Reputation and Signaling in Asset Sales: Internet Appendix

Barney Hartman-Glaser*

September 1, 2016

Appendix D. Non-Markov Perfect Equilibrium

In this appendix, I consider the game when there is no honest-type issuer under the definition of equilibrium consisting of conditions 2, 4, and 5 of Definition 1. I however maintain equilibrium selection implied by Definition 2, which essentially states that an issuer can always separate when she has a high-quality asset by issuing the quantity $\hat{q}$. A result of maintaining this assumption is that the lowest stage game payoff that is available to the issuer with a high-quality asset is $\hat{q} + \gamma(1 - \hat{q})$ and the "worst" equilibrium, that is the equilibrium with the lowest issuer value, is the repeated LCSE. By issuer value, I mean the time zero expected present value of all stage game payoffs to the issuer before learning the quality of the asset she has to sell at time zero. The goal here is to determine to what extent the set of equilibria under this relaxed definition of equilibrium given $\phi_0 = 0$ resembles that of the model discussed above.

First, I show that relaxing the Markov restriction on equilibrium allows for the existence of truth-telling equilibria without reputational concerns; however, the parameter restriction required for their existence coincides with that of Proposition 2. In Proposition D.1, I give a condition for the existence of a truth-telling equilibrium.

Proposition D.1. *Suppose $\phi_0 = 0$, then there exists a truth-telling equilibrium with $Q^O_{hh}(\mathcal{H}_t) > \hat{q}$ for some $\mathcal{H}_t$ if and only if $\delta \geq \frac{1-\gamma\ell}{1-\gamma\ell+\lambda(1-\gamma)}$.*

Proof. The proof is identical to that of Proposition 2 of the main text. □

*Anderson School of Management, UCLA. Phone: 310-825-4083, Email: bhglaser@anderson.ucla.edu.

Proposition D.1 shows that although allowing for path-dependent strategies does mean that a truth-telling equilibrium is supportable without the type of issuer reputation considered in this paper, the restriction on the parameters required for truth-telling does not change. The intuition is that the equilibrium punishment for misreporting is bounded below by Assumption **??**. Thus, although allowing for path dependence does mean that investors can punish the issuer even without a reputation, the harshest punishment remains the repeated LCSE.

Dropping the Markov restriction from the definition of equilibrium does however allow a repeated pooling equilibrium which is not part of the set of equilibria of the reputation game. Indeed, suppose an equilibrium calls for the issuer to always sell the entire asset at the pooled price $(1-\delta)(\lambda+(1-\lambda)\ell)$. If the issuer ever retains part of the asset, the investors revert to the separating equilibrium for all time. In this case, the issuer will never want to deviate when she has a low-quality asset to sell. When she has a high-quality asset, she will not deviate if:

$$(1-\delta)(\hat{q}+\gamma(1-\hat{q}))+\delta(\lambda(\hat{q}+\gamma(1-\hat{q}))+(1-\lambda)\ell) \leq \lambda+(1-\lambda)\ell. \tag{D.1}$$

In Proposition D.2, I give a simplified version of this restriction.

Proposition D.2. *Suppose* $\phi_0 = 0$ *and* $\delta \geq \frac{\gamma(1-(1-\lambda)\ell)-\lambda}{\gamma(1-\ell)(1-\lambda)}$, *then there exists an equilibrium with issuer strategies* $Q^O_{hh} = 1$, $\pi_h = 1$, *and* $\pi_\ell = 0$, *investor beliefs* $\mu_h = \lambda$ *and* $\mu_\ell = 0$, *and prices* $P_h(q) = \lambda + (1-\lambda)\ell$ *and* $P_\ell(q) = \ell$.

Proof. The condition on δ guarantees the issuer will never deviate when she has a high-quality asset. When the issuer has a low-quality asset, her maximum gain from deviating is weakly less than when she has a high-quality asset. As a result, she will not deviate either. Thus condition 2 of Definition 1 holds. Conditions 4 and 5 of Definition 1 are satisfied by construction. $\square$

One aspect of the repeated pooling equilibrium that is worth noting is that it delivers the same value to the issuer as the truth-telling equilibrium $V = \lambda + (1-\lambda)\ell$. Indeed, this equilibrium delivers perfect allocative efficiency. It is also straightforward to check that the restriction given in Proposition D.2 is weaker than the necessary condition for the existence of a truth-telling equilibrium. Thus, perfect allocative efficiency can be achieved in the repeated games model for a larger set of parameters than in the reputation model.

Although the repeated pooling equilibrium is clearly dominant in the sense that it delivers full information payoffs, it does not always exist. When parameters are such that neither the truth-telling equilibrium nor the repeated pooling equilibrium exist, it is because the single period gains to the low-type issuer from emulating a high-type outweigh the benefits of maintaining a repeated pooling equilibrium. At the same time the value of playing a repeated pooling equilibrium turns out to always be greater than a repeated *partial* pooling equilibrium. Thus, if it is impossible to sustain pooling, it is also impossible to sustain partial pooling. This reasoning leads directly to Proposition D.3.

Proposition D.3. *If the repeated pooling equilibrium does not exist, that is if* $\delta < \frac{\gamma(1-(1-\lambda)\ell)-\lambda}{\gamma(1-\ell)(1-\lambda)}$, *then the only equilibrium is the repeated LCSE.*

Proof. The first step is to find the highest possible value the issuer can obtain in equilibrium. It is useful to define the following quantities:

$$V^{hS} = \text{issuer continuation payoff when asset is high quality and issuer separates},$$
$$V^{hP} = \text{issuer continuation payoff when asset is high quality and issuer pools},$$
$$V^{\ell S} = \text{issuer continuation payoff when asset is low quality and issuer separates}, \textit{and}$$
$$V^{\ell P} = \text{issuer continuation payoff when asset is low quality and issuer pools}.$$

By "issuer pools" I mean reports a quality and sells a quantity that both high- and low-quality issuers choose in equilibrium with some positive probability. The only restriction I place on the above quantities is that they each be the issuer value for some equilibrium that satisfies conditions 2, 4, and 5 of Definition 1. Suppose $\tilde{Q}$ is some quantity that both high- and low-quality issuers sell in equilibrium following an identical report, and let $\tilde{P}$ be the equilibrium price following such a report-quantity pair. The single deviation principle then states that:

$$(1-\delta)(\tilde{P}\tilde{Q} + \gamma(1-\tilde{Q})) + \delta V^{hP} \geq (1-\delta)(\hat{q} + \gamma(1-\hat{q})) + \delta V^{hS}, \tag{D.2}$$

$$(1-\delta)(\tilde{P}\tilde{Q} + \gamma(1-\tilde{Q})\ell) + \delta V^{\ell P} \geq (1-\delta)\ell + \delta V^{\ell S}. \tag{D.3}$$

Note that Eq. (D.3) must bind, otherwise an issuer selling a low-quality asset would strictly prefer to separate and a truth-telling equilibrium would exist, which would in turn violate Proposition D.1 given the propositions assumption on δ. Together, Eqs. (D.2) and (D.3) imply that the issuer receives the value:

$$V = (1-\delta)(\ell + \gamma\lambda(1-\tilde{Q}))(1-\ell)) + \delta(V^{\ell S} + \lambda(V^{hP} - V^{\ell P})). \tag{D.4}$$

Now fix $V^{\ell S}, V^{hP}$, and $V^{\ell P}$, and note that I am free to choose $\tilde{Q}$ to be any quantity between $\hat{q}$ and $\overline{Q}$, where:

$$\overline{Q} = \frac{\ell(1-\delta)(1-\gamma) + \delta((V^{\ell S} - V^{\ell P}) - (V^{hS} - V^{hP}) - \hat{q}(1-\gamma)(1-\delta)}{\gamma(1-\ell)(1-\delta)}, \tag{D.5}$$

is the largest quantity such that it is possible to satisfy both Equations (D.2) and (D.3). Eq. (D.4) implies that, fixing $V^{\ell S}, V^{hP}$, and $V^{\ell P}$, the equilibrium that delivers the highest value to the issuer is the one with the smallest $\tilde{Q}$. Fixing $\tilde{Q}$ at the lowest quantity, I am free to choose $V^{\ell S}$, V^{hP}, and $V^{\ell P}$, so long as they are issuer values from some equilibrium. Thus the equilibrium with the highest value to the issuer is delivered by setting $\tilde{Q}$ and $V^{\ell P}$ as low as possible and $V^{\ell S}$ and V^{hP} as high as possible. Recall that the issuer value for the repeated LCSE is the lowest value obtainable. Let $V^{\max}$ be the largest possible equilibrium value for the issuer, I then have:

$$V^{\max} = (1-\delta)(\ell + \gamma\lambda(1-\tilde{Q}))(1-\ell)) + \delta(V^{\max} + \lambda(V^{\max} - V^{\text{LCSE}})), \tag{D.6}$$

which simplifies to:

$$V^{\max} = V^{\text{LCSE}}. \tag{D.7}$$

But this means that the largest issuer value available in equilibrium is V^{LCSE}.

Now note that the lowest possible equilibrium value is also V^{LCSE}, since the definition of equilibrium requires that the issuer can always separate when she has a high-quality asset. Thus, $V = V^{\text{LCSE}}$ for any equilibrium. Moreover, the only equilibrium that satisfies Eqs. (D.2) and (D.3) with $V^{\ell S} = V^{hP} = V^{\ell P} = V^{\text{LCSE}}$ is the LCSE. $\square$

Proposition D.3 highlights the difference between the pure repeated games model and the reputation model I present in the main text. In the pure repeated games model, partial pooling never obtains, while in the model I present in the text, one of the main equilibria of interest is a partial pooling equilibrium. This difference arises because reputation of the kind considered in the main text allows for partial pooling to have a different effect on prices for different levels of reputation. In other words, the effect of opportunistic behavior on prices decreases as reputation improves. In the pure repeated games model, equilibrium opportunistic behavior has the same effect on prices regardless of the history of past play. In summary, while allowing for path-dependent strategies does indeed expand the set of equilibria to include a repeated pooling equilibrium, removing reputation concerns rules out partial pooling, which is a primary focus of the model.