Internet Appendix for

"Gold, Platinum, and Expected Stock Returns" *

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A1. Econometric Inference for Predictive Regressions

Stambaugh (1999) shows that predictive regressions using persistent predictors are biased in finite samples. The standard return predictability regression is:

$$r_{t+1}^{e} = \alpha + \beta x_t + \epsilon_{t+1}$$

where $r_{t+1}^{e} = \log\left(\frac{P_{t+1}+D_{t+1}}{P_t}\right) - r_t^f$ is the log excess return from time t to time $t+1$, and x_t is some predictor known at time t such as the log price-dividend ratio or the log GP ratio. If x_t is a persistent predictor, we can model it as an AR(1) process:

$$x_{t+1} = \mu + \rho x_t + u_{t+1}$$

For predictors such as the price-dividend ratio, $cov(\epsilon, u) \neq 0$, since a positive return shock typically means prices increased, which also increases the price-dividend ratio. Letting $\gamma = \frac{cov(\epsilon_{t+1}, u_{t+1})}{var(u_{t+1})}$, the bias in the estimate of the predictive beta can be written as:

$$\mathbb{E}[\hat{\beta} - \beta] = \gamma \underbrace{\mathbb{E}[\hat{\rho} - \rho]}_{\approx -\frac{(1+3\rho)}{T} < 0} \tag{1}$$

The degree of bias is proportional to γ, which can be estimated as the slope of the regression of residuals from the predictive regression on the residuals from the AR(1) regression of the predictor variable. For the price-dividend ratio, the correlation between ϵ and u in the data is 0.94, while it is only -0.17 for the GP ratio. Also note that there is no mechanical correlation between the residuals as is the case for the PD ratio. More formally, we project $\hat{\epsilon}_t$ on $\hat{u}_t$ and estimate $\hat{\gamma}$ to be 10.55 for the PD ratio, whereas for the GP ratio $\hat{\gamma}$ is only -1.78. Evaluating at the maximum bias ($\rho = 1$) estimates an upper bound of -0.090 for PD ratio bias, which is enough to change the sign, whereas it is only 0.015 for the GP ratio, which is small compared to the predictive beta of 0.237. The evidence suggests that the GP ratio predictability is not driven by finite sample bias.

Predictor persistence also potentially affects the size of tests (see e.g., Torous, Valkanov, and Yan, 2004). For $\delta = corr(\epsilon, u)$, the test statistic for β has a non-standard limiting distribution:

$$t_\beta \Longrightarrow \delta\tau_\rho + \sqrt{(1-\delta^2)}\, z$$

where τ_ρ is non-normal and z is normal. We follow Elliot and Stock (1994) and use Monte Carlo simulations to asses the magnitude of these size distortions. We run 100,000 simulations of length equal to our sample size at a monthly frequency by simulating the above dynamics, evaluating all parameters using their sample values. When $\delta = 0.94$ (which is the case for the PD ratio), a 5% test has a true rejection rate of 17%. For the GP ratio, where $\delta = -0.167$, a 5% test has a true rejection rate of 6%, which is very close to the true size. Since the absolute value of δ is small in the case of the GP ratio, the significance of the predictability tests is not affected by potential size distortions due to predictor persistence.

A2. Stationary Mean of λ_t

We compute the stationary mean of the λ_t process. The process is given by:

$$d\lambda_t = \kappa_\lambda(\xi_t - \lambda_t)dt + \sigma_\lambda\sqrt{\lambda_t}dW_t^\lambda + J_t^\lambda dN_t^\lambda$$

which implies that

$$\begin{aligned} \frac{\mathbb{E}\left[\lambda_t\right]}{dt} &= \kappa_\lambda(n(t) - m(t)) + \mu_\lambda\rho_0 + \mu_\lambda\rho_1 m(t) \\ \frac{\mathbb{E}\left[\xi_t\right]}{dt} &= \kappa_\xi\bar{\xi} - \kappa_\xi n(t) \end{aligned} \tag{2}$$

where $m(t) = \mathbb{E}[\lambda_t]$ and $n(t) = \mathbb{E}[\xi_t]$, with $n(t) \to \bar{\xi}$. Solving the ordinary differential equation for $m(t)$ implied by (2) results in the stationary mean of λ_t:

$$\mathbb{E}[\lambda_\infty] = \lim_{t\to\infty} m(t) = \frac{\kappa_\lambda \bar{\xi} + \mu_\lambda \rho_0}{\kappa_\lambda - \mu_\lambda \rho_1} \tag{3}$$

with necessary conditions $\kappa_\lambda > \mu_\lambda \rho_1$ and $\kappa_\lambda \bar{\xi} + \mu_\lambda \rho_0 > 0$, which are satisfied under the model calibration.

A3. Relative Lease Rates and Expected Returns

The model also predicts a relation between the relative lease rates of gold and platinum. The question is whether relative lease rates would serve as a better proxy for risk premia compared to relative prices. Using the approximation in Appendix A5, we can write the relative lease rate as:

$$\begin{aligned}\log \frac{Q_{x,t}/P_{x,t}}{Q_{g,t}/P_{g,t}} &= \log \frac{G^g(\lambda_t, \xi_t)}{G^x(\lambda_t, \xi_t, \log Z_t)} \\ &\approx \text{ constants } + (b^*_{g,\lambda} - b^*_{x,\lambda})\,\lambda_t + (b^*_{g,\xi} - b^*_{x,\xi})\,\xi_t.\end{aligned}$$

When the instantaneous disaster risk represented by λ increases, both the price of gold P_g and platinum P_x decrease, consistent with the fact that gold and platinum are risky assets. Our calibration implies that P_g falls less than P_x making GP increasing in λ. The impact of ξ on P_g and P_x is qualitatively similar to that of λ.

However, when λ increases, the rental income of gold relative to platinum increases as well because of the preference for gold in times of high λ. Under our calibration, the gold lease rate increases more than the platinum lease rate upon an increase in λ_t which implies $b^*_{g,\lambda} - b^*_{x,\lambda} < 0$. In other words, the rental income effect of a change in λ dominates the effect of λ on P_g and P_x. The mechanism looks different for ξ. When the long-run average disaster risk represented by ξ increases, the price of both the price of gold P_g and platinum P_x decrease similar to the effect of λ; gold falls less than platinum. However, as we show in the user costs of gold and platinum, ξ has no direct effect on the rental income from gold and platinum. As a result, we have $b^*_{g,\xi} - b^*_{x,\xi} > 0$ and the relative lease rate given by $\log \frac{Q_{x,t}/P_{x,t}}{Q_{g,t}/P_{g,t}}$ has a negative loading on λ_t and a positive loading on ξ_t which is easy to verify in model simulations. Because the equity premium loadings on λ_t and ξ_t are both positive, the relative lease rate is not an ideal return predictor as opposed to $\log GP_t$ which is given by

$$\begin{aligned} \log GP_t &= \log \frac{P_{g,t}}{P_{x,t}} \\ &= \text{cons} + \underbrace{(\frac{1}{\epsilon} - b^*_{x,Z})}_{>0} \log Z_t + \underbrace{(a_2 - b_2 + b^*_{g,\lambda} - b^*_{x,\lambda})}_{>0} \lambda_t + \underbrace{(b^*_{g,\xi} - b^*_{x,\xi})}_{>0} \xi_t. \end{aligned} \tag{4}$$

The price ratio $\log GP_t$ isolates the impact of disaster probabilities on gold and platinum prices, and abstracts from the effect on the gold rental income. The term $a_2 - b_2$, which is assumed to be positive in our calibration, ensures that $\log GP_t$ is increasing in both λ_t and ξ_t making it a better candidate for return predictability compared to relative lease rate.

Table 1 reports empirical estimates and model simulation results for equity return predictability by the relative lease rate.[1] The empirical slope estimates are not statistically different from zero at any horizon, and the predictive power is very low with R^2 values of 0.03% at the 1 year and 2.19% at the 5 year horizon. The model is consistent with the lack of return predictability using relative lease rates. The median estimates of slope coefficients are close to zero and the 90% confidence bands include a wide range of positive and negative values both among all and no disaster simulations. This is in stark contrast to strong the predictive power of $\log GP_t$ in the model where the 90% confidence band across no-disaster simulations includes only positive values and the R^2 values are significantly higher. In sum, relative lease

[1] Similar results obtain for predicting gold and platinum returns using the relative lease rate which we do not report for brevity.

rates do not serve as a good empirical return predictor and our model is consistent with this result.

A4. Contemporaneous Relation between Returns and GP shocks

This section discusses the contemporenous relation between shocks to GP and returns, and whether this relation can justify the predictability evidence presented in the paper.

We can write the expected excess return as

$$r^e_{t+1} = \mathbb{E}_t\left[r^e_{t+1}\right] + \nu_{t+1}, \tag{5}$$

where $\mathbb{E}_t\left[r^e_{t+1}\right]$ is the conditional expected return and ν_{t+1} is the unexpected return. Assume that the expected return is given by

$$\mathbb{E}_t\left[r^e_{t+1}\right] = \gamma_0 + \gamma_1'\, x_t. \tag{6}$$

where x is a vector of predictor variables. In the present paper, x_t is $\log GP_t$. This simple characterization gives rise to the predictability regressions in the paper. Assume that the predictors follow processes such that the conditional risk premium follows an AR(1) process:

$$\mathbb{E}_{t+1}\left[r^e_{t+2}\right] = \overline{\gamma} + \kappa_1\,\mathbb{E}_t\left[r^e_{t+1}\right] + u_{t+1}, \tag{7}$$

which would, for example, obtain if all predictors follow AR(1) processes. Following Campbell (1990), we write the unexpected component of the realized return as:

$$\nu_{t+1} = \underbrace{(\mathbb{E}_{t+1} - \mathbb{E}_t)\sum_{j=1}^{\infty}\rho^{j-1}\Delta d_{t+j}}_{=\eta^d_{t+1}\text{ cash-flow news}} - \underbrace{(\mathbb{E}_{t+1} - \mathbb{E}_t)\sum_{j=1}^{\infty}\rho^{j} r_{t+j}}_{=\eta^r_{t+1}\text{ discount rate news}}, \tag{8}$$

where the first component is an unexpected shock to expected future cash-flows and the second component is the shock to expected returns. We can further decompose the expected return component into riskless rate and excess return components:

$$\nu_{t+1} = (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \rho^{j} r^{f}_{t+1+j} - (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \rho^{j} r^{e}_{t+1+j}. \tag{9}$$

Let η^r denote the expected excess return component of the unexpected return:

$$\eta^r_{t+1} = (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \rho^{j} r^{e}_{t+1+j}. \tag{10}$$

We can rewrite η^r_{t+1} as

$$\eta^r_{t+1} = \sum_{j=1}^{\infty} \rho^{j} \left(\mathbb{E}_{t+1} \left[r^e_{t+1+j} \right] - \mathbb{E}_t \left[r^e_{t+1+j} \right] \right). \tag{11}$$

Using Equation (7) and law of iterated expectations, we can write the components as

$$\mathbb{E}_t \left[r^e_{t+1+j} \right] = \mathbb{E}_t \left[\mathbb{E}_{t+j} \left[r^e_{t+1+j} \right] \right] = \kappa_1^j \, \mathbb{E}_t \left[r^e_{t+1} \right] + \sum_{i=1}^{j} \overline{\gamma} \, \kappa_1^{i-1}, \tag{12}$$

and

$$\mathbb{E}_{t+1} \left[r^e_{t+1+j} \right] = \mathbb{E}_{t+1} \left[\mathbb{E}_{t+j} \left[r^e_{t+1+j} \right] \right] = \kappa_1^{j-1} \, \mathbb{E}_{t+1} \left[r^e_{t+2} \right] + \sum_{i=1}^{j-1} \overline{\gamma} \, \kappa_1^{i-1}. \tag{13}$$

As a result, the revision in the expectation can be written as

$$\begin{aligned} \mathbb{E}_{t+1} \left[r^e_{t+1+j} \right] - \mathbb{E}_t \left[r^e_{t+1+j} \right] &= -\overline{\gamma} \, \kappa_1^{j-1} + \kappa_1^{j-1} \left(\mathbb{E}_{t+1} \left[r^e_{t+2} \right] - \kappa_1 \, \mathbb{E}_t \left[r^e_{t+1} \right] \right) \\ &= \kappa_1^{j-1} \, u_{t+1}, \end{aligned} \tag{14}$$

where the second equation follows from (7). Using Equation (11), we can write

$$\begin{aligned}\eta^r_{t+1} &= \sum_{j=1}^{\infty} \rho^j \, \kappa_1^{j-1} \, u_{t+1} \\ &= \frac{1}{\kappa_1} \, u_{t+1} \, \sum_{j=1}^{\infty} (\rho \kappa_1)^j \\ &= \frac{\rho}{1-\rho\kappa_1} \, u_{t+1},\end{aligned} \tag{15}$$

where ρ is a number slightly below one. Campbell (1990) reports that the impact of riskless rate news is negligible. Therefore, we assume that discount rate news and excess return news are approximately equal.

Consider the correlation between news $corr(\eta^d, \eta^r) = \rho_c$ which is likely to be negative giving rise to excess volatility. Then, we can write the unexpected return as

$$\eta^d = \rho_c \, \eta^r + \sqrt{1-\rho_c^2} \, \tilde{\eta}^d, \tag{16}$$

where $corr(\tilde{\eta}^d, \eta^r) = 0$. We can write the unexpected components of returns as

$$\nu = \sqrt{1-\rho_c^2} \, \tilde{\eta}^d - (1-\rho_c) \, \eta^r. \tag{17}$$

Plugging (15) into (17), we can write

$$\nu_{t+1} = \sqrt{1-\rho_c^2} \, \tilde{\eta}^d_{t+1} - (1-\rho_c) \, \frac{\rho}{1-\rho \, \kappa_1} u_{t+1}. \tag{18}$$

Because we assumed $corr(\tilde{\eta}^d, \eta^r) = 0$, the regression coefficient of ν_t on u_t should be $-(1-\rho_c) \, \frac{\rho}{1-\rho \, \kappa_1}$ if the predictor x captures all the discount rate variation.

Campbell (1990) reports $\rho_c = -0.16$ for the period 1952 to 1988, and $\rho \approx 0.95$ for annual data. Based on this and using the annual persistence of $\log GP$, the theoretical value of the regression coefficient is

$$-\left(1-\rho_c\right)\frac{\rho}{1-\rho\,\kappa_1}=-3.84. \tag{19}$$

Note that this value obtains under the assumption that $\log GP$ is the only driver of expected returns. We acknowledge that this is unlikely to be the case.[2] But we also have shown that $\log GP$ is likely to proxy for a substantial portion of persistent component of expected equity returns. Hence, we would like to see that the empirical estimate is not different from the theoretical value by too many orders of magnitude.

The OLS estimate of the regression coefficient of ν_t on u_t is -1.50 with a standard error of 1.10. In other words, the theoretical value is around the 95% of the empirical estimate's distribution. This regression has an adjusted R^2 of 15% at the annual frequency. The negative value confirms that discount rate shocks represented by shocks to $\log GP$ have a negative effect on stock market valuations leading to low contemporaneous returns.

We repeat the same exercise in model simulations and obtain a theoretical value of -5.45. However, in simulated short samples with the same length as the data, we obtain a median value of -4.09 for the empirical estimate for the coefficient of ν_t on u_t, while the 90% confidence band is given by $[-16.84; 1.29]$ which includes the data value.

Finally, we also run the contemporaneous regression of excess equity return on $\log GP_t$, namely $\log R_t - R_t^f = \beta_0 + \beta_1 \log GP_t + \epsilon_t$, and obtain an estimate of $\beta_1 = 0.19$ (t=2.02) in the data. The sign of the estimate is consistent with the findings of Le and Zhu (2013) using gold only. The population value from the model is 0.13, while the median value across simulated short samples is 0.44 with a 90% confidence band given by $[0.10; 1.14]$.[3]

[2] As shown in Table 5, most other predictors are not significant once we control for GP but there are few exceptions such as the variance risk premium.

[3] Consistent with Le and Zhu (2013), we find a positive coefficient estimate when we regress gold lease rates on the market return contemporaneously. However, the estimated coefficients in contemporaneous regressions of lease rates on market returns are not statistically significant. In the model, regressing market returns on the relative gold-platinum lease rate delivers a wide range of estimates with the 90% confidence band $[-28.26; 1.75]$ while the empirical estimate is essentially zero. This evidence is supportive of the weak relation between equity returns and lease rates discussed in Appendix A6.

Table 1: Simulation Results: Predictability using Relative Lease Rates

Table 1 shows results from model simulations for aggregate equity return predictability. State variables are simulated at a monthly frequency and aggregated to an annual frequency. The population moments are computed from a 1,000,000 year simulation. The model confidence intervals are computed from 100,000 simulations of length equal to the length of the data. We run the regression:

$$\frac{12}{h}\sum_{i=1}^{h}\log R_{t+i} - \log R^{f}_{t+i} = \beta_0 + \beta_1 \log\frac{Q_{g,t}/P_{g,t}}{Q_{x,t}/P_{x,t}} + \epsilon_{t+h}$$

and the period is annual.

	Data	All Simulations				No Disaster Simulations		
	Est.	5%	50%	95%	Pop.	5%	50%	95%
$\beta_{1y,gp}$	0.005	-0.572	0.001	0.335	0.030	-0.808	-0.065	0.295
$\beta_{3y,gp}$	0.001	-0.447	-0.015	0.275	0.020	-0.561	-0.107	0.229
$\beta_{5y,gp}$	0.002	-0.369	-0.02	0.199	0.014	-0.435	-0.108	0.157
$R^2_{1y,gp}$	0.03	0.00	2.57	18.67	0.15	0.00	2.81	19.20
$R^2_{3y,gp}$	0.52	0.00	6.38	38.71	0.21	0.00	7.01	41.15
$R^2_{5y,gp}$	2.19	0.07	8.08	46.41	0.18	0.05	10.94	46.76

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