)	(0.54)	(-2.71)	(-5.36)	(0.29)	(-1.38)	(-0.05)
boot <i>p</i> -val	[0.116]	[0.109]	[0.618]	[0.023]	[0.001]	[0.762]	[0.155]	
$R^2$	0.00	0.84	0.72	0.94	0.64	0.00	0.58	0.40
SPEC TEST	79.21	69.73	10.01	50.67	77.48	29.76	18.30	
asy <i>p</i> -val	[0.000]	[0.000]	[0.040]	[0.000]	[0.000]	[0.097]	[0.050]	
boot <i>p</i> -val	[0.001]	[0.038]	[0.186]	[0.197]	[0.823]	[0.444]	[0.435]	
RANK TEST	3.47	8.06	9.69	3.59	6.41	2.05	2.80	
fs <i>p</i> -val	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.011]	[0.003]	
$N$	25	20	6	18	20	23	12	124
$T$	172	148	65	103	47	105	135	172

  

Panel B: GLS							
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
<i>CPTLT</i>	-3.48	3.96	4.08	7.50	6.05	1.21	12.28
$t\text{-stat}_c$	(-1.91)	(2.50)	(1.38)	(3.05)	(2.43)	(0.53)	(3.99)
$t\text{-stat}_m$	(-1.72)	(2.10)	(1.21)	(2.25)	(1.88)	(0.37)	(2.54)
boot <i>p</i> -val	[0.091]	[0.045]	[0.253]	[0.121]	[0.198]	[0.738]	[0.088]
<i>INT</i>	3.49	0.08	0.96	-2.28	-0.09	-0.06	-1.16
$t\text{-stat}_c$	(4.49)	(6.39)	(1.75)	(-2.83)	(-3.75)	(-0.17)	(-2.37)
$t\text{-stat}_m$	(3.69)	(5.97)	(1.47)	(-2.06)	(-2.37)	(-0.15)	(-2.14)
boot <i>p</i> -val	[0.002]	[0.000]	[0.197]	[0.120]	[0.210]	[0.892]	[0.051]
$R^2$	0.05	0.09	0.16	0.16	0.07	0.01	0.30
SPEC TEST	70.76	71.60	11.35	63.25	76.85	29.59	29.63
asy <i>p</i> -val	[0.000]	[0.000]	[0.023]	[0.000]	[0.000]	[0.101]	[0.001]
boot <i>p</i> -val	[0.003]	[0.014]	[0.100]	[0.053]	[0.720]	[0.446]	[0.114]
$N$	25	20	6	18	20	23	12
$T$	172	148	65	103	47	105	135

**Table D.1**

Summary statistics, Sharpe ratio analysis, and GRS.

Panel A reports factor means (Fac. Mean), standard deviations (Fac. SD), and correlation (Fac. Corr). In Panel B, we report bias-adjusted squared Sharpe ratios ( $Sh^2$ ) for the CAPM, the two-factor model of HKM (HKM), and the single-factor model of HKM (HKMSF). Panel C is for differences in bias-adjusted sample squared Sharpe ratios between models. Finally, Panel D reports a conditional heteroskedastic version of the GRS test. *MKT* and *CPTLT* denote the market and traded capital factors, respectively.  $N$  and  $T$  represent the number of assets and time series observations, respectively. Panels A–C are based on  $T = 516$ .  $p$ -values are in square brackets.

Panel A: Summary statistics		
	<i>MKT</i>	<i>CPTLT</i>
Fac. Mean	0.005 [0.022]	0.006 [0.062]
Fac. SD	0.047	0.069
Fac. Corr.	0.819	

Panel B: Squared Sharpe ratios			
	CAPM	HKM	HKMSF
$Sh^2$	0.008 [0.040]	0.006 [0.202]	0.005 [0.117]

Panel C: Squared Sharpe ratio comparisons		
	HKM	HKMSF
CAPM	0.002 [0.986]	0.003 [0.513]
HKM		0.001 [0.194]

Panel D: GRS							
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
CAPM	4.39 [0.000]	6.20 [0.000]	2.28 [0.038]	4.03 [0.000]	3.47 [0.000]	1.33 [0.147]	7.40 [0.000]
HKM	4.48 [0.000]	6.21 [0.000]	2.19 [0.046]	4.01 [0.000]	3.47 [0.000]	1.32 [0.149]	7.39 [0.000]
HKMSF	4.54 [0.000]	6.52 [0.000]	2.14 [0.051]	4.01 [0.000]	3.53 [0.000]	1.31 [0.156]	7.41 [0.000]
$N$	25	20	6	18	20	23	12
$T$	516	445	196	310	143	316	407

**Table D.2**

OLS cross-sectional asset-pricing tests by asset class (nontraded capital factor).

The table presents the OLS estimates of the prices of beta and covariance risks for HKM's two-factor model. *MKT* and *CPTL* denote the market and nontraded capital factors, respectively. *INT* is the cross-sectional intercept estimate. For each parameter estimate, we report the Jagannathan and Wang (1998) *t*-ratio ( $t\text{-stat}_c$ ), the KRS model misspecification-robust *t*-ratio ( $t\text{-stat}_m$ ), and the bootstrap *p*-value (boot *p*-val). In addition, we present the sample OLS cross-sectional  $R^2$  ( $R^2$ ) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) *p*-values. Finally, RANK TEST and fs *p*-val are the (approximate) finite-sample rank test discussed in Appendix F and its finite-sample *p*-value.  $N$  and  $T$  represent the number of assets and time series observations, respectively. *p*-values are in square brackets.

Panel A: Price of beta risk

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
<i>CPTL</i>	1.38	1.32	1.80	22.68	5.50	-0.51	6.84	3.10
$t\text{-stat}_c$	(1.16)	(0.72)	(1.35)	(0.80)	(3.09)	(-0.39)	(3.52)	(2.10)
$t\text{-stat}_m$	(0.44)	(0.42)	(0.84)	(0.97)	(0.99)	(-0.23)	(2.46)	(1.26)
boot <i>p</i> -val	[0.646]	[0.630]	[0.408]	[0.604]	[0.388]	[0.828]	[0.024]	
<i>MKT</i>	0.07	1.50	1.75	2.08	-0.13	0.43	3.03	0.78
$t\text{-stat}_c$	(0.17)	(1.80)	(2.16)	(0.74)	(-0.15)	(0.60)	(1.76)	(1.52)
$t\text{-stat}_m$	(0.06)	(1.03)	(2.06)	(0.67)	(-0.03)	(0.40)	(0.48)	(1.56)
boot <i>p</i> -val	[0.957]	[0.264]	[0.035]	[0.386]	[0.970]	[0.646]	[0.627]	
<i>INT</i>	0.59	0.11	0.02	-2.26	-0.16	-0.04	-0.34	-0.19
$t\text{-stat}_c$	(1.68)	(3.81)	(0.06)	(-0.77)	(-3.92)	(-0.21)	(-1.30)	(-1.06)
$t\text{-stat}_m$	(0.49)	(2.14)	(0.08)	(-0.75)	(-2.06)	(-0.14)	(-0.47)	(-0.91)
boot <i>p</i> -val	[0.618]	[0.010]	[0.941]	[0.398]	[0.179]	[0.898]	[0.614]	
$R^2$	0.27	0.79	0.71	0.96	0.72	0.04	0.32	0.70
SPEC TEST	68.49	47.89	5.94	4.58	29.30	28.22	27.29	
asy <i>p</i> -val	[0.000]	[0.000]	[0.115]	[0.995]	[0.032]	[0.104]	[0.001]	
boot <i>p</i> -val	[0.000]	[0.002]	[0.207]	[0.993]	[0.744]	[0.187]	[0.037]	
RANK TEST	1.89	2.50	8.30	0.68	2.18	1.21	1.21	
fs <i>p</i> -val	[0.008]	[0.001]	[0.000]	[0.811]	[0.007]	[0.243]	[0.284]	
$N$	25	20	6	18	20	23	12	124
$T$	516	445	196	310	143	316	407	516

Panel B: Price of covariance risk

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
<i>CPTL</i>	6.30	-1.82	-0.39	101.47	32.03	-5.02	17.36	10.81
$t\text{-stat}_c$	(1.48)	(-0.14)	(-0.11)	(0.73)	(2.14)	(-0.88)	(1.63)	(1.76)
$t\text{-stat}_m$	(0.66)	(-0.08)	(-0.05)	(0.88)	(0.51)	(-0.52)	(0.55)	(0.97)
boot <i>p</i> -val	[0.500]	[0.931]	[0.965]	[0.619]	[0.625]	[0.577]	[0.552]	
<i>MKT</i>	-6.69	9.12	8.38	-105.69	-36.74	7.78	-4.53	-8.48
$t\text{-stat}_c$	(-1.58)	(0.50)	(1.58)	(-0.72)	(-1.92)	(1.00)	(-0.26)	(-1.14)
$t\text{-stat}_m$	(-1.06)	(0.27)	(0.80)	(-0.88)	(-0.40)	(0.62)	(-0.07)	(-0.65)
boot <i>p</i> -val	[0.304]	[0.762]	[0.358]	[0.636]	[0.693]	[0.509]	[0.944]	

**Table D.3**

OLS cross-sectional asset-pricing tests by asset class (traded capital factor).

The table presents the OLS estimates of the prices of beta and covariance risks for HKM's two-factor model. *MKT* and *CPTLT* denote the market and traded capital factors, respectively. *INT* is the cross-sectional intercept estimate. For each parameter estimate, we report the Jagannathan and Wang (1998) *t*-ratio ( $t\text{-stat}_c$ ), the KRS model misspecification-robust *t*-ratio ( $t\text{-stat}_m$ ), and the bootstrap *p*-value (boot *p*-val). In addition, we present the sample OLS cross-sectional  $R^2$  ( $R^2$ ) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) *p*-values. Finally, RANK TEST and fs *p*-val are the (approximate) finite-sample rank test discussed in Appendix F and its finite-sample *p*-value.  $N$  and  $T$  represent the number of assets and time series observations, respectively. *p*-values are in square brackets.

Panel A: Price of beta risk

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
<i>CPTLT</i>	0.80	2.40	0.37	54.97	5.89	-0.41	6.48	2.64
$t\text{-stat}_c$	(0.88)	(1.96)	(0.22)	(0.42)	(2.90)	(-0.31)	(3.32)	(2.22)
$t\text{-stat}_m$	(0.37)	(1.36)	(0.12)	(0.70)	(2.28)	(-0.15)	(1.83)	(1.20)
boot <i>p</i> -val	[0.697]	[0.152]	[0.893]	[0.930]	[0.099]	[0.870]	[0.080]	
<i>MKT</i>	-0.03	1.00	2.02	6.25	-0.11	0.58	2.72	0.78
$t\text{-stat}_c$	(-0.06)	(1.37)	(2.29)	(0.39)	(-0.11)	(0.76)	(1.36)	(1.54)
$t\text{-stat}_m$	(-0.03)	(0.97)	(2.42)	(0.47)	(-0.05)	(0.45)	(0.34)	(1.56)
boot <i>p</i> -val	[0.981]	[0.306]	[0.019]	[0.819]	[0.971]	[0.612]	[0.748]	
<i>INT</i>	0.69	0.08	0.11	-7.12	-0.16	-0.05	-0.40	-0.20
$t\text{-stat}_c$	(2.06)	(3.90)	(0.55)	(-0.39)	(-3.37)	(-0.24)	(-1.70)	(-1.07)
$t\text{-stat}_m$	(0.71)	(1.53)	(0.47)	(-0.53)	(-4.17)	(-0.14)	(-0.52)	(-0.89)
boot <i>p</i> -val	[0.472]	[0.116]	[0.600]	[0.887]	[0.010]	[0.888]	[0.537]	
$R^2$	0.26	0.81	0.76	0.97	0.76	0.04	0.31	0.63
SPEC TEST	66.92	48.70	5.52	0.67	24.89	28.79	26.52	
asy <i>p</i> -val	[0.000]	[0.000]	[0.137]	[1.000]	[0.097]	[0.092]	[0.002]	
boot <i>p</i> -val	[0.000]	[0.001]	[0.209]	[0.000]	[0.842]	[0.166]	[0.043]	
RANK TEST	3.27	2.73	2.94	0.51	1.57	0.59	0.90	
fs <i>p</i> -val	[0.000]	[0.000]	[0.022]	[0.939]	[0.078]	[0.922]	[0.531]	
$N$	25	20	6	18	20	23	12	124
$T$	516	445	196	310	143	316	407	516

Panel B: Price of covariance risk

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
<i>CPTLT</i>	5.24	7.57	-12.10	321.18	42.20	-7.75	20.90	10.75
$t\text{-stat}_c$	(1.41)	(0.64)	(-1.25)	(0.42)	(2.24)	(-0.90)	(1.69)	(1.80)
$t\text{-stat}_m$	(0.74)	(0.42)	(-0.65)	(0.73)	(1.14)	(-0.37)	(0.43)	(0.90)
boot <i>p</i> -val	[0.446]	[0.653]	[0.516]	[0.925]	[0.318]	[0.704]	[0.633]	
<i>MKT</i>	-6.44	-4.53	23.94	-373.34	-55.73	12.51	-12.07	-9.38
$t\text{-stat}_c$	(-1.55)	(-0.26)	(1.73)	(-0.41)	(-2.10)	(1.00)	(-0.54)	(-1.24)
$t\text{-stat}_m$	(-1.15)	(-0.17)	(1.01)	(-0.75)	(-0.97)	(0.42)	(-0.12)	(-0.63)
boot <i>p</i> -val	[0.248]	[0.853]	[0.300]	[0.921]	[0.373]	[0.669]	[0.900]	

**Table D.4**

GLS cross-sectional asset-pricing tests by asset class (nontraded capital factor).

The table presents the GLS estimates of the prices of beta and covariance risks for HKM's two-factor model. *MKT* and *CPTL* denote the market and nontraded capital factors, respectively. *INT* is the cross-sectional intercept estimate. For each parameter estimate, we report the Jagannathan and Wang (1998) *t*-ratio ( $t\text{-stat}_c$ ), the KRS model misspecification-robust *t*-ratio ( $t\text{-stat}_m$ ), and the bootstrap *p*-value (boot *p*-val). In addition, we present the sample GLS cross-sectional  $R^2$  ( $R^2$ ) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) *p*-values. *N* and *T* represent the number of assets and time series observations, respectively. *p*-values are in square brackets.

Panel A: Price of beta risk							
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
<i>CPTL</i>	-0.75	1.90	2.14	0.72	1.37	1.24	4.14
$t\text{-stat}_c$	(-0.80)	(2.56)	(1.71)	(0.46)	(1.68)	(1.19)	(2.89)
$t\text{-stat}_m$	(-0.53)	(2.10)	(1.46)	(0.15)	(1.46)	(0.97)	(2.28)
boot <i>p</i> -val	[0.577]	[0.032]	[0.176]	[0.884]	[0.154]	[0.380]	[0.036]
<i>MKT</i>	-0.87	1.66	1.63	0.80	0.40	-0.05	3.95
$t\text{-stat}_c$	(-2.55)	(3.71)	(2.00)	(1.62)	(0.77)	(-0.10)	(2.75)
$t\text{-stat}_m$	(-2.07)	(3.02)	(1.90)	(1.18)	(0.66)	(-0.09)	(1.56)
boot <i>p</i> -val	[0.027]	[0.009]	[0.062]	[0.220]	[0.509]	[0.920]	[0.233]
<i>INT</i>	1.36	0.03	0.09	-0.40	-0.04	0.11	-0.59
$t\text{-stat}_c$	(4.93)	(7.06)	(0.41)	(-1.08)	(-4.53)	(0.83)	(-2.72)
$t\text{-stat}_m$	(3.64)	(6.46)	(0.45)	(-0.60)	(-3.18)	(0.75)	(-1.76)
boot <i>p</i> -val	[0.000]	[0.000]	[0.634]	[0.546]	[0.034]	[0.466]	[0.153]
$R^2$	0.12	0.26	0.48	0.04	0.06	0.07	0.26
SPEC TEST	68.96	46.84	5.85	68.08	58.34	29.60	33.46
asy <i>p</i> -val	[0.000]	[0.000]	[0.119]	[0.000]	[0.000]	[0.077]	[0.000]
boot <i>p</i> -val	[0.000]	[0.005]	[0.221]	[0.000]	[0.073]	[0.148]	[0.008]
<i>N</i>	25	20	6	18	20	23	12
<i>T</i>	516	445	196	310	143	316	407

  

Panel B: Price of covariance risk							
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
<i>CPTL</i>	1.10	0.17	1.49	-0.93	5.23	6.52	-1.40
$t\text{-stat}_c$	(0.32)	(0.04)	(0.45)	(-0.14)	(1.08)	(1.31)	(-0.20)
$t\text{-stat}_m$	(0.21)	(0.03)	(0.36)	(-0.04)	(0.79)	(0.97)	(-0.10)
boot <i>p</i> -val	[0.829]	[0.970]	[0.701]	[0.971]	[0.446]	[0.381]	[0.924]
<i>MKT</i>	-5.25	7.69	5.82	4.74	-4.11	-7.68	21.02
$t\text{-stat}_c$	(-1.57)	(1.36)	(1.08)	(0.61)	(-0.63)	(-1.17)	(1.56)
$t\text{-stat}_m$	(-1.10)	(0.94)	(0.94)	(0.21)	(-0.45)	(-0.86)	(0.78)
boot <i>p</i> -val	[0.272]	[0.344]	[0.320]	[0.836]	[0.664]	[0.420]	[0.489]

**Table D.5**

GLS cross-sectional asset-pricing tests by asset class (traded capital factor).

The table presents the GLS estimates of the prices of beta and covariance risks for HKM's two-factor model. *MKT* and *CPTLT* denote the market and traded capital factors, respectively. *INT* is the cross-sectional intercept estimate. For each parameter estimate, we report the Jagannathan and Wang (1998) *t*-ratio ( $t\text{-stat}_c$ ), the KRS model misspecification-robust *t*-ratio ( $t\text{-stat}_m$ ), and the bootstrap *p*-value (boot *p*-val). In addition, we present the sample GLS cross-sectional  $R^2$  ( $R^2$ ) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) *p*-values. *N* and *T* represent the number of assets and time series observations, respectively. *p*-values are in square brackets.

Panel A: Price of beta risk							
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
<i>CPTLT</i>	-1.61	2.36	1.54	3.58	1.63	0.39	4.58
$t\text{-stat}_c$	(-2.13)	(3.72)	(1.01)	(1.40)	(2.03)	(0.38)	(3.16)
$t\text{-stat}_m$	(-1.58)	(3.06)	(0.75)	(0.68)	(1.73)	(0.22)	(2.51)
boot <i>p</i> -val	[0.118]	[0.002]	[0.444]	[0.668]	[0.111]	[0.807]	[0.028]
<i>MKT</i>	-1.03	1.57	1.70	1.06	0.32	0.00	3.97
$t\text{-stat}_c$	(-3.02)	(3.43)	(2.14)	(1.74)	(0.62)	(0.01)	(2.56)
$t\text{-stat}_m$	(-2.59)	(2.81)	(2.05)	(1.24)	(0.51)	(0.01)	(1.19)
boot <i>p</i> -val	[0.014]	[0.012]	[0.040]	[0.288]	[0.598]	[0.994]	[0.366]
<i>INT</i>	1.52	0.03	0.12	-0.72	-0.04	0.09	-0.59
$t\text{-stat}_c$	(5.56)	(7.31)	(0.62)	(-1.48)	(-4.40)	(0.68)	(-2.82)
$t\text{-stat}_m$	(4.36)	(6.62)	(0.60)	(-0.88)	(-3.19)	(0.52)	(-1.53)
boot <i>p</i> -val	[0.000]	[0.000]	[0.532]	[0.513]	[0.028]	[0.604]	[0.218]
$R^2$	0.12	0.27	0.48	0.07	0.07	0.00	0.26
SPEC TEST	68.99	47.41	6.58	59.62	57.28	31.88	32.87
asy <i>p</i> -val	[0.000]	[0.000]	[0.087]	[0.000]	[0.000]	[0.045]	[0.000]
boot <i>p</i> -val	[0.000]	[0.004]	[0.156]	[0.001]	[0.067]	[0.083]	[0.008]
<i>N</i>	25	20	6	18	20	23	12
<i>T</i>	516	445	196	310	143	316	407

  

Panel B: Price of covariance risk							
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
<i>CPTLT</i>	-2.32	2.79	-3.04	15.38	8.48	2.61	-1.75
$t\text{-stat}_c$	(-0.74)	(0.70)	(-0.47)	(1.01)	(1.54)	(0.36)	(-0.18)
$t\text{-stat}_m$	(-0.54)	(0.45)	(-0.31)	(0.50)	(0.98)	(0.18)	(-0.07)
boot <i>p</i> -val	[0.576]	[0.651]	[0.733]	[0.737]	[0.378]	[0.842]	[0.951]
<i>MKT</i>	-1.94	4.00	11.43	-14.37	-9.66	-3.27	21.70
$t\text{-stat}_c$	(-0.58)	(0.64)	(1.35)	(-0.79)	(-1.18)	(-0.31)	(1.20)
$t\text{-stat}_m$	(-0.45)	(0.42)	(0.96)	(-0.39)	(-0.74)	(-0.17)	(0.49)
boot <i>p</i> -val	[0.650]	[0.677]	[0.264]	[0.784]	[0.505]	[0.862]	[0.679]

**Table D.6**

Constrained OLS cross-sectional asset-pricing tests by asset class (nontraded capital factor).

The table presents the constrained OLS estimates of the prices of beta and covariance risks for HKM's two-factor model. *MKT* and *CPTL* denote the market and nontraded capital factors, respectively. For each parameter estimate, we report the Jagannathan and Wang (1998) *t*-ratio ( $t\text{-stat}_c$ ), the KRS model misspecification-robust *t*-ratio ( $t\text{-stat}_m$ ), and the bootstrap *p*-value (boot *p*-val). In addition, we present the sample uncentered OLS cross-sectional  $R^2$  ( $R_U^2$ ) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) *p*-values. Finally, RANK TEST and fs *p*-val are the (approximate) finite-sample rank test discussed in Appendix F and its finite-sample *p*-value. *N* and *T* represent the number of assets and time series observations, respectively. *p*-values are in square brackets.

Panel A: Price of beta risk								
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
<i>CPTL</i>	2.79	5.18	1.83	41.47	1.26	-0.60	6.70	3.54
$t\text{-stat}_c$	(3.45)	(1.31)	(1.34)	(0.39)	(1.26)	(-0.44)	(2.77)	(2.63)
$t\text{-stat}_m$	(3.35)	(1.32)	(0.84)	(0.46)	(0.26)	(-0.26)	(2.16)	(1.45)
boot <i>p</i> -val	[0.001]	[0.261]	[0.402]	[0.837]	[0.804]	[0.774]	[0.067]	
<i>MKT</i>	0.63	0.38	1.79	-1.52	1.82	0.30	0.26	0.49
$t\text{-stat}_c$	(2.81)	(0.25)	(2.02)	(-0.34)	(2.20)	(0.38)	(0.14)	(0.94)
$t\text{-stat}_m$	(2.82)	(0.23)	(1.99)	(-0.30)	(0.32)	(0.37)	(0.11)	(0.98)
boot <i>p</i> -val	[0.008]	[0.816]	[0.080]	[0.865]	[0.749]	[0.680]	[0.898]	
$R_U^2$	0.92	0.92	0.94	0.70	0.64	0.08	0.74	0.47
SPEC TEST	85.65	53.21	6.93	1.07	67.67	29.11	24.38	
asy <i>p</i> -val	[0.000]	[0.000]	[0.139]	[1.000]	[0.000]	[0.111]	[0.007]	
boot <i>p</i> -val	[0.000]	[0.096]	[0.267]	[1.000]	[0.166]	[0.195]	[0.103]	
RANK TEST	3.04	2.45	6.64	0.71	2.24	1.21	2.59	
fs <i>p</i> -val	[0.000]	[0.001]	[0.000]	[0.790]	[0.005]	[0.242]	[0.003]	
<i>N</i>	25	20	6	18	20	23	12	124
<i>T</i>	516	445	196	310	143	316	407	516

Panel B: Price of covariance risk								
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
<i>CPTL</i>	10.12	24.26	-0.43	215.73	-4.46	-4.70	32.24	14.50
$t\text{-stat}_c$	(2.76)	(0.89)	(-0.12)	(0.38)	(-0.63)	(-0.85)	(2.73)	(2.39)
$t\text{-stat}_m$	(2.56)	(0.87)	(-0.05)	(0.45)	(-0.07)	(-0.51)	(2.84)	(1.26)
boot <i>p</i> -val	[0.017]	[0.407]	[0.951]	[0.828]	[0.942]	[0.612]	[0.035]	
<i>MKT</i>	-8.40	-25.44	8.59	-252.17	13.24	6.78	-34.85	-13.93
$t\text{-stat}_c$	(-1.92)	(-0.68)	(1.49)	(-0.38)	(1.20)	(0.89)	(-1.96)	(-1.78)
$t\text{-stat}_m$	(-1.79)	(-0.66)	(0.82)	(-0.44)	(0.14)	(0.64)	(-2.05)	(-0.98)
boot <i>p</i> -val	[0.089]	[0.506]	[0.385]	[0.827]	[0.887]	[0.510]	[0.070]	



**Table D.7**

Constrained GLS cross-sectional asset-pricing tests by asset class (nontraded capital factor).

The table presents the constrained GLS estimates of the prices of beta and covariance risks for HKM's two-factor model. *MKT* and *CPTL* denote the market and nontraded capital factors, respectively. For each parameter estimate, we report the Jagannathan and Wang (1998) *t*-ratio ( $t\text{-stat}_c$ ), the KRS model misspecification-robust *t*-ratio ( $t\text{-stat}_m$ ), and the bootstrap *p*-value (boot *p*-val). In addition, we present the sample uncentered GLS cross-sectional  $R^2$  ( $R_U^2$ ) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) *p*-values.  $N$  and  $T$  represent the number of assets and time series observations, respectively. *p*-values are in square brackets.

Panel A: Price of beta risk							
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
<i>CPTL</i>	2.74	1.25	2.32	-0.20	0.67	1.15	3.90
$t\text{-stat}_c$	(4.22)	(1.70)	(1.82)	(-0.15)	(0.87)	(1.11)	(3.64)
$t\text{-stat}_m$	(2.98)	(1.26)	(1.59)	(-0.05)	(0.70)	(0.90)	(2.10)
boot <i>p</i> -val	[0.023]	[0.208]	[0.143]	[0.955]	[0.487]	[0.420]	[0.048]
<i>MKT</i>	0.48	1.86	1.77	0.38	0.53	-0.01	1.10
$t\text{-stat}_c$	(2.30)	(3.97)	(2.10)	(1.33)	(1.01)	(-0.01)	(1.32)
$t\text{-stat}_m$	(2.30)	(2.93)	(2.06)	(1.19)	(0.85)	(-0.01)	(0.72)
boot <i>p</i> -val	[0.032]	[0.016]	[0.070]	[0.236]	[0.428]	[0.990]	[0.465]
$R_U^2$	0.18	0.17	0.56	0.02	0.02	0.06	0.18
SPEC TEST	83.91	92.88	6.65	72.51	76.83	30.64	51.93
asy <i>p</i> -val	[0.000]	[0.000]	[0.156]	[0.000]	[0.000]	[0.080]	[0.000]
boot <i>p</i> -val	[0.000]	[0.000]	[0.310]	[0.000]	[0.023]	[0.147]	[0.000]
$N$	25	20	6	18	20	23	12
$T$	516	445	196	310	143	316	407

  

Panel B: Price of covariance risk							
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
<i>CPTL</i>	10.71	-4.28	1.63	-3.14	0.39	5.77	13.45
$t\text{-stat}_c$	(3.73)	(-1.13)	(0.49)	(-0.47)	(0.09)	(1.21)	(2.56)
$t\text{-stat}_m$	(2.38)	(-0.64)	(0.40)	(-0.16)	(0.06)	(0.89)	(1.34)
boot <i>p</i> -val	[0.065]	[0.574]	[0.669]	[0.872]	[0.953]	[0.412]	[0.220]
<i>MKT</i>	-9.75	13.60	6.27	5.30	1.99	-6.61	-9.66
$t\text{-stat}_c$	(-2.84)	(2.37)	(1.13)	(0.68)	(0.33)	(-1.05)	(-1.24)
$t\text{-stat}_m$	(-1.91)	(1.38)	(1.00)	(0.23)	(0.21)	(-0.78)	(-0.62)
boot <i>p</i> -val	[0.138]	[0.236]	[0.297]	[0.818]	[0.844]	[0.453]	[0.514]

**Table D.8**

OLS cross-sectional asset-pricing tests by asset class (price of beta risk in single factor models).

The table presents the OLS estimates of the prices of beta risk for the CAPM and HKM's single-factor model (HKMSF). *MKT*, *CPTL*, and *CPTLT* denote the market and the nontraded and traded capital factors, respectively. *INT* is the cross-sectional intercept estimate. For each parameter estimate, we report the Jagannathan and Wang (1998) *t*-ratio ( $t\text{-stat}_c$ ), the KRS model misspecification-robust *t*-ratio ( $t\text{-stat}_m$ ), and the bootstrap *p*-value (boot *p*-val). In addition, we present the sample OLS cross-sectional  $R^2$  ( $R^2$ ) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) *p*-values. Finally, RANK TEST and fs *p*-val are the (approximate) finite-sample rank test discussed in Appendix F and its finite-sample *p*-value.  $N$  and  $T$  represent the number of assets and time series observations, respectively. *p*-values are in square brackets.

Panel A: CAPM

	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
<i>MKT</i>	-0.71	1.39	1.73	3.37	2.30	0.37	5.19	0.53
$t\text{-stat}_c$	(-1.48)	(3.18)	(2.03)	(2.63)	(3.38)	(0.53)	(2.98)	(1.07)
$t\text{-stat}_m$	(-1.45)	(3.08)	(1.97)	(2.74)	(3.08)	(0.32)	(1.56)	(1.09)
boot <i>p</i> -val	[0.157]	[0.005]	[0.061]	[0.016]	[0.003]	[0.708]	[0.196]	
<i>INT</i>	1.42	0.11	0.02	-2.56	-0.11	0.02	-0.66	0.07
$t\text{-stat}_c$	(3.26)	(1.99)	(0.07)	(-2.47)	(-4.20)	(0.10)	(-2.68)	(0.45)
$t\text{-stat}_m$	(3.12)	(1.98)	(0.08)	(-2.58)	(-5.27)	(0.06)	(-2.53)	(0.43)
boot <i>p</i> -val	[0.003]	[0.060]	[0.944]	[0.024]	[0.000]	[0.947]	[0.062]	
$R^2$	0.22	0.79	0.71	0.91	0.67	0.01	0.26	0.26
SPEC TEST	70.59	49.10	6.38	41.87	48.35	31.70	25.27	
asy <i>p</i> -val	[0.000]	[0.000]	[0.173]	[0.000]	[0.000]	[0.063]	[0.005]	
boot <i>p</i> -val	[0.000]	[0.002]	[0.308]	[0.092]	[0.460]	[0.145]	[0.135]	
RANK TEST	17.54	12.31	15.39	7.77	12.30	5.24	2.99	
fs <i>p</i> -val	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	
$N$	25	20	6	18	20	23	12	124
$T$	516	445	196	310	143	316	407	516

**Table D.8 (cont'd)**

OLS cross-sectional asset-pricing tests by asset class (price of beta risk in single factor models).

Panel B: HKMSF (nontraded capital factor)								
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
<i>CPTL</i>	-1.92	2.19	2.89	6.29	3.27	-0.09	6.84	1.09
<i>t-stat<sub>c</sub></i>	(-1.36)	(3.14)	(1.92)	(2.50)	(3.57)	(-0.08)	(3.53)	(1.09)
<i>t-stat<sub>m</sub></i>	(-1.09)	(3.07)	(1.84)	(2.58)	(3.26)	(-0.04)	(2.44)	(1.10)
boot <i>p-val</i>	[0.258]	[0.004]	[0.102]	[0.018]	[0.003]	[0.963]	[0.042]	
<i>INT</i>	1.71	0.10	0.06	-2.54	-0.13	0.10	-0.39	0.06
<i>t-stat<sub>c</sub></i>	(2.58)	(1.79)	(0.23)	(-2.49)	(-4.91)	(0.55)	(-2.18)	(0.35)
<i>t-stat<sub>m</sub></i>	(2.00)	(1.80)	(0.26)	(-2.59)	(-6.02)	(0.40)	(-2.24)	(0.33)
boot <i>p-val</i>	[0.053]	[0.079]	[0.821]	[0.018]	[0.000]	[0.702]	[0.025]	
<i>R</i> <sup>2</sup>	0.12	0.79	0.67	0.92	0.69	0.00	0.32	0.29
SPEC TEST	70.70	51.94	6.12	38.18	53.64	31.75	29.59	
asy <i>p-val</i>	[0.000]	[0.000]	[0.190]	[0.001]	[0.000]	[0.062]	[0.001]	
boot <i>p-val</i>	[0.000]	[0.002]	[0.344]	[0.127]	[0.343]	[0.109]	[0.054]	
RANK TEST	5.38	10.58	12.56	3.79	11.67	3.88	4.68	
fs <i>p-val</i>	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	
Panel C: HKMSF (traded capital factor)								
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
<i>CPTLT</i>	-1.71	2.13	2.81	6.27	3.52	0.23	6.67	1.02
<i>t-stat<sub>c</sub></i>	(-1.38)	(3.22)	(1.78)	(2.39)	(3.40)	(0.21)	(3.54)	(1.10)
<i>t-stat<sub>m</sub></i>	(-1.14)	(3.14)	(1.69)	(2.48)	(3.14)	(0.12)	(2.31)	(1.11)
boot <i>p-val</i>	[0.253]	[0.004]	[0.111]	[0.034]	[0.003]	[0.888]	[0.041]	
<i>INT</i>	1.68	0.09	0.00	-2.75	-0.12	0.07	-0.49	0.04
<i>t-stat<sub>c</sub></i>	(2.65)	(1.68)	(0.02)	(-2.34)	(-4.75)	(0.39)	(-2.62)	(0.26)
<i>t-stat<sub>m</sub></i>	(2.10)	(1.68)	(0.02)	(-2.44)	(-6.06)	(0.26)	(-2.73)	(0.25)
boot <i>p-val</i>	[0.042]	[0.111]	[0.983]	[0.036]	[0.000]	[0.789]	[0.013]	
<i>R</i> <sup>2</sup>	0.12	0.80	0.65	0.91	0.70	0.00	0.31	0.29
SPEC TEST	69.96	50.87	6.46	40.44	50.04	32.07	29.20	
asy <i>p-val</i>	[0.000]	[0.000]	[0.168]	[0.001]	[0.000]	[0.058]	[0.001]	
boot <i>p-val</i>	[0.000]	[0.004]	[0.317]	[0.087]	[0.369]	[0.119]	[0.056]	
RANK TEST	7.15	11.85	10.89	4.31	11.89	4.49	4.58	
fs <i>p-val</i>	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	

**Table D.9**

GLS cross-sectional asset-pricing tests by asset class (price of beta risk in single factor models).

The table presents the GLS estimates of the prices of beta risk for the CAPM and HKM's single-factor model (HKMSF). *MKT*, *CPTL*, and *CPTLT* denote the market and the nontraded and traded capital factors, respectively. *INT* is the cross-sectional intercept estimate. For each parameter estimate, we report the Jagannathan and Wang (1998) *t*-ratio (*t-stat<sub>c</sub>*), the KRS model misspecification-robust *t*-ratio (*t-stat<sub>m</sub>*), and the bootstrap *p*-value (boot *p-val*). In addition, we present the sample GLS cross-sectional  $R^2$  ( $R^2$ ) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) *p*-values. *N* and *T* represent the number of assets and time series observations, respectively. *p*-values are in square brackets.

Panel A: CAPM							
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
<i>MKT</i>	-0.93	1.67	1.67	0.81	0.61	0.08	3.79
<i>t-stat<sub>c</sub></i>	(-3.03)	(3.98)	(2.03)	(1.66)	(1.20)	(0.16)	(3.23)
<i>t-stat<sub>m</sub></i>	(-2.83)	(3.59)	(1.93)	(1.40)	(1.16)	(0.15)	(2.38)
boot <i>p-val</i>	[0.003]	[0.001]	[0.076]	[0.169]	[0.254]	[0.888]	[0.086]
<i>INT</i>	1.41	0.03	0.09	-0.42	-0.04	0.07	-0.57
<i>t-stat<sub>c</sub></i>	(6.22)	(7.13)	(0.45)	(-1.19)	(-4.61)	(0.58)	(-3.37)
<i>t-stat<sub>m</sub></i>	(5.48)	(6.67)	(0.50)	(-0.89)	(-3.35)	(0.53)	(-3.19)
boot <i>p-val</i>	[0.000]	[0.000]	[0.626]	[0.405]	[0.022]	[0.631]	[0.019]
$R^2$	0.11	0.26	0.46	0.03	0.03	0.00	0.26
SPEC TEST	69.22	46.78	6.45	68.53	54.88	32.16	34.68
asy <i>p-val</i>	[0.000]	[0.000]	[0.168]	[0.000]	[0.000]	[0.056]	[0.000]
boot <i>p-val</i>	[0.000]	[0.011]	[0.288]	[0.000]	[0.097]	[0.108]	[0.014]
<i>N</i>	25	20	6	18	20	23	12
<i>T</i>	516	445	196	310	143	316	407

**Table D.9 (cont'd)**

GLS cross-sectional asset-pricing tests by asset class (price of beta risk in single factor models).

Panel B: HKMSF (nontraded capital factor)							
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
<i>CPTL</i>	-1.76	2.43	2.51	1.47	1.18	0.58	4.11
<i>t-stat<sub>c</sub></i>	(-2.57)	(3.69)	(2.00)	(1.48)	(1.60)	(0.71)	(3.65)
<i>t-stat<sub>m</sub></i>	(-2.09)	(3.36)	(1.84)	(0.96)	(1.56)	(0.64)	(2.41)
boot <i>p-val</i>	[0.043]	[0.002]	[0.096]	[0.380]	[0.121]	[0.498]	[0.039]
<i>INT</i>	1.48	0.03	0.12	-0.42	-0.04	0.07	-0.36
<i>t-stat<sub>c</sub></i>	(5.34)	(7.48)	(0.54)	(-1.13)	(-4.69)	(0.59)	(-3.11)
<i>t-stat<sub>m</sub></i>	(4.35)	(6.69)	(0.60)	(-0.68)	(-3.37)	(0.54)	(-3.12)
boot <i>p-val</i>	[0.000]	[0.000]	[0.580]	[0.538]	[0.024]	[0.624]	[0.005]
<i>R</i> <sup>2</sup>	0.08	0.23	0.39	0.03	0.05	0.02	0.21
SPEC TEST	71.40	50.59	6.54	70.42	55.91	31.68	44.89
asy <i>p-val</i>	[0.000]	[0.000]	[0.162]	[0.000]	[0.000]	[0.063]	[0.000]
boot <i>p-val</i>	[0.000]	[0.004]	[0.316]	[0.000]	[0.098]	[0.130]	[0.001]
Panel C: HKMSF (traded capital factor)							
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
<i>CPTLT</i>	-1.85	2.53	2.59	1.79	1.27	0.18	4.36
<i>t-stat<sub>c</sub></i>	(-3.02)	(4.14)	(1.70)	(1.88)	(1.62)	(0.23)	(3.66)
<i>t-stat<sub>m</sub></i>	(-2.53)	(3.86)	(1.58)	(1.36)	(1.59)	(0.20)	(2.59)
boot <i>p-val</i>	[0.016]	[0.000]	[0.144]	[0.230]	[0.124]	[0.829]	[0.028]
<i>INT</i>	1.55	0.03	0.07	-0.57	-0.04	0.07	-0.42
<i>t-stat<sub>c</sub></i>	(5.70)	(7.41)	(0.30)	(-1.49)	(-4.60)	(0.60)	(-3.42)
<i>t-stat<sub>m</sub></i>	(4.69)	(6.51)	(0.33)	(-1.00)	(-3.34)	(0.55)	(-3.40)
boot <i>p-val</i>	[0.000]	[0.000]	[0.766]	[0.376]	[0.024]	[0.606]	[0.002]
<i>R</i> <sup>2</sup>	0.12	0.26	0.38	0.05	0.05	0.00	0.23
SPEC TEST	69.39	48.77	6.70	66.93	54.77	32.10	41.71
asy <i>p-val</i>	[0.000]	[0.000]	[0.152]	[0.000]	[0.000]	[0.057]	[0.000]
boot <i>p-val</i>	[0.000]	[0.008]	[0.301]	[0.002]	[0.097]	[0.110]	[0.001]

**Table D.10**

Constrained OLS cross-sectional asset-pricing tests by asset class (price of beta risk in single factor models).

The table presents the constrained OLS estimates of the prices of beta risk for the CAPM and HKM's single-factor model (HKMSF) with nontraded capital. *MKT* and *CPTL* denote the market and nontraded capital factors, respectively. For each parameter estimate, we report the Jagannathan and Wang (1998) *t*-ratio ( $t\text{-stat}_c$ ), the KRS model misspecification-robust *t*-ratio ( $t\text{-stat}_m$ ), and the bootstrap *p*-value (boot *p*-val). In addition, we present the sample uncentered OLS cross-sectional  $R^2$  ( $R_U^2$ ) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) *p*-values. Finally, RANK TEST and fs *p*-val are the (approximate) finite-sample rank test discussed in Appendix F and its finite-sample *p*-value.  $N$  and  $T$  represent the number of assets and time series observations, respectively. *p*-values are in square brackets.

Panel A: CAPM								
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
<i>MKT</i>	0.61	2.08	1.77	0.49	1.42	0.43	-4.06	0.46
$t\text{-stat}_c$	(2.75)	(3.49)	(1.93)	(1.76)	(2.22)	(0.53)	(-1.93)	(0.92)
$t\text{-stat}_m$	(2.75)	(3.50)	(1.91)	(1.79)	(2.11)	(0.54)	(-2.40)	(0.97)
boot <i>p</i> -val	[0.009]	[0.000]	[0.133]	[0.076]	[0.043]	[0.567]	[0.048]	
$R_U^2$	0.84	0.89	0.94	0.53	0.63	0.04	0.40	0.46
SPEC TEST	115.05	92.81	7.50	74.10	71.47	31.82	48.99	
asy <i>p</i> -val	[0.000]	[0.000]	[0.186]	[0.000]	[0.000]	[0.080]	[0.000]	
boot <i>p</i> -val	[0.000]	[0.000]	[0.353]	[0.000]	[0.146]	[0.175]	[0.001]	
RANK TEST	2312.99	11.69	17.18	132.41	11.70	5.05	3.60	
fs <i>p</i> -val	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	
$N$	25	20	6	18	20	23	12	124
$T$	516	445	196	310	143	316	407	516
Panel B: HKMSF								
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX	All
<i>CPTL</i>	1.22	3.15	3.13	0.93	1.93	0.57	1.31	0.95
$t\text{-stat}_c$	(2.66)	(3.31)	(1.78)	(1.68)	(2.29)	(0.38)	(0.68)	(0.95)
$t\text{-stat}_m$	(2.66)	(3.33)	(1.76)	(1.71)	(2.18)	(0.37)	(0.08)	(0.98)
boot <i>p</i> -val	[0.013]	[0.001]	[0.169]	[0.086]	[0.041]	[0.687]	[0.929]	
$R_U^2$	0.88	0.90	0.93	0.53	0.63	0.02	0.00	0.47
SPEC TEST	108.58	101.97	6.59	75.74	78.30	31.99	77.09	
asy <i>p</i> -val	[0.000]	[0.000]	[0.253]	[0.000]	[0.000]	[0.077]	[0.000]	
boot <i>p</i> -val	[0.000]	[0.000]	[0.468]	[0.000]	[0.081]	[0.159]	[0.000]	
RANK TEST	32.39	10.10	13.32	21.22	11.59	3.71	4.31	
fs <i>p</i> -val	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	

**Table D.11**

Constrained GLS cross-sectional asset-pricing tests by asset class (price of beta risk in single factor models).

The table presents the constrained GLS estimates of the prices of beta risk for the CAPM and HKM's single-factor model (HKMSF) with nontraded capital. *MKT* and *CPTL* denote the market and nontraded capital factors, respectively. For each parameter estimate, we report the Jagannathan and Wang (1998) *t*-ratio ( $t\text{-stat}_c$ ), the KRS model misspecification-robust *t*-ratio ( $t\text{-stat}_m$ ), and the bootstrap *p*-value (boot *p*-val). In addition, we present the sample uncentered GLS cross-sectional  $R^2$  ( $R_U^2$ ) and the conditional heteroskedastic model specification test (SPEC TEST) of Shanken (1985) with asymptotic (asy) and bootstrap (boot) *p*-values. *N* and *T* represent the number of assets and time series observations, respectively. *p*-values are in square brackets.

Panel A: CAPM							
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
<i>MKT</i>	0.49	1.69	1.81	0.35	0.55	0.10	1.74
$t\text{-stat}_c$	(2.36)	(4.02)	(2.14)	(1.27)	(1.09)	(0.20)	(2.19)
$t\text{-stat}_m$	(2.36)	(3.43)	(2.09)	(1.28)	(1.05)	(0.19)	(1.20)
boot <i>p</i> -val	[0.028]	[0.002]	[0.079]	[0.198]	[0.299]	[0.858]	[0.265]
$R_U^2$	0.05	0.15	0.55	0.02	0.02	0.00	0.06
SPEC TEST	115.10	99.34	7.46	76.25	76.52	32.45	73.28
asy <i>p</i> -val	[0.000]	[0.000]	[0.188]	[0.000]	[0.000]	[0.070]	[0.000]
boot <i>p</i> -val	[0.000]	[0.000]	[0.374]	[0.000]	[0.018]	[0.148]	[0.000]
<i>N</i>	25	20	6	18	20	23	12
<i>T</i>	516	445	196	310	143	316	407

  

Panel B: HKMSF (nontraded capital factor)							
	FF25	US bonds	Sov. bonds	Options	CDS	Commod.	FX
<i>CPTL</i>	1.33	2.16	2.80	0.61	0.76	0.58	3.84
$t\text{-stat}_c$	(3.27)	(3.38)	(2.14)	(1.13)	(1.04)	(0.70)	(3.47)
$t\text{-stat}_m$	(3.13)	(2.99)	(1.98)	(1.04)	(0.97)	(0.63)	(2.12)
boot <i>p</i> -val	[0.002]	[0.009]	[0.088]	[0.303]	[0.338]	[0.530]	[0.065]
$R_U^2$	0.10	0.10	0.48	0.02	0.01	0.02	0.16
SPEC TEST	108.38	112.04	6.93	78.07	79.04	31.99	56.35
asy <i>p</i> -val	[0.000]	[0.000]	[0.226]	[0.000]	[0.000]	[0.078]	[0.000]
boot <i>p</i> -val	[0.000]	[0.000]	[0.452]	[0.000]	[0.020]	[0.154]	[0.000]