OS.1. **Supplement to Section 3 – Model extensions**

**OS.1.1. Intermediate goods**

**Environment.** We now consider an environment in which final goods are produced using capital and intermediate goods as inputs. For simplicity, labor is no longer a factor of production. The production of final goods is given by the production function \((\theta_{j,s}K_{j,s})^aX_{j,s}^{1-a}\), where \(X_{j,s}\) denotes the use of intermediate goods by firm \(j\) in state \(s\). Let \(Q\) denote the price of intermediate goods, then we obtain expressions for the demand for intermediates and final-good producers profits that are analogous to the ones with labor input:

\[
X_{j,s} = \left[1 - \frac{\alpha}{Q_s}\right]^\frac{1}{\alpha} \theta_{j,s}K_{j,s}, \quad \pi_{j,s} = \alpha \theta_{j,s} \left[1 - \frac{\alpha}{Q_s}\right]^{1-a}. \tag{OS.1}
\]

Intermediate goods are produced using a decreasing returns to scale technology. In particular, to produce \(X_s\) units of the intermediate good, \(\frac{X_s^{1+\nu}}{1+\nu}\) units of the final good are needed, where \(\nu > 0\). The problem of the intermediate-goods firm is:

\[
\pi_{X,s} = \max_{X_s} \left[Q_sX_s - \frac{X_s^{1+\nu}}{1+\nu}\right]. \tag{OS.2}
\]

We allow for a flexible ownership structure, assuming that a fraction \(\omega_{X,1}\) of the profits of
the intermediate-goods sector goes to investors and the fraction \(1 - \omega X, I\) goes to hand-to-mouth intermediate-goods entrepreneurs.

The first-order condition for the profit maximization problem (OS.2) is:

\[ Q_s = X_s^\nu. \]  \hspace{1cm} (OS.3)

Notice that the parameter \(\nu\) is the inverse supply elasticity of intermediate goods. The market-clearing condition for intermediate goods is given by \(\sum_j X, s = X_s\). Plugging the demand and supply for intermediate goods into the market clearing condition, we obtain

\[ X_s = (1 - \alpha) \frac{1}{1+\nu} (\Theta K_s)^{\frac{\alpha}{\alpha + 1}}, \quad Q_s = (1 - \alpha) \frac{\nu}{1+\nu} (\Theta K_s)^{\frac{\alpha}{\alpha + 1}}. \]  \hspace{1cm} (OS.4)

The profit of intermediate-goods producers is given by

\[ \pi_{X,s} = \frac{\nu}{1+\nu} (1 - \alpha) \frac{1+\nu}{\alpha + 1} (\Theta K_s)^{\frac{\alpha(1+\nu)}{\alpha + 1}}. \]  \hspace{1cm} (OS.5)

The return on assets of a final-good producer can be written as

\[ R_{j,s} = 1 - \delta + \alpha \theta_{j,s} (1 - \alpha) \frac{1+\nu}{\alpha + 1} (\Theta K_s)^{\frac{\alpha(1+\nu)}{\alpha + 1}}, \]  \hspace{1cm} (OS.6)

and, as in the main text, \(R_{j,s}^a\) denotes its average over \(P_i\).

Notice that as \(\nu \to \infty\), we recover the expression we obtained for the case with a inelastic labor supply in the main text.

**Idiosyncratic risk externalities.** Consider the impact on the welfare of investors of a perturbation of investment

\[ V(\Delta) = u \left( E_0 - \sum_{k=0}^{1} I_k(\Delta) \right) + \beta E \left[ u \left( \frac{R_{i,s}^a(\Delta) K_s + \omega X, I \pi_{X,s} + T_s}{N} \right) \right], \]  \hspace{1cm} (OS.7)

where

\[ T_s = (1 - \omega X, I) \pi_{X,s} - C_{X,s}, \]  \hspace{1cm} (OS.8)
and $C_{X,s}$ denotes the consumption by intermediate-goods entrepreneurs in laissez-faire.

The derivative of $V(\Delta)$ is given by

$$V'(0) = \beta \mathbb{E} \left[ u'(C_{i,s}) \left( \frac{\partial R_{i,s}^a K_s}{\partial \Delta} \frac{1}{N} + \frac{\partial \pi_{X,s}}{\partial \Delta} \frac{1}{N} \right) \right]. \tag{OS.9}$$

The derivative of the ROA and intermediate-goods profits with respect to $\Delta$ are

$$\frac{\partial R_{i,s}^a}{\partial \Delta} K_s \bigg|_{\Delta=0} = -\frac{\nu (1 - \alpha)}{\alpha + \nu} \Theta_K (\Theta K_s)^{(\alpha - 1)\nu} (\kappa_0 + \kappa_1 \psi_1) N \tag{OS.10}$$

$$\frac{\partial \pi_{X,s}}{\partial \Delta} \bigg|_{\Delta=0} = \frac{\nu (1 - \alpha)}{\alpha + \nu} \Theta (\Theta K_s)^{(\alpha - 1)\nu} (\kappa_0 + \kappa_1 \psi_1) N. \tag{OS.11}$$

Hence, we can write the derivative of the value function as

$$V'(0) = -\frac{\nu (1 - \alpha)}{\alpha + \nu} \beta \mathbb{E} \left[ \text{Cov}_{s} \left( u'(C_{i,s}), R_{i,s}^a \right) (\kappa_0 + \kappa_1 \psi_1) \right]. \tag{OS.12}$$

The expression above is analogous to the one we derived in the main text. The only difference is the constant of proportionality which is not $1 - \alpha$ but instead $\frac{\nu (1 - \alpha)}{\alpha + \nu}$. Hence, allowing for an elastic response of the variable input dampens the effect. For instance, if we set $\nu = 1$ and $\alpha = 0.3$, this implies a reduction in the effect of roughly 23%. The next section shows that this reasoning follows through when labor is itself an intermediate input in elastic supply.

OS.1.2. Elastic labor supply

Let us now consider a version of the model with an elastic labor supply. Let worker utility in state $s \in S$ be given by

$$u_{w,s} = U \left( C_{w,s} - L_{s}^{\frac{1+\nu}{1+\nu}} \right), \tag{OS.13}$$

for some strictly increasing, concave function $U : \mathbb{R} \rightarrow \mathbb{R}.$ Workers solve, state by state,

$$v_{w,s} := \max_{\left\{ C_{w,s}, L_s \right\}} C_{w,s} - L_{s}^{\frac{1+\nu}{1+\nu}}, \tag{OS.14}$$

s.t. $C_{w,s} = W_s L_s - T_s$.

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1This GHH preference specification (after Greenwood et al., 1988) makes labor supply free of wealth effects.
Labor supply is then given by $W_s = L_s^{\nu}$, with the parameter $\nu$ representing the inverse labor supply elasticity. Labor market equilibrium requires that

$$L_s = (1 - \alpha)\frac{1}{\nu + \alpha} (\Theta K_s)^{\frac{\nu + \alpha}{\nu}} \quad \text{and} \quad W_s = (1 - \alpha)\frac{\nu}{\nu + \alpha} (\Theta K_s)^{\frac{\nu}{\nu + \alpha}}.$$  \hspace{1cm} (OS.15)

As a consequence, the value achieved in problem OS.14 is $v_{W,s} = \frac{\nu}{1 + \nu} (1 - \alpha)\frac{1}{\nu + \alpha} (\Theta K_s)^{\frac{\nu + \alpha}{\nu}}$ and profits are $\pi_{j,s} = \alpha \theta_{j,s} (1 - \alpha) (\frac{1}{\nu + \alpha}) (\Theta K_s)^{\frac{\nu + \alpha}{\nu}}$. The former repeats the expression obtained for intermediate-goods producer profits, while the latter replicates the expression for the profits of final-good producers from the previous section. So, returns on assets are the same as before.

The impact on the welfare of investors of a perturbation of investment is

$$V(\Delta) = u \left( E_0 - \sum_{k=0}^{1} I^k(\Delta) \right) + \beta \mathbb{E} \left[ u \left( \frac{R^a_{j,s}(\Delta)K_s + T_s}{N} \right) \right],$$  \hspace{1cm} (OS.16)

where $T_s = \frac{\nu}{1 + \nu} (1 - \alpha)\frac{1}{\nu + \alpha} (\Theta K_s)^{\frac{\nu + \alpha}{\nu}} - v_{W,s}$ and $U(v_{w,s})$ is the state-contingent utility obtained by workers under laissez-faire. Notice that the tax $T_s$ only differs from its counterpart from the previous extension by a constant. We then recover Eq. OS.12 as the welfare gain from an investment perturbation.

Therefore, an elastic labor supply dampens the welfare gain of investment perturbations by a factor of $\frac{\nu}{\alpha + \nu}$. Intuitively, first, idiosyncratic risk externalities work through any input prices. Second, the more hours supplied respond to possible wage changes, the lower is the level and the smaller is the cyclicality of idiosyncratic risk externalities.

**OS.1.3. CES production function**

We now assume that capital and labor are combined according to a CES production function. The problem of a firm in period 1 is then given by

$$\max_{L} \left[ \alpha(\theta_{j,s}K_{j,s})^{\frac{\nu}{\nu + \alpha}} + (1 - \alpha)L^{\frac{\nu}{\nu + \alpha}} \right]^{\frac{\nu + \alpha}{\nu}} - W_s L.$$  \hspace{1cm} (OS.17)
The demand for labor is given by

\[ W_s = (1 - \alpha) \left( \alpha \left( \frac{\theta_{j,s}K_{j,s}}{L_{j,s}} \right)^{\frac{\epsilon - 1}{\epsilon}} + 1 - \alpha \right)^{\frac{1}{1 - \epsilon}}. \] (OS.18)

The (effective) capital-labor ratio is then equalized across firms. Profit per unit of capital for firm \( j \) can be written as

\[ \pi_{j,s} = \alpha \theta_{j,s} W_s (\Theta K_s)^{-\frac{1}{\epsilon}}. \] (OS.19)

The wage and profit per unit of capital can be written as

\[ W_s = (1 - \alpha) \left( \alpha (\Theta K_s)^{\frac{\epsilon - 1}{\epsilon}} + 1 - \alpha \right)^{\frac{1}{1 - \epsilon}}, \quad \pi_{j,s} = \alpha \theta_{j,s} \left[ \alpha (\Theta K_s)^{\frac{\epsilon - 1}{\epsilon}} + 1 - \alpha \right]^{\frac{1}{1 - \epsilon}} (\Theta K_s)^{-\frac{1}{\epsilon}}. \] (OS.20)

The derivative of the wage with respect to \( K_s \) is given by

\[ \frac{\partial W_s}{\partial K_s} = (1 - \alpha) \frac{\alpha \Theta}{\epsilon} \left[ \alpha (\Theta K_s)^{\frac{\epsilon - 1}{\epsilon}} + 1 - \alpha \right]^{\frac{1}{1 - \epsilon}} (\Theta K_s)^{-\frac{1}{\epsilon}}, \] (OS.21)

and the derivative of \( \pi_{j,s} \) is given by

\[ \frac{\partial \pi_{j,s}}{\partial K_s} = -(1 - \alpha) \frac{\alpha \theta_{j,s}}{\epsilon} \left[ \alpha (\Theta K_s)^{\frac{\epsilon - 1}{\epsilon}} + 1 - \alpha \right]^{\frac{1}{1 - \epsilon}} (\Theta K_s)^{-\frac{1}{\epsilon}}. \] (OS.22)

Following similar steps to the case with Cobb-Douglas production function, we find that the derivative of \( V(\Delta) \) is given by

\[ V'(0) = \beta \mathbb{E} \left[ u'(C_{i,s}) \left( \frac{\partial R_{i,s}^a K_s}{\partial \Delta N} + \frac{\partial W_s}{\partial \Delta N} \right) \right] \] (OS.23)

\[ = -\beta \mathbb{E} \left[ \frac{1 - \bar{\alpha}_s}{\epsilon} \text{Cov}_s(C_{i,s}^{\gamma} R_{i,s}^a) (\kappa_0 + \kappa_1 q_1^1) \right], \] (OS.24)

where \( 1 - \bar{\alpha}_s \equiv (1 - \alpha) \left[ \alpha (\Theta K_s)^{\frac{\epsilon - 1}{\epsilon}} + 1 - \alpha \right]^{-1} \) is the labor share in state \( s \).

We obtain two differences with respect to the formula in the baseline model. First, the labor share varies across states in the CES case. Second, the welfare impact of the intervention is amplified if the elasticity of substitution \( \epsilon \) is less than one, while the effect is dampened if \( \epsilon > 1 \). For instance, for an estimate of 0.7 from Oberfield and Raval (2021), one obtains an amplification of OS.5.
around 40%.

OS.1.4. **Endogenous Participation**

We consider next the case in which the participation set \( P \) is endogenous. For simplicity, we study the case in which there is a continuum of firms and the productivity distribution is derived from the gamma process as introduced in Section A.1.

Investors can now choose any level of diversification, \( \phi \in [0,1] \), subject to a cognitive cost. This cost can reflect the burden of information acquisition and processing or simply a disutility associated with having to pay attention to a larger set of firms.

Formally, we introduce the cost function \( I : [0,1] \to \mathbb{R} \), which is increasing, convex, and satisfies \( I'(0) = 0 \) and \( \lim_{\phi \to 1} I'(\phi) = \infty \). The cognitive cost then increases with the fraction of firms the investor pays attention to.

The investor’s problem can now be written in two steps, as a nested optimization problem. First, the optimal portfolio and savings choice for a given participation set. Denote the value function obtained at this stage by \( W(\phi) \). Second, as the outer part of the nested problem, the market participation choice consists of maximizing \( W(\phi) - I(\phi) \).

In equilibrium, an equally weighted portfolio within \( P \) is optimal, and \( W(\phi) \) satisfies

\[
W(\phi) = \max_{C_{i,0}} u \left( C_{i,0} \right) + \beta \mathbb{E} \left[ u \left( (E - C_{i,0}) R^{\phi}_{i,s} \bar{q}_{s} \right) \right],
\]

where \( R^{\phi}_{i,s} = \left( 1 - \delta + \alpha \Theta^{\phi} K_{s}^{\alpha - 1} \frac{x_{i,s}}{\phi} \right) \) and \( x_{i,s} \sim \text{Be} \left( \nu \phi, \nu (1 - \phi) \right) \).

Consider next the welfare impact of a given perturbation:

\[
V(\Delta) = \max_{\phi} u \left( E_{0} - \sum_{s=0}^{1} I^{k}(\Delta) \right) + \beta \mathbb{E} \left[ u \left( \left( R^{\phi}_{i,s}(\Delta, \phi) K_{s}(\Delta) + \tau_{s}(\Delta) \right) \right) \right] - I(\phi).
\]

Applying an envelope argument on \( \phi \), the derivative \( V'(0) \) is identical to the one in the case where \( \phi \) is exogenous. Hence, our results apply directly to this case as well. Moreover, the value of \( \phi \) that solves the problem above for \( \Delta = 0 \) coincides with the one in laissez-faire. Therefore, absent the use of additional instruments (i.e. around laissez-faire), the planner has no incentives to distort the investor’s participation decision.
OS.1.5. Epstein-Zin preferences

We assume now that investors have Epstein-Zin preferences with elasticity of intertemporal substitution $\eta^{-1}$ and coefficient of risk aversion $\gamma$. The investor’s problem is

$$\max_{C_{i,0}, \{\omega_{i,j}\}} \frac{C_{i,0}^{1-\eta}}{1-\eta} + \beta \frac{U_{1,1}^{1-\eta}}{1-\eta},$$ \hspace{1cm} (OS.27)

subject to a non-negativity condition on consumption, limited participation $\omega_{i,j} = 0$, for $j \notin P_i$, and

$$C_{i,s} = R_{i,s}^p (E_0 - C_{i,0}), \quad R_{i,s}^p := \sum_{j \in P_i} \omega_{i,j} \frac{R_{j,s}^a K_{j,s}}{P_j}, \quad U_{i,1} := E[C_{i,s}^{1-\gamma}]^{\frac{1}{1-\gamma}},$$ \hspace{1cm} (OS.28)

$$\sum_{j \in J} \omega_{i,j} = 1,$$

where $R_{i,s}^p$ is the return on investor $i$’s portfolio and $P_j$ is the price of a share in firm $j$.

The optimality conditions for initial consumption and portfolio weights are given by

$$1 = E[M_{i,s} R_{i,s}^p], \quad P_j = E[M_{i,s} R_{j,s}^a K_{j,s}],$$ \hspace{1cm} (OS.29)

for all $j \in P_i$, where $M_{i,s}$ denotes the stochastic discount factor (SDF) for investor $i$ and it is given by

$$M_{i,s} = \beta E \left[ \left( \frac{C_{i,s}}{C_{i,0}} \right)^{1-\gamma} \right]^{\frac{\gamma-\eta}{\gamma-\eta}} \left( \frac{C_{i,s}}{C_{i,0}} \right)^{-\gamma}.$$ \hspace{1cm} (OS.30)

Asset pricing. The log SDF can be written as

$$m_{i,s} = \log \beta + \frac{\gamma - \eta}{1 - \gamma} \log E[e^{(1-\gamma)c_{i,s}}] - \gamma c_{i,s} + \eta c_0$$ \hspace{1cm} (OS.31)

$$\approx \log \beta - \gamma (c_{i,s} - E[c_{i,s}]) - \eta (E[c_{i,s}] - c_0) + (\gamma - \eta)(1 - \gamma) \frac{Var[c_{i,s}]}{2}.$$ \hspace{1cm} (OS.32)

The risk-free interest rate is given by

$$r_f := -\log E[\exp(m_{i,s})] \approx -E[m_{i,s}] - \frac{Var[m_{i,s}]}{2}$$ \hspace{1cm} (OS.33)

$$= -\log \beta + \eta \left( E[c_{i,s}] + \frac{\sigma_c^2}{2} - c_0 \right) - \frac{\gamma(1 + \eta)}{2} \left( Var[c_s] + \phi_u E[\psi^2 \sigma^2 \theta] \right).$$ \hspace{1cm} (OS.34)
The excess return on firm \( j \) is given by

\[
1 = \mathbb{E}[M_{i,s}R_{j,s}] \Rightarrow 0 = \mathbb{E}[m_{i,s}] + \mathbb{E}[r_{j,s}] + \frac{1}{2} \text{Var}[m_{i,s}] + \text{Cov}(m_{i,s}, r_{j,s}) + \frac{1}{2} \text{Var}[r_{j,s}],
\]  

(OS.35)

where \( r_{j,s} := \log R_{j,s} \).

Rearranging the expression above, we obtain

\[
\mathbb{E}[r_{j,s}] - r_f + \frac{\sigma_r^2}{2} = \gamma \text{Cov}(c_{i,s}, r_{j,s}) = \gamma \text{Cov}(\tilde{c}_s, \tilde{r}_s) + \gamma \phi_u \mathbb{E}[\sigma_s^2],
\]

(OS.36)

using the fact that \( c_{s,i} = \tau_s + \psi_s \tilde{c}_{i,s}, r_{j,s} = \tau_s + \psi_s \theta_{j,s} \), and \( \text{Cov}(\tilde{\theta}_p, \theta_{j,s}) = \phi_u \sigma_{\theta}^2 \).

**Investment.** We consider the impact of idiosyncratic risk on investment in the special case with no aggregate risk, \( \phi_s^1 = 1 \) for all \( s \in S \). The Euler equation for investment is given by

\[
1 = \beta \mathbb{E} \left[ \left( \frac{C_{i,s}}{C_0} \right)^{1-\gamma} \right]^{1-\eta} \mathbb{E} \left[ \left( \frac{C_{i,s}}{C_0} \right)^{-\gamma} R_{j,s} \right].
\]

(OS.37)

In the benchmark case \( \sigma_\theta = 0 \), we obtain the standard Euler equation

\[
1 = \beta \left( \frac{C^*_i}{C^*_0} \right)^{-\eta} R^*_j.
\]

(OS.38)

Computing the asymptotic expansion of the investment Euler equation, we obtain

\[
0 = \frac{\gamma(\eta - 1)}{2} (\psi^*)^2 \phi_u - \eta \frac{\hat{C}_1}{C_1^*} + \frac{\hat{R}^a}{R^a,*} + \eta \frac{\hat{C}_0}{C_0^*}.
\]

(OS.39)

Rearranging the expression above, we obtain

\[
\eta \left( \frac{\hat{C}_1}{C_1^*} - \frac{\hat{C}_0}{C_0^*} \right) - \frac{\hat{R}^a}{R^a,*} = \frac{\gamma(\eta - 1)}{2} (\psi^*)^2 \phi_u.
\]

(OS.40)

The left-hand side is increasing in the amount of investment. The right-hand side is positive if \( \eta > 1 \).
Idiosyncratic risk externalities. Welfare given intervention of size $\Delta$ with Epstein-Zin preferences is given by

$$V(\Delta) = \left( \frac{E_0 - \sum_{k=0}^{1} I^k(\Delta)}{1 - \eta} \right)^{1 - \eta} + \frac{\beta U_1(\Delta)^{1 - \eta}}{1 - \eta}, \quad (OS.41)$$

where

$$U_1(\Delta) = \mathbb{E} \left[ \left( \frac{R_{i,s}(\Delta)K_s(\Delta) + (1 - \alpha)(\Theta K_s(\Delta))^{\alpha} - \bar{C}_{w,s}}{N} \right)^{1 - \gamma} \right]^{\frac{1}{1 - \gamma}}. \quad (OS.42)$$

Taking the derivative of $V(\Delta)$, we obtain

$$V'(0) = -C_0^{-\eta}(\kappa_0 + \kappa_1) + \beta U_1^{-\eta} \mathbb{E} \left[ C_{i,s}^{-\gamma} \left( R_{i,s}(\kappa_0 + \kappa_1 \phi_s^\gamma) + \frac{\partial R_{i,s}^{\alpha} K_s}{\partial \Delta} \frac{K_s}{N} + \alpha (1 - \alpha) \Theta^\alpha K_s^{\alpha - 1} (\kappa_0 + \kappa_1 \phi_s^\gamma) \right) \right]$$

$$= \beta U_1^{-\eta} \mathbb{E} \left[ C_{i,s}^{-\gamma} \left( -\alpha (1 - \alpha) (\bar{R}_{i,s} - \Theta) \Theta^{\alpha - 1} K_s^{\alpha - 1} \right) (\kappa_0 + \kappa_1 \phi_s^\gamma) \right]$$

$$= -(1 - \alpha) C_0^{-\eta} \mathbb{E} \left[ \text{Cov}_s(M_{i,s}, R_{i,s}^{\alpha}) (\kappa_0 + \kappa_1 \phi_s^\gamma) \right]. \quad (OS.43)$$

The covariance above can be written

$$\text{Cov}_s(M_{i,s}, R_{i,s}^{\alpha}) = -\gamma M_{i,s}^{\alpha} R_{i,s}^{\alpha} \phi_u(\psi_s^\gamma)^2 \sigma_\theta^2 + o(\sigma_\theta^2). \quad (OS.44)$$

The expression for $V'(0)$ is then given by

$$\frac{V'(0)}{C_0^{-\eta}} = (1 - \alpha) \mathbb{E}^Q \left[ \gamma \phi_u \sigma_\theta^2 (\kappa_0 + \kappa_1 \phi_s^\gamma) \right] + o(\sigma_\theta^2). \quad (OS.45)$$

The expression above coincides with the one obtained in the CRRA case. It remains to show that the idiosyncratic variance risk premium is positive:

$$\mathbb{E}^Q[\sigma_\theta^2] - \mathbb{E}[\sigma_\theta^2] = \text{Cov} \left( M_{i,s} R_{i,s}^{\alpha}, \sigma_\theta^2 \right)$$

$$= \beta U_1^{-\eta} C_0^{\gamma} \text{Cov} \left( C_{i,s}^{-\gamma} R_{i,s}^{\alpha}, \sigma_\theta^2 \right) > 0, \quad (OS.46)$$

as $C_{i,s}^{-\gamma} R_{i,s}^{\alpha}$ and $\sigma_\theta^2$ are both decreasing in $K_s$. 

OS.9
Supplement to Section 4

OS.2. Alternative interpretations of underinvestment and excessive risk-taking

We provide two alternative interpretations of the welfare impact of underinvestment. The first interpretation considers a "social insurance" effect. Notice that expression (36) can be written as

\[
IRE = -1 + \beta \frac{u'(C_{i,s})}{u'(C_0)} R_{i,s}^a - (1 - \alpha) \beta \frac{u'(C_{i,s})}{u'(C_0)} \left( R_{i,s}^a - \overline{R}^a_s \right)
\]

The first term captures the private trade-off which, by the Euler equation, is equal to zero. The planner internalizes an additional effect that acts as insurance: it is negative if the firm’s profitability is above average and positive otherwise. The planner effectively perceives the return risk as only a fraction \( \alpha \) of what private investors perceive. The externality value of 3% can then be interpreted as a price of three cents for an "insurance policy" of \( 1 - \alpha \) for each dollar of notional value.

The second interpretation is that the social cost of capital is smaller than the private cost. As seen above, the social value of one unit of capital is

\[
Q^{\text{social}} = 1 + IRE = 1 + \beta \frac{u'(C_{i,s})}{u'(C_0)} \left( \alpha R_{i,s}^a + (1 - \alpha) \overline{R}^a_s \right)
\]

A high social value of capital implies an expected return on the investment perceived by the planner that is smaller than the private return. As the expected return on the firm, or equivalently its cost of capital, is related to the amount of capital in the economy, the capital stock seems too low from a planner’s perspective. Assuming for simplicity there is no aggregate risk, we can relate the capital stock to the cost of capital using (29):

\[
\alpha \Theta^a K^{a-1} \approx r_f + \gamma \Phi_0 \sigma^2 + \delta \Rightarrow \frac{\Delta Y}{Y} \approx -\frac{\alpha}{1 - \alpha} \frac{\Delta r^{cc}}{r^{cc} + \delta}.
\]

The above expression signifies the impact on capital stock of a reduction in the cost of capital. Using the estimate of 18% for the user cost \( r^{cc} + \delta \) by Barro and Furman (2018), a reduction of 3% in \( r^{cc} \) would imply an increase in aggregate output of 8%.

\[\text{To put these numbers in perspective, Barro and Furman (2018) expected, as a consequence of the 2017 tax reform,}\]
Also, we provide another perspective for interpreting the welfare impact of excessive risk-taking. Notice that we can write the expression for the idiosyncratic risk externality on aggregate risk-taking as follows

\[
IRE_{\chi} = -\mathbb{E}\left[\beta \frac{u'(C_{i,s})}{u'(C_0)} R_{i,s} \frac{\phi^{e}_s}{\sigma_{\phi}}\right] + (1 - \alpha)\mathbb{E}\left[\beta \frac{u'(C_{i,s})}{u'(C_0)} (R_{i,s} - \bar{R}_s) \frac{\phi^{e}_s}{\sigma_{\phi}}\right],
\]

(OS.49)

where \(\sigma_{\phi} \equiv \sqrt{\text{Var}[\phi_{s}^e]}\).

The term capturing the private trade-off is equal to zero. Expanding the first term to account for covariances, we obtain, after some rearrangements, an expression representing the share invested in the risky technology

\[
\chi = \frac{\mathbb{E}[\phi^{e}_s]}{\gamma \sigma_{\phi}^{2}} - \left(1 - \frac{1}{\gamma}\right) \left[\frac{\text{Cov}(\log R_{s}^{2}, \phi^{e}_s)}{\sigma_{\phi}^{2}} - \frac{\gamma \phi_{u}\text{Cov}(\sigma_{s}^{2}, \phi^{e}_s)}{2 \sigma_{\phi}^{2}}\right].
\]

(OS.50)

Analogous to the financial portfolio decisions studied in Merton (1973), we can divide the share invested in the risky technology into a myopic and a hedging component. The myopic component captures the usual (static) risk-return trade-off, while the second component captures the fact that the ROA varies across states. Importantly, the covariance between idiosyncratic variance and the payoff of the risky technology is negative, consistent with the result expressed in Proposition 1, where we find that the presence of idiosyncratic risk reduces aggregate risk-taking relative to a first-best economy.

In contrast to private agents, a social planner internalizes the fact that an increase in aggregate risk-taking would raise idiosyncratic volatility in bad times and reduce it in good times. This makes \(\text{Cov}(\sigma_{s}^{2}, \phi^{e}_{s})\) effectively more negative, indicating that the planner would choose a smaller share \(\chi\) than the one chosen by private agents.

Here the externality can be interpreted as reducing the effective Sharpe ratio perceived by the planner. The planner values the risky investment as if the Sharpe ratio on the risky technology is effectively \(\frac{\mathbb{E}[\phi^{e}_s]}{\sigma_{\phi}} - IRE_{\chi}\). Given an externality value of 1.2% and Sharpe ratio of, say, 0.30, the social Sharpe ratio is \(\frac{0.012}{0.30} = 4\%\) below the private one.

if the provision were made permanent, an expansion of aggregate output of roughly 5%.

OS.11
OS.3. **Derivation of the share invested in the risky technology**

The optimality condition for the risky technology can be written as

$$
E \left[ \beta \frac{u'(C_{i,s})}{u'(C_0)} R_{j,s} \varphi_s^e \right] = 0 \Rightarrow E[\varphi_s^e] = -\text{Cov} \left( E_s \left[ \beta \frac{(C_{i,s})^{-\gamma}}{C_0^{-\gamma}} R_{j,s} \right], \varphi_s^e \right). \quad (OS.51)
$$

From equation (25) and $C_s = R_s^a K_s / N$, we obtain the approximate expression

$$
E[\varphi_s^e] \approx \text{Cov} \left( \gamma k_s + (\gamma - 1) \log R_s^a - \frac{\gamma(\gamma - 1)}{2} \phi_u \sigma_s^2, \varphi_s^e \right). \quad (OS.52)
$$

Using $k_s = \log(1 + \chi \varphi_s^e) + \log I \approx \chi \varphi_s^e + \log I$, we obtain

$$
E[\varphi_s^e] = \chi \gamma \sigma^2 + (\gamma - 1) \text{Cov} \left( \log R_s^a - \frac{\gamma}{2} \phi_u \sigma_s^2, \varphi_s^e \right). \quad (OS.53)
$$

Rearranging the expression above, we can solve for $\chi$

$$
\chi = \frac{E[\varphi_s^e]}{\gamma \sigma^2} - \left( 1 - \frac{1}{\gamma} \right) \left[ \text{Cov} \left( \log R_s^a, \varphi_s^e \right) - \frac{\gamma}{2} \frac{\phi_u \sigma_s^2}{\sigma^2} \right]. \quad (OS.54)
$$

**Supplement to Section 5**

OS.4. **Equilibrium Definition**

An allocation is given by consumption and portfolio decisions for investors, $(C_{i,0}, \Psi_i, \{\omega_{i,j}\}_{j \in \mathcal{J}})$ for $i \in \mathcal{I}$, investment and labor demand decisions for firms, $(I_{j,0}^0, I_j, L_{i,j}, L_{h,j})$ for $j \in \mathcal{J}$, and workers’ consumption, $(C_{w,l}, C_{w,h})$. A competitive equilibrium is defined as an allocation, asset prices $(P_{e,j}, P_{d,j})$ for each firm $j$, and wages $W_s$ for each state $s$ such that:

i. Consumption and portfolio decisions, $(C_{i,0}, \Psi_i, \{\omega_{i,j}\}_{j \in \mathcal{J}})$, solve Problem (D.3) for each $i \in \mathcal{I}$.

ii. Investment and debt issuance decisions solve Problem (D.1) given $M_{j,s} = N_j^{-1} \sum_{i \in \mathcal{P}_j} \beta \frac{u'(C_{i,s})}{u'(C_0)}$, where $N_j = \# \{i | j \in \mathcal{P}_i\}$, and labor demand is given by Eq. 5.
iii. Asset markets for equity and debt clear, i.e.,

\[ \Psi (E_0 - T - C_0) = P_e \] (OS.55)

and

\[ (1 - \Psi) (E_0 - T - C_0) = IP_{dd}. \] (OS.56)

iv. The government’s budget at \( t = 0 \) is balanced, with \( T = \tau_d D \).

v. Worker consumption in each state \( s \in S \) is given by \( C_{w,s} = W_s - T_{w,s} \).

vi. The labor market clears at each \( s \in S \), i.e.

\[ \sum_{j \in J} L_{j,s} = 1. \]

vii. Consumption goods markets clear, i.e.,

\[ \sum_{i \in I} C_{i,0} + \sum_{j \in J} (I^0_j + I^1_j) = NE_0, \] at \( t = 0 \) and at each \( s \in S \)

\[ C_{w,s} + \sum_{i \in I} C_{i,s} = \sum_{j \in J} (Y_{s,j} + (1 - \delta)K_{s,j}). \] (OS.57)

OS.5. Implementation with regulatory instruments

**Definition 1.** An allocation features an implicit investment subsidy whenever, for each \( j \in J \),

\[ 1 \geq \mathbb{E} \left[ M_{j,s} \left( R^2_{j,s} (1 + \chi \varphi_s^c) \right) \right]. \] (OS.58)

An allocation features an implicit risk-taking tax whenever, for each \( j \in J \)

\[ \mathbb{E} \left[ M_{j,s} R^2_{j,s} \varphi_s^1 \right] \geq \mathbb{E} \left[ M_{j,s} R^2_{j,s} \varphi_s^0 \right]. \] (OS.59)

**Proposition 1** (Implementation). A symmetric allocation \( \left( C_0, \{ I^k \}_{k=0,1}, \{ \omega_{i,j} \}_{j \in J}, C_{i,s}, C_{w,s} \right) \) with an implicit investment subsidy and an implicit risk-taking tax can be implemented with a set of subsidies \( \{ \tau^k_s, \tau^d \} \) and financial regulation with risk weights \( \{ \omega^k \}_{k=0,1} \) whenever it satisfies:

i. Feasibility with \( E_0 = \sum_k I^k + C_0 \) and \( K_s = N \sum_k \varphi_s^1 I^k \).

ii. Equally weighted portfolios within each agent’s participation set, i.e., \( \omega_{i,j} = \frac{1}{P}, \) whenever \( j \in P_1 \), and \( \omega_{i,j} = 0 \), otherwise.
iii. The distribution of $t = 1$ consumption is given by

$$C_{i,s} = \sum_j \omega_{i,j} \frac{R_{i,s}^a K_{j,s}^a}{\sum_k l^k s} K_s + \frac{T_{w,s}}{N},$$  \hspace{1cm} (OS.60)

and

$$C_{w,s} = (1 - \alpha) \Theta^a K_s^a - T_{w,s},$$  \hspace{1cm} (OS.61)

for some $T_{w,s}$.

Furthermore, $\tau^d > 0$ and $\omega_1 > \omega_0$.

Proof of Proposition 1. Take an allocation that satisfies the requirements of the proposition. Define $I = \sum I^k$ and $\chi = I^1/I$. Let $d = \frac{D}{I}$ and take any $0 < d \leq (1 - \delta) \left(1 - \chi \left(1 - \phi^1_l\right)\right)$. We verify that we can find a system of subsidies and risk weights that satisfies all the conditions for an equilibrium.

**Investor optimality.** From the investor’s side, we obtain, for savings,

$$1 = \beta E \left[ \frac{u'(C_{i,s})}{u(C_0)} R_{i,s} \right],$$

for the portfolio shares

$$\beta E \left[ \frac{u'(C_{i,s})}{u(C_0)} R_e \right] = \frac{1}{F_d} \beta E \left[ \frac{u'(C_{i,s})}{u(C_0)} \right],$$

where $R_e$ is the optimal equity portfolio’s (random) return. Together, these are equivalent to

$$P_e = \beta E \left[ \frac{u'(C_{i,s})}{u(C_0)} \right] (R_{i,s}^a (1 + \chi \phi^e_s) - d) I,$$ \hspace{1cm} (OS.62)

for each $j \in \mathcal{P}_i$ and

$$P_d = \beta E \left[ \frac{u'(C_{i,s})}{u(C_0)} \right].$$ \hspace{1cm} (OS.63)

**Firm optimality.** As discussed in Section 5, investment and capital structure decisions are made to maximize the joint surplus of the intermediary-firm relationship. We seek to construct an allocation in which the debt constraint in (D.2) is not binding in the firm’s problem. In such a situation,
the problem can be rewritten as

$$\max_{d, I, \chi \geq 0} \left\{ P_d \left( 1 + \tau^d \right) d - 1 + \mathbb{E} \left[ M_{j,s} \left( R_{j,s}^\alpha \left( 1 + \chi \varphi^e_s \right) - d \right) \right] \right\} I,$$

s.t.

$$1 - P_d d \geq \omega^0 (1 - \chi) + \omega^1 \chi.$$

Its first-order conditions give us, for $I$, $\chi$ and $d$, respectively,

$$P_d \left( 1 + \tau^d \right) d + \mathbb{E} \left[ M_{j,s} \left( R_{j,s}^\alpha \left( 1 + \chi \varphi^e_s \right) - d \right) \right] = 1,$$  \hspace{1cm} (OS.64)

$$\frac{\mathbb{E} \left[ M_{j,s} R_{j,s}^\alpha \varphi^e_s \right]}{\omega^1 - \omega^0} = \frac{\mu^rw}{I} \geq 0,$$  \hspace{1cm} (OS.65)

and

$$P_d \left( 1 + \tau^d \right) - \mathbb{E} \left[ M_{j,s} \right] = \frac{\mu^rw}{I} \geq 0.$$  \hspace{1cm} (OS.66)

Additionally, it is required that

$$1 - P_d d = \omega^0 (1 - \chi) + \omega^1 \chi,$$  \hspace{1cm} (OS.67)

and

$$(1 - \delta) \left( 1 - \chi \left( 1 - \varphi^1_L \right) \right) \geq d.$$  \hspace{1cm} (OS.68)

**Labor market equilibrium and worker consumption.** Similarly to laissez-faire, optimality and labor market clearing can be ensured under $W_s = (1 - \alpha) \Theta^s K_s^\alpha$. In the presence of the lump-sum tax, we have

$$C_{w,s} = (1 - \alpha) \Theta^s K_s^\alpha - T_{w,s}.$$  \hspace{1cm} (OS.69)

**Market-clearing for equity and debt.** Market clearing for equity requires that, for aggregates, both Eqs. (OS.55) and (OS.56) hold.
Verification of implementability. We seek to find $P_e, P_d, \tau^d, \{\omega^k\}$ that support the candidate allocation and $d > 0$ as an equilibrium. Notice first that, from (OS.62) and (OS.63), asset prices are given as a function of the allocation. Eq. (OS.63) together with Equation (OS.64) and the fact that $E [M_{j,s}] = P_d$ delivers

\[ \tau^d d + E \left[ M_{j,s} \left( R^a_{j,s} (1 + \chi \varphi^e_s) \right) \right] = 1, \tag{OS.70} \]

which pins down $\tau^d$. Notice that, because the allocation features an implicit investment subsidy, $\tau^d > 0$. Eq. (OS.66) implies that $\frac{\mu^w}{T} = \tau^d P_d \geq 0$. Then, we can use Eq. (OS.65) to establish that

\[ \omega^1 - \omega^0 = \frac{E \left[ M_{j,s} R^a_{j,s} \varphi^e_s \right]}{\tau^d P_d} \geq 0. \]

Lastly, we can obtain $\omega_0$ from Eq. (OS.67). Set $T = \tau^d d$ and use Eq. (OS.55) to solve for $\Psi$. It then follows that, adding (OS.55) and (OS.56), we obtain

\[ (E_0 - T - C_0) = P_e + IP_d d. \]

Using feasibility at date $t = 0$ and (OS.70), we verify that this equation holds, proving equality in Equation OS.56. \qed
References


