

Internet appendix to “Tax distortions and bond issue pricing”

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Abstract

This Internet Appendix contains supplemental material that was not included in Landoni (2018). References to figures and tables contained in the main paper use the same numbering scheme as the paper (e.g., “Table 1”). References to figures and tables contained within this Internet Appendix are prepended with “IA” (e.g., “Table IA.1”).

Keywords: Municipal, Bonds, Security design, Tax arbitrage, Coupon rate, Issue price

JEL: G12, G32, G35, G38, H2

1. Model

1.1. Overview

This section describes the model that produces the predictions shown in Section 4 of the paper. The model is similar to Constantinides and Ingersoll (1984), in that it prices taxable and tax-exempt bonds when a taxable marginal investor trades optimally (i.e., realizes gains and losses to maximize its after-tax expected return) and bids competitively (i.e., does not have market power, and therefore pays the full value of the bonds it buys). I assume that the issuer and the investor are risk-neutral and they agree on a pretax short rate process $\{r_t\}$. I allow for a realistic tax code and, optionally, nonzero transaction costs. Unlike Constantinides and Ingersoll (1984), I explicitly model the tax effects of different issue prices, which requires me to keep track of an extra state variable, the original issue yield (y_0). Like them, I solve the model for different sets of tax rates, corresponding to different possible marginal investors, although I only report results when the marginal investor is a taxable individual. I also allow for an exogenous probability λ that the investor must trade (a liquidity shock); however, if realizing gains or losses is optimal, the investor will trade with certainty. Upon a trade, the investor may face a proportional transaction cost K . Bonds pay a coupon only once a year and investors can trade them once a year, immediately after the coupon has been paid.

The model is solved as follows. First, for every combination of the coupon, c , the time to maturity, T , and the state of the interest rate process at the time of issue, r_0 , I use backward induction to derive the investor's optimal trading strategy (Sell/Do Not Sell) and the exact market price of the bond at any time t prior to maturity. Given T , the trading strategy is a function of the current state of the interest rate process, r_t , and the investor's book yield, b . The price is a function of r_t only.

This first step pins down the issue price, P_0 , as a function of the coupon rate, c . Next, I

derive c^* , the optimal coupon rate for the issuer. A fraction W of the issue price is paid by the investor in anticipation of future tax benefits that are not offset by a cost for the issuer. I define the optimal coupon c^* as the coupon that maximizes the subsidy ratio W , i.e., the tax subsidy per dollar of principal transferred from the government to the issuer. Note that W can be negative when the investor has tax costs that are not offset by tax benefits for the issuer, but it can still be maximized.

1.2. Assumptions

1.2.1. The marginal investor

Investor Type	τ	τ_G	τ_{MD}	τ_I	λ	K
<i>Tax-exempt bonds</i>						
Benchmark	0%	20%	40%	0%	0%	0.0%
Rare Trading, Low Cost	0%	20%	40%	0%	1%	0.5%
Frequent Trading, Low Cost	0%	20%	40%	0%	5%	0.5%
Rare Trading, High Cost	0%	20%	40%	0%	1%	1.0%
<i>Taxable bonds</i>						
Benchmark	40%	20%	40%	35%	0%	0.0%
Rare Trading, Low Cost	40%	20%	40%	35%	1%	0.5%
Frequent Trading, Low Cost	40%	20%	40%	35%	5%	0.5%
Rare Trading, High Cost	40%	20%	40%	35%	1%	1.0%

Table IA.1: The trading strategy for the investor and the related issuance strategy for the issuer are optimized for different investor-asset-issuer combinations. Different investor types face different probabilities of a forced sale λ , costs of trading K , and tax codes. A tax code consists of four tax rates (τ , τ_G , τ_{MD} , τ_I): respectively, a tax rate on interest income, a tax rate on capital gains, a tax rate on market discount income and a tax rate on the issuer’s interest expense.

I assume that, from issuance to maturity, a taxable individual investor is indifferent between buying the coupon bond or investing in a one-period bond.

The individual faces a tax code that is a realistic approximation to the actual U.S. federal income tax code. It includes all rules about original issue discount and market discount, both regular and *de minimis*, as well as acquisition premium and premium amortization. I assume

that investors in taxable bonds choose to amortize premium instead of taking a capital loss when the bond matures or is disposed of. This assumption is standard in the literature (see also Strnad, 1995) because choosing to amortize premium dominates the alternative for virtually every taxpayer.

Once determined the respective amounts, interest income, capital gains/losses, and market discount income are taxed respectively at flat tax rates τ , τ_G , and τ_{MD} . The model allows for different types of taxpayer. For instance, for individual investors, τ is zero for tax-exempt bonds and 40% for taxable bonds. For nonlife insurance companies, τ is 5.25% for tax-exempt bonds and 35% for taxable bonds (Burstein, 2007). In addition, interest is deemed a deductible expense for the issuer at a tax rate τ_I (zero for tax-exempt issuers, and 35% for taxable issuers). Different investors may face different costs of trading K , and may have different liquidity shocks λ . Although the paper only reports the results for individual investors with no liquidity shocks and transaction costs, in order to study the sensitivity of the results to different sets of assumptions, I derive the optimal trading strategy for different investor types, listed in Table IA.1. All the findings in this paper hold qualitatively for all types of investor.

For a given tax code $(\tau, \tau_G, \tau_{MD})$, total tax liability is calculated using two functions, $\text{Tax}_t(\cdot)$, the tax on current income, and $\text{Tax}_t^S(\cdot)$, the tax on realized gains or losses realized upon a sale. These functions are defined in 1.3.1 and 1.3.2 below.

1.2.2. The interest rate process

Similar to Constantinides and Ingersoll (1984), I assume that the long-run distribution of the interest rate is uniform. In particular, the interest rate r is modeled as a Markov process with $N_r = 21$ discrete values ($r \in \{0\%, 0.5\%, \dots, r_{max} = 10\%\}$). Time is divided into discrete one-year ticks. Each year, the interest rate has a positive probability of either

staying at the same level, or jumping by up to two percentage points (e.g., if $r_t = 5\%$, $r_{t+1} \in \{3\%, 3.5\%, \dots, 6.5\%, 7\%\}$). The following table shows the probability distribution of rate changes.

$\Delta = r_{t+1} - r_t$	-2%	-1 $\frac{1}{2}$ %	-1%	-1/2%	0	+1/2%	+1%	+1 $\frac{1}{2}$ %	+2%
Pr(Δ)	.05	.07	.11	.15	.24	.15	.11	.07	.05

When the current rate is less than two percentage points away from the reflecting boundaries of 0% and 10%, an adjustment is needed because some events in the table are impossible. For instance, if the interest rate is 8.5%, a +2% change would bring the interest rate to 10.5%, which is higher than the upper bound. In this case, the vector is simply truncated and rescaled to sum to one. This adjustment leaves the long-run distribution uniform.

This simple calibration is meant to roughly mimic the dynamic of the U.S. municipal and taxable short rates. The benefits of this choice are simplicity and the ability to compare the timing option value obtained directly against Constantinides and Ingersoll (1984), who use essentially an identical setup. The benefit of having a more realistic interest rate model (mainly, an upward sloping yield curve) is outweighed by the additional complexity.

1.2.3. Other assumptions

I make several additional assumptions for simplicity. These assumptions can be relaxed without qualitatively affecting the model's predictions.

Investors are assumed to be risk-neutral. This assumption slightly affects the value of tax timing options. The direction of this effect depends on what we assume about the investor's stochastic discount factor. However, for tax-exempt bonds, the tax-timing option is "in the money" when the bond price has fallen, and therefore the assumption of risk neutrality likely *understates* the value of the tax-timing option, which reduces the bond's overall risk. Consistent with this view, Longstaff (2011) demonstrates the existence of a negative tax risk

premium using municipal rate swaps. For taxable bonds, much of the tax advantage comes from realizing gains. In this case, the assumption of risk neutrality is likely to overstate the value of the tax-timing option. In any case, the importance of this assumption should not be overstated.

I also assume that investors trade optimally and all the incremental value is captured by the issuer. Part of the value could be captured by the investors who initially bid for the bonds, if they are not bidding against one another in a fully competitive way. Moreover, investors could anticipate that they will not be able to time their trading optimally, but perhaps only realize a loss once in a while if it's large enough. In this case, part of the value will not be recognized at all. Even so, the choice of coupon is assumed to be costless for the issuer, and therefore if any part of the value is captured by the issuer, the issuer has an incentive to issue at a premium.

1.3. The investor's problem, or pricing the bond

A bond of face value 1 paying coupon c matures at time T . At time $t < T$, a risk-neutral investor maximizes the present value of after-tax final wealth. Taking the bond characteristics as given, the investor's utility is

$$V_t(c, T, t_{buy}, b, r_t, y_0), \tag{1}$$

a function of four state variables: the time of purchase t_{buy} , the investor's book yield b (i.e. the yield to maturity of the bond at t_{buy}), the short rate r_t , and the original issue yield y_0 .

At every period, the investor can choose to hold on to the bond (Do Not Sell), or to realize the position's unrealized gain or loss by selling the bond and buying it back (Sell). If the investor decides to hold, however, the bond will still be sold (and the gain or loss realized) with a probability λ . Each sale, voluntary or involuntary, incurs a proportional

transaction cost K . This yields the following definition for the value function:

$$V_t(c, T, t_{buy}, b, r_t, y_0) = \max_{i \in \{\text{Sell}, \text{Do Not Sell}\}} \frac{c + \mathbb{E}_t [V_{t+1}^i] - \text{Tax}_{t+1}(T, b^i, c, y_0)}{1 + r_t} + \text{Tax}_t^S(c, T, t_{buy}, b, P_t, y_0) + KP_t \mathbf{1}_{(i=\text{Sell})} \quad (2)$$

where P_t is the market price of the bond, and $\mathbf{1}_{(i=\text{Sell})}$ is an indicator function that is 1 if the investor decides to sell and λ otherwise. The tax code is represented by two functions. $\text{Tax}_{t+1}(c, T, b^i, y_0)$ is the tax due on the (taxable, or tax-exempt) interest income received from the bond at time $t + 1$. b^i , the book yield, is indexed by i because it depends on the current choice. If the investor holds on to the bond, it stays the same ($b^{\text{Do Not Sell}} = b$); if the investor sells the bond, the book yield resets to the current market yield ($b^{\text{Sell}} = y_t$). $\text{Tax}_t^S(c, T, t_{buy}, b, P_t, y_0)$ is the tax due if the bond is sold at time t . These functions are described in a separate subsection below. In the special case that $t = T$, at the time of maturity, V_T is equal to

$$V_T(c, T, t_{buy}, b, r_T, y_0) = 1 - \text{Tax}_T^S(c, T, t_{buy}, b, 1, y_0). \quad (3)$$

The choice-dependent, next-period value functions V_{t+1}^i are defined as:

$$V_{t+1}^{\text{Sell}} = V_{t+1}(c, T, t, y_t, r_{t+1}, y_0); \quad (4)$$

$$V_{t+1}^{\text{Do Not Sell}} = \lambda V_{t+1}^{\text{Sell}} + (1 - \lambda) V_{t+1}(c, T, t_{buy}, b, r_{t+1}, y_0). \quad (5)$$

Note that in the definition of V_{t+1}^{Sell} , b and t_{buy} are replaced by y_t and t , because selling the bond resets the book yield to the current yield and the purchase time to the current time.

So far we have taken the current price P_t and yield y_t as given. The price is defined as a

special case of the value function:

$$P_t(c, T, r_t) = V_t(c, T, t, y_t, r_t, y_0). \quad (6)$$

This definition is recursive: because there is a one-to-one mapping between P_t and y_t , P_t depends on itself. Any guess for P_t quickly converges to the unique fixed point. Once obtained P_t , it is easy to calculate V_t for all $b \neq y_t$ using Eq. (2). In the special case $t = 0$, P_0 depends on itself via the additional channel of the original issue yield, y_0 . Once again, any guess for y_0 converges by iterating the entire backward induction process a few times.

1.3.1. *Tax_t: the tax on accrued income*

For notational convenience, define the function $P(c, T, y)$ as the price of a bond with coupon rate c , remaining time to maturity T , and yield to maturity y .

- Define $dB = P(c, T - t, b) - P(c, T - t + 1, b)$. dB is the hypothetical change in tax basis using book yield.
- Define dRP , the change in revised issue price RP .
 - If the bond was issued at premium, par, or de minimis OID for taxable bonds, $RP_t = 1 \forall t$ and therefore $dRP = 0$.
 - If the bond was issued with OID, revised issue price equals original issue price plus accrued OID: $dRP = P(c, T - t, y_0) - P(c, T - t + 1, y_0)$
- Define accrued income as $I = C + \min(dRP, dB)$.

Note that accrued income is equal to the coupon plus the lesser of (i) the change in revised price and (ii) the hypothetical change tax basis. Note that $dRP \geq 0$ by construction. If

dB is negative, the bond is a premium bond and there is premium amortization. If dB is positive, the increase in book value is included only up to the extent of OID accrual. If $dB > dRP$ there is either de minimis OID or any market discount, which does not accrue until the bond is sold.

Accrued income is then multiplied by the appropriate income tax rate τ .

1.3.2. Tax_t^S : the tax on gains and losses upon a sale

For notational convenience, define $T_{buy} = T - t_{buy}$ as the remaining maturity when the bond was bought.

- Calculate the revised issue price at the time of issue (RP_0), the time of purchase (RP_{buy}) and the time of sale, i.e. the current period (RP_t). As above, $RP_s = P(c, T - s, y_0)$ for an OID bond and $RP_s = 1$ otherwise, for all s .
- Calculate MD , the market discount at the time of purchase.
 - If bond was bought at issue ($T_{buy} = T$), $MD = 0$.
 - Otherwise, calculate purchase price $B_{buy} = P(c, T_{buy}, b)$. Note that the purchase price is equal to the book value using the book yield and the original maturity. Then, market discount is equal to any positive difference between revised price and actual purchase price. $MD = \max\{RP_{buy} - B_{buy}, 0\}$
- Calculate current tax basis B_t .
 - If the bond was bought at issue:
 - * If the bond is taxable and was issued with de minimis discount ($1 - P_0 \in (0, 0.0025\text{int}(T)]$), the basis is just equal to the issue price ($B_t = P_0$).
 - * Otherwise, just use the original issue yield: $B_t = P(c, T - t, y_0)$

- If the bond was bought after issue:
 - * If there is no market discount ($MD = 0$), calculate current tax basis using book yield: $B_t = P(c, T - t, b)$. This includes bonds with an acquisition premium, i.e., bonds trading at a discount that is less than the original issue discount.
 - * If there is any market discount ($MD > 0$), current basis is equal to purchase price plus accrued OID: $B_t = B_{buy} + (RP_t - RP_{buy})$.
- Calculate sale gain as the difference between sale price and tax basis: $Gain = P_t - B_t$.
- Calculate tax.
 - If market discount is more than de minimis ($MD > 0.0025 \text{int}(T_{buy})$), calculate the fraction of market discount that has accrued: $AMD = MD(t - t_{buy}) / T_{buy}$. Note that market discount accrues linearly, not using the constant yield method.
 - If the sale gain is negative, it is treated as a capital loss (taxed at rate τ_G). If it is positive, it is taxed at the market discount rate (τ_{MD}) up to the amount of accrued market discount, and any excess gain is treated as capital gain: $\text{Tax}^S = \tau_G \text{Gain} + (\tau_{MD} - \tau_G) \max\{0, \min\{\text{Gain}, AMD\}\}$.

1.4. The issuer's problem, or choosing the optimal coupon rate

The issuer's problem is simple. $P_0(c, T, r_0)$ as defined in the previous subsection is the issue price of a T -year bond with coupon c when the current short rate is r_0 . Define \hat{P}_0 as the after-tax cost to the issuer of all future cash flows from the bond:

$$\hat{P}_0(c, T, r_0) = \mathbb{E}_0 \left[\frac{c}{1 + r_0(1 - \tau_I)} + \frac{c}{(1 + r_0(1 - \tau_I))(1 + r_1(1 - \tau_I))} + \cdots + \frac{1 + c}{(1 + r_0(1 - \tau_I)) \cdots (1 + r_{T-1}(1 - \tau_I))} \right]. \quad (7)$$

For a given amount that it needs to borrow, the issuer wishes to minimize the present value of the cash flows that it promises to pay back. In other words, the issuer wants to maximize the subsidy ratio W , i.e., the fraction of issued amount that is not compensation for future cash outflows:

$$\max_{c \geq 0} W \equiv \frac{P_0 - \hat{P}_0}{P_0}. \quad (8)$$

The optimal coupon c^* is simply found by solving the investor's problem for set values of $c \in \{0\%, .125\%, .25\%, \dots 10\%\}$ and picking the one that gives the highest W .

This rough grid search method yields a less-precise optimum compared to hill-climbing or annealing algorithms. However, a finite optimum may not always exist. In these cases, grid search will simply yield $c^* = 10\%$, the maximum value available, while more sophisticated algorithms will fail. In practice, an interior optimal coupon (i.e., below 10%) occurs very often.

1.5. Solution: optimal issue price

Fig. IA.1 plots the subsidy ratio W as a function of the coupon rate, over three different values for the initial state of the interest rate process r_0 (2%, 5%, 8%). W is expressed in percentage points. Tax-exempt bonds are in the left column, and taxable bonds in the right column.

In the case of tax-exempt bonds (left side of Fig. IA.1), W has one or two local maximums. Premium is always preferred to par, as the slope is steep and positive immediately to the right of par. A local maximum is always reached when the marginal benefit from reducing the likelihood of market discount equals the marginal cost of reducing the bond's price volatility. The other potential local maximum is zero coupon, a corner solution.

The premium solution usually prevails. The zero-coupon solution can only exist and prevail when the bond's maturity is long enough and the interest rate is high enough (e.g.,

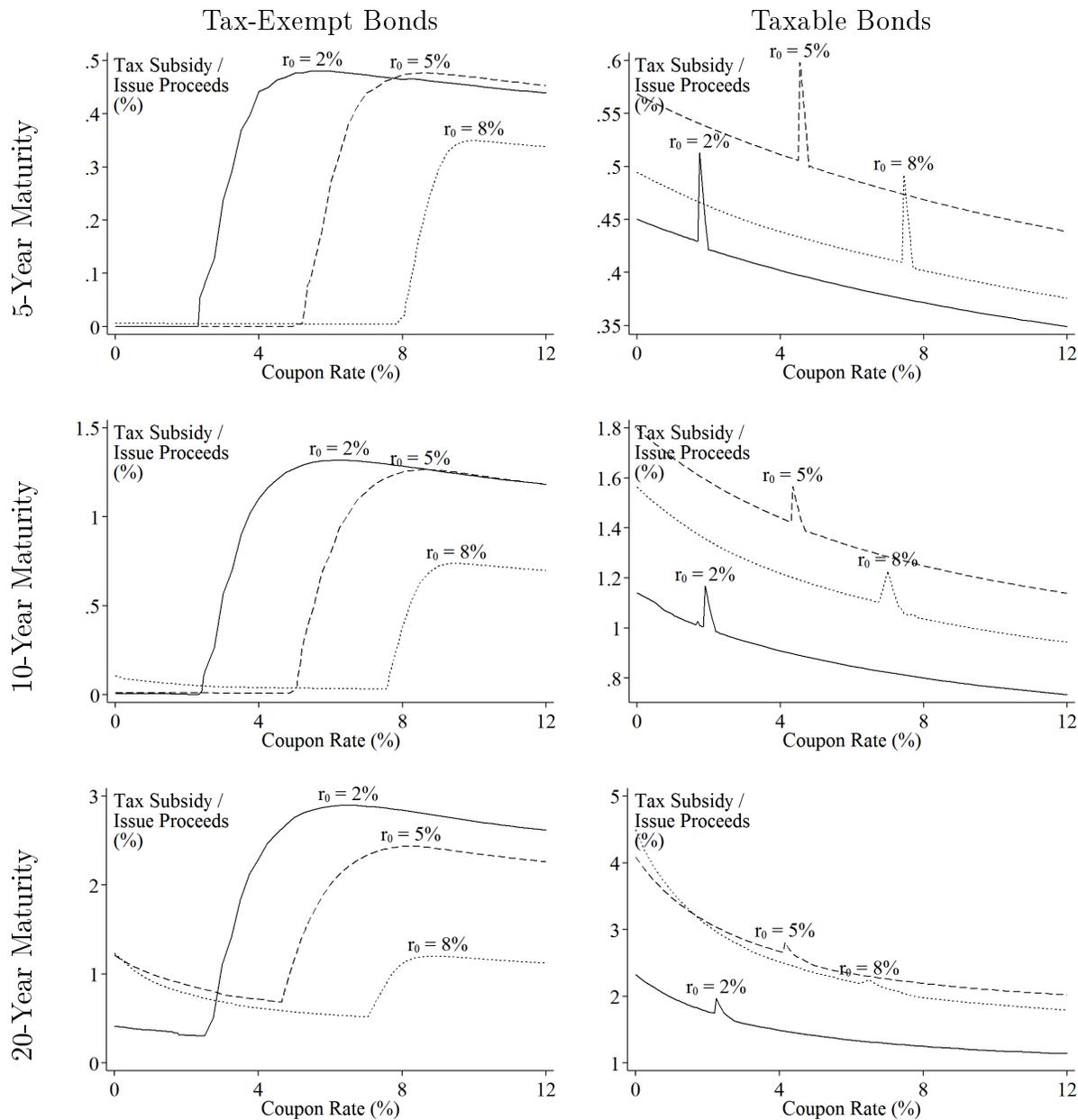


Figure IA.1: Model results: W , value of the tax subsidy to issuers as a fraction of total amount issued, for tax-exempt bonds (left) and taxable bonds (right) of 5-, 10-, and 20-year maturity (top, middle and bottom row, respectively). W is plotted for different values of r_0 , the short rate at the time of issue (2%, 5% and 8%). The marginal investor is a taxable individual. Transaction costs are assumed to be zero.

in the case of a 20-year bond when $r_0 = 8\%$, as in the bottom panel).¹ If these conditions are not verified, realizing losses is not profitable. Realizing losses is the only tax arbitrage strategy available to par and OID tax-exempt bond investors. For instance, for a 10-year bond (middle panel), realizing losses is profitable only when $r_0 = 8\%$. Already when $r_0 = 5\%$, the volatility benefit is muted because there are no profitable strategies at all, and W is 0 for all discount bonds, regardless of coupon.

For taxable bonds (right side of Fig. IA.1), W also has two local maximums. One is, once again, zero coupon. The other is the de minimis boundary. De minimis discount is defined as a discount of less than 0.25 times the number of whole years remaining until maturity. For the investor, de minimis discount is treated favorably, regardless of whether it is original issue discount or market discount: first, it is taxed at the lower capital gains tax rate, and second, it is deferred until maturity or when the bond is sold (Internal Revenue Code §1273(a)(3)). For the issuer, however, an offsetting rule does not exist: de minimis discount is counted as ordinary interest expense (Treasury Regulations §1.163-4(a)(1)). The net result of issuing bonds with de minimis OID is to convert some ordinary income into capital gains. Because the de minimis boundary is set rather generously, this amount is small but significant.²

To see why the de minimis boundary is a local optimum, consider the case of a 5-year bond. The de minimis boundary is 98.75. If the issue price is below 98.75, the bond becomes an OID bond and all interest income (discount and coupon) is taxable as ordinary income.

¹The model predicts that very long-term noncallable bonds are issued with zero coupon. In the data, 80% of noncallable bonds with maturity 20 years or longer are issued with zero coupon. However, noncallable long-term bonds are highly unusual. Because of this potential for sample selection, I am not able to verify whether these bonds are issued with zero coupon for tax reasons or for other reasons.

²No tax arbitrage is possible for tax-exempt bonds because original issue discount is treated as tax-exempt interest, regardless of discount size (Internal Revenue Code §1288(b)(1)). De minimis rules also exist for *market* discount (§1288(a)(2)(C)). Like the rest of market discount rules, they apply to both taxable and tax-exempt bonds, but per se they do not affect the issue price decision. These rules are the subject of Ang et al. (2010).

If the price is 100 or higher, all interest is also taxable as ordinary income. If the price is between 98.75 and 100, however, an amount of interest equal to 100 minus the price is converted into capital gains. Clearly, issuing the bond at a price of exactly 98.75 has to be a local optimum. Compared to issuing at par, this strategy increases the 5-year bond's value by about 0.24 per 100 face value, dominating any other tax effects.

Unlike in the case of tax-exempt bonds, the zero-coupon optimum always exists and it often prevails, as in the middle and bottom panels (10- and 20-year bonds). This difference occurs for two reasons. First, by definition of “de minimis”, the amount of interest that can be converted into capital gains is relatively small. Second, for taxable bonds there are multiple tax timing strategies. For instance, it can be profitable to realize gains *and* losses (Constantinides and Ingersoll, 1984). Thus, price volatility is more valuable because it increases the value of multiple strategies.

2. A brief discussion of the effect of ignoring state taxes

The model ignores state taxes. This is a conservative assumption because it reduces the value of realizing losses in most cases. This section contains a brief discussion of the most common cases.

Let's initially consider the case of a taxable individual holding bonds issued by the State of California. It is well-known that the market is segmented (Schultz, 2012) and investors have a bias for own-state bonds. This bias may be bred by familiarity, but it is certainly helped by the fact that many states exempt own-state bonds from state income tax. Thus, the most common case is that the individual is also a resident of California. In this case, state tax rules mirror and magnify the effect of federal ones: investors get a tax deduction for capital losses (both at the state and federal level), but they do not pay tax on coupon income (either at the state or federal level). Thus, in the most common case, ignoring state taxes underestimates the value of tax timing.

If the investor is, instead, a resident of Oregon, coupon income is taxable at the state level. However, Oregon (like almost all other states) does not have a preferential tax rate for capital gains. Thus, in Oregon, realizing losses is particularly valuable. Suppose our investor's marginal tax rate is 5%, and that she realizes \$100 of losses. This will give her an immediate benefit of \$5. The investor then buys back the bond immediately with a \$100 lower tax basis, and therefore realizes \$100 of additional interest income spread over the life of the bond. This additional income will cause her to pay additional taxes of \$5. In other words, it's as if the investor received a \$5 interest-free loan from the government of Oregon. To be sure, the present value benefit of this "loan" is small, but to the extent that it exists, it further adds to the investor's benefit and the taxpayer's burden. Thus, in the second most common case, ignoring state taxes is either harmless or it causes a slight underestimate of the value of tax timing.

Some states, like Texas, do not tax income at all. Thus, if the investor is a resident of Texas, ignoring state taxes is entirely appropriate. It is also appropriate to ignore state taxes for bonds issued under those U.S. jurisdictions (such as Puerto Rico) whose bonds cannot be taxed by U.S. states.

Finally, to the best of my knowledge, the case of a Hawaii investor holding non-Hawaii bonds is the only example in which ignoring state taxes *may* slightly overestimate the effect of tax timing. Hawaii taxes interest on out-of-state bonds *and* has a preferential tax rate for capital gains. Suppose the investor has a 9% marginal tax rate on interest and a 7.5% marginal tax rate on capital gains. By realizing \$100 of losses and immediately repurchasing the bond, the investor will receive an immediate \$7.5 tax benefit, but will face \$9 in additional taxes spread over the life of the bond. For a reasonable range of discount rates, the present value of realizing losses is small in absolute value.

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