

Empirical Analysis of Corporate Tax Reforms:
What is the Null and Where Did It Come From?

ONLINE APPENDIX *

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Abstract

This is the Online Appendix to our paper “Empirical Analysis of Corporate Tax Reforms: What is the Null and Where Did It Come From?” In this Online Appendix, we show how to derive contingent claim prices and exploit the recursive relationships derived in the body of the paper to pin down the components of firm value.

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1 Appendix

1.1 Canonical ODE

As a preparation, we solve the following ODE:

$$\rho v = \mu x v' + \frac{1}{2} \sigma^2 x^2 v'' + z x + Z \quad (1)$$

We know the solution is of the form:

$$v = x^{B_1} K_1 + x^{B_2} K_2 + \frac{z x}{\rho - \mu} + \frac{Z}{\rho} \quad (2)$$

where the exponents are the negative and positive roots of

$$\frac{1}{2} \sigma^2 B^2 + \left(\mu - \frac{1}{2} \sigma^2 \right) B - \rho = 0. \quad (3)$$

It will be useful to keep in mind this solution as we seek to price the various contingent claims.

1.2 Canonical ODE System

Moreover, the following system of ODEs will also appear frequently in our valuations:

$$\begin{aligned} r v(x, l) &= \mu x v_x(x, l) + \frac{1}{2} \sigma^2 x^2 v_{xx}(x, l) + \lambda_l [v(x, h) - v(x, l)] \\ r v(x, h) &= \mu x v_x(x, h) + \frac{1}{2} \sigma^2 x^2 v_{xx}(x, h) + \lambda_h [v(x, l) - v(x, h)]. \end{aligned} \quad (4)$$

We conjecture solutions of the form:

$$\begin{aligned} v(x, l) &= L x^\beta \\ v(x, h) &= H x^\beta \end{aligned} \quad (5)$$

Substituting these derivatives back into the ODEs one obtains:

$$\begin{aligned} (r + \lambda_l) L x^\beta &= \mu x \beta L x^{\beta-1} + \frac{1}{2} \sigma^2 x^2 (\beta^2 - \beta) L x^{\beta-2} + \lambda_l H x^\beta \\ (r + \lambda_h) H x^\beta &= \mu x \beta H x^{\beta-1} + \frac{1}{2} \sigma^2 x^2 (\beta^2 - \beta) H x^{\beta-2} + \lambda_h L x^\beta, \end{aligned} \quad (6)$$

which is equivalent to

$$\begin{aligned} \left[(r + \lambda_l) - \left(\mu - \frac{1}{2} \sigma^2 \right) \beta - \frac{1}{2} \sigma^2 \beta^2 \right] L &= \lambda_l H \\ \left[(r + \lambda_h) - \left(\mu - \frac{1}{2} \sigma^2 \right) \beta - \frac{1}{2} \sigma^2 \beta^2 \right] H &= \lambda_h L. \end{aligned} \quad (7)$$

Thus we demand:

$$\left[(r + \lambda_l) - \left(\mu - \frac{1}{2}\sigma^2 \right) \beta - \frac{1}{2}\sigma^2\beta^2 \right] \left[(r + \lambda_h) - \left(\mu - \frac{1}{2}\sigma^2 \right) \beta - \frac{1}{2}\sigma^2\beta^2 \right] = \lambda_l\lambda_h. \quad (8)$$

Letting

$$g_l(\beta) \equiv (r + \lambda_l) - \left(\mu - \frac{1}{2}\sigma^2 \right) \beta - \frac{1}{2}\sigma^2\beta^2 \quad (9)$$

$$g_h(\beta) \equiv (r + \lambda_h) - \left(\mu - \frac{1}{2}\sigma^2 \right) \beta - \frac{1}{2}\sigma^2\beta^2,$$

we then demand that any candidate exponent β must satisfy the following characteristic equation:

$$g_l(\beta)g_h(\beta) = \lambda_l\lambda_h. \quad (10)$$

Thus, the general form of the solution is:

$$v(x, l) = L_1x^{\beta_1} + L_2x^{\beta_2} + L_3x^{\beta_3} + L_4x^{\beta_4} \quad (11)$$

$$v(x, h) = H_1x^{\beta_1} + H_2x^{\beta_2} + H_3x^{\beta_3} + H_4x^{\beta_4},$$

where the β_n are the roots of the characteristic equation, with:

$$\beta_1 < \beta_2 < 0 < \beta_3 < \beta_4. \quad (12)$$

We know the respective constants are linked via:

$$H_n = \left[\frac{g_l(\beta_n)}{\lambda_l} \right] L_n. \quad (13)$$

So the Canonical ODE System has solutions of the form:

$$v(x, l) = L_1x^{\beta_1} + L_2x^{\beta_2} + L_3x^{\beta_3} + L_4x^{\beta_4} \quad (14)$$

$$\begin{aligned} v(x, h) &= H_1x^{\beta_1} + H_2x^{\beta_2} + H_3x^{\beta_3} + H_4x^{\beta_4} \\ &= L_1 \left[\frac{g_l(\beta_1)}{\lambda_l} \right] x^{\beta_1} + L_2 \left[\frac{g_l(\beta_2)}{\lambda_l} \right] x^{\beta_2} + L_3 \left[\frac{g_l(\beta_3)}{\lambda_l} \right] x^{\beta_3} + L_4 \left[\frac{g_l(\beta_4)}{\lambda_l} \right] x^{\beta_4}. \end{aligned}$$

Or for brevity we can write:

$$v(x, l) = \sum_{n=1}^4 x^{\beta_n} L_n \quad (15)$$

$$v(x, h) = \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n. \quad (16)$$

With these solutions in mind, we can now price each of the contingent claims.

1.3 Contingent Down Claim

This claim pays one if and when default occurs in state i unless it has been knocked out by call or default in the other tax state. Let $d^i(x, j, \Omega)$ denote the price of this claim when EBIT is x and the current tax state is j . By the Feynman-Kac formula, this function must satisfy the following system of ODEs:

$$\begin{aligned}
rd^i(x, l, \Omega) &= \mu x d_x^i(x, l, \Omega) + \frac{1}{2} \sigma^2 x^2 d_{xx}^i(x, l, \Omega) + \lambda_l [d^i(x, h, \Omega) - d^i(x, l, \Omega)] & (17) \\
rd^i(x, h, \Omega) &= \mu x d_x^i(x, h, \Omega) + \frac{1}{2} \sigma^2 x^2 d_{xx}^i(x, h, \Omega) + \lambda_h [d^i(x, l, \Omega) - d^i(x, h, \Omega)] \\
d^i(x, i, \Omega) &= 1 & \text{if } x \in (0, c] \\
d^i(x, j, \Omega) &= 0 & \text{if } x \in (0, c] \\
d^i(x, l, \Omega) &= 0 & \text{if } x \in [\gamma^l, \infty) \\
d^i(x, h, \Omega) &= 0 & \text{if } x \in [\gamma^h, \infty).
\end{aligned}$$

To solve this system, we need to consider three cases separately depending on the ranking between γ^l and γ^h .

- Case 1: $\gamma^l > \gamma^h$.

First, we assume that the refinancing threshold is higher in the low tax state. On the region of $[c, \gamma^h]$, equation (17) reduces to a Canonical ODE System with a solution

$$\begin{aligned}
d^i(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n & (18) \\
d^i(x, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n.
\end{aligned}$$

On the other hand, when $x \in [\gamma^h, \gamma^l]$, equation (17) reduces to a Canonical ODE:

$$\begin{aligned}
d^i(x, h) &= 0 & (19) \\
(r + \lambda^l) \tilde{d}^i(x, l) &= \mu x \tilde{d}_x^i(x, l) + \frac{1}{2} \sigma^2 x^2 \tilde{d}_{xx}^i(x, l) \\
&\Rightarrow \tilde{d}^i(x, l) = x^{B_1} K_1 + x^{B_2} K_2.
\end{aligned}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned}
d^i(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n = \Phi(i = l) \\
d^i(c, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n = \Phi(i = h) \\
d^i(\gamma^h, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] (\gamma^h)^{\beta_n} L_n = 0 \\
\tilde{d}^i(\gamma^l, l) &= (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 = 0 \\
d^i(\gamma^h, l) &= \tilde{d}^i(\gamma^h, l) \Rightarrow \sum_{n=1}^4 (\gamma^h)^{\beta_n} L_n = (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 \\
d_x^i(\gamma^h, l) &= \tilde{d}_x^i(\gamma^h, l) \Rightarrow \sum_{n=1}^4 \beta_n (\gamma^h)^{\beta_n - 1} L_n = B_1 (\gamma^h)^{B_1 - 1} K_1 + B_2 (\gamma^h)^{B_2 - 1} K_2.
\end{aligned}$$

Here $\Phi(i = l)$ and $\Phi(i = h)$ are indicator variables. By solving these linear equations, we can pin down the constants and then compute a contingent down claim price.

- Case 2: $\gamma^h > \gamma^l$.

Next, we consider the case where the refinancing threshold is higher in the high tax state. On the region of $[c, \gamma^l]$, equation (17) again reduces to the Canonical ODE System with the solution of form

$$\begin{aligned}
d^i(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n \\
d^i(x, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n.
\end{aligned} \tag{20}$$

On the other hand, when $x \in [\gamma^l, \gamma^h]$, equation (17) reduces to the Canonical ODE:

$$\begin{aligned}
d^i(x, l) &= 0 \\
(r + \lambda^h) \tilde{d}^i(x, h) &= \mu x \tilde{d}_x^i(x, h) + \frac{1}{2} \sigma^2 x^2 \tilde{d}_{xx}^i(x, h) \\
&\Rightarrow \tilde{d}^i(x, h) = x^{B_1} K_1 + x^{B_2} K_2.
\end{aligned} \tag{21}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned}
d^i(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n = \Phi(i = l) \\
d^i(c, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n = \Phi(i = h) \\
d^i(\gamma^l, l) &= \sum_{n=1}^4 (\gamma^l)^{\beta_n} L_n = 0 \\
\tilde{d}^i(\gamma^h, h) &= (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 = 0 \\
d^i(\gamma^l, h) &= \tilde{d}^i(\gamma^l, h) \Rightarrow \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] (\gamma^l)^{\beta_n} L_n = (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 \\
d_x^i(\gamma^l, h) &= \tilde{d}_x^i(\gamma^l, h) \Rightarrow \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] \beta_n (\gamma^l)^{\beta_n - 1} L_n = B_1 (\gamma^l)^{B_1 - 1} K_1 + B_2 (\gamma^l)^{B_2 - 1} K_2.
\end{aligned}$$

By solving these linear equations, we can pin down the constants and then compute contingent down claim prices.

- Case 3: $\gamma^h = \gamma^l = \gamma$.

In this case, equation (17) is simply the Canonical ODE System has a solution of the form:

$$\begin{aligned}
d^i(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n \\
d^i(x, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n.
\end{aligned} \tag{22}$$

The boundary conditions give us four linear equations in the four constants, (L_1, L_2, L_3, L_4) :

$$\begin{aligned}
d^i(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n = \Phi(i = l) \\
d^i(c, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n = \Phi(i = h) \\
d^i(\gamma, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] (\gamma)^{\beta_n} L_n = 0 \\
d^i(\gamma, l) &= \sum_{n=1}^4 (\gamma)^{\beta_n} L_n = 0.
\end{aligned}$$

By solving these linear equations, we can pin down the constants and then compute contingent down claim prices.

1.4 Contingent Up Claim

This claim pays $x(T)/\gamma^i$, where T is the time of the call, if and when call occurs under tax regime i unless it has been knocked out by default or call in the other tax state j . Let $u^i(x, j, \Omega)$ denote the price of this claim when EBIT is x and the current tax state is j . By the Feynman-Kac formula, this function must satisfy the following system of ODEs:

$$\begin{aligned}
ru^i(x, l, \Omega) &= \mu xu_x^i(x, l, \Omega) + \frac{1}{2}\sigma^2 x^2 u_{xx}^i(x, l, \Omega) + \lambda_l [u^i(x, h, \Omega) - u^i(x, l, \Omega)] & (23) \\
ru^i(x, h, \Omega) &= \mu xu_x^i(x, h, \Omega) + \frac{1}{2}\sigma^2 x^2 u_{xx}^i(x, h, \Omega) + \lambda_h [u^i(x, l, \Omega) - u^i(x, h, \Omega)] \\
u^i(x, i, \Omega) &= 0 \quad \text{if } x \in (0, c^i] \\
u^i(x, j, \Omega) &= 0 \quad \text{if } x \in (0, c^j] \\
u^i(x, i, \Omega) &= \frac{x}{\gamma^i} \quad \text{if } x \in [\gamma^i, \infty) \\
u^i(x, j, \Omega) &= 0 \quad \text{if } x \in [\gamma^j, \infty).
\end{aligned}$$

Note that this formulation ensures that the call (and thus restructuring) might occur due to a jump in a tax regime. Similarly, we consider two cases separately with different payoff states.

1.4.1 Contingent Up Claim: 1-State Payoff

- Case 1: $\gamma^l > \gamma^h$.

On the region of $[c, \gamma^h]$, equation (23) reduces to a Canonical ODE System with a solution

$$\begin{aligned}
u^l(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n & (24) \\
u^l(x, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n.
\end{aligned}$$

When $x \in [\gamma^h, \gamma^l]$, the claim is worth 0 in the high tax rate state and so the Canonical ODE System Reduces to a Canonical ODE

$$\begin{aligned}
u^l(x, h) &= 0 \\
(r + \lambda_l)\tilde{u}^l(x, l) &= \mu x \tilde{u}_x^l(x, l) + \frac{1}{2}\sigma^2 x^2 \tilde{u}_{xx}^l(x, l) \\
\Rightarrow \tilde{u}^l(x, l) &= x^{B_1} K_1 + x^{B_2} K_2.
\end{aligned}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned}
u^l(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n = 0 \\
u^l(c, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n = 0 \\
u^l(\gamma^h, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] (\gamma^h)^{\beta_n} L_n = 0 \\
\tilde{u}^l(\gamma^l, l) &= (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 = 1 \\
u^l(\gamma^h, l) &= \tilde{u}^l(\gamma^h, l) \Rightarrow \sum_{n=1}^4 (\gamma^h)^{\beta_n} L_n = (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 \\
u_x^l(\gamma^h, l) &= \tilde{u}_x^l(\gamma^h, l) \Rightarrow \sum_{n=1}^4 \beta_n (\gamma^h)^{\beta_n - 1} L_n = B_1 (\gamma^h)^{B_1 - 1} K_1 + B_2 (\gamma^h)^{B_2 - 1} K_2.
\end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the contingent up claim price.

- Case 2: $\gamma^h > \gamma^l$.

On the region of $[c, \gamma^h]$, equation (23) reduces to a Canonical ODE System with a solution of the form

$$\begin{aligned}
u^l(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n \\
u^l(x, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n.
\end{aligned} \tag{25}$$

When $x \in [\gamma^l, \gamma^h]$, the claim is worth x/γ^l in the low tax rate state and so the Canonical ODE System Reduces to a Canonical ODE

$$\begin{aligned}
u^l(x, l) &= \frac{x}{\gamma^l} \\
(r + \lambda_h) \tilde{u}^l(x, h) &= \mu x \tilde{u}_x^l(x, h) + \frac{1}{2} \sigma^2 x^2 \tilde{u}_{xx}^l(x, h) + \frac{\lambda_h x}{\gamma^l} \\
\Rightarrow \tilde{u}^l(x, h) &= x^{B_1} K_1 + x^{B_2} K_2 + \frac{\lambda_h x}{\gamma^l (r + \lambda_h - \mu)}.
\end{aligned}$$

The boundary conditions give us six linear equations in the six constants $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned}
u^l(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n = 0 \\
u^l(c, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n = 0 \\
u^l(\gamma^l, l) &= \sum_{n=1}^4 (\gamma^l)^{\beta_n} L_n = 1 \\
\tilde{u}^l(\gamma^h, h) &= (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 + \frac{\lambda_h \gamma^h}{\gamma^l(r + \lambda_h - \mu)} = 0 \\
u^l(\gamma^l, h) &= \tilde{u}^l(\gamma^l, h) \Rightarrow \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] (\gamma^l)^{\beta_n} L_n = (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 + \frac{\lambda_h}{r + \lambda_h - \mu} \\
u_x^h(\gamma^l, h) &= \tilde{u}_x^h(\gamma^l, h) \Rightarrow \sum_{n=1}^4 \beta_n \left[\frac{g_l(\beta_n)}{\lambda_l} \right] (\gamma^l)^{\beta_n - 1} L_n = B_1 (\gamma^l)^{B_1 - 1} K_1 + B_2 (\gamma^l)^{B_2 - 1} K_2 + \frac{\lambda_h}{\gamma^l(r + \lambda_h - \mu)}.
\end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the contingent up claim price.

- Case 3: $\gamma^h = \gamma^l = \gamma$.

In this case, equation (23) is simply a Canonical ODE System with a solution of the form

$$\begin{aligned}
u^l(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n \\
u^l(x, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n.
\end{aligned} \tag{26}$$

The boundary conditions give us four linear equations in the four constants, (L_1, L_2, L_3, L_4) :

$$\begin{aligned}
u^l(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n = 0 \\
u^l(c, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n = 0 \\
u^l(\gamma, l) &= \sum_{n=1}^4 (\gamma)^{\beta_n} L_n = 1 \\
u^l(\gamma, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n = 0.
\end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the contingent up claim price.

1.4.2 Contingent Up Claim: h-State Payoff

- Case 1: $\gamma^l > \gamma^h$.

On the region of $[c, \gamma^h]$, equation (23) reduces to a Canonical ODE System with a solution of the form

$$\begin{aligned} u^h(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n \\ u^h(x, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n. \end{aligned} \quad (27)$$

When $x \in [\gamma^h, \gamma^l]$, the claim is worth x/γ^h in the high tax rate state and so the Canonical ODE System Reduces to a Canonical ODE

$$\begin{aligned} u^h(x, h) &= \frac{x}{\gamma^h} \\ (r + \lambda_l) \tilde{u}^h(x, l) &= \mu x \tilde{u}_x^h(x, l) + \frac{1}{2} \sigma^2 x^2 \tilde{u}_{xx}^h(x, l) + \frac{\lambda_l x}{\gamma^h} \\ \Rightarrow \tilde{u}^h(x, l) &= x^{B_1} K_1 + x^{B_2} K_2 + \frac{\lambda_l x}{\gamma^h (r + \lambda_l - \mu)}. \end{aligned}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned} u^h(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n = 0 \\ u^h(c, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n = 0 \\ u^h(\gamma^h, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] (\gamma^h)^{\beta_n} L_n = 1 \\ \tilde{u}^h(\gamma^l, l) &= (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 + \frac{\lambda_l \gamma^l}{\gamma^h (r + \lambda_l - \mu)} = 0 \\ u^h(\gamma^h, l) &= \tilde{u}^h(\gamma^h, l) \Rightarrow \sum_{n=1}^4 (\gamma^h)^{\beta_n} L_n = (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 + \frac{\lambda_l}{r + \lambda_l - \mu} \\ u_x^h(\gamma^h, l) &= \tilde{u}_x^h(\gamma^h, l) \Rightarrow \sum_{n=1}^4 \beta_n (\gamma^h)^{\beta_n - 1} L_n = B_1 (\gamma^h)^{B_1 - 1} K_1 + B_2 (\gamma^h)^{B_2 - 1} K_2 + \frac{\lambda_l}{\gamma^h (r + \lambda_l - \mu)}. \end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the contingent up claim price.

- Case 2: $\gamma^h > \gamma^l$.

On the region of $[c, \gamma^l]$, equation (23) reduces to a Canonical ODE System with a solution of the form

$$\begin{aligned} u^h(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n \\ u^h(x, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n. \end{aligned} \tag{28}$$

When $x \in [\gamma^l, \gamma^h]$, the claim is worth 0 in the low tax rate state and so the Canonical ODE System Reduces to a Canonical ODE

$$\begin{aligned} u^h(x, l) &= 0 \\ (r + \lambda_h) \tilde{u}^h(x, h) &= \mu x \tilde{u}_x^h(x, h) + \frac{1}{2} \sigma^2 x^2 \tilde{u}_{xx}^h(x, h) \\ \Rightarrow \tilde{u}^h(x, h) &= x^{B_1} K_1 + x^{B_2} K_2. \end{aligned}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned} u^h(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n = 0 \\ u^h(c, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n = 0 \\ u^h(\gamma^l, l) &= \sum_{n=1}^4 (\gamma^l)^{\beta_n} L_n = 0 \\ \tilde{u}^h(\gamma^h, h) &= (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 = 1 \\ u^h(\gamma^l, h) &= \tilde{u}^h(\gamma^l, h) \Rightarrow \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] (\gamma^l)^{\beta_n} L_n = (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 \\ u_x^h(\gamma^l, h) &= \tilde{u}_x^h(\gamma^l, h) \Rightarrow \sum_{n=1}^4 \beta_n \left[\frac{g_l(\beta_n)}{\lambda_l} \right] (\gamma^l)^{\beta_n - 1} L_n = B_1 (\gamma^l)^{B_1 - 1} K_1 + B_2 (\gamma^l)^{B_2 - 1} K_2. \end{aligned}$$

By solving these linear equations, we can pin down the constants and then compute contingent up claim prices.

- Case 3: $\gamma^h = \gamma^l = \gamma$.

In this case, equation (23) is simply a Canonical ODE System with a solution of the form

$$\begin{aligned} u^h(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n \\ u^h(x, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n. \end{aligned} \quad (29)$$

The boundary conditions give us four linear equations in the four constants, (L_1, L_2, L_3, L_4) :

$$\begin{aligned} u^h(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n = 0 \\ u^h(c, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n = 0 \\ u^h(\gamma, l) &= \sum_{n=1}^4 (\gamma^l)^{\beta_n} L_n = 0 \\ u^h(\gamma, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] \gamma^{\beta_n} L_n = 1. \end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the contingent up claim price.

1.5 Adjusted Contingent Up Claim

This claim pays one if and when call occurs under tax regime i unless it has been knocked out by default or upward restructuring in the other tax state j . Let $m^i(x, j, \Omega)$ denote the price of this claim when EBIT is x and the current tax state is j . By the Feynman-Kac formula, this function must satisfy the following system of ODEs:

$$\begin{aligned} rm^i(x, l, \Omega) &= \mu x m_x^i(x, l, \Omega) + \frac{1}{2} \sigma^2 x^2 m_{xx}^i(x, l, \Omega) + \lambda_l [m^i(x, h, \Omega) - m^i(x, l, \Omega)] \\ rm^i(x, h, \Omega) &= \mu x m_x^i(x, h, \Omega) + \frac{1}{2} \sigma^2 x^2 m_{xx}^i(x, h, \Omega) + \lambda_h [m^i(x, l, \Omega) - m^i(x, h, \Omega)] \\ m^i(x, i, \Omega) &= 0 \quad \text{if } x \in (0, c] \\ m^i(x, j, \Omega) &= 0 \quad \text{if } x \in (0, c] \\ m^i(x, i, \Omega) &= 1 \quad \text{if } x \in [\gamma^i, \infty) \\ m^i(x, j, \Omega) &= 0 \quad \text{if } x \in [\gamma^j, \infty). \end{aligned} \quad (30)$$

In the following, we derive a price expression for each payoff state ($i = l, h$).

1.5.1 Adjusted Contingent Up Claim: l-State Payoff

- Case 1: $\gamma^l > \gamma^h$.

On the region of $[c, \gamma^h]$: equation (30) reduces to a Canonical ODE System with a solution

$$\begin{aligned} m^l(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n \\ m^l(x, h) &= \sum_{n=1}^4 \left[\frac{gl(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n. \end{aligned} \quad (31)$$

When $x \in [\gamma^h, \gamma^l]$, the claim is worth 0 in the high tax rate state and so the Canonical ODE System Reduces to a Canonical ODE

$$\begin{aligned} m^l(x, h) &= 0 \\ (r + \lambda_l)\tilde{m}^l(x, l) &= \mu x \tilde{m}_x^l(x, l) + \frac{1}{2} \sigma^2 x^2 \tilde{m}_{xx}^l(x, l) \\ \Rightarrow \tilde{m}^l(x, l) &= x^{B_1} K_1 + x^{B_2} K_2. \end{aligned}$$

The boundary conditions give us six linear equations in the six constants, ($L_1, L_2, L_3, L_4, K_1, K_2$):

$$\begin{aligned} m^l(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n = 0 \\ m^l(c, h) &= \sum_{n=1}^4 \left[\frac{gl(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n = 0 \\ m^l(\gamma^h, h) &= \sum_{n=1}^4 \left[\frac{gl(\beta_n)}{\lambda_l} \right] (\gamma^h)^{\beta_n} L_n = 0 \\ \tilde{m}^l(\gamma^l, l) &= (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 = 1 \\ m^l(\gamma^h, l) &= \tilde{m}^l(\gamma^h, l) \Rightarrow \sum_{n=1}^4 (\gamma^h)^{\beta_n} L_n = (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 \\ m_x^l(\gamma^h, l) &= \tilde{m}_x^l(\gamma^h, l) \Rightarrow \sum_{n=1}^4 \beta_n (\gamma^h)^{\beta_n - 1} L_n = B_1 (\gamma^h)^{B_1 - 1} K_1 + B_2 (\gamma^h)^{B_2 - 1} K_2. \end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the adjusted contingent up claim price.

- Case 2: $\gamma^h > \gamma^l$.

On the region of $[c, \gamma^l]$, equation (30) reduces to a Canonical ODE System with a solution

$$\begin{aligned} m^l(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n \\ m^l(x, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n. \end{aligned} \quad (32)$$

When $x \in [\gamma^l, \gamma^h]$, the claim is worth one in the low tax rate state and so the Canonical ODE System Reduces to a Canonical ODE

$$\begin{aligned} \tilde{m}^l(x, l) &= 1 \\ (r + \lambda_h) \tilde{m}^l(x, h) &= \mu x \tilde{m}_x^l(x, h) + \frac{1}{2} \sigma^2 x^2 \tilde{m}_{xx}^l(x, h) + \lambda_h \\ \Rightarrow \tilde{m}^l(x, h) &= x^{B_1} K_1 + x^{B_2} K_2 + \frac{\lambda_h}{r + \lambda_h}. \end{aligned}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned} m^l(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n = 0 \\ m^l(c, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n = 0 \\ \tilde{m}^l(\gamma^h, h) &= (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 + \frac{\lambda_h}{r + \lambda_h} = 0 \\ m^l(\gamma^l, l) &= \sum_{n=1}^4 (\gamma^l)^{\beta_n} L_n = 1 \\ m^l(\gamma^l, h) &= \tilde{m}^l(\gamma^l, h) \Rightarrow \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] (\gamma^l)^{\beta_n} L_n = (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 + \frac{\lambda_h}{r + \lambda_h} \\ m_x^l(\gamma^l, h) &= \tilde{m}_x^l(\gamma^l, h) \Rightarrow \sum_{n=1}^4 \beta_n \left[\frac{g_l(\beta_n)}{\lambda_l} \right] (\gamma^l)^{\beta_n - 1} L_n = B_1 (\gamma^l)^{B_1 - 1} K_1 + B_2 (\gamma^l)^{B_2 - 1} K_2. \end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the adjusted contingent up claim price.

- Case 3: $\gamma^h = \gamma^l = \gamma$.

In this case, equation (30) is simply a Canonical ODE System with a solution of the form:

$$\begin{aligned} m^l(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n \\ m^l(x, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n. \end{aligned} \quad (33)$$

The boundary conditions give us four linear equations in the four constants, (L_1, L_2, L_3, L_4) :

$$\begin{aligned} m^l(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n = 0 \\ m^l(c, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n = 0 \\ m^l(\gamma, l) &= \sum_{n=1}^4 \gamma^{\beta_n} L_n = 1 \\ m^l(\gamma, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] \gamma^{\beta_n} L_n = 0. \end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the adjusted contingent up claim price.

1.5.2 Adjusted Contingent Up Claim: h-State Payoff

We next consider the pricing of an adjusted contingent up claim with payoff state is h .

- Case 1: $\gamma^l > \gamma^h$.

On the region of $[c, \gamma^h]$, equation (30) reduces to the Canonical ODE System with the solution form

$$\begin{aligned} m^h(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n \\ m^h(x, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n. \end{aligned} \quad (34)$$

When $x \in [\gamma^h, \gamma^l]$, equation (30) instead reduces to the Canonical ODE

$$\begin{aligned} m^h(x, h) &= 1 \\ (r + \lambda_l) \tilde{m}^h(x, l) &= \mu x \tilde{m}_x^h(x, l) + \frac{1}{2} \sigma^2 x^2 \tilde{m}_{xx}^h(x, l) + \lambda_l \\ \Rightarrow \tilde{m}^h(x, l) &= x^{B_1} K_1 + x^{B_2} K_2 + \frac{\lambda_l}{r + \lambda_l}. \end{aligned}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned}
m^h(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n = 0 \\
m^h(c, h) &= \sum_{n=1}^4 \left[\frac{gl(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n = 0 \\
m^h(\gamma^l, h) &= \sum_{n=1}^4 \left[\frac{gl(\beta_n)}{\lambda_l} \right] (\gamma^l)^{\beta_n} L_n = 1 \\
\tilde{m}^h(\gamma^l, l) &= (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 + \frac{\lambda_l}{r + \lambda_l} = 0 \\
m^h(\gamma^h, l) &= \tilde{m}^h(\gamma^h, l) \Rightarrow \sum_{n=1}^4 (\gamma^h)^{\beta_n} L_n = (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 + \frac{\lambda_l}{r + \lambda_l} \\
m_x^h(\gamma^h, l) &= \tilde{m}_x^h(\gamma^h, l) \Rightarrow \sum_{n=1}^4 \beta_n (\gamma^h)^{\beta_n - 1} L_n = B_1 (\gamma^h)^{B_1 - 1} K_1 + B_2 (\gamma^h)^{B_2 - 1} K_2.
\end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the adjusted contingent up claim price.

- Case 2: $\gamma^h > \gamma^l$.

On the region of $[c, \gamma^l]$, equation (30) reduces to a Canonical ODE System with a solution

$$\begin{aligned}
m^h(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n \\
m^h(x, h) &= \sum_{n=1}^4 \left[\frac{gl(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n.
\end{aligned} \tag{35}$$

When $x \in [\gamma^l, \gamma^h]$, the claim is worth 0 in the low tax rate state and so the Canonical ODE System Reduces to a Canonical ODE:

$$\begin{aligned}
\tilde{m}^h(x, l) &= 0 \\
(r + \lambda_h) \tilde{m}^h(x, h) &= \mu x \tilde{m}_x^h(x, h) + \frac{1}{2} \sigma^2 x^2 \tilde{m}_{xx}^h(x, h) \\
&\Rightarrow \tilde{m}^h(x, h) = x^{B_1} K_1 + x^{B_2} K_2.
\end{aligned}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned}
m^h(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n = 0 \\
m^h(c, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n = 0 \\
\tilde{m}^h(\gamma^h, h) &= (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 = 1 \\
m^h(\gamma^l, l) &= \sum_{n=1}^4 (\gamma^l)^{\beta_n} L_n = 0 \\
m^h(\gamma^l, h) &= \tilde{m}^h(\gamma^l, h) \Rightarrow \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] (\gamma^l)^{\beta_n} L_n = (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 \\
m_x^h(\gamma^l, h) &= \tilde{m}_x^h(\gamma^l, h) \Rightarrow \sum_{n=1}^4 \beta_n \left[\frac{g_l(\beta_n)}{\lambda_l} \right] (\gamma^l)^{\beta_n - 1} L_n = B_1 (\gamma^l)^{B_1 - 1} K_1 + B_2 (\gamma^l)^{B_2 - 1} K_2.
\end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the adjusted contingent up claim price.

- Case 3: $\gamma^h = \gamma^l = \gamma$.

In this case, equation (30) is simply the Canonical ODE System with a solution of form:

$$\begin{aligned}
m^h(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n \\
m^h(x, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n.
\end{aligned} \tag{36}$$

The boundary conditions give us four linear equations in the four constants, (L_1, L_2, L_3, L_4) :

$$\begin{aligned}
m^h(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n = 0 \\
m^h(c, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n = 0 \\
m^h(\gamma, l) &= \sum_{n=1}^4 \gamma^{\beta_n} L_n = 0 \\
m^h(\gamma, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] \gamma^{\beta_n} L_n = 1.
\end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the adjusted contingent up claim price.

1.6 Contingent Occupation Claims

This claim delivers an instantaneous unit flow of dt whenever the tax regime is equal to i . The claim is knocked by default or call. Let $a^i(x, j, \Omega)$ denote the price of this claim when EBIT is x and the current tax state is j . By the Feynman-Kac formula, this function must satisfy the following system of ODEs:

$$\begin{aligned}
ra^i(x, i, \Omega) &= 1 + \mu xa_x^i(x, i, \Omega) + \frac{1}{2}\sigma^2 x^2 a_{xx}^i(x, i, \Omega) + \lambda_i[a^i(x, j, \Omega) - a^i(x, i, \Omega)] \\
ra^i(x, j, \Omega) &= \mu xa_x^i(x, j, \Omega) + \frac{1}{2}\sigma^2 x^2 a_{xx}^i(x, j, \Omega) + \lambda_j[a^i(x, i, \Omega) - a^i(x, j, \Omega)] \\
a^i(x, i, \Omega) &= 0 \quad \text{if } x \in (0, c] \\
a^i(x, j, \Omega) &= 0 \quad \text{if } x \in (0, c] \\
a^i(x, i, \Omega) &= 0 \quad \text{if } x \in [\gamma^i, \infty) \\
a^i(x, j, \Omega) &= 0 \quad \text{if } x \in [\gamma^j, \infty).
\end{aligned} \tag{37}$$

In the following, we derive a price expression for each payoff state ($i = l, h$).

1.6.1 Contingent Occupation Claims: 1-State Payoff

- Case 1: $\gamma^l > \gamma^h$.

On the region of $[c, \gamma^h]$, equation (37) reduces to a Canonical ODE System with additional constant terms:

$$\begin{aligned}
ra^l(x, l) &= 1 + \mu xa_x^l(x, l) + \frac{1}{2}\sigma^2 x^2 a_{xx}^l(x, l) + \lambda_l[a^l(x, h) - a^l(x, l)] \\
ra^l(x, h) &= \mu xa_x^l(x, h) + \frac{1}{2}\sigma^2 x^2 a_{xx}^l(x, h) + \lambda_h[a^l(x, l) - a^l(x, h)].
\end{aligned} \tag{38}$$

We can also find a closed-form solution:

$$\begin{aligned}
a^l(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r + \lambda_l + \lambda_h)} \\
a^l(x, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n + \frac{\lambda_h}{r(r + \lambda_l + \lambda_h)}.
\end{aligned} \tag{39}$$

When $x \in [\gamma^h, \gamma^l]$, equation (37) reduces to a Canonical ODE:

$$\begin{aligned} a^l(x, h) &= 0 \\ (r + \lambda_l)\tilde{a}^l(x, l) &= 1 + \mu x \tilde{a}_x^l(x, l) + \frac{1}{2}\sigma^2 x^2 \tilde{a}_{xx}^l(x, l) \\ \tilde{a}^l(x, l) &= x^{B_1} K_1 + x^{B_2} K_2 + \frac{1}{r + \lambda_l}. \end{aligned}$$

The boundary conditions give us six linear equations in the six constants $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned} a^l(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r + \lambda_l + \lambda_h)} = 0 \tag{40} \\ a^l(c, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n + \frac{\lambda_h}{r(r + \lambda_l + \lambda_h)} = 0 \\ a^l(\gamma^h, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] (\gamma^h)^{\beta_n} L_n + \frac{\lambda_h}{r(r + \lambda_l + \lambda_h)} = 0 \\ \tilde{a}^l(\gamma^l, l) &= (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 + \frac{1}{r + \lambda_l} = 0 \\ a^l(\gamma^h, l) &= \tilde{a}^l(\gamma^h, l) \Rightarrow \sum_{n=1}^4 (\gamma^h)^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r + \lambda_l + \lambda_h)} = (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 + \frac{1}{r + \lambda_l} \\ a_x^l(\gamma^h, l) &= \tilde{a}_x^l(\gamma^h, l) \Rightarrow \sum_{n=1}^4 \beta_n (\gamma^h)^{\beta_n - 1} L_n = B_1 (\gamma^h)^{B_1 - 1} K_1 + B_2 (\gamma^h)^{B_2 - 1} K_2. \end{aligned}$$

By solving these linear equations, we can pin down the constants and then compute contingent occupation claim prices.

- Case 2: $\gamma^h > \gamma^l$

We have the following differential equations that must be satisfied in the interval (c, γ^l) :

$$\begin{aligned} r a^l(x, l) &= 1 + \mu x a_x^l(x, l) + \frac{1}{2}\sigma^2 x^2 a_{xx}^l(x, l) + \lambda_l [a^l(x, h) - a^l(x, l)] \tag{41} \\ r a^l(x, h) &= \mu x a_x^l(x, h) + \frac{1}{2}\sigma^2 x^2 a_{xx}^l(x, h) + \lambda_h [a^l(x, l) - a^l(x, h)]. \end{aligned}$$

The solution is:

$$\begin{aligned} a^l(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r + \lambda_l + \lambda_h)} \tag{42} \\ a^l(x, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n + \frac{\lambda_h}{r(r + \lambda_l + \lambda_h)}. \end{aligned}$$

And on the interval $[\gamma^l, \gamma^h]$ we have:

$$\begin{aligned} a^l(x, l) &= 0 \\ (r + \lambda_h)\tilde{a}^l(x, h) &= \mu x \tilde{a}_x^l(x, h) + \frac{1}{2}\sigma^2 x^2 \tilde{a}_{xx}^l(x, h) \\ \tilde{a}^l(x, h) &= x^{B_1} K_1 + x^{B_2} K_2. \end{aligned}$$

The boundary conditions give us six linear equations in the six constants $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned} a^l(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r + \lambda_l + \lambda_h)} = 0 \\ a^l(c, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n + \frac{\lambda_h}{r(r + \lambda_l + \lambda_h)} = 0 \\ a^l(\gamma^l, l) &= \sum_{n=1}^4 (\gamma^l)^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r + \lambda_l + \lambda_h)} = 0 \\ \tilde{a}^l(\gamma^h, h) &= (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 = 0 \\ a^l(\gamma^l, h) &= \tilde{a}^l(\gamma^l, h) \Rightarrow \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] (\gamma^l)^{\beta_n} L_n + \frac{\lambda_h}{r(r + \lambda_l + \lambda_h)} = (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 \\ a_x^l(\gamma^l, h) &= \tilde{a}_x^l(\gamma^l, h) \Rightarrow \sum_{n=1}^4 \beta_n \left[\frac{g_l(\beta_n)}{\lambda_l} \right] (\gamma^l)^{\beta_n - 1} L_n = B_1 (\gamma^l)^{B_1 - 1} K_1 + B_2 (\gamma^l)^{B_2 - 1} K_2. \end{aligned} \tag{43}$$

By solving these linear equations, we can pin down the constants and then compute contingent occupation claim prices.

- Case 3: $\gamma^h = \gamma^l = \gamma$

We have the following differential equations that must be satisfied in the interval (c, γ) :

$$\begin{aligned} r a^l(x, l) &= 1 + \mu x a_x^l(x, l) + \frac{1}{2}\sigma^2 x^2 a_{xx}^l(x, l) + \lambda_l [a^l(x, h) - a^l(x, l)] \\ r a^l(x, h) &= \mu x a_x^l(x, h) + \frac{1}{2}\sigma^2 x^2 a_{xx}^l(x, h) + \lambda_h [a^l(x, l) - a^l(x, h)]. \end{aligned} \tag{44}$$

The solution is:

$$\begin{aligned} a^l(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r + \lambda_l + \lambda_h)} \\ a^l(x, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n + \frac{\lambda_h}{r(r + \lambda_l + \lambda_h)} \end{aligned} \tag{45}$$

The boundary conditions give us four linear equations in the four constants, (L_1, L_2, L_3, L_4) :

$$\begin{aligned}
a^l(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r + \lambda_l + \lambda_h)} = 0 \\
a^l(c, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n + \frac{\lambda_h}{r(r + \lambda_l + \lambda_h)} = 0 \\
a^l(\gamma, l) &= \sum_{n=1}^4 \gamma^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r + \lambda_l + \lambda_h)} = 0 \\
a^l(\gamma, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] \gamma^{\beta_n} L_n + \frac{\lambda_h}{r(r + \lambda_l + \lambda_h)} = 0.
\end{aligned} \tag{46}$$

By solving these linear equations, we can pin down the constants and then compute contingent occupation claim prices.

1.6.2 Contingent Occupation Claims: h-State Payoff

- Case 1: $\gamma^l > \gamma^h$

We have the following differential equations that must be satisfied in the interval (c, γ^h) :

$$\begin{aligned}
ra^h(x, l) &= \mu xa_x^h(x, l) + \frac{1}{2} \sigma^2 x^2 a_{xx}^h(x, l) + \lambda_l [a^h(x, h) - a^h(x, l)] \\
ra^h(x, h) &= 1 + \mu xa_x^h(x, h) + \frac{1}{2} \sigma^2 x^2 a_{xx}^h(x, h) + \lambda_h [a^h(x, l) - a^h(x, h)].
\end{aligned} \tag{47}$$

The solution is:

$$\begin{aligned}
a^h(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n + \frac{\lambda_l}{r(r + \lambda_l + \lambda_h)} \\
a^h(x, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r + \lambda_l + \lambda_h)}.
\end{aligned} \tag{48}$$

And on the interval $[\gamma^h, \gamma^l]$ we have:

$$\begin{aligned}
a^h(x, h) &= 0 \\
(r + \lambda_l) \tilde{a}^h(x, l) &= \mu x \tilde{a}_x^h(x, l) + \frac{1}{2} \sigma^2 x^2 \tilde{a}_{xx}^h(x, l) \\
\tilde{a}^h(x, l) &= x^{B_1} K_1 + x^{B_2} K_2
\end{aligned}$$

Boundary conditions:

$$\begin{aligned}
a^h(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n + \frac{\lambda_l}{r(r + \lambda_l + \lambda_h)} = 0 \\
a^h(c, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r + \lambda_l + \lambda_h)} = 0 \\
a^h(\gamma^h, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] (\gamma^h)^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r + \lambda_l + \lambda_h)} = 0 \\
\tilde{a}^h(\gamma^l, l) &= (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 = 0 \\
a^h(\gamma^h, l) &= \tilde{a}^h(\gamma^h, l) \Rightarrow \sum_{n=1}^4 (\gamma^h)^{\beta_n} L_n + \frac{\lambda_l}{r(r + \lambda_l + \lambda_h)} = (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 \\
a_x^h(\gamma^h, l) &= \tilde{a}_x^h(\gamma^h, l) \Rightarrow \sum_{n=1}^4 \beta_n (\gamma^h)^{\beta_n - 1} L_n = B_1 (\gamma^h)^{B_1 - 1} K_1 + B_2 (\gamma^h)^{B_2 - 1} K_2.
\end{aligned} \tag{49}$$

- Case 2: $\gamma^h > \gamma^l$

We have the following differential equations that must be satisfied in the interval (c, γ^l) :

$$\begin{aligned}
ra^h(x, l) &= \mu x a_x^h(x, l) + \frac{1}{2} \sigma^2 x^2 a_{xx}^h(x, l) + \lambda_l [a^h(x, h) - a^h(x, l)] \\
ra^h(x, h) &= 1 + \mu x a_x^h(x, h) + \frac{1}{2} \sigma^2 x^2 a_{xx}^h(x, h) + \lambda_h [a^h(x, l) - a^h(x, h)].
\end{aligned} \tag{50}$$

The solution is:

$$\begin{aligned}
a^h(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n + \frac{\lambda_l}{r(r + \lambda_l + \lambda_h)} \\
a^h(x, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r + \lambda_l + \lambda_h)}.
\end{aligned} \tag{51}$$

And on the interval $[\gamma^l, \gamma^h]$ we have:

$$\begin{aligned}
a^h(x, l) &= 0 \\
(r + \lambda_h) \tilde{a}^h(x, h) &= 1 + \mu x \tilde{a}_x^h(x, h) + \frac{1}{2} \sigma^2 x^2 \tilde{a}_{xx}^h(x, h) \\
\tilde{a}^h(x, h) &= x^{B_1} K_1 + x^{B_2} K_2 + \frac{1}{r + \lambda_h}
\end{aligned}$$

Boundary conditions:

$$\begin{aligned}
a^h(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n + \frac{\lambda_l}{r(r + \lambda_l + \lambda_h)} = 0 \\
a^h(c, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r + \lambda_l + \lambda_h)} = 0 \\
a^h(\gamma^l, l) &= \sum_{n=1}^4 (\gamma^l)^{\beta_n} L_n + \frac{\lambda_l}{r(r + \lambda_l + \lambda_h)} = 0 \\
\tilde{a}^h(\gamma^h, h) &= (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 + \frac{1}{r + \lambda_h} = 0 \\
a^h(\gamma^l, h) &= \tilde{a}^h(\gamma^l, h) \Rightarrow \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] (\gamma^l)^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r + \lambda_l + \lambda_h)} = (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 + \frac{1}{r + \lambda_h} \\
a_x^h(\gamma^l, h) &= \tilde{a}_x^h(\gamma^l, h) \Rightarrow \sum_{n=1}^4 \beta_n \left[\frac{g_l(\beta_n)}{\lambda_l} \right] (\gamma^l)^{\beta_n - 1} L_n = B_1 (\gamma^l)^{B_1 - 1} K_1 + B_2 (\gamma^l)^{B_2 - 1} K_2.
\end{aligned} \tag{52}$$

- Case 3: $\gamma^h = \gamma^l = \gamma$

We have the following differential equations that must be satisfied in the interval (c, γ) :

$$\begin{aligned}
ra^h(x, l) &= \mu x a_x^h(x, l) + \frac{1}{2} \sigma^2 x^2 a_{xx}^h(x, l) + \lambda_l [a^h(x, h) - a^h(x, l)] \\
ra^h(x, h) &= 1 + \mu x a_x^h(x, h) + \frac{1}{2} \sigma^2 x^2 a_{xx}^h(x, h) + \lambda_h [a^h(x, l) - a^h(x, h)].
\end{aligned} \tag{53}$$

The solution is:

$$\begin{aligned}
a^h(x, l) &= \sum_{n=1}^4 x^{\beta_n} L_n + \frac{\lambda_l}{r(r + \lambda_l + \lambda_h)} \\
a^h(x, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] x^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r + \lambda_l + \lambda_h)}.
\end{aligned} \tag{54}$$

Boundary conditions:

$$\begin{aligned}
a^h(c, l) &= \sum_{n=1}^4 c^{\beta_n} L_n + \frac{\lambda_l}{r(r + \lambda_l + \lambda_h)} = 0 \\
a^h(c, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] c^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r + \lambda_l + \lambda_h)} = 0 \\
a^h(\gamma, l) &= \sum_{n=1}^4 \gamma^{\beta_n} L_n + \frac{\lambda_l}{r(r + \lambda_l + \lambda_h)} = 0 \\
a^h(\gamma, h) &= \sum_{n=1}^4 \left[\frac{g_l(\beta_n)}{\lambda_l} \right] \gamma^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r + \lambda_l + \lambda_h)} = 0.
\end{aligned} \tag{55}$$