Online Appendix to

"Interest Rate Volatility, the Yield Curve, and the Macroeconomy"

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These notes supplement Joslin and Konchitchki (2017). Joslin and Konchitchki (2017) considers the residual variance – the portion of the variance of the five-year zero rate that is uncorrelated with the level, slope, and curvature of the yield curve. In the macro-finance model described in the paper, there are stochastic correlations among all of the risk factors.

Figure 1 plots the time series of the local correlation of the residual variance with the total variance of the 5-year yield for the $A_1(4)$ model. Note that a high correlation between the variance and residual variance indicates that the yield curve, as summarized by its level, slope, and curvature, explains a small part of the variation in the yield volatility. The correlation typically ranges from 40% to 60%, thus indicating that about half of the variation in the variance of the 5-year yield is accounted for by factors unrelated to the level slope and curvature of the yield curve. A notable exception it that in late 1998, around the time of the Russian default and the LTCM bailout, the model indicates that the variance was nearly perfectly correlated with the residual variance. This indicates that in this period movements in the volatility of the yield curve were almost completely uncorrelated to movements in the level, slope, and curvature of the yield curve.

In our model and its estimation, we study the properties of interest rate volatility as if interest rate volatility is a given construct. We also examine a source of time series variation in interest rate volatility. Because of the major role of the corporate sector in the macroeconomy including in affecting monetary policy (Fischer and Merton, 1984). As we describe, we employ a real options model of investment. In a real options model of investment, higher uncertainty can make real options more valuable and increase asset values with a concurrent increase in volatility. Given that higher uncertainty can be driven by higher cross-sectional dispersion in corporate unexpected performance, we operationalize the real options investment

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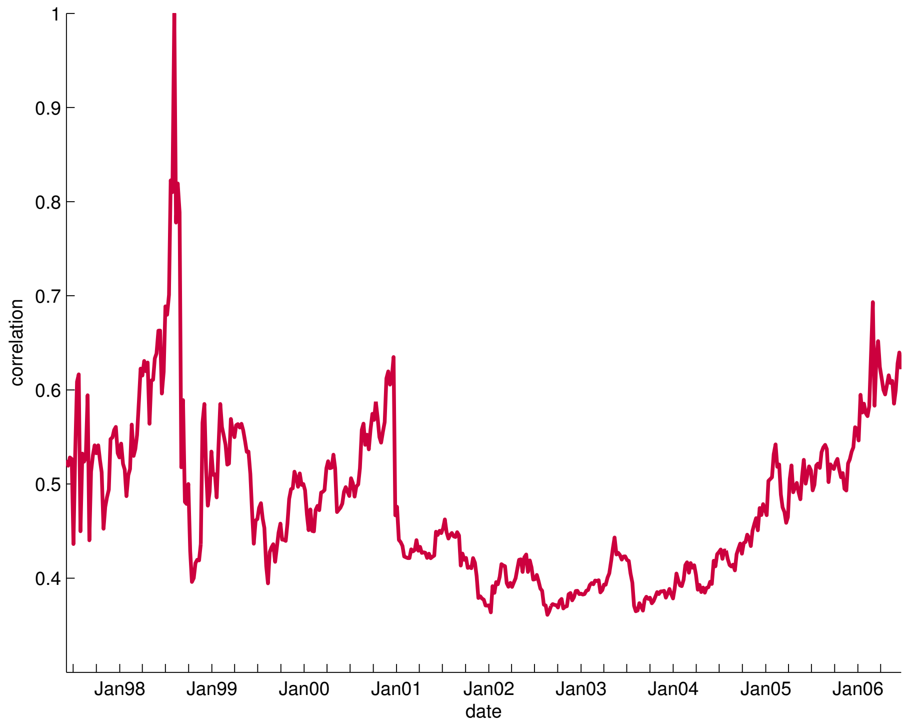

Figure 1: Correlation of variance with residual variance

This figure plots the correlation of the variance of the 5-year yield with the residual variance for the $A_1(4)$ model. The residual variance is defined as the risk which is locally uncorrelated with the 6-month, 2-year, and 10-year yields. A correlation of 1 between the variance and residual variance indicates that 6-month, 2-year, and 10-year yields are uncorrelated with volatility.

channel by examining how interest rate volatility is linked to cross-sectional dispersion in firms earnings information. Indeed, in standard economics models, both the risk free rate and the overall discount rate (or cost of capital) are endogenously determined in equilibrium. Higher uncertainty can lead to lower interest rates through the precautionary savings channel (decreased inflationary pressures stemming from reduced consumption and increased saving). On the other hand, higher uncertainty may lead to higher interest rates and higher cost of capital through changes in investment opportunity sets. In sum the empirical analysis focuses on how the corporate sector is linked to interest rate volatility.

First, Figure 2 plots time series of interest rate volatilities calculated as the monthly standard deviation of daily interest rate yields on Treasury bonds and bills for all horizons available from the Federal Reserve Board of Governors H.15 Reports. The figure shows that the volatilities across different horizons are highly correlated, with slightly higher volatilities for short-term maturity horizons. The figure also shows that volatilities spike around recessions, demonstrating the sensitivity of yields to economic uncertainty such as recessions.

The next analysis focuses on the accounting-volatility link, where again we operationalize the real options investment channel by examining how interest rate volatility is linked to cross- sectional dispersion in firms' earnings information. Cross-sectional dispersion in firms' earnings information is the standard deviation of each quarterly cross section of all quarterly earnings changes in the period. We measure accounting earnings for firm i in quarter q ($Earn_{i,q}$) as scaled quarterly income before extraordinary items, and earnings change ($\Delta Earn_{i,q}$) as the year-over-year change in $Earn_{i,q}$. To mitigate possible outlier effects, we trim each firm's $Earn_{i,q}$ and $\Delta Earn_{i,q}$ based on the top and bottom one percentile of each periodic cross-section. To avoid negative denominator problems, we scale earnings by sales.

Time series of dispersion in earnings information are calculated every month throughout the year based on the cross-sectional standard deviation across all firms with fiscal quarter ends falling in that month. We then estimate regression models of future one-month and two-months interest rate volatility for each horizon in the Fed H.15 reports on current period dispersion in earnings information.

Table 1 reports these regressions results. The major finding is that there is a strong link between corporate accounting information and future interest rate volatilities, especially over the short to middle terms. In particular, the cross-sectional earnings information dispersion is significantly tied to subsequent interest rate volatility,which is consistent with the real options investment channel.

We also conduct two additional analyses. First, in the regressions we employ additional subsequent horizons of up to four periods ahead. Second, we use IBES data to measure the dispersion in earnings information. In this case we obtain analyst earnings forecasts from the IBES Summary History Summary Statistics with Actuals (EPS for U.S. Region) dataset.

We use a firm's analyst median earnings-per-share consensus forecast as the analyst forecast of the firm, which is then multiplied by the number of shares outstanding, adjusted for stock splits and stock dividends, on the last day of the period for which the forecast is calculated. We then subtract the actual earnings minus its expectation for each firm. The inferences are unchanged as a result of these additional analyses.

The results of Joslin and Konchitchki (2017) (and related results of Cochrane and Piazzesi (2008), Ludvigson and Ng (2000), Rudebusch et al. (2006), Wright and Zhou (2009) and others) suggest that the cross-section of bond yields may poorly identify expected excess returns for holding long maturity bonds. This raises the question of whether expected excess returns may be accurately identified using all fixed income security prices – including fixed income derivatives. To see why it may be the case that expected excess returns are not identified from prices, consider the simple two-factor short rate model where the short rate, r_t, is simply the sum of observable inflation, i_t, and a nominal-real spread, s_t. Suppose that the state variable $X_t = (i_t, s_t)$ follows the diffusion

$$dX_t = \begin{bmatrix} .2 & 0 \\ 0 & .1 \end{bmatrix} \Big(\theta - X_t\Big)dt + \begin{bmatrix} \sigma_i & 0 \\ 0 & \sigma_s \end{bmatrix} dB_t^P \tag{1}$$

Suppose also that the degree to which agents are risk averse to inflation risk depends on the current level of inflation ($\lambda_t = \lambda_0 + \lambda_1 i_t$) so that under Q:

$$dX_t = \begin{bmatrix} .1 & 0 \\ 0 & .1 \end{bmatrix} \Big(\theta - X_t\Big)dt + \begin{bmatrix} \sigma_i & 0 \\ 0 & \sigma_s \end{bmatrix} dB_t^Q \tag{2}$$

In this case, risk-neutrally, i_t and s_t affect fixed income security prices only through their sum and as far as prices are concerned this is a one-factor model. However, knowing both i_t and s_t are informative for learning about expected excess returns for holding long maturity bonds. That is, fixed income security prices (both bonds and derivatives) *do not* determine the state of the economy and *do not* identify expected excess returns.

Indeed, Ludvigson and Ng (2000) find evidence that real and inflation risk factors have the ability to predict variation in bond excess returns above and beyond the level of interest rates. Excess returns will be weakly identified from bond prices when there is a risk factor (after a change of variables) which does not affect Q-expectation of future interest rates but does affect P-expectations of future interest rates. In the example, we change the variable to $Y_t = (r_t, i_t)$. Then fixing r_t and changing i_t do not affect future Q-expectations of interest rates but does

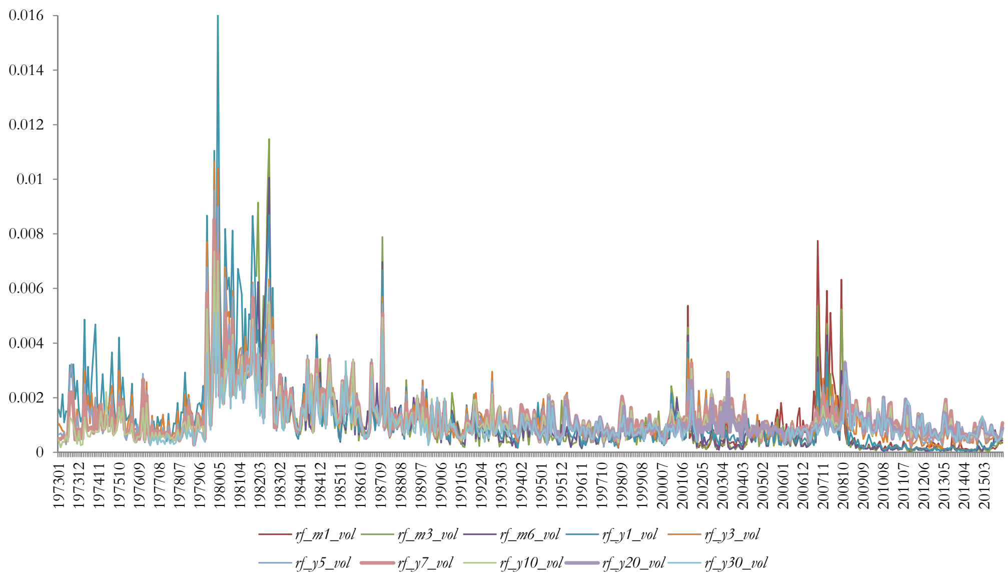

Figure 2: Time series of interest rate volatilities

This figure plots time series of yield volatilities of constant maturity Treasury bonds and bills for all horizons available from the Federal Reserve Board of Governors H15 Reports. We obtain data from the Federal Reserve Board of Governors H15 Reports available on Wharton Research Data Services (WRDS). We use all data with consecutive observations when considering the intersection of Fed's yields and Compustat's accounting earnings (January 1973 to January 2016). The available horizons are for the 1-month (rf_m1), 3-month (rf_m3), 6-month (rf_m6), 1-year (rf_y1), 3-year (rf_y3), 5-year (rf_y5), 7-year (rf_y7), 10-year (rf_y10), 20-year (rf_y20), and 30-year (rf_y30) yields. The suffix "_vol" refers to volatility, calculated as the monthly standard deviation of the daily interest rate yields during that month.

(a) Corporate earnings effect on one month ahead volatilities for different horizons of constant maturity Treasury bills and bonds

	Estimate	StdErr	t-statistic	Adj.R^2
1 Month	0.00052**	0.00025	2.11253	0.08378
3 Month	0.00044**	0.00020	2.19679	0.10889
6 Month	0.00027**	0.00011	2.45221	0.06559
3 Year	0.00018**	0.00005	3.78813	0.04870
5 Year	0.00015**	0.00005	3.01000	0.04082
7 Year	0.00013**	0.00005	2.55691	0.02877
10 Year	0.00011**	0.00004	2.53991	0.02220
20 Year	0.00007*	0.00004	1.93235	0.00712
30 Year	0.00005	0.00004	1.44384	0.00107

(b) Corporate earnings effect on two months ahead volatilities for different horizons of constant maturity Treasury bills and bonds

	Estimate	StdErr	t-statistic	Adj.R^2
1 Month	0.00040**	0.00014	2.76060	0.04413
3 Month	0.00036**	0.00013	2.82923	0.07172
6 Month	0.00032**	0.00011	2.86632	0.09705
1 Year	0.00030**	0.00009	3.36788	0.11335
3 Year	0.00018**	0.00008	2.42936	0.04794
5 Year	0.00015**	0.00006	2.58699	0.03804
7 Year	0.00013**	0.00005	2.59619	0.02833
10 Year	0.00013**	0.00005	2.59764	0.03052
20 Year	0.00009**	0.00004	1.95786	0.01323
30 Year	0.00004	0.00004	1.10180	-0.00311

Table 1: Accounting information and interest rate volatilities. This table provides results from regression models of future interest rate volatilities on cross sectional dispersion in earnings information. Accounting data items are from the Compustat North America Fundamentals Quarterly dataset and yield volatilities are from the Federal Reserve Board of Governors H15 Reports dataset, both available on Wharton Research Data Services (WRDS). Yield volatilities are the constant maturity Treasury bonds and bills for all horizons available from the Federal Reserve Board of Governors H15 Reports. We use all data with consecutive observations when considering the intersection of Fed's yields and Compustat's accounting earnings (January 1973 to January 2016). The available horizons are for the 1-, 3-, and 6-month as well as for the 1-, 3-, 5-, 7-, 10-, 20-, and 30-year yields. We calculate yield volatilities as the monthly standard deviation of the daily interest rate yields during that month, and earnings dispersion information as the periodic cross sectional dispersion in earnings changes.

affect future P-expectations. This is because the drift of r_t is $\kappa\theta - .2i_t - .1s_t = \kappa\theta - .1i_t - .1r_t$.[1] Additionally, there is the possibility that excess returns may be identified accurately not through bond prices but through derivative prices. This will be precluded when there another orthogonality condition on the volatility is obtained.

The econometric method used in Joslin and Konchitchki (2017) precludes such identification of risk premia since it is assumed that the state of the economy can be inferred from fixed income security prices. To allow for such flexibility would require the use of a filtering technique which is complicated both by computational burden and by the non-linearity in the option prices.

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[1]Precisely, the condition in an affine model is the existence of an eigenvector of κ^Q which is orthogonal to ρ_1 but which is not an eigenvector of κ^P