Internet Appendix

A Proofs

A.1 Proof of Proposition 1

See Liptser and Shiryaev (2001).

A.2 Proof of Proposition 2

Let the dynamics of the price of the dividend strip with maturity i and the corresponding equity yield be given by

$$\frac{dP(t,i)}{P(t,i)} = \mu_P(t,i)dt + \sigma_P(t,i)d\widehat{B}_t,$$
$$de(t,i) = \mu_e(t,i)dt + \sigma_e(t,i)d\widehat{B}_t.$$

From the definition of the dividend strip in (16), we can interpret S_t^c as the value process for the self-financing portfolio that consists of a continuum of dividend strips:

$$S_t^c = n_t S_t = \int_t^\infty n_t P(t,i)di,$$

where

$$n_t \equiv \exp\left(\int_0^t \frac{D_u}{S_u}du\right)$$

is the number of dividend strips of each maturity held during the period $[t, t+dt]$. The corresponding relative weight $w_t(i)$ is given by

$$w_t(i) \equiv \frac{P(t,i)}{\int_t^\infty P(t,i)di}, \quad \text{with} \quad \int_t^\infty w_t(i)di = 1.$$

$w_t(i)$ tells us the relative proportion of the total portfolio value invested at time t in the dividend strip that pays $D_i di$ in the interval $[i, i + di]$. As in van Binsbergen and Koijen (2017), the stock return is the instantaneous return on the value-weighted portfolio of strips

$$\frac{dS_t^c}{S_t^c} = \int_t^\infty \frac{dP(t,i)}{P(t,i)} w_t(i) di.$$

Applying Ito's lemma yields

$$\begin{aligned} \frac{dS_t^c}{S_t^c} &= \int_t^\infty \left(d\log P(t,i) + \frac{1}{2}\sigma_P^2(t,i)dt \right) w_t(i)di \\ &= \int_t^\infty d\log P(t,i) w_t(i)di + \frac{1}{2}\int_t^\infty \sigma_P^2(t,i) w_t(i) di dt. \end{aligned} \tag{A.1}$$

Moreover, we have

$$d\log S_t^c = \frac{dS_t^c}{S_t^c} - \frac{1}{2}\left(\frac{dS_t^c}{S_t^c}\right)^2 = \frac{dS_t^c}{S_t^c} - \frac{1}{2} v_t dt, \tag{A.2}$$

where

$$v_t dt = \left(\frac{dS_t^c}{S_t^c}\right)^2 = \int_t^\infty \int_t^\infty \sigma_P(t,i)\sigma_P(t,j) w_t(i) w_t(j) di dj dt. \tag{A.3}$$

Putting (A.1), (A.2), and (A.3) together provides the relation between the instantaneous stock return and dividend strip returns

$$\begin{aligned} d\log S_t^c = &\int_t^\infty d\log P(t,i) w_t(i) di \\ &+ \frac{1}{2}\int_t^\infty \left(\sigma_P^2(t,i) - \int_t^\infty \sigma_P(t,i)\sigma_P(t,j) w_t(j) dj \right) w_t(i) di dt. \end{aligned} \tag{A.4}$$

The HPR is by definition given by

$$\log R(t,T) = \int_t^T d\log S_s^c.$$

Using (A.4) allows us to write

$$\log R(t,T) = \int_t^T \int_s^\infty d\log P(s,i) w_t(i) di + \frac{1}{2}\int_t^T \int_s^\infty \left(\sigma_P^2(s,i) - \int_s^\infty \sigma_P(s,i)\sigma_P(s,j)w_s(j)dj\right) w_s(i) di ds.$$

Moreover, from the definitions of the dividend strip price and the equity yield, we know that

$$d\log P(t,T) = d\log D_t + e(t,T)dt - de(t,T)(T-t)$$
$$\sigma_P(t,T) = \sigma_D - \sigma_e(t,T)(T-t).$$

□

A.3 Proof of Proposition 3

The conditional variance of the holding period return is

$$\mathrm{Var}_t\left[\log R(t,T)\right] = \mathrm{Cov}_t\left(\int_t^T d\log S_s^c, \int_t^T d\log S_u^c\right) = \int_t^T\int_t^T \mathrm{Cov}_t\left(d\log S_s^c, d\log S_u^c\right) \quad (A.5)$$

For $s = u$, the integrand in (A.5) is the instantaneous variance of log returns. Using (A.4) and Fubini's theorem, we can write it as follows:

$$\mathrm{Cov}_t\left(d\log S_s^c, d\log S_s^c\right) = \mathrm{Var}_t\left(d\log S_s^c\right) = E_t\left[\left(d\log S_s^c - E_t\left[d\log S_s^c\right]\right)^2\right]$$
$$= E_t\left[\left(\int_s^\infty \sigma_P(s,i)w_s(i)di\right)^2\right] ds = E_t\left[\int_s^\infty\int_s^\infty \sigma_P(s,i)\sigma_P(s,j)w_s(i)w_s(j)didj\right] ds$$
$$= \int_s^\infty\int_s^\infty E_t\left[\sigma_P(s,i)\sigma_P(s,j)w_s(i)w_s(j)\right] didjds.$$

For $s \neq u$, we approximate (A.4) by abstracting from the second-order volatility adjustment term. We obtain

$$\begin{aligned}
&\mathrm{Cov}_t\left(d\log S_s^c, d\log S_u^c\right) \approx \mathrm{Cov}_t\left(\int_s^\infty d\log P(s,i)w_s(i)di, \int_u^\infty d\log P(u,j)w_u(j)dj\right) \\
&= \int_s^\infty\int_u^\infty \mathrm{Cov}_t\left(d\log P(s,i)w_s(i), d\log P(u,j)w_u(j)\right) di dj.
\end{aligned}$$

□

A.4 Proof of Proposition 4

To prove the statements in the first part of the proposition we start by computing the HPR variance at a τ-year horizon in the model with only the permanent component. For this, we use the fact that the moment generating function of the logarithm of the stock price is given by

$$\widehat{MGF}(t,\tau,u) = E_t\left[\left(S_{t+\tau}^c\right)^u\right] = e^{u\log(S_t^c)+\widehat{h}_0(\tau;u)+\widehat{h}_1(\tau;u)\widehat{x}_t}, \tag{A.6}$$

where $\widehat{h}_0$ and $\widehat{h}_1$ solve

$$\begin{aligned}
\widehat{h}_1'(\tau) &= u - \kappa_x \widehat{h}_1(\tau), \\
\widehat{h}_0'(\tau) &= \frac{1}{2}\overline{v}u^2 + u\left(m - \frac{\overline{v}}{2}\right) + u\overline{\gamma}_x\widehat{h}_1(\tau) + \frac{1}{2}\frac{\overline{\gamma}_x^2}{\overline{v}}\widehat{h}_1^2(\tau)
\end{aligned}$$

with initial conditions $\widehat{h}_0(0) = \widehat{h}_1(0) = 0$. Solving these two ODEs and using (A.6) together with the definition of $\mathrm{vol}_{S,t}(\tau)$ in (24) yields

$$\mathrm{vol}_{S,t}(\tau)^2 = \frac{(\overline{\gamma}_x + \overline{v}\kappa_x)^2}{\overline{v}\kappa_x^2} - \frac{1}{\tau}\frac{e^{-2\kappa_x\tau}(1-e^{\kappa_x\tau})((1-3e^{\kappa_x\tau})\overline{\gamma}_x - 4e^{\kappa_x\tau}\overline{v}\kappa_x)\overline{\gamma}_x}{2\overline{v}\kappa_x^3}. \tag{A.7}$$

Taking the limits of (A.7) and using the fact that the steady-state posterior variance $\overline{\gamma}_x$ is a (nonnegative) solution to

$$\sigma_x^2 - 2\kappa_x \overline{\gamma}_x - \frac{1}{\overline{v}} \overline{\gamma}_x^2 = 0$$

we obtain

$$\lim_{\tau \to 0^+} \text{vol}_{S,t}(\tau)^2 = \overline{v},$$
$$\lim_{\tau \to \infty} \text{vol}_{S,t}(\tau)^2 = \overline{v} + \frac{\sigma_x^2}{\kappa_x^2}.$$

The derivative of (A.7) with respect to the horizon is

$$\partial_\tau \text{vol}_{S,t}(\tau)^2 = \frac{\overline{\gamma}_x^2 e^{-\kappa_x \tau}}{2\kappa_x^3 \tau^2 \overline{v}} \left(\left(3 + 4\kappa_x \frac{\overline{v}}{\overline{\gamma}_x}\right) e^{\kappa_x \tau} + (1 + 2\kappa_x \tau) e^{-\kappa_x \tau} - 4\left(1 + \kappa_x \tau\right) \left(1 + \kappa_x \frac{\overline{v}}{\overline{\gamma}_x}\right) \right). \tag{A.8}$$

With parameters κ_x, $\overline{\gamma}_x$, and $\overline{v}$ being nonnegative, in order to show that the term structure of HPR volatility is nondecreasing for $\tau > 0$ it is enough to show that the expression in the parentheses in (A.8) is nonnegative, i.e.

$$\left(3 + 4\kappa_x \frac{\overline{v}}{\overline{\gamma}_x}\right) e^{\kappa_x \tau} + (1 + 2\kappa_x \tau) e^{-\kappa_x \tau} - 4\left(1 + \kappa_x \tau\right) \left(1 + \kappa_x \frac{\overline{v}}{\overline{\gamma}_x}\right) \geq 0.$$

The left hand side of the above inequality can be rearranged into a sum of two parts

$$e^{-\kappa_x \tau} \left[1 + 2\kappa_x \tau + 3e^{2\kappa_x \tau} - 4e^{\kappa_x \tau}\left(1 + \kappa_x \tau\right)\right] + \left[4e^{\kappa_x \tau} \kappa_x \frac{\overline{v}}{\overline{\gamma}_x} - 4\left(1 + \kappa_x \tau\right) \kappa_x \frac{\overline{v}}{\overline{\gamma}_x}\right]. \tag{A.9}$$

The second bracket is positive because $e^{\kappa_x \tau} > 1 + \kappa_x \tau$ and it is enough to prove that the first bracket of (A.9) is nonnegative:

$$1 + 2\kappa_x \tau + 3e^{2\kappa_x \tau} - 4e^{\kappa_x \tau}\left(1 + \kappa_x \tau\right) \geq 0.$$

Rearranging the left-hand side of the above inequality gives

$$
\begin{aligned}
&1 + 2\kappa_x\tau + 3e^{2\kappa_x\tau} - 4e^{\kappa_x\tau}\left(1 + \kappa_x\tau\right) \\
&\quad = \left(e^{\kappa_x\tau} - (1 + \kappa_x\tau)\right)^2 + 2e^{2\kappa_x\tau} - 2e^{\kappa_x\tau}(1 + \kappa_x\tau) - (\kappa_x\tau)^2 \\
&\quad > \left(e^{\kappa_x\tau} - (1 + \kappa_x\tau)\right)^2 + 2e^{\kappa_x\tau}(1 + \kappa_x\tau + \frac{(\kappa_x\tau)^2}{2}) - 2e^{\kappa_x\tau}(1 + \kappa_x\tau) - (\kappa_x\tau)^2 \\
&\quad = \left(e^{\kappa_x\tau} - (1 + \kappa_x\tau)\right)^2 + (\kappa_x\tau)^2(e^{\kappa_x\tau} - 1) \\
&\quad \geq 0.
\end{aligned}
$$

This proves that the term structure of HPR volatility in the P-CV model is monotonically increasing.

To prove the statements in the second part of the proposition, we need the moment generating function of the logarithm of the stock price in the PT-CV model.

$$
\widehat{MGF}(t, \tau, u) = E_t\left[\left(S^c_{t+\tau}\right)^u\right] = e^{u\log(S^c_t) + \widehat{g}_0(\tau;u) + \widehat{g}_1(\tau;u)\widehat{x}_t + \widehat{g}_2(\tau;u)\widehat{x}_t}, \tag{A.10}
$$

where $\widehat{g}_0$, $\widehat{g}_1$ and $\widehat{g}_2$ solve

$$
\begin{aligned}
\widehat{g}_1'(\tau) &= u - \kappa_x\widehat{g}_1(\tau), \\
\widehat{g}_2'(\tau) &= -\kappa_z\left(u + \widehat{g}_2(\tau)\right), \\
\widehat{g}_0'(\tau) &= \frac{1}{2}\overline{v}u^2 + u(m - \frac{\overline{v}}{2}) + u\left((\overline{\gamma}_{zx} - \overline{\gamma}_z\kappa_z + \sigma_z^2)\widehat{g}_2(\tau) + (\overline{\gamma}_x - \overline{\gamma}_z x\kappa_z)\widehat{g}_1(\tau)\right) \\
&\quad + \frac{1}{2\overline{v}}\left((\overline{\gamma}_{zx} - \overline{\gamma}_z\kappa_z + \sigma_z^2)\widehat{g}_2(\tau) + (\overline{\gamma}_x - \overline{\gamma}_z x\kappa_z)\widehat{g}_1(\tau)\right)^2,
\end{aligned}
$$

with initial conditions $\widehat{g}_0(0) = \widehat{g}_1(0) = \widehat{g}_2(0) = 0$. Solving these two ODEs and using (A.10) together with the definition of $\mathrm{vol}_{S,t}(\tau)$ gives us the expression for the HPR volatility in the PT-CV model. Using the obtained expression and the fact that the elements of the steady-state posterior variance-covariance matrix solve

$$\sigma_x^2 - 2\kappa_x\overline{\gamma}_x - \frac{1}{\overline{v}}\left(\overline{\gamma}_x - \kappa_z\overline{\gamma}_{zx}\right)^2 = 0,$$
$$\sigma_z^2 - 2\kappa_z\overline{\gamma}_z - \frac{1}{\overline{v}}\left(\sigma_z^2 - \kappa_z\overline{\gamma}_z + \overline{\gamma}_{zx}\right)^2 = 0,$$
$$-\overline{\gamma}_{zx}(\kappa_x + \kappa_z) - \frac{1}{\overline{v}}\left(\overline{\gamma}_x - \kappa_z\overline{\gamma}_{zx}\right)\left(\sigma_z^2 - \kappa_z\overline{\gamma}_z + \overline{\gamma}_{zx}\right) = 0,$$

we can derive the limiting expressions for $\mathrm{vol}_{S,t}(\tau)^2$:

$$\lim_{\tau\to 0^+} \mathrm{vol}_{S,t}(\tau)^2 = \overline{v},$$
$$\lim_{\tau\to \infty} \mathrm{vol}_{S,t}(\tau)^2 = \overline{v} + \frac{\sigma_x^2}{\kappa_x^2} - \sigma_z^2.$$

We see that the long-term variance (volatility) of HPRs is smaller than the short-term variance (volatility) if

$$\frac{\sigma_x^2}{\kappa_x^2} - \sigma_z^2 < 0.$$

□

A.5 Proof of Proposition 5

To compute the price of the dividend strip we recall that

$$P(t,T) = E_t\left[\frac{\xi_T}{\xi_t}D_T\right] = \frac{1}{\xi_t}E_t\left[e^{\log\xi_T + \log D_T}\right].$$

For the conditional expectation

$$H(t,\xi_t,D_t,\theta_t;T) \equiv E_t\left[e^{\log\xi_T + \log D_T}\right] \tag{A.11}$$

we guess an (approximate) exponential affine solution of the form

$$H(t, \xi_t, D_t, \theta_t; T) = e^{\log \xi_t + \log D_t + a(T-t) + \theta_t^\top b(T-t)}, \tag{A.12}$$

where $\theta_t = [\widehat{x}_t, \widehat{z}_t, v_t, \gamma_{x,t}, \gamma_{z,t}, \gamma_{zx,t}]^\top$ is the vector of state variables. Given the dynamics of the state-price density and the dividend process in (12) and (15), as well as the dynamics of the state variables provided in (6)–(10) and (29), it follows from the Feynman-Kac Theorem that H, as given in (A.11), needs to satisfy the following PDE

$$\begin{aligned}
&H_t - r_f \xi H_\xi + \mu_D D H_D - \kappa_x x H_x - \kappa_z z H_z + \kappa_v(\bar{v} - v) H_v \\
&\quad + \left(\sigma_x^2 - 2\kappa_x\gamma_x - \frac{1}{v}\left(\gamma_x - \gamma_{zx}\kappa_z\right)^2\right) H_{\gamma_x} + \left(\sigma_z^2 - 2\kappa_z\gamma_z - \frac{1}{v}\left(\gamma_{zx} - \gamma_z\kappa_z + \sigma_z^2\right)^2\right) H_{\gamma_z} \\
&\quad + \left(-\left(\kappa_x + \kappa_z\right)\gamma_{zx} - \frac{1}{v}\left(\gamma_x - \gamma_{zx}\kappa_z\right)\left(\gamma_{zx} - \gamma_z\kappa_z + \sigma_z^2\right)\right) H_{\gamma_{zx}} \\
&\quad + \frac{1}{2v}\left(m + x - \kappa_z z - r_f\right)^2 \xi^2 H_{\xi\xi} + \frac{1}{2}\sigma_D^2 D^2 H_{DD} + \frac{1}{2v}(\gamma_x - \kappa_z\gamma_{zx})^2 H_{xx} \\
&\quad + \frac{1}{2v}(\gamma_{zx} - \kappa_z\gamma_z + \sigma_z^2)^2 H_{zz} + \frac{1}{2}\sigma_v^2 v H_{vv} - \frac{1}{\sqrt{v}}(m + x - \kappa_z z - r_f)\sigma_D H_{\xi D} + \sigma_D\sigma_v\sqrt{v} H_{Dv} \\
&\quad - \frac{1}{v}(m + x - \kappa_z z - r_f)(\gamma_x - \kappa_z\gamma_{zx}) H_{\xi x} - \frac{1}{v}(m + x - \kappa_z z - r_f)(\gamma_{zx} - \kappa_z\gamma_z + \sigma_z^2) H_{\xi z} \\
&\quad - (m + x - \kappa_z z - r_f)\sigma_v H_{\xi v} + \frac{1}{\sqrt{v}}\sigma_D(\gamma_x - \kappa_z\gamma_{zx}) H_{Dx} + \frac{1}{\sqrt{v}}\sigma_D(\gamma_{zx} - \kappa_z\gamma_z + \sigma_z^2) H_{Dz} \\
&\quad + \frac{1}{v}\left(\gamma_{zx} - \kappa_z\gamma_z + \sigma_z^2\right)\left(\gamma_x - \kappa_z\gamma_{zx}\right) H_{zx} + (\gamma_x - \kappa_z\gamma_{zx})\sigma_v H_{xv} + (\gamma_{zx} - \kappa_z\gamma_z + \sigma_z^2)\sigma_v H_{zv} = 0.
\end{aligned}$$

Plugging the guess (A.12) (and the resulting partial derivatives from the guess solution) into the PDE and performing a first-order Taylor expansion around the long-term means of the state variables $\theta_t = [\widehat{x}_t, \widehat{z}_t, v_t, \gamma_{x,t}, \gamma_{z,t}, \gamma_{zx,t}]^\top$ allows us to obtain a system of ODEs that determines the function $a(\cdot)$ and the vector valued function $b(\cdot)$, with the first-order conditions $a(0) = b_i(0) = 0$ for $i = 1, ..., 6$. Finally, equity yields are given by

$$e(t, T) = -\frac{1}{T - t}\left(a(T - t) + \theta_t^\top b(T - t)\right). \tag{A.13}$$

The diffusion of the equity yield is recovered from the expression for $e(t,T)$ in (A.13) by applying Ito's lemma:

$$\sigma_e(t,T) = -\frac{1}{T-t}\left[\frac{1}{\sqrt{v_t}}\left(\gamma_{x,t} - \kappa_z\gamma_{zx,t}\right), \frac{1}{\sqrt{v_t}}\left(\gamma_{zx,t} - \kappa_z\gamma_{z,t} + \sigma_z^2\right), \sigma_v\sqrt{v_t}, 0, 0, 0\right] b(T-t).$$

In the P-CV model ($S_t^c = Y_t$ and constant instantaneous volatility, $v_t = \overline{v}$), the volatility of equity yields is given by

$$\text{vol}_{e,t}(\tau) = \frac{1}{\tau}\left|\frac{\overline{\gamma}_x}{\sqrt{\overline{v}}}b(\tau)\right|,$$

where $\overline{\gamma}_x$ solves $\dfrac{d\gamma_{x,t}}{dt} = 0$. The function $b_1(\tau)$ is given by

$$b(\tau) = -\frac{c_2}{c_1}\left(\exp(-\tau c_1/c_3) - 1\right),$$

with

$$\begin{aligned}
c_1 &= (a_x-1)^2\sqrt{\overline{v}}\frac{\overline{\gamma}_x^2}{\overline{v}} + \left(\frac{\overline{\gamma}_x}{\sqrt{\overline{v}}} + \sqrt{\overline{v}}\kappa_x\right)(m-m_s)^2, \\
c_2 &= (a_x-1)\left((m-m_s)^2\sqrt{\overline{v}} - (a_x-1)(r_f+\overline{v})\eta_x + m(a_x\eta_x + \sqrt{\overline{v}}\kappa_x) - m_s(\eta_x + \sqrt{\overline{v}}\kappa_x)\right), \\
c_3 &= (m-m_s)^2\sqrt{\overline{v}}.
\end{aligned}$$

For $a_x \to 1$, we have $b(\tau) = 0$ and $\text{vol}_{e,t}(\tau) = 0$.

In the PT-CV model ($S_t^c = Y_t Z_t$ and constant instantaneous volatility, $v_t = \overline{v}$), the volatility of equity yields is given by

$$\text{vol}_{e,t}(\tau) = \frac{1}{\tau}\left|\eta_x b_1(\tau) + \eta_z b_2(\tau)\right|,$$

where

$$\eta_x \equiv \frac{\overline{\gamma}_x - \kappa_z \overline{\gamma}_{zx}}{\sqrt{\overline{v}}}, \quad \eta_z \equiv \frac{\overline{\gamma}_{zx} - \kappa_z \overline{\gamma}_z + \sigma_z^2}{\sqrt{\overline{v}}} \tag{A.14}$$

and the steady-state posterior variance-covariance matrix

$$\overline{\Gamma} \equiv \begin{pmatrix} \overline{\gamma}_z & \overline{\gamma}_{zx} \\ \overline{\gamma}_{zx} & \overline{\gamma}_x \end{pmatrix}$$

is obtained by solving the following system of equations $\frac{d\gamma_{z,t}}{dt} = 0$, $\frac{d\gamma_{x,t}}{dt} = 0$, and $\frac{d\gamma_{zx,t}}{dt} = 0$. Functions $b_1(\tau)$ and $b_2(\tau)$ solve the system of ODEs

$$A_1 b_1'(\tau) + B_1 b_2(\tau) + C_1 b_1(\tau) + D_1 = 0,$$
$$A_2 b_2'(\tau) + B_2 b_1(\tau) + C_2 b_2(\tau) + D_2 = 0,$$

where

$$A_1 = -(m - m_s)^2 \sqrt{\overline{v}}, \quad A_2 = -A_1,$$
$$B_1 = \eta_z \left(-(m - m_s)^2 + (a_x - 1)\sqrt{\overline{v}}(-(a_x - 1)\eta_x + (a_z - 1)\eta_z \kappa_z) \right),$$
$$B_2 = -\eta_x \kappa_z \left((m - m_s)^2 + (a_z - 1)\sqrt{\overline{v}}((a_x - 1)\eta_x - (a_z - 1)\eta_z \kappa_z) \right),$$
$$C_1 = -\left((m - m_s)^2(\eta_x + \sqrt{\overline{v}}\kappa_x) + (a_x - 1)\sqrt{\overline{v}}\eta_x((a_x - 1)\eta_x - (a_z - 1)\eta_z \kappa_z) \right),$$
$$C_2 = \kappa_z \left((m - m_s)^2(\sqrt{\overline{v}} - \eta_z) + (a_z - 1)\sqrt{\overline{v}}\eta_z(-(a_x - 1)\eta_x + (a_z - 1)\eta_z \kappa_z) \right),$$
$$D_1 = (a_x - 1)\left((m - m_s)^2\sqrt{\overline{v}} - (a_x - 1)(r_f + \overline{v})\eta_x - m_s(\eta_x + \sqrt{\overline{v}}\kappa_x) + m(a_x \eta_x + \sqrt{\overline{v}}\kappa_x) \right)$$
$$+ (a_z - 1)(m_s - r_f - \overline{v} + a_x(-m + r_f + \overline{v}))\eta_z \kappa_z,$$
$$D_2 = \kappa_z \Big((a_z - 1)(m - m_s)^2\sqrt{\overline{v}} - (a_x - 1)m_s \eta_x - (a_z - 1)m_s(\sqrt{\overline{v}} - \eta_z)\kappa_z$$
$$+ (a_z - 1)(r_f + \overline{v})(-(a_x - 1)\eta_x + (a_z - 1)\eta_z \kappa_z) + m((a_x - 1)a_z \eta_x + (a_z - 1)(\sqrt{\overline{v}} - a_z \eta_z)\kappa_z) \Big).$$

For $a_x \to 1$, we have

$$\begin{aligned}\lim_{\tau\to 0^+} \text{vol}_{e,t}(\tau) &= \frac{1}{A_2}\left|D_1\eta_x - D_2\eta_z\right| \\ &= \frac{1}{(m-m_s)^2\sqrt{\overline{v}}}\Big|(a_z-1)\eta_z\kappa_z\Big((m-m_s)\big((m-m_s)\sqrt{\overline{v}}+\eta_x\big) \\ &+ \big((m-m_s)\sqrt{\overline{v}} + (m_s - r_f - \overline{v} + a_z(-m+r_f+\overline{v}))\eta_z\big)\kappa_z\Big)\Big|.\end{aligned}$$

The short-term volatility of the equity yield is strictly greater than zero as long as the term within the absolute value is non-zero. Moreover, the long-term equity yield volatility satisfies[24]

$$\lim_{\tau\to\infty} \text{vol}_{e,t}(\tau) = 0.$$

We see that

$$\lim_{\tau\to\infty} \text{vol}_{e,t}(\tau) \leq \lim_{\tau\to 0^+} \text{vol}_{e,t}(\tau),$$

with strict inequality if

$$\begin{aligned}&(a_z-1)\eta_z\kappa_z\Big((m-m_s)\big((m-m_s)\sqrt{\overline{v}}+\eta_x\big) \\ &+ \big((m-m_s)\sqrt{\overline{v}} + (m_s - r_f - \overline{v} + a_z(-m+r_f+\overline{v}))\eta_z\big)\kappa_z\Big) \neq 0.\end{aligned}$$

[24]Note that the following parameter restrictions need to hold for the long-term moments of equity yields to be determinate

$$\begin{aligned}A_2 &> 0, \\ C_2 - C_1 &> 0, \\ (C_2 - C_1) - \sqrt{(C_1+C_2)^2 - 4B_1B_2} &> 0.\end{aligned}$$

□

A.6 Proof of Proposition 6

The optimal value function is of the form $J(t, W, x, z)$, and the HJB equation is as follows:

$$J_t - \kappa_z z J_z + \frac{1}{2}\eta_z^2 J_{zz} - \kappa_x x J_x + \frac{1}{2}\eta_x^2 J_{xx} + \eta_z \eta_x J_{zx} + \sup_{\pi} \Big\{ r_f W J_W + \pi W \left(m + x - \kappa_z z - r_f\right) J_W + \frac{1}{2}\pi^2 W^2 \bar{v} J_{WW} + \pi W \sqrt{\bar{v}} \eta_x J_{Wx} + \pi W \sqrt{\bar{v}} \eta_z J_{Wz} \Big\} = 0, \tag{A.15}$$

where η_x and η_z are as defined in (A.14).

Solving the first-order condition yields the following optimal portfolio weight

$$\pi^\star = -\frac{J_W}{W J_{WW}} \frac{m + x - \kappa_z z - r_f}{\bar{v}} - \frac{J_{Wx}}{W J_{WW}} \frac{\eta_x}{\sqrt{\bar{v}}} - \frac{J_{Wz}}{W J_{WW}} \frac{\eta_z}{\sqrt{\bar{v}}}. \tag{A.16}$$

Inspired by results in Kim and Omberg (1996) and Zariphopoulou (2001), we make the conjecture

$$J(t, W, x, z) = e^{-\delta t} \frac{W^{1-\gamma}}{1-\gamma} F(t, x, z)^\gamma. \tag{A.17}$$

Plugging this conjecture back into the HJB equation (A.15) implies that the function $F(t, x, z)$ solves the linear second-order PDE

$$F_t + \left[-\kappa_x x + \frac{(1-\gamma)(m + x - \kappa_z z - r_f)\eta_x}{\gamma \sqrt{\bar{v}}} \right] F_x + \left[-\kappa_z z + \frac{(1-\gamma)(m + x - \kappa_z z - r_f)\eta_z}{\gamma \sqrt{\bar{v}}} \right] F_z$$

$$+ \frac{1}{2}\eta_x^2 F_{xx} + \frac{1}{2}\eta_z^2 F_{zz} + \eta_x \eta_z F_{xz} + \frac{1-\gamma}{\gamma} \left[r_f - \frac{\delta}{1-\gamma} + \frac{\left(m + x - \kappa_z z - r_f\right)^2}{2\gamma \bar{v}} \right] F = 0. \tag{A.18}$$

The solution to PDE (A.18) is given by

$$F(\tau, \theta) = \exp\left(\frac{1}{2}\theta^\top A(\tau)\theta + \theta^\top B(\tau) + C(\tau)\right), \tag{A.19}$$

where $\theta = [x, z]^\top$, $A(\tau)$ is a symmetric 2×2 matrix-valued function, $B(\tau)$ is a 2×1 matrix-valued function and $C(\tau) \in \mathbb{R}$ with A, B, and C solving the system of matrix Riccati differential equations

$$\begin{aligned}
\dot{A}(\tau) &= \frac{1}{2}A(\tau)q_1^\top q_1 A(\tau) + (q_2 + q_3 q_1)A(\tau) + A(\tau)(q_2 + (q_3 q_1)^\top) + q_4 \\
\dot{B}(\tau) &= \frac{1}{2}A(\tau)q_1^\top q_1 B(\tau) + (q_2 + q_3 q_1)B(\tau) + A(\tau)q_1^\top q_5 + q_6 \\
\dot{C}(\tau) &= \frac{1}{4}B^\top(\tau)q_1^\top q_1 B(\tau) + \frac{1}{2}q_7^\top A(\tau)q_7 + B(\tau)^\top q_1^\top q_5 + q_8
\end{aligned}$$

with coefficients

$$\begin{aligned}
q_1 &= \begin{pmatrix} \eta_x & \eta_z \\ \eta_x & \eta_z \end{pmatrix}, \quad q_2 = \begin{pmatrix} -\kappa_x & 0 \\ 0 & -\kappa_z \end{pmatrix}, \\
q_3 &= \frac{1-\gamma}{\gamma}\frac{1}{\sqrt{\bar{v}}}\begin{pmatrix} 1 & 0 \\ 0 & -\kappa_z \end{pmatrix}, \quad q_4 = \frac{1-\gamma}{\gamma}\frac{1}{\gamma\bar{v}}\begin{pmatrix} 1 & -\kappa_z \\ -\kappa_z & \kappa_z^2 \end{pmatrix}, \\
q_5 &= \frac{1-\gamma}{\gamma}\frac{(m - r_f)}{\sqrt{\bar{v}}}\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad q_6 = \frac{1-\gamma}{\gamma}\frac{(m - r_f)}{\gamma\bar{v}}\begin{pmatrix} 1 \\ -\kappa_z \end{pmatrix}, \\
q_7 &= \begin{pmatrix} \eta_x \\ \eta_z \end{pmatrix}, \quad q_8 = \frac{1-\gamma}{\gamma}\left[r_f - \frac{\delta}{1-\gamma} + \frac{(m - r_f)^2}{2\gamma\bar{v}}\right]
\end{aligned}$$

The boundary conditions are $A_{ij}(0) = B_i(0) = C(0) = 0$, where A_{ij} and B_i denote the components of A and B. Substituting Equation (A.19) in Equation (A.17) and plugging the result in Equation (A.16) yields the optimal portfolio weight.

□

A.7 Proof of Proposition 7

For the problem with stochastic return volatility v_t, the optimal value function is of the form $J(t, W, x, z, v, \gamma_x, \gamma_z, \gamma_{zx})$, and the HJB equation is as follows:

$$\begin{aligned}
&J_t - \kappa_x x J_x - \kappa_z z J_z + \kappa_v(\bar{v} - v)J_v \\
&+ \left(\sigma_x^2 - 2\kappa_x\gamma_x - \frac{1}{v}\left(\gamma_x - \gamma_{zx}\kappa_z\right)^2\right) J_{\gamma_x} + \left(\sigma_z^2 - 2\kappa_z\gamma_z - \frac{1}{v}\left(\gamma_{zx} - \gamma_z\kappa_z + \sigma_z^2\right)^2\right) J_{\gamma_z} \\
&+ \left(-\left(\kappa_x + \kappa_z\right)\gamma_{zx} - \frac{1}{v}\left(\gamma_x - \gamma_{zx}\kappa_z\right)\left(\gamma_{zx} - \gamma_z\kappa_z + \sigma_z^2\right)\right) J_{\gamma_{zx}} \\
&+ \frac{1}{2v}(\gamma_{zx} - \kappa_z\gamma_z + \sigma_z^2)^2 J_{zz} + \frac{1}{2v}(\gamma_x - \kappa_z\gamma_{zx})^2 J_{xx} + \frac{1}{2}\sigma_v^2 v J_{vv} \\
&+ \frac{1}{v}\left(\gamma_{zx} - \kappa_z\gamma_z + \sigma_z^2\right)\left(\gamma_x - \kappa_z\gamma_{zx}\right) J_{zx} + (\gamma_x - \kappa_z\gamma_{zx})\sigma_v J_{xv} + (\gamma_{zx} - \kappa_z\gamma_z + \sigma_z^2)\sigma_v J_{zv} \\
&+ r_f W J_W + \sup_\pi \Big\{\pi W\left(m + x - \kappa_z z - r_f\right) J_W + \frac{1}{2}\pi^2 W^2 v J_{WW} + \pi W(\gamma_x - \kappa_z\gamma_{zx})J_{Wx} \\
&+ \pi W(\gamma_{zx} - \kappa_z\gamma_z + \sigma_z^2)J_{Wz} + \pi W v\sigma_v J_{Wv}\Big\} = 0. \qquad \text{(A.20)}
\end{aligned}$$

Solving the first-order condition yields the following optimal portfolio weight

$$\begin{aligned}
\pi^\star = &-\frac{J_W}{WJ_{WW}}\frac{(m + x - \kappa_z z - r_f)}{v} - \frac{J_{Wx}}{WJ_{WW}}\frac{\gamma_x - \kappa_z\gamma_{zx}}{v} \\
&- \frac{J_{Wz}}{WJ_{WW}}\frac{\gamma_{zx} - \kappa_z\gamma_z + \sigma_z^2}{v} - \frac{J_{Wv}}{WJ_{WW}}\sigma_v. \qquad \text{(A.21)}
\end{aligned}$$

After plugging Equation (A.21) into the HJB equation (A.20), and guessing that

$$J(t, W, x, z, v_t, \gamma_x, \gamma_z, \gamma_{zx}) = e^{-\delta t}\frac{W^{1-\gamma}}{1-\gamma}F(T - t, x, z, v, \gamma_x, \gamma_z, \gamma_{zx})^\gamma,$$

we obtain a nonlinear PDE for the F function.

We look for an approximate analytic solution to this PDE of the form

$$F(\tau, \theta) = \exp\left(\frac{1}{2}\theta^\top A(\tau)\theta + \theta^\top B(\tau) + C(\tau)\right), \qquad \text{(A.22)}$$

where $\theta = [x, z, v, \gamma_x, \gamma_z, \gamma_{zx}]^\top$, $A(\tau)$ is a symmetric 6×6 matrix-valued function, $B(\tau)$ is a 6×1 matrix-valued function, and $C(\tau) \in \mathbb{R}$. Plugging (A.22) in the PDE and performing a second order Taylor expansion around the long-run means of the state variables allows us to use the variable separation method to obtain a coupled system of ODEs for coefficient functions A, B, and C.

□

B Maximum Likelihood Estimation

Based on the dynamics provided in (5)-(7), (9)-(11), and (29), the monthly log-return r_t, the transitory component $\widehat{z}_t$, the stochastic drift $\widehat{x}_t$, the return variance v_t, and the posterior variance-covariances γ_z, γ_x, and γ_{zx} are discretized as follows:

$$
\begin{aligned}
r_{t+\Delta} &= (m - v_t/2 + \widehat{x}_t - \kappa_z \widehat{z}_t)\,\Delta + \sqrt{v_t \Delta}\epsilon_{t+\Delta}, \\
\widehat{z}_{t+\Delta} &= e^{-\kappa_z \Delta}\widehat{z}_t + \left(\gamma_{zx} - \gamma_z \kappa_z + \sigma_z^2\right)\sqrt{\frac{1 - e^{-2\kappa_z \Delta}}{2\kappa_z v_t}}\epsilon_{t+\Delta}, \\
\widehat{x}_{t+\Delta} &= e^{-\kappa_x \Delta}\widehat{x}_t + (\gamma_x - \gamma_{zx}\kappa_z)\sqrt{\frac{1 - e^{-2\kappa_x \Delta}}{2\kappa_x v_t}}\epsilon_{t+\Delta}, \\
v_{t+\Delta} &= e^{-\kappa_v \Delta} v_t + \overline{v}\left(1 - e^{-\kappa_v \Delta}\right) + \operatorname{sign}(\sigma_v)\sqrt{l_t}\epsilon_{t+\Delta}, \\
\gamma_{z,t+\Delta} &= \gamma_{z,t} + \left[-\frac{\left(\gamma_{zx,t} - \gamma_{z,t}\kappa_z + \sigma_z^2\right)^2}{v_t} - 2\gamma_{z,t}\kappa_z + \sigma_z^2\right]\Delta, \\
\gamma_{x,t+\Delta} &= \gamma_{x,t} + \left[-\frac{\left(\gamma_{x,t} - \gamma_{zx,t}\kappa_z\right)^2}{v_t} - 2\gamma_{x,t}\kappa_x + \sigma_x^2\right]\Delta, \\
\gamma_{zx,t+\Delta} &= \gamma_{zx,t} + \left[-\frac{\left(\gamma_{x,t} - \gamma_{zx,t}\kappa_z\right)\left(\gamma_{zx,t} - \gamma_{z,t}\kappa_z + \sigma_z^2\right)}{v_t} - \left(\kappa_x + \kappa_z\right)\gamma_{zx,t}\right]\Delta,
\end{aligned}
$$

where $\Delta = 1/12 = 1$ month, $\epsilon_t \sim \mathrm{N}(0, 1)$ follows a normal distribution with mean 0 and variance 1, and $l_t \equiv \frac{\sigma_v^2}{\kappa_v} v_t (e^{-\kappa_v \Delta} - e^{-2\kappa_v \Delta}) + \frac{\sigma_v^2}{2\kappa_v}\overline{v}(1 - e^{-\kappa_v \Delta})^2$. Conditional on knowing the parameters and the initial values $\widehat{z}_0$, $\widehat{x}_0$, v_0, $\gamma_{z,0}$, $\gamma_{x,0}$, and $\gamma_{zx,0}$, observing monthly returns r_t allows us to sequentially back out ϵ_t and therefore $\widehat{z}_t$, $\widehat{x}_t$, v_t, $\gamma_{z,t}$, $\gamma_{x,t}$, and $\gamma_{zx,t}$.

The 8-dimensional vector of parameters is $\Theta \equiv (m, \sigma_z, \kappa_z, \sigma_x, \kappa_x, \overline{v}, \sigma_v, \kappa_v)$ and the log-likelihood function $\mathcal{L}(\Theta; r_\Delta, r_{2\Delta}, \ldots, r_{N\Delta})$ satisfies

$$\mathcal{L}(\Theta; r_\Delta, r_{2\Delta}, \ldots, r_{N\Delta}) = \sum_{i=1}^{N} \log\left(\frac{1}{\sqrt{2\pi v_{(i-1)\Delta}\Delta}}\right) - \frac{1}{2}\frac{\left(r_{i\Delta} - \left(m - \frac{1}{2}v_{(i-1)\Delta} + \widehat{x}_{(i-1)\Delta} - \kappa_z \widehat{z}_{(i-1)\Delta}\right)\Delta\right)^2}{v_{(i-1)\Delta}\Delta}, \quad \text{(B.1)}$$

where N is the number of observations. The vector of parameters Θ is chosen to maximize the log-likelihood function provided in Equation (B.1).

C Predictive Regressions

Table C1 reports the estimates obtained by regressing cumulative dividend growth rates on either the labour share or the financial leverage from the nonfinancial corporate sector in the post-war sample (1946-2014). Actual dividend growth rates are predicted by both the labour share and the financial leverage with positive and significant slopes increasing with the forecasting horizon (Belo et al., 2015; Marfè, 2016). Consistent with actual data, model-implied dividend growth rates are positively and significantly predicted by the financial leverage, with slopes increasing with the forecasting horizon. The explanatory power, however, is larger than in actual data. The slope of the relation between the model-implied dividend growth rate and the labour share is positive and increases with the forecasting horizon, in line with actual data. This positive relation, however, is insignificant when using model-implied data, whereas it is strongly significant when using actual data. Although *(i)* the labour share and the financial leverage are variables that are external to our model and *(ii)* our estimation methodology is not set to capture any of these dividend growth predictability patterns, model-implied dividend growth rates preserve to some extent the predictability observed in actual data.

Table C1

Dividend Growth Predictability by External Variables.

This table reports the estimates obtained by regressing the cumulative log dividend growth rate, g, over several horizons on either the labour share, LS, or the financial leverage, FL:

$$\sum_{i=1}^{n} g_{t+i} = a + b\,x_t + \epsilon_t,$$

where $n = \{1, 2, 3, 5, 7, 10\}$ years and $x = \{LS, FL\}$. The actual log dividend growth rate is that of the S&P 500, while the model-implied counterpart satisfies $\tilde{y}_t = \log(m - m_s + (1 - a_x)\widehat{x}_t + (a_z - 1)\kappa_z \widehat{z}_t)$ and $D_t = P_t \exp(\tilde{y}_t)$ (see Section 3.3). The labour share (labour compensations over value added) and the financial leverage (debt over equity) are from the nonfinancial corporate sector (Flows of Funds). Newey-West t-statistics are reported in parentheses and 10%, 5%, and 1% significance levels are denoted with *, **, and ***, respectively. Monthly data are aggregated at yearly frequency over the 1946-2014 sample. P, PT, CV, and SV, stand for permanent component with stochastic drift, permanent component with stochastic drift and transitory component, constant volatility, and stochastic volatility, respectively.

Predictability by labor share: $\sum_{i=1}^{n} g_{t+i}$	1	2	3	5	7	10
Data						
LS	-0.428	-0.264	1.224	4.598***	4.885***	5.004*
t-stat	(-1.03)	(-0.31)	(0.93)	(4.50)	(3.05)	(1.81)
adj-R^2	0.00	-0.01	0.01	0.20	0.20	0.12
P-CV Model ($k_z = 0$)						
LS	-0.402	0.142	0.836	3.597	4.869	4.581
t-stat	(-0.41)	(0.08)	(0.35)	(1.17)	(1.10)	(0.55)
adj-R^2	-0.01	-0.02	-0.01	0.01	0.01	-0.01
P-SV Model ($k_z = 0$)						
LS	-0.374	0.112	0.791	3.356	4.722	4.230
t-stat	(-0.41)	(0.06)	(0.34)	(1.07)	(1.09)	(0.52)
adj-R^2	-0.01	-0.02	-0.01	0.00	0.01	-0.01
PT-CV Model						
LS	0.448	0.939	1.196	0.821	-0.526	-2.634
t-stat	(1.28)	(1.29)	(0.99)	(0.34)	(-0.19)	(-0.87)
adj-R^2	0.02	0.03	0.02	-0.01	-0.02	0.00
PT-SV Model						
LS	-0.004	0.238	0.273	0.425	0.623	3.007
t-stat	(-0.01)	(0.48)	(0.37)	(0.34)	(0.36)	(0.74)
adj-R^2	-0.02	-0.01	-0.01	-0.01	-0.01	0.00

Predictability by financial leverage: $\sum_{i=1}^{n} g_{t+i}$	1	2	3	5	7	10
Data						
FL	0.202	0.607***	0.912***	0.985***	0.663**	0.693**
t-stat	(1.60)	(3.58)	(4.33)	(4.35)	(2.28)	(2.14)
adj-R^2	0.02	0.10	0.16	0.17	0.07	0.07
P-CV Model ($k_z = 0$)						
FL	1.208***	2.115***	2.511***	3.886***	4.877***	6.953***
t-stat	(4.60)	(4.47)	(3.94)	(5.71)	(7.01)	(8.06)
adj-R^2	0.19	0.31	0.34	0.46	0.53	0.71
P-SV Model ($k_z = 0$)						
FL	1.097***	2.013***	2.429***	3.732***	4.705***	6.683***
t-stat	(4.55)	(4.52)	(4.02)	(5.61)	(6.94)	(8.11)
adj-R^2	0.18	0.31	0.34	0.46	0.53	0.71
PT-CV Model						
FL	-0.076	-0.152	-0.244	-0.135	0.092	0.800
t-stat	(-0.52)	(-0.58)	(-0.72)	(-0.30)	(0.18)	(1.30)
adj-R^2	-0.01	-0.00	0.01	-0.01	-0.02	0.04
PT-SV Model						
FL	0.525***	0.873***	0.952***	1.487***	2.022***	2.927***
t-stat	(4.80)	(4.47)	(4.38)	(5.37)	(4.72)	(5.97)
adj-R^2	0.17	0.28	0.30	0.40	0.47	0.65

D Actual and Model-Implied Dividend Yield

Table D1 reports the correlation between the actual S&P 500 log dividend yield and the model-implied log dividend yield, between the actual log dividend yield and the state variables $\widehat{x}$ and $\widehat{z}$, and between the model-implied log dividend yield and the state variables $\widehat{x}$ and $\widehat{z}$. Correlations are computed over the full sample (1872-2015) as well as over the 1872-1945 and 1946-2015 sub-samples.

Table D1

Actual and Model-Implied Dividend Yields.

This table reports the correlations and p-values between the actual S&P 500 log dividend yield and the model-implied log dividend yield, between the actual log dividend yield and the state variables $\widehat{x}$ and $\widehat{z}$, and between the model-implied log dividend yield and the state variables $\widehat{x}$ and $\widehat{z}$. The model-implied log dividend yield satisfies $\tilde{y}_t = \log\left(m - m_s + (1 - a_x)\widehat{x}_t + (a_z - 1)\kappa_z \widehat{z}_t\right)$ (see Section 3.3). Correlations are computed over the full sample (1872-2015) as well as over the 1872-1945 and 1946-2015 sub-samples. P, PT, CV, and SV, stand for permanent component with stochastic drift, permanent component with stochastic drift and transitory component, constant volatility, and stochastic volatility, respectively.

	1872–2015				1872–1945				1946–2015			
	P-CV	P-SV	PT-CV	PT-SV	P-CV	P-SV	PT-CV	PT-SV	P-CV	P-SV	PT-CV	PT-SV
corr($\tilde{y}^{\text{actual}}, \tilde{y}^{\text{model}}$)	0.09	0.13	0.62	0.83	0.23	0.32	0.43	0.29	0.01	0.02	0.42	0.95
p-value	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.80)	(0.65)	(0.00)	(0.00)
corr($\tilde{y}^{\text{actual}}, \widehat{x}$)	-0.09	-0.13	-0.12	-0.23	-0.23	-0.32	-0.24	-0.33	-0.01	-0.02	-0.02	-0.17
p-value	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.80)	(0.63)	(0.49)	(0.00)
corr($\tilde{y}^{\text{actual}}, \widehat{z}$)	0.00	0.00	-0.61	-0.81	0.00	0.00	-0.45	-0.31	0.00	0.00	-0.38	-0.96
p-value	(1.00)	(1.00)	(0.00)	(0.00)	(1.00)	(1.00)	(0.00)	(0.00)	(1.00)	(1.00)	(0.00)	(0.00)
corr($\tilde{y}^{\text{model}}, \widehat{x}$)	-1.00	-1.00	-0.15	-0.20	-1.00	-1.00	-0.15	-0.18	-1.00	-1.00	-0.15	-0.19
p-value	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
corr($\tilde{y}^{\text{model}}, \widehat{z}$)	0.00	0.00	-0.59	-0.98	0.00	0.00	-0.98	-1.00	0.00	0.00	-0.99	-0.99
p-value	(1.00)	(1.00)	(0.00)	(0.00)	(1.00)	(1.00)	(0.00)	(0.00)	(1.00)	(1.00)	(0.00)	(0.00)

E In-sample Portfolio Performance

In this section, we compare the in-sample performance of different investment strategies. Specifically, we use the parameters estimated over the entire sample (see Table 2) to implement strategies that invest in the S&P 500 and in a riskless asset with constant return. In-sample, strategies that consider the transitory component of stock returns significantly outperform those that ignore it.

Consider an investor with an investment horizon equal to T and an initial wealth equal to \$1. The investor starts implementing her strategy at time $t = 0$ and rebalances every month. Upon reaching the horizon $t = T$, the investor reiterates the process until reaching the terminal date of the sample. Since data are at the monthly frequency, monthly portfolio excess returns are provided over the entire sample.

Table E1 shows that considering a stochastic drift (P-CV) increases the mean, volatility, Sharpe ratio, skewness, and kurtosis of portfolio returns compared to the model with constant mean and constant volatility (CM-CV). Because the increase in the mean is large, terminal wealth is several orders of magnitude larger when expected stock returns are assumed to be stochastic.

In contrast, considering stochastic return variance (P-SV) decreases the mean and Sharpe ratio of portfolio returns but increases their volatility, skewness, and kurtosis (see Table E1). Although both volatility and kurtosis increase, it is important to note that it is in fact good volatility and good kurtosis that increase, while bad volatility and bad kurtosis remain at levels observed under constant stock return variance (see Table E2). That is, when considering stochastic stock return variance, the investor enjoys the increase in good return volatility, larger return spikes because of higher skewness, and a larger number of return spikes because of higher good kurtosis.

When the investor accounts for the presence of the transitory component (PT-CV, PT-SV), the mean, volatility, skewness, and kurtosis of her portfolio returns increase (see Table E1). Since increases in mean and volatility are of similar magnitudes, the Sharpe ratio remains the same. Importantly, the increase in volatility does not hurt the investor because the increase in good volatility is larger than that in bad volatility (see Table E2). Moreover, the increase in kurtosis is purely beneficial to the investor because good kurtosis increases and bad kurtosis decreases. In other words, the investor enjoys higher returns, larger return spikes, a larger number of return spikes, a lower number of extreme negative returns, and is not hurt by the increase in volatility when accounting for the transitory component of stock returns.

These results show that accounting for the transitory component of stock returns has no significant impact on the left tail of the distribution of portfolio returns and mainly impacts its right tail through higher good volatility, skewness, and good kurtosis. Properly modeling the behaviour of short-term equity returns implies surges in portfolio returns, which are captured by measures of performance such as good volatility, skewness, and good kurtosis. This shows that modeling the transitory component of stock returns is beneficial to the investor, and even more so when return variance is considered to be stochastic. Furthermore, it is worth noting that these benefits are not captured by the most common measure of portfolio performance, namely the Sharpe ratio.

Tables E1 and E2 show that the portfolio performance benefits of considering the transitory component are robust to changes in both investment horizon and risk aversion. An increase in the investment horizon has a weak impact on the mean, volatility, and Sharpe ratio, but it increases skewness and good kurtosis across all strategies. An increase in risk aversion has no impact on the Sharpe ratio because it decreases the mean and volatility by the same percentage across all strategies. Furthermore, an increase in risk aversion impacts skewness and kurtosis only when return volatility is stochastic. In this case, an increase in risk aversion increases skewness and good kurtosis, while it decreases bad kurtosis.

Table E3 confirms our previous statement that modeling the transitory component of equity returns is particularly beneficial to the investor when return volatility is stochastic. Indeed, the certainty equivalent return obtained by considering the transitory component is about 1% and 10% larger than that obtained by ignoring it when return volatility is constant and stochastic, respectively. Since the fraction of wealth invested in the stock increases with the horizon and decreases with risk aversion for all investment strategies (see Table 6), all strategies converge to riskless strategies when the investment horizon decreases and when risk aversion increases. For this reason, the ratio of certainty equivalent returns increases with the horizon and decreases with risk aversion.

Table E1

In-Sample Portfolio Moments vs. Investment Horizon and Risk Aversion.

Mean, volatility, and Sharpe ratio are in annualized terms. In columns 1 to 3, the investment horizon is set to 1 year. In columns 4 to 6, risk aversion is $\gamma = 5$ and the investment horizon is set to 1 month, 1 year, and 5 years, respectively. P, PT, CM, CV, and SV stand for permanent component with stochastic drift, permanent component with stochastic drift and transitory component, constant mean, constant volatility, and stochastic volatility, respectively. Statistics are computed using monthly S&P 500 data from 02/1871 to 02/2016.

	Risk Aversion			Horizon		
	5	7	10	1m	1y	5y
			Mean			
CM-CV	2.11%	1.51%	1.05%	2.11%	2.11%	2.11%
P-CV	21.93%	15.57%	10.85%	21.95%	21.93%	21.93%
PT-CV	22.31%	15.83%	11.03%	22.29%	22.31%	22.40%
P-SV	17.87%	12.67%	8.82%	18.49%	17.87%	17.70%
PT-SV	19.34%	13.71%	9.55%	19.94%	19.34%	19.35%
			Volatility			
CM-CV	6.52%	4.65%	3.26%	6.52%	6.52%	6.52%
P-CV	34.81%	24.72%	17.23%	34.83%	34.81%	34.81%
PT-CV	36.66%	26.03%	18.14%	36.62%	36.66%	37.01%
P-SV	42.59%	30.35%	21.20%	43.10%	42.59%	42.65%
PT-SV	46.41%	33.08%	23.12%	46.81%	46.41%	46.61%
			Sharpe Ratio			
CM-CV	0.32	0.32	0.32	0.32	0.32	0.32
P-CV	0.63	0.63	0.63	0.63	0.63	0.63
PT-CV	0.61	0.61	0.61	0.61	0.61	0.61
P-SV	0.42	0.42	0.42	0.43	0.42	0.42
PT-SV	0.42	0.41	0.41	0.43	0.42	0.42
			Skewness			
CM-CV	0.55	0.55	0.55	0.55	0.55	0.55
P-CV	9.10	9.10	9.10	9.10	9.10	9.10
PT-CV	9.73	9.73	9.72	9.72	9.73	9.87
P-SV	25.68	25.86	25.99	24.85	25.68	25.84
PT-SV	26.84	27.03	27.16	26.02	26.84	26.71
			Kurtosis			
CM-CV	20.71	20.71	20.71	20.71	20.71	20.71
P-CV	124.24	124.20	124.16	124.33	124.24	124.25
PT-CV	141.47	141.32	141.21	141.20	141.47	146.44
P-SV	881.43	890.23	896.66	841.50	881.43	889.25
PT-SV	942.14	951.18	957.76	901.42	942.14	935.33
			Terminal Wealth			
CM-CV	2.2×10^{4}	1.0×10^{4}	5.9×10^{3}	2.2×10^{4}	2.2×10^{4}	2.2×10^{4}
P-CV	1.7×10^{14}	2.8×10^{11}	1.5×10^{9}	1.7×10^{14}	1.7×10^{14}	1.7×10^{14}
PT-CV	1.8×10^{14}	3.2×10^{11}	1.7×10^{9}	1.8×10^{14}	1.8×10^{14}	1.9×10^{14}
P-SV	4.8×10^{11}	3.9×10^{9}	7.2×10^{7}	8.7×10^{11}	4.8×10^{11}	3.9×10^{11}
PT-SV	2.0×10^{12}	1.1×10^{10}	1.6×10^{8}	3.5×10^{12}	2.0×10^{12}	1.9×10^{12}

Table E2

In-Sample Portfolio Return Volatility and Kurtosis Decomposition vs. Investment Horizon and Risk Aversion.

Volatility is in annualized terms. In columns 1 to 3, the investment horizon is set to 1 year. In columns 4 to 6, risk aversion is $\gamma = 5$ and the investment horizon is set to 1 month, 1 year, and 5 years, respectively. P, PT, CM, CV, and SV stand for permanent component with stochastic drift, permanent component with stochastic drift and transitory component, constant mean, constant volatility, and stochastic volatility, respectively. Statistics are computed using monthly S&P 500 data from 02/1871 to 02/2016.

	Risk Aversion			Horizon		
	5	7	10	1m	1y	5y
			Good Volatility			
CM-CV	4.50%	3.22%	2.25%	4.50%	4.50%	4.50%
P-CV	32.37%	22.99%	16.02%	32.39%	32.37%	32.37%
PT-CV	34.23%	24.30%	16.93%	34.19%	34.23%	34.54%
P-SV	40.64%	28.97%	20.25%	40.98%	40.64%	40.72%
PT-SV	44.40%	31.67%	22.15%	44.64%	44.40%	44.57%
			Bad Volatility			
CM-CV	4.71%	3.37%	2.36%	4.71%	4.71%	4.71%
P-CV	12.80%	9.09%	6.33%	12.82%	12.80%	12.80%
PT-CV	13.13%	9.32%	6.50%	13.12%	13.13%	13.30%
P-SV	12.76%	9.03%	6.28%	13.36%	12.76%	12.69%
PT-SV	13.50%	9.55%	6.64%	14.09%	13.50%	13.64%
			Good Kurtosis			
CM-CV	16.09	16.09	16.09	16.09	16.09	16.09
P-CV	123.75	123.70	123.67	123.84	123.75	123.76
PT-CV	141.00	140.85	140.74	140.73	141.00	145.96
P-SV	880.82	889.64	896.09	840.77	880.82	888.66
PT-SV	941.67	950.73	957.32	900.89	941.67	934.86
			Bad Kurtosis			
CM-CV	4.62	4.62	4.62	4.62	4.62	4.62
P-CV	0.50	0.49	0.49	0.50	0.50	0.49
PT-CV	0.47	0.47	0.47	0.47	0.47	0.49
P-SV	0.61	0.59	0.57	0.73	0.61	0.60
PT-SV	0.47	0.45	0.44	0.54	0.47	0.47

Table E3

In-Sample Ratio of Certainty Equivalent Returns.

CER stands for certainty equivalent return. In columns 1 to 3, the investment horizon is set to 1 year. In columns 4 to 6, risk aversion is $\gamma = 5$ and the investment horizon is set to 1 month, 1 year, and 5 years, respectively. P, PT, CV, and SV stand for permanent component with stochastic drift, permanent component with stochastic drift and transitory component, constant volatility, and stochastic volatility, respectively. Statistics are computed using monthly S&P 500 data from 02/1871 to 02/2016.

	Risk Aversion			Horizon		
	5	7	10	1m	1y	5y
			Ratio of Mean CER			
PT-CV/P-CV	1.0127	1.0110	1.0092	1.0126	1.0127	1.0195
PT-SV/P-SV	1.1037	1.0880	1.0717	1.0950	1.1037	1.1170
			Ratio of Median CER			
PT-CV/P-CV	1.0096	1.0084	1.0072	1.0030	1.0096	1.0170
PT-SV/P-SV	1.0927	1.0771	1.0618	1.0770	1.0927	1.0939