

# Internet Appendix

## Are Disagreements Agreeable? Evidence from Information Aggregation

### Appendix A. Six LASSO Methods

In this section, for each method we explain how to construct the out-of-sample forecast in month  $t$  for the return in month  $t + 1$ .

**Equal-weight LASSO** In month  $t$ , we choose  $J$  out of  $K$  individual disagreement measures via the following LASSO optimization problem:

$$\max_{\beta} \sum_{j=1}^{t-1} (R_{t+1} - \sum_{k=1}^K \beta_k D_t^k)^2 + \lambda \sum_{k=1}^K |\beta_k|, \quad (\text{A1})$$

where  $D_t^k$  is the observation of individual disagreement measure  $k$  ( $k = 1, \dots, K$ ) in month  $t$ . Then we construct an equal-weight disagreement index as

$$D_t^{\text{EW}} = \sum_{j=1}^J \tilde{D}_t^j, \quad (\text{A2})$$

where  $\tilde{D}_t^1$  through  $\tilde{D}_t^J$  are the selected individual disagreement measures in month  $t$ . Based on the predictive regression (7), we estimate the expected market return as

$$\hat{R}_{t+1}^{\text{EW-LASSO}} = \hat{\alpha}_t + \hat{\beta}_t D_t^{\text{EW}}. \quad (\text{A3})$$

Empirically, [Chinco, Clark-Joseph, and Ye \(2019\)](#) find that the LASSO performs well in identifying sparse and high-frequency return predictors in a cross-sectional framework.

**Combination LASSO** To reduce model instability and uncertainty, [Han, He, Rapach, and Zhou \(2019\)](#) propose a combination LASSO method to improve the forecasting power of individual stock return predictors, which directly combines individual stock return forecasts. In this paper,

suppose  $\hat{R}_{t+1}^k$  is the market return forecast based on disagreement measure  $D_t^k$  and  $M$  is the initial sample size for parameter training. In month  $t$ , the combination LASSO estimates the expected market return as

$$\hat{R}_{t+1}^{\text{C-LASSO}} = \sum_{k=1}^K \hat{\beta}_k \hat{R}_{t+1}^k, \quad (\text{A4})$$

where  $\hat{\beta} = (\hat{\beta}_1, \dots, \hat{\beta}_K)$  is the estimate via the following LASSO optimization problem,

$$\max_{\beta} \sum_{j=M+1}^t (R_{t+1} - \sum_{k=1}^K \beta_k \hat{R}_{t+1}^k)^2 + \lambda \sum_{k=1}^K |\beta_k|. \quad (\text{A5})$$

**Encompassing LASSO** Suppose  $\hat{R}_{t+1}$  is the market return forecast based on all the individual disagreement measures via a multivariate predictive regression. [Han, He, Rapach, and Zhou \(2019\)](#) propose an encompassing LASSO method as

$$\hat{R}_{t+1}^{\text{E-LASSO}} = \theta_t \hat{R}_{t+1} + (1 - \theta_t) \hat{R}_{t+1}^{\text{C-LASSO}}, \quad (\text{A6})$$

where  $\theta_t$  is estimated with the [Harvey, Leybourne, and Newbold \(1998\)](#) forecast encompassing test.

**Adaptive LASSO** As in [Freyberger, Neuhierl, and Weber \(2020\)](#), the adaptive LASSO weights the terms in the penalty of (A5) to encourage small first-round coefficient estimates to be set to zero,

$$\max_{\beta} \sum_{j=M+1}^t (R_{t+1} - \sum_{k=1}^K \beta_k \hat{R}_{t+1}^k)^2 + \lambda \sum_{k=1}^K w_k |\beta_k|. \quad (\text{A7})$$

and estimate the expected market return as

$$\hat{R}_{t+1}^{\text{A-LASSO}} = \sum_{k=1}^K \hat{\beta}_k \hat{R}_{t+1}^k, \quad (\text{A8})$$

where  $w_i = 1/|\hat{\beta}_k|^v$ ,  $\hat{\beta}_k$  is the univariate predictive regression estimate, and  $v > 0$ .

**Egalitarian LASSO** Instead of shrinking the coefficient to zero, [Diebold and Shin \(2019\)](#) propose to shrink it to the simple average,

$$\max_{\beta} \sum_{j=M+1}^t (R_{t+1} - \sum_{k=1}^K \beta_k \hat{R}_{t+1}^k)^2 + \lambda \sum_{k=1}^K |\beta_k - \frac{1}{K}|. \quad (\text{A9})$$

Then the expected market return can be estimated as

$$\hat{R}_{t+1}^{\text{Eg-LASSO}} = \sum_{k=1}^K \hat{\beta}_k \hat{R}_{t+1}^k. \quad (\text{A10})$$

**Elastic net** To handle the potential highly correlated return forecasts, one may solve for the following optimization problem,

$$\max_{\beta} \sum_{j=1}^t (R_t - \sum_{i=1}^N \beta_i \hat{R}_{i,t})^2 + \lambda_1 \sum_{i=1}^N |\beta_i| + \lambda_2 \sum_{i=1}^N \beta_i^2, \quad (\text{A11})$$

and estimate the expected market return as

$$\hat{R}_{t+1}^{\text{EN-LASSO}} = \sum_{k=1}^K \hat{\beta}_k \hat{R}_{t+1}^k. \quad (\text{A12})$$

Empirically, [Kozak, Nagel, and Santosh \(2020\)](#) show that the elastic net is powerful in predicting stock returns in a cross-sectional framework.

In all the six LASSO-related methods, the tuning parameter  $\lambda$  is chosen via the corrected version of the Akaike information criterion (AICc). [Han, He, Rapach, and Zhou \(2019\)](#) show that the AICc performs quantitatively similar as alternative cross validation criteria.

## Appendix B. Forecasting economic activities

This section shows that the disagreement index negatively predicts future economic activities. Specifically, we consider six macro variables as the proxy of economic activities, including the CFNAI, industrial production growth, unemployment rate, aggregate equity issuance, total business inventory, and capacity utilization.

The macro variables are adjusted for seasonality and annualized for ease of exposition. To control for the autocorrelations, we follow [Allen, Bali, and Tang \(2012\)](#) and run the following regression:

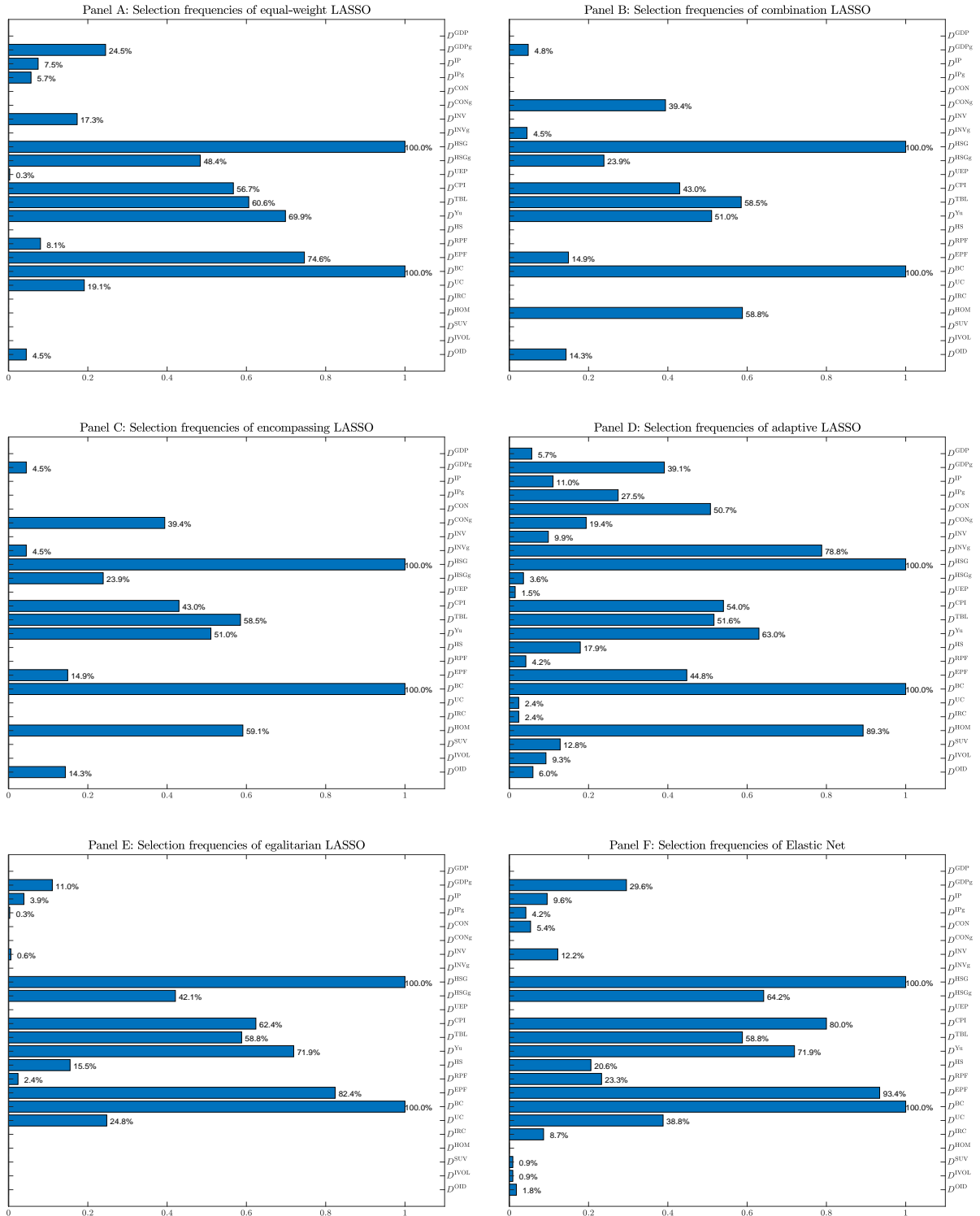
$$y_{t+1} = \alpha + \beta D_t + \sum_{i=1}^{12} \lambda_i y_{t-i+1} + \varepsilon_{t+1}, \quad (\text{A1})$$

where  $y_{t+1}$  is one of the macro variables.

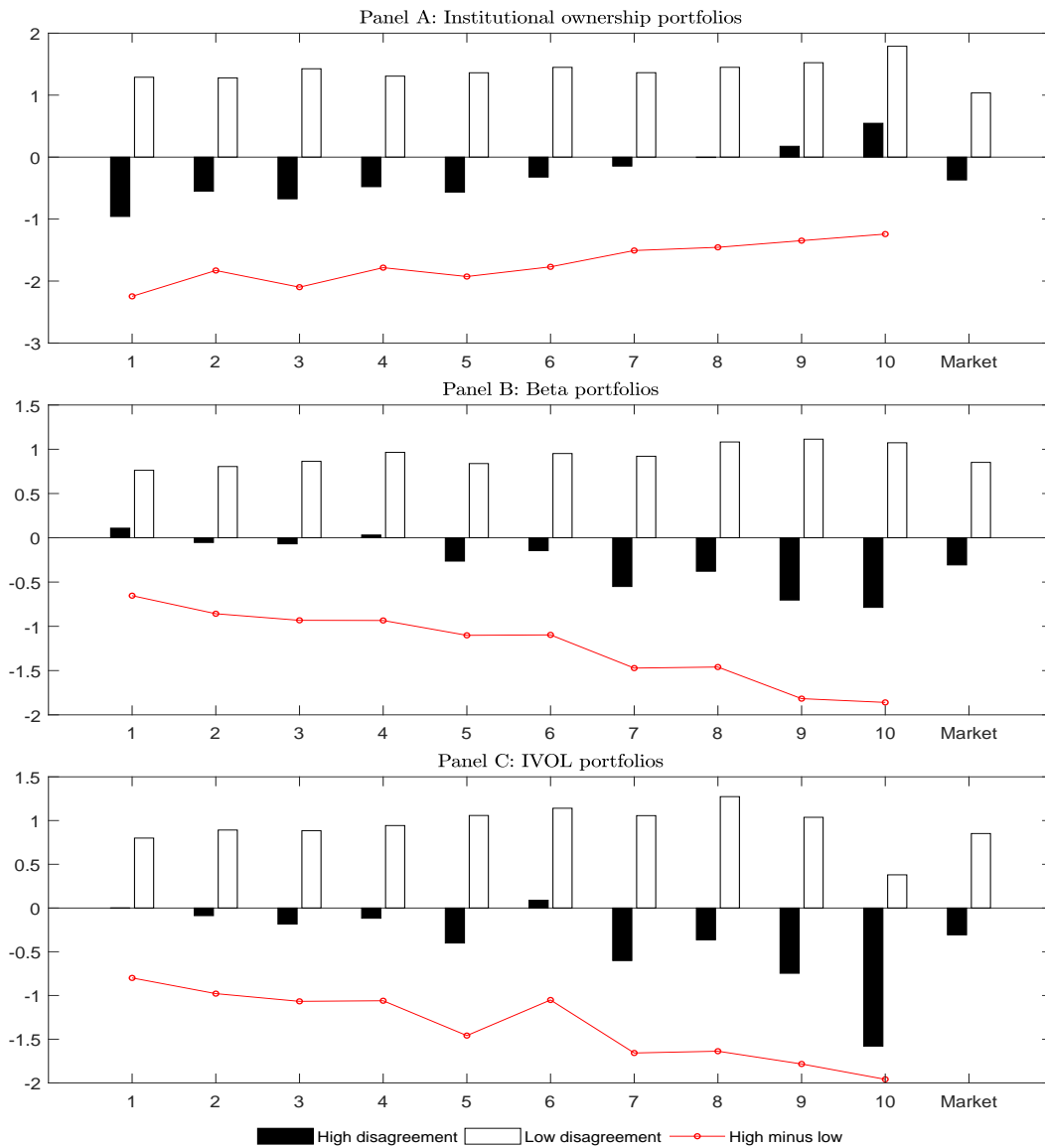
Table [A4](#) shows that the disagreement index negatively predicts future economic activities. For instance, a one-standard deviation increase in the disagreement index predicts a 0.93% decrease in the CFNAI and a 0.22% increase in unemployment, respectively.



**Fig. A1.** This figure plots the individual disagreement measures selected by the LASSO-related techniques at each point in time when conducting out-of-sample forecasting over 1991:02–2018:12.



**Fig. A2.** This figure plots the selection frequency of each individual disagreement measure over the 1991:02–2018:12 out-of-sample period.



**Fig. A3.** This figure plots the average monthly excess returns of decile portfolios in high and low disagreement periods, where a month is in a high disagreement period if  $D^{PLS}$  in month  $t - 1$  is above its previous 24-month moving average, and otherwise in a low disagreement period.

**Table A1 Correlations between individual disagreement measures**

This table reports the pairwise correlations of 24 individual disagreement measures used in this paper. The first 13 measures are obtained from the survey of professional forecasters (SPF) at a quarterly frequency, each of which is defined by the level or growth difference between the 75th and 25th percentiles of the forecasts.  $D^{Yu}$  and  $D^{HS}$  are value- and beta-weighted analyst forecast dispersions (Yu, 2011; Hong and Sraer, 2016). The next six are household belief dispersions on macroeconomic conditions from the Michigan survey of consumers attitudes.  $D^{SUV}$  is a disagreement measure based on the standardized unexplained trading volume of NYSE stocks (Garfinkel, 2009).  $D^{IVOL}$  is the value-weighted idiosyncratic volatility proposed by Boehme, Danielsen, and Sorescu (2006) as a measure of investor disagreement.  $D^{OID}$  is a disagreement measure defined by the open interest difference of OEX call and put options (Ge, Lin, and Pearson, 2016).

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	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
1 $D^{GDP}$	1.00																								
2 $D^{GDPg}$	0.80	1.00																							
3 $D^{IP}$	0.54	0.56	1.00																						
4 $D^{IPg}$	0.56	0.67	0.83	1.00																					
5 $D^{CON}$	0.62	0.61	0.49	0.51	1.00																				
6 $D^{CONg}$	0.66	0.71	0.54	0.60	0.82	1.00																			
7 $D^{INV}$	0.45	0.42	0.38	0.34	0.49	0.42	1.00																		
8 $D^{INVg}$	0.42	0.52	0.50	0.58	0.32	0.49	0.68	1.00																	
9 $D^{HSG}$	0.38	0.35	0.22	0.31	0.24	0.26	-0.06	0.04	1.00																
10 $D^{HSGg}$	0.50	0.54	0.32	0.40	0.28	0.37	0.16	0.35	0.51	1.00															
11 $D^{UEP}$	0.47	0.50	0.61	0.54	0.42	0.54	0.26	0.41	0.19	0.43	1.00														
12 $D^{CPI}$	0.43	0.39	0.32	0.45	0.26	0.36	0.14	0.26	0.43	0.44	0.31	1.00													
13 $D^{TBL}$	0.38	0.35	0.37	0.29	0.13	0.19	0.10	0.12	0.29	0.15	0.16	0.35	1.00												
14 $D^{Yu}$	0.26	0.39	0.35	0.26	0.26	0.33	0.32	0.33	0.16	0.21	0.22	0.15	0.08	1.00											
15 $D^{HS}$	0.23	0.25	0.42	0.30	0.20	0.18	0.21	0.27	0.08	0.01	0.16	0.07	0.06	0.64	1.00										
16 $D^{RPF}$	0.01	0.00	-0.03	0.07	-0.09	-0.05	-0.07	0.00	0.08	0.04	-0.14	0.05	0.16	-0.30	-0.13	1.00									
17 $D^{EPF}$	0.10	0.13	0.20	0.16	-0.01	0.07	-0.01	0.13	0.05	0.03	0.07	-0.03	0.19	-0.02	0.12	0.16	1.00								
18 $D^{BC}$	-0.22	-0.23	-0.28	-0.19	-0.22	-0.24	-0.08	-0.11	-0.28	-0.19	-0.41	-0.36	-0.14	-0.31	-0.25	0.23	-0.02	1.00							
19 $D^{UC}$	0.14	0.18	0.24	0.32	-0.10	0.09	-0.08	0.22	0.05	0.23	0.15	0.21	0.17	-0.13	0.06	0.27	0.45	0.01	1.00						
20 $D^{IRC}$	0.36	0.28	0.34	0.33	0.24	0.25	0.09	0.08	0.30	0.34	0.28	0.17	0.31	0.04	0.14	0.07	0.20	-0.14	0.26	1.00					
21 $D^{HOM}$	0.06	0.03	-0.04	-0.03	-0.05	0.07	-0.11	-0.10	0.13	-0.05	-0.04	0.25	0.34	0.17	0.09	-0.07	-0.03	-0.14	-0.07	-0.01	1.00				
22 $D^{SUV}$	0.00	0.00	-0.17	0.00	0.10	0.10	0.03	0.05	0.18	0.09	-0.11	0.14	-0.10	-0.19	-0.19	0.19	-0.12	0.19	-0.10	-0.09	0.06	1.00			
23 $D^{IVOL}$	0.32	0.34	0.36	0.29	0.37	0.31	0.50	0.41	0.20	0.19	0.27	0.00	-0.03	0.53	0.69	-0.23	0.03	-0.24	-0.11	0.15	-0.07	-0.09	1.00		
24 $D^{OID}$	0.22	0.19	0.13	0.17	0.15	0.17	0.24	0.31	0.11	0.21	0.17	0.00	-0.01	0.12	0.16	-0.05	0.13	-0.04	0.21	0.00	0.07	0.08	0.20	1.00	



**Table A2 Forecasting market returns with different moment PLS disagreement indexes**

This table presents the regression slopes, Newey-West  $t$ -values, in-sample  $R^2$ s, and out-of-sample  $R^2_{OS}$  of predicting market returns with the first to sixth moment PLS disagreement indexes, respectively. Statistical significance for  $R^2_{OS}$  is based on the  $p$ -value of the [Clark and West \(2007\)](#) MSFE-adjusted statistic for testing  $H_0 : R^2_{OS} \leq 0$  against  $H_A : R^2_{OS} > 0$ . The in- and out-of-sample periods are 1969:12–2018:12 and 1991:02–2018:12, respectively. \*\*\*, \*\*, and \* indicate significance at the 1%, 5%, and 10% levels, respectively.

Moment	$\beta$	$t$ -value	$R^2$	$R^2_{OS}$
1st	-0.83***	-3.96	2.52	1.56**
2nd	-0.49	-1.06	0.21	-0.87
3rd	-0.20	-0.56	0.05	-0.29
4th	-0.01	-0.02	0.00	-0.24
5th	-0.10	-0.56	0.06	-0.16
6th	-0.12	-1.27	0.29	-0.08

**Table A3 Disagreement with market volatility and trading volume: Robustness check**

Panel A presents the results of predicting the volume-volatility correlation with the disagreement index:

$$\text{Correlation}_{t+1} = \alpha + \beta D_t + \varepsilon_{t+1},$$

where the correlation in month  $t + 1$  refers to the correlation between the daily change in turnover of NYSE stocks and the daily change in volatility within month  $t + 1$ . Realized volatility, realized semi-volatility, and median realized volatility are estimated based on the S&P 500 index returns from 5-minute intervals (Andersen, Dobrev, and Schaumburg, 2012), and futures realized volatility is estimated based on the S&P 500 index futures contract returns from 5-minute intervals (Johnson, 2019). Panel B presents the results of the following regression:

$$\text{Volatility}_{t+1} = \alpha + \beta_1 \text{D\_Volume}_t + \beta_2 \text{Volume}_t^\circ + \varepsilon_{t+1}.$$

D\_Volume is the disagreement-related volume and extracted with the PLS method, and Volume<sup>°</sup> is the residual of regressing volume on D\_Volume. D\_Volatility is the disagreement-related volatility. Following Hamilton (2018), we apply AR(4) to both trading volume and market volatility to remove potential trends and expected information. Reported are regression coefficient, Newey-West  $t$ -value, and  $R^2$ . The sample period is 2000:01–2018:12 for the first three volatility measures and 1990:01–2015:12 for the last one. \*\*\*, \*\*, and \* indicate significance at the 1%, 5%, and 10% levels, respectively.

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Panel A: Predicting volatility-volume correlation

	$\beta$	$t$ -value	$R^2$
Realized volatility	5.22***	3.36	4.02
Realized semi-volatility	3.25	1.51	1.27
Median realized volatility	3.21**	2.05	1.57
Futures realized volatility	5.30***	3.97	4.68

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Panel B: Predicting market volatility

	$\beta_1$	$t$ -value	$\beta_2$	$t$ -value	$R^2$	Corr(D_Volume, D_Volatility)
Realized volatility	3.17*	1.80	0.74	0.60	1.45	0.45
Realized semi-volatility	3.29*	1.79	0.71	0.50	1.32	0.44
Median realized volatility	4.19**	2.33	1.15	0.77	2.20	0.59
Futures realized volatility	5.84***	4.90	1.22	1.13	5.43	0.62

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**Table A4 Forecasting economic activities with disagreement**

The table presents the regression slope, Newey-West  $t$ -value, and  $R^2$  of predicting economic activities with the disagreement index as

$$y_{t+1} = \alpha + \beta D_t + \sum_{i=1}^{12} \lambda_i y_{t-i+1} + \varepsilon_{t+1}.$$

Economic activities include Chicago Fed National Activity Index (CFNAI), industrial production growth, unemployment, aggregate equity issuance (Baker and Wurgler, 2000), business inventory, and capacity utilization. \*\*\*, \*\*, \* indicate significance at the 1%, 5%, and 10% levels, respectively.

Economic activity	$\beta$	$t$ -value	$R^2$
CFNAI	-0.93**	-2.05	27.13
Industrial production	-1.04***	-2.68	20.92
Unemployment	0.22**	2.22	18.06
Equity issuance	-4.73**	-2.46	29.35
Business inventory	-0.57***	-3.49	58.52
Capacity utilization	-0.72***	-2.30	20.00

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