Internet Appendix for "Advising Shareholders in Takeovers"

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B. Auxiliary results

Lemma 4 ***[Auxiliary result for Proposition 1].*** *The the intersection of $\tau(\underline{q}) < p_1 < \tau(\overline{q})$ and*

$$\mathbb{E}\left[\widetilde{q}|\tau(\widetilde{s}) \leq p_1\right] + \Delta \leq p_1 < \mathbb{E}\left[\widetilde{q}|\tau(\widetilde{s}) > p_1\right] + \Delta \tag{75}$$

is equivalent to the intersection of $\tau(\underline{q}) < p_1$ and $p_1 \in [\tau(q_L), \tau(q_H))$.

Proof. Based on (9),

$$\tau(x) = \lambda(1 - \frac{\delta}{2})x + \lambda\frac{\delta}{2}\overline{q} + (1-\lambda)\,\mathbb{E}\left[\widetilde{q}\right] + \delta\Delta + (1-\delta)\,\beta \tag{76}$$

and

$$\tau^{-1}(p_1) = \frac{p_1 - \lambda\frac{\delta}{2}\overline{q} - (1-\lambda)\,\mathbb{E}\left[\widetilde{q}\right] - \delta\Delta - (1-\delta)\,\beta}{\lambda\left(1 - \frac{\delta}{2}\right)}. \tag{77}$$

Therefore,

$$\tau(\underline{q}) < p_1 < \tau(\overline{q}) \Leftrightarrow \tag{78}$$

$$\lambda\underline{q} + \delta\lambda\frac{\overline{q} - \underline{q}}{2} + (1-\lambda)\,\mathbb{E}\left[\widetilde{q}\right] + \delta\Delta + (1-\delta)\,\beta < p_1 < \lambda\overline{q} + (1-\lambda)\,\mathbb{E}\left[\widetilde{q}\right] + \delta\Delta + (1-\delta)\,\beta \tag{79}$$

and

$$\mathbb{E}\left[\widetilde{q}|\tau(\widetilde{s}) \leq p_1\right] + \Delta \leq p_1 < \mathbb{E}\left[\widetilde{q}|\tau(\widetilde{s}) > p_1\right] + \Delta \Leftrightarrow \tag{80}$$

$$\lambda\frac{\underline{q} + \tau^{-1}(p_1)}{2} + (1-\lambda)\,\mathbb{E}\left[\widetilde{q}\right] + \Delta \leq p_1 < \lambda\frac{\overline{q} + \tau^{-1}(p_1)}{2} + (1-\lambda)\,\mathbb{E}\left[\widetilde{q}\right] + \Delta \Leftrightarrow \tag{81}$$

$$\lambda\underline{q} - \lambda\frac{\delta}{1-\delta}\frac{\overline{q} - \underline{q}}{2} + (1-\lambda)\,\mathbb{E}\left[\widetilde{q}\right] + 2\Delta - \beta \leq p_1 < \lambda\overline{q} + (1-\lambda)\,\mathbb{E}\left[\widetilde{q}\right] + 2\Delta - \beta \tag{82}$$

The intersection of the conditions $\tau(\underline{q}) < p_1 < \tau(\overline{q})$ and (75) requires

$$\begin{aligned} & \lambda \underline{q} + \lambda\delta\frac{\overline{q}-\underline{q}}{2} + (1-\lambda)\,\mathbb{E}\left[\widetilde{q}\right] + \delta\Delta + (1-\delta)\,\beta + (2-\delta)\max\{0, \Delta - \beta - \lambda\frac{\delta}{1-\delta}\frac{\overline{q}-\underline{q}}{2}\} \\ \leq \; & p_1 < \lambda\overline{q} + (1-\lambda)\,\mathbb{E}\left[\widetilde{q}\right] + \delta\Delta + (1-\delta)\,\beta + (2-\delta)\min\{0, \Delta - \beta\} \end{aligned} \tag{83}$$

where the weak inequality must be strict if $\beta > \Delta - \lambda\frac{\delta}{1-\delta}\frac{\overline{q}-\underline{q}}{2}$. It can be verified that this condition is equivalent to $p_1 \in [\tau(q_L), \tau(q_H))$ and $\tau(\underline{q}) < p_1$. ∎

Lemma 5 ***[Auxiliary result for Proposition 2].*** *If the initial offer is rejected and revised by the bidder on the equilibrium path, then* $p_1^* < p_2^*$.

Proof. First, I argue that if condition (14) holds then $\tau(q_L) < \mathbb{E}\left[\widetilde{q}\right] + \Delta < \tau(q_H)$. Note that

$$\tau(x) < \mathbb{E}\left[\widetilde{q}\right] + \Delta \Leftrightarrow x < \frac{\underline{q} + (1-\delta)\,\overline{q} + \frac{2}{\lambda}\,(\Delta-\beta)\,(1-\delta)}{2-\delta}. \tag{84}$$

The following three cases establish this result:

1. If $\Delta \leq \beta$ then $q_L = \underline{q}$ and $q_H = \overline{q} + \frac{2}{\lambda}\,(\Delta - \beta)$. Therefore, $\tau(\underline{q}) < \mathbb{E}\left[\widetilde{q}\right] + \Delta < \tau(\overline{q} + \frac{2}{\lambda}\,(\Delta - \beta))$ requires $\beta < \Delta + \lambda\frac{\overline{q}-\underline{q}}{2}$, which holds if (14) holds.

2. If $\Delta - \lambda\frac{\delta}{1-\delta}\frac{\overline{q}-\underline{q}}{2} \leq \beta < \Delta$ then $q_L = \underline{q}$ and $q_H = \overline{q}$. Therefore, $\tau(\underline{q}) < \mathbb{E}\left[\widetilde{q}\right] + \Delta < \tau(\overline{q})$ requires $\Delta - \lambda\frac{1}{1-\delta}\frac{\overline{q}-\underline{q}}{2} < \beta < \Delta + \lambda\frac{\overline{q}-\underline{q}}{2}$, which holds if (14) holds.

3. If $\beta < \Delta - \lambda\frac{\delta}{1-\delta}\frac{\overline{q}-\underline{q}}{2}$ then $q_L = \underline{q} + \frac{2}{\lambda}(\Delta - \beta - \lambda\frac{\delta}{1-\delta}\frac{\overline{q}-\underline{q}}{2})$ and $q_H = \overline{q}$. Therefore, $\tau(\underline{q} + 2(\Delta - \beta - \lambda\frac{\delta}{1-\delta}\frac{\overline{q}-\underline{q}}{2})) < \mathbb{E}\left[\widetilde{q}\right] + \Delta < \tau(\overline{q})$ requires $\Delta - \lambda\frac{1}{1-\delta}\frac{\overline{q}-\underline{q}}{2} < \beta$, which holds if (14) holds.

Next, consider the statement itself. The bidder revises the initial offer on the equilibrium path only if condition (14) holds and $b < \overline{b}$. If $b \in [0, \underline{b})$ then the initial offer is $p_1^* < \tau\left(p_L\right)$ and the revised offer is $p_2^* = \mathbb{E}\left[\widetilde{q}\right] + \Delta$. Based on the first part of this proof, $\tau\left(p_L\right) < \mathbb{E}\left[\widetilde{q}\right] + \Delta$, and therefore, $p_1^* < p_2^*$. Suppose $b \in (\underline{b}, \overline{b})$. The board is influential with respect to offer p_1^*. Based on Lemma 1 and part (i) of Proposition 1, if the initial offer is rejected then it must be $p_1^* < \tau(\widetilde{s})$, $p_1^* < \mathbb{E}\left[\widetilde{q}|\tau(\widetilde{s}) > p_1^*\right] + \Delta$, and $p_2^* = \mathbb{E}\left[\widetilde{q}|\tau(\widetilde{s}) > p_1^*\right] + \Delta$. Therefore, $p_1^* < p_2^*$ as required. ∎

Lemma 6 ***[Auxiliary result for Proposition 3].*** *The statement in Proposition 3 holds with respect to* λ, $\overline{q} - \underline{q}$, *and* δ.

Proof. Let $\kappa \equiv \lambda\frac{\overline{q}-\underline{q}}{2}$. I first consider part (ii) of Proposition 3. According to expression (24) in the proof of Proposition 2,

$$\begin{aligned}\Delta - \frac{\delta}{1-\delta}\kappa &< \beta < \Delta + \kappa \text{ and } b \in [0, \underline{b}) \Leftrightarrow & (85)\\ \max\{(b + \Delta - \beta)\frac{1-\delta}{\delta}, (\beta - \Delta)\frac{2-\delta}{1-\delta}\} &< \kappa \\ \text{or } b(1-\delta) + \beta - \Delta &< \kappa \leq (\beta - \Delta)\frac{2-\delta}{1-\delta}. & (86)\end{aligned}$$

Since

$$\frac{\beta - \Delta}{(1-\delta)^2} < (>)\ b \Rightarrow (b + \Delta - \beta)\frac{1-\delta}{\delta} > (<)\ b(1-\delta) + \beta - \Delta > (<)\ (\beta - \Delta)\frac{2-\delta}{1-\delta}, \quad (87)$$

we require

$$\max\{\frac{b(1-\delta) + \Delta - \beta}{\delta}, b(1-\delta)\} + \beta - \Delta < \kappa, \quad (88)$$

which establishes part (ii) with respect to κ. Note that if $\Delta - \beta \geq 0$ then (88) becomes $\frac{b+\Delta-\beta}{\kappa+b} < \delta$. If $\Delta - \beta < 0$ then (88) becomes

$$\frac{b + \Delta - \beta}{\kappa + \Delta - \beta + b} < \delta < 1 - \sqrt{\frac{\beta - \Delta}{b}} \text{ or } \max\{1 - \sqrt{\frac{\beta - \Delta}{b}}, 1 - \frac{\kappa + \Delta - \beta}{b}\} < \delta. \quad (89)$$

Since

$$\sqrt{b} > (<)\frac{\kappa + \Delta - \beta}{\sqrt{\beta - \Delta}} \Rightarrow 1 - \sqrt{\frac{\beta - \Delta}{b}} < (>)\frac{b + \Delta - \beta}{\kappa + \Delta - \beta + b} < (>) 1 - \frac{\kappa + \Delta - \beta}{b}, \quad (90)$$

(88) is equivalent to

$$\max\{\frac{b + \Delta - \beta}{\kappa + \Delta - \beta + b}, 1 - \frac{\kappa + \Delta - \beta}{b}\} < \delta. \quad (91)$$

Whether $\Delta - \beta \geq 0$ or $\Delta - \beta < 0$, this argument establishes part (ii) with respect to δ. Next, consider part (i) of Proposition 3. According to expression (24),

$$\begin{aligned}
\beta &\in (\Delta - \frac{\kappa}{1-\delta}, \Delta + \kappa) \text{ and } b \in \left[\overline{b}, \infty\right), \text{ or } \beta \notin (\Delta - \frac{\kappa}{1-\delta}, \Delta + \kappa) \Leftrightarrow \\
\beta &\leq \Delta \text{ and } \kappa \leq (\Delta - \beta)(1-\delta), \text{or} \\
\beta &\leq \Delta \text{ and } (\Delta - \beta)(1-\delta) < \kappa \leq (b + \Delta - \beta)\frac{1-\delta}{2-\delta}, \text{or} \\
\Delta &< \beta \text{ and } (\beta - \Delta)\frac{2-\delta}{1-\delta} < \kappa \leq (\sqrt{b} + \sqrt{\beta - \Delta})^2 \frac{1-\delta}{2-\delta}, \text{or} \\
\Delta &< \beta \text{ and } \beta - \Delta < \kappa \leq \min\{b(1-\delta) + \beta - \Delta, (\beta - \Delta)\frac{2-\delta}{1-\delta}\}, \text{or} \\
\Delta &< \beta \text{ and } \kappa \leq \beta - \Delta.
\end{aligned}$$

Since

$$b < \frac{\beta - \Delta}{(1-\delta)^2} \Rightarrow (\sqrt{b} + \sqrt{\beta - \Delta})^2 \frac{1-\delta}{2-\delta} < b(1-\delta) + \beta - \Delta < (\beta - \Delta)\frac{2-\delta}{1-\delta} \tag{92}$$

$$b > \frac{\beta - \Delta}{(1-\delta)^2} \Rightarrow b(1-\delta) + \beta - \Delta > (\sqrt{b} + \sqrt{\beta - \Delta})^2 \frac{1-\delta}{2-\delta} > (\beta - \Delta)\frac{2-\delta}{1-\delta}, \tag{93}$$

we have

$$\kappa \leq \begin{cases} \max\{\Delta - \beta, \frac{b+\Delta-\beta}{2-\delta}\}(1-\delta) & \textit{if } \beta - \Delta \leq 0 \\ (\sqrt{b} + \sqrt{\beta - \Delta})^2 \frac{1-\delta}{2-\delta} & \textit{if } 0 < \beta - \Delta \leq b(1-\delta)^2 \\ b(1-\delta) + \beta - \Delta & \textit{if } b(1-\delta)^2 < \beta - \Delta, \end{cases} \tag{94}$$

which establishes part (i) with respect to κ. Note that (94) is equivalent to

$$\begin{cases} \kappa \leq \max\{\Delta - \beta, \frac{b+\Delta-\beta}{2-\delta}\}(1-\delta) & \textit{if } \beta - \Delta \leq 0 \\ \begin{array}{l} \kappa \leq [(\sqrt{b} + \sqrt{\beta - \Delta})^2 - \kappa](1-\delta) \;\; \textit{and} \; \sqrt{\frac{\beta - \Delta}{b}} \leq 1 - \delta, \; \textit{or} \\ \frac{\kappa + \Delta - \beta}{b} \leq 1 - \delta < \sqrt{\frac{\beta - \Delta}{b}} \end{array} & \textit{if } 0 < \beta - \Delta \end{cases} \tag{95}$$

which is also equivalent to

$$\begin{cases} \frac{\kappa}{b+\Delta-\beta-\kappa} \leq 1-\delta < \frac{b}{\Delta-\beta} \text{ or } \frac{\max\{\kappa,b\}}{\Delta-\beta} \leq 1-\delta & \textit{if } 0 \leq \Delta-\beta \textit{ and } \kappa-b < \Delta-\beta \\ \frac{\max\{\kappa,b\}}{\Delta-\beta} \leq 1-\delta & \textit{if } 0 \leq \Delta-\beta \textit{ and } \Delta-\beta \leq \kappa-b \\ \begin{array}{l} \max\{\frac{\kappa}{(\sqrt{b}+\sqrt{\beta-\Delta})^2-\kappa}, \sqrt{\frac{\beta-\Delta}{b}}\} \leq 1-\delta \\ \textit{or } \frac{\kappa+\Delta-\beta}{b} \leq 1-\delta < \sqrt{\frac{\beta-\Delta}{b}} \end{array} & \textit{if } 0 < \beta-\Delta \textit{ and } \sqrt{\kappa}-\sqrt{b} < \sqrt{\beta-\Delta} \\ \frac{\kappa+\Delta-\beta}{b} \leq 1-\delta < \sqrt{\frac{\beta-\Delta}{b}} & \textit{if } 0 < \beta-\Delta \textit{ and } \sqrt{\beta-\Delta} \leq \sqrt{\kappa}-\sqrt{b} \end{cases} \tag{96}$$

Suppose $0 \leq \Delta - \beta$. Since

$$\frac{\kappa}{b+\Delta-\beta-\kappa} < \frac{b}{\Delta-\beta} \Leftrightarrow \kappa < b \text{ and } \frac{\kappa}{b+\Delta-\beta-\kappa} < \frac{\kappa}{\Delta-\beta} \Leftrightarrow \kappa < b,$$

in this range (94) is equivalent to

$$1-\delta \geq \begin{cases} \min\{\frac{\kappa}{b+\Delta-\beta-\kappa}, \frac{\kappa}{\Delta-\beta}\} & \textit{if } \kappa-b < \Delta-\beta \\ \frac{\max\{\kappa,b\}}{\Delta-\beta} & \textit{if } \Delta-\beta \leq \kappa-b, \end{cases} \tag{97}$$

which establishes part (i) with respect to δ when $0 \leq \Delta - \beta$. Suppose $0 > \Delta - \beta$. Since

$$\sqrt{\beta-\Delta} \leq \sqrt{\kappa}-\sqrt{b} \Rightarrow \sqrt{\frac{\beta-\Delta}{b}} \leq \frac{\kappa+\Delta-\beta}{b} \tag{98}$$

and

$$\frac{\kappa+\Delta-\beta}{b} < \sqrt{\frac{\beta-\Delta}{b}} \Leftrightarrow \frac{\kappa}{\sqrt{\beta-\Delta}} - \sqrt{\beta-\Delta} < \sqrt{b} \tag{99}$$

$$\frac{\kappa}{(\sqrt{b}+\sqrt{\beta-\Delta})^2-\kappa} < \sqrt{\frac{\beta-\Delta}{b}} \Leftrightarrow \frac{\kappa}{\sqrt{\beta-\Delta}} - \sqrt{\beta-\Delta} < \sqrt{b}, \tag{100}$$

when $0 > \Delta - \beta$ condition (94) is equivalent to

$$1-\delta \geq \begin{cases} \frac{\kappa+\Delta-\beta}{b} & \textit{if } \max\left\{\sqrt{\kappa}-\sqrt{\beta-\Delta}, \frac{\kappa}{\sqrt{\beta-\Delta}}-\sqrt{\beta-\Delta}\right\} < \sqrt{b} \\ \frac{\kappa}{(\sqrt{b}+\sqrt{\beta-\Delta})^2-\kappa} & \textit{if } \sqrt{\kappa}-\sqrt{\beta-\Delta} < \sqrt{b} < \frac{\kappa}{\sqrt{\beta-\Delta}}-\sqrt{\beta-\Delta}, \end{cases} \tag{101}$$

which establishes part (i) with respect to δ when $0 > \Delta - \beta$. ■

Lemma 7 ***[Auxiliary result for Proposition 4].*** *There is a configuration of parameters such that $H^{**} > 0$ and a configuration such that $H^{**} < 0$, where H^{**} is given by (19).*

Proof. Suppose $\Delta - \frac{1}{1-\delta}\kappa < \beta < \Delta - \frac{\delta}{1-\delta}\kappa$. Note that $b < \bar{b}$ implies $b < \beta - \Delta + \frac{2-\delta}{1-\delta}\kappa$. Then the minimum of (43) is obtained when $\beta = \Delta - \frac{1}{1-\delta}\kappa$ and $b = 0$. In this case, the minimum is $\Delta - \frac{\kappa}{1-\delta}$. The maximum of (43) is obtained when $\beta = \Delta - \frac{\delta}{1-\delta}\kappa$ and $b = \beta - \Delta + \frac{2-\delta}{1-\delta}\kappa$. In this case the maximum is $\Delta - \frac{\delta}{1-\delta}\kappa$. Therefore, if $\Delta - \frac{1}{1-\delta}\kappa < 0 < \Delta - \frac{\delta}{1-\delta}\kappa$ then there is a configuration of parameters such that $H^{**} > 0$ and a configuration such that $H^{**} < 0$. ■

Lemma 8 ***[Auxiliary result for Proposition 6 and Proposition 7].*** *Suppose condition (14) holds and $b \in \left[\underline{b}, \bar{b}\right)$. The expected shareholder value is given by*

$$V = \begin{cases} \psi_1 & \text{if in addition } \Delta - \beta - \frac{\delta}{1-\delta}\kappa \leq b \\ \psi_2 & \text{if in addition } b < \Delta - \beta - \frac{\delta}{1-\delta}\kappa, \end{cases} \tag{102}$$

where $\kappa \equiv \lambda\frac{\bar{q}-\underline{q}}{2}$,

$$\psi_1 \equiv \mathbb{E}\left[\widetilde{q}\right] + \delta\Delta + \frac{1-\delta}{\kappa}\frac{(\Delta+b)^2 - \left(\beta + \frac{\delta}{1-\delta}\kappa\right)^2}{4} \tag{103}$$

and

$$\psi_2 \equiv \mathbb{E}\left[\widetilde{q}\right] + \delta\Delta + \frac{1-\delta}{\kappa}\left(\Delta - \beta - \frac{\delta}{1-\delta}\kappa\right)\Delta. \tag{104}$$

Moreover, in the relevant range, ψ_1 and ψ_2 are decreasing in λ, ψ_2 is decreasing in β and ψ_1 is decreasing in β if and only if $\beta > \frac{\delta}{1-\delta}\kappa$.

Proof. Suppose condition (14) holds and $b \in \left[\underline{b}, \bar{b}\right)$. Based on (24), which is given, in the proof of Proposition 2, $\underline{b} < \bar{b}$ requires $\beta < \Delta + \frac{1-\delta}{2-\delta}\kappa$. Therefore, we restrict attention to $\Delta - \frac{1}{1-\delta}\kappa < \beta < \Delta + \frac{1-\delta}{2-\delta}\kappa$. According to part (ii) of Proposition 2, $p_1^* = \tau(\max\{q_L, s^{**}\})$ and

$$\begin{aligned} V &= \Pr\left[\tau\left(\widetilde{s}\right) \leq p_1^*\right] p_1^* + \Pr\left[\tau\left(\widetilde{s}\right) > p_1^*\right]\left(\mathbb{E}\left[\widetilde{q} | \tau\left(\widetilde{s}\right) > p_1^*\right] + \delta\Delta\right) \\ &= \Pr\left[\widetilde{s} \leq x\right]\tau\left(x\right) + \Pr\left[\widetilde{s} > x\right]\left(\mathbb{E}\left[\widetilde{q} | \widetilde{s} > x\right] + \delta\Delta\right) \\ &= (1-\delta)\frac{x-\underline{q}}{\bar{q}-\underline{q}}\left(\beta - \lambda\frac{\bar{q}-x}{2}\right) + \lambda\frac{x-\underline{q}}{2} + \mathbb{E}\left[\widetilde{q}\right] + \delta\Delta \\ &= (1-\delta)\frac{x-\underline{q}}{\bar{q}-\underline{q}}\left(\beta - \lambda\frac{\bar{q}-x}{2} + \frac{\bar{q}-\underline{q}}{1-\delta}\frac{\lambda}{2}\right) + \mathbb{E}\left[\widetilde{q}\right] + \delta\Delta, \end{aligned}$$

where $x = \max\{q_L, s^{**}\}$. Recall $s^{**} \geq q_L \Leftrightarrow b \geq \left|\Delta - \beta - \frac{\delta}{1-\delta}\kappa\right|$. Notice that if $\Delta - \frac{\delta}{1-\delta}\kappa \leq \beta$ then $\underline{b} \leq b$ requires $\beta - \Delta + \frac{\delta}{1-\delta}\kappa < b$, and hence, $s^{**} \geq q_L$. Therefore, the intersection of

$b \geq \left|\Delta - \beta - \frac{\delta}{1-\delta}\kappa\right|$ with $b \in \left[\underline{b}, \overline{b}\right)$ and (14) requires $\Delta - b - \frac{\delta}{1-\delta}\kappa \leq \beta < \Delta + \frac{1-\delta}{2-\delta}\kappa$. We consider two cases:

1. If in addition $\Delta - \beta - \frac{\delta}{1-\delta}\kappa \leq b$ then $p_1^* = \tau(s^{**})$ where s^{**} as given by (15). Letting $x = s^{**}$ in the expression above yields ψ_1 as given (103).

2. If in addition $b < \Delta - \beta - \frac{\delta}{1-\delta}\kappa$ then $q_L > \underline{q}$ and $p_1^* = \tau(q_L)$ where q_L as given by (13). Letting $x = q_L$ in the expression above yields ψ_2 as given (104).

The comparative statics with respect to β follow directly from (103) and (104). Note that $\frac{\partial \psi_2}{\partial \lambda} \propto \beta - \Delta$. Since $V = \psi_2$ only if $\beta < \Delta$ then $\frac{\partial \psi_2}{\partial \lambda} < 0$. Note that $\frac{\partial \psi_2}{\partial \lambda} \propto \beta^2 - (\frac{\delta}{1-\delta}\kappa)^2 - (\Delta + b)^2$. Thus, if $\beta \leq \Delta$ then $\frac{\partial \psi_2}{\partial \lambda} < 0$. Suppose $\Delta < \beta < \Delta + \frac{1-\delta}{2-\delta}\kappa$. Note that $b \geq \underline{b}$ implies $\beta \leq -\frac{\delta}{1-\delta}\kappa + \Delta + b$, which implies $\frac{\partial \psi_1}{\partial \lambda} < 0$, as required. ■

C. Supplemental results for Section 3.5

Proposition 8. *Suppose the board can issue a second recommendation after the bidder revises the initial offer but before target shareholders make their final decision. Suppose also* $b = 0$, $\lambda = 1$, *and* $\beta \in (\Delta - \frac{\overline{q}-\underline{q}}{2}, \Delta + \frac{\overline{q}-\underline{q}}{2})$, *then:*

(i) *If* $\delta < \frac{1}{2}$ *and* $\Delta - \frac{\overline{q}-\underline{q}}{2} < \beta \leq \Delta - \frac{1}{2}\frac{1}{1-\delta}\frac{\overline{q}-\underline{q}}{2}$ *then the board is influential with respect to the initial offer but not with respect to the revised offer. The bidder's initial offer is* $p_1^* = \tau\left(q^{**}\right)$ *where*

$$q^{**} = \underline{q} - 2\left(\frac{\delta}{1-\delta}\frac{\overline{q}-\underline{q}}{2} + \Delta - \beta\right), \tag{105}$$

and it is accepted by shareholders if and only if $\widetilde{q} < q^{**}$. *If the initial offer is rejected, the revised offer is* $p_2^* = \mathbb{E}\left[\widetilde{q}|\widetilde{q} > q^*\right] + \Delta$, *and it is always accepted by shareholders.*

(ii) *If* $\delta < \frac{1}{2}$ *and* $\Delta - \frac{1}{2}\frac{1}{1-\delta}\frac{\overline{q}-\underline{q}}{2} < \beta \leq \Delta$ *then the board is influential with respect to the initial and the revised offer. The bidder's initial offer is* $p_1^* = \eta\left(q^{***}\right)$ *where*

$$\eta\left(x\right) = \left(1-\delta\right)\left(x+\beta\right) + \delta\left(x + 2\Delta - \beta\right) \tag{106}$$

and

$$q^{***} = \underline{q} + 2\left(\Delta - \beta\right)\left(1 - 2\delta\right), \tag{107}$$

and it is accepted by shareholders if and only if $\widetilde{q} < q^{***}$. *If the initial offer is rejected, the revised offer is* $p_2^* = q^{***} + 2\Delta - \beta$, *and it is accepted by shareholders if and only if* $\widetilde{q} < p_2^* - \beta$.

(iii) *If $\delta \geq \frac{1}{2}$ and $\Delta - \frac{\overline{q}-\underline{q}}{2} < \beta \leq \Delta$ then the board is not influential with respect to the initial offer but it is influential with respect to the revised offer. The bidder's initial offer is rejected for sure, the revised offer is $p_2^* = \underline{q} + 2\Delta - \beta$ and it is accepted by shareholders if and only if $\widetilde{q} + \beta \leq p_2^*$.*

(iv) *If $\Delta < \beta$ then the board is not influential with respect to the initial and the revised offer. Both the initial offer and the revised offer are low such that they are always rejected by shareholders.*

Proof. If the board is not influential with respect to the initial offer, then no information about $\widetilde{q}$ is revealed in the first round. Therefore, the outcome of the second round is given by Proposition 2, applied to cases where $\delta = 0$. Since $b = 0$ the expected profit of the bidder in this case is zero.

We consider two main cases. First, suppose $\Delta - \frac{\overline{q}-\underline{q}}{2} < \beta \leq \Delta$. Suppose also that the board is influential with respect to the initial offer p_1, and that the initial offer is rejected if and only $\widetilde{q} \geq \chi \in (\underline{q}, \overline{q})$. The analysis of the second round is given by Proposition 2 where $\delta = 0$ and $\widetilde{q}$ is distributed uniformly on $[\chi, \overline{q}]$. Consider the following cases:

1. If $\overline{q} - 2(\Delta - \beta) \leq \chi$ then $\beta \leq \Delta - \frac{\overline{q}-\chi}{2}$, and based on Corollary 1, the board is not influential with respect to any revised offer. Therefore, the bidder offers shareholders $p_2 = \mathbb{E}[q|q > \chi] + \Delta$ and acquires the firm with probability one.

2. If $\chi < \overline{q} - 2(\Delta - \beta)$ then $\Delta - \frac{\overline{q}-\chi}{2} < \beta \leq \Delta$ based on Proposition 2 the bidder offers shareholders $p_2 = \chi + 2\Delta - \beta$, and the board is influential with respect to this offer. The revised offer is accepted if and only if $p_2 \geq \widetilde{q} + \beta$.

Overall, given χ and q, the board's utility in the second round is

$$W_{board}(\chi, q) = \begin{cases} \max\{\chi + 2\Delta - \beta, q + \beta\} & \text{if } \chi < \overline{q} - 2(\Delta - \beta) \\ \Delta + \frac{\overline{q}+\chi}{2} & \text{if } \overline{q} - 2(\Delta - \beta) \leq \chi. \end{cases} \tag{108}$$

If the board is influential in the first round, it recommends accepting the offer p_1 if and only if

$$p_1 \geq \delta W_{board}(\chi, q) + (1 - \delta)(q + \beta). \tag{109}$$

Notice that the right hand side ("RHS") strictly increases in q (when $\delta < 1$). Therefore, $\chi \in (\underline{q}, \overline{q})$ requires in equilibrium the board to be indifferent between approving and rejecting offer p_1 when $q = \chi$. Given p_1, $\chi(p_1)$ is determined such that

$$p_1 = \delta W_{board}(\chi, \chi) + (1-\delta)(\chi + \beta) \Leftrightarrow \tag{110}$$

$$p_1 = (1-\delta)(\chi+\beta) + \delta \times \begin{cases} \chi + 2\Delta - \beta & \textit{if } \chi < \overline{q} - 2(\Delta - \beta) \\ \frac{\overline{q}+\chi}{2} + \Delta & \textit{if } \overline{q} - 2(\Delta-\beta) \leq \chi, \end{cases} \tag{111}$$

and note that $\overline{q} - 2(\Delta - \beta) \in (\underline{q}, \overline{q})$. Also notice that the RHS is continuous and strictly increasing in χ. Therefore, χ is uniquely defined by p_1.

The board is influential with respect to the initial offer p_1 only if in addition

$$\underline{q} < \chi(p_1) < \overline{q} \text{ and } \mathbb{E}[\widetilde{q}|\widetilde{q} \leq \chi(p_1)] + \Delta \leq p_1 < \mathbb{E}[\widetilde{q}|\widetilde{q} > \chi(p_1)] + \Delta. \tag{112}$$

The intersection of these conditions is

$$\begin{aligned} \underline{q} &< \chi < \overline{q} - 2(\Delta - \beta) \text{ and} \\ \frac{\chi + \underline{q}}{2} + \Delta &\leq (1-\delta)(\chi+\beta) + \delta(\chi + 2\Delta - \beta) < \frac{\chi + \overline{q}}{2} + \Delta, \end{aligned} \tag{113}$$

or

$$\overline{q} - 2(\Delta - \beta) \leq \chi < \overline{q} \text{ and } \frac{\chi + \underline{q}}{2} + \Delta \leq \tau(\chi) < \frac{\chi + \overline{q}}{2} + \Delta, \tag{114}$$

which is given by

$$\underline{q} + 2(\Delta - \beta)\max\{1 - 2\delta, 0\} \leq \chi < \overline{q} - 2(\Delta - \beta) \tag{115}$$

and

$$\max\left\{\overline{q} - 2(\Delta-\beta), \underline{q} - 2\frac{\delta}{1-\delta}\frac{\overline{q}-\underline{q}}{2} + 2(\Delta - \beta)\right\} \leq \chi < \overline{q}. \tag{116}$$

Note

$$\underline{q} + 2(\Delta - \beta)\max\{1-2\delta, 0\} < \overline{q} - 2(\Delta-\beta) \Leftrightarrow \Delta - \frac{1}{\max\{2(1-\delta), 1\}}\frac{\overline{q}-\underline{q}}{2} < \beta \tag{117}$$

and

$$\underline{q} - 2\frac{\delta}{1-\delta}\frac{\overline{q}-\underline{q}}{2} + 2(\Delta-\beta) < \overline{q} - 2(\Delta - \beta) \Leftrightarrow \Delta - \frac{1}{2(1-\delta)}\frac{\overline{q}-\underline{q}}{2} < \beta, \tag{118}$$

where

$$\Delta - \frac{1}{2(1-\delta)} \frac{\overline{q}-\underline{q}}{2} < \Delta - \frac{1}{\max\{2(1-\delta),1\}} \frac{\overline{q}-\underline{q}}{2}. \tag{119}$$

There are two sub-cases:

1. Suppose $\Delta - \frac{\overline{q}-\underline{q}}{2} < \beta < \Delta - \frac{1}{\max\{2(1-\delta),1\}} \frac{\overline{q}-\underline{q}}{2}$. This condition requires $\delta < \frac{1}{2}$. In this case, condition 115 is empty, that is, the board cannot be influential in both rounds. Moreover, $\Delta - \frac{1}{2(1-\delta)} \frac{\overline{q}-\underline{q}}{2} = \Delta - \frac{1}{\max\{2(1-\delta),1\}} \frac{\overline{q}-\underline{q}}{2}$, and thus, $\beta < \Delta - \frac{1}{\max\{2(1-\delta),1\}} \frac{\overline{q}-\underline{q}}{2}$ implies $\overline{q} - 2(\Delta - \beta) < \underline{q} - 2\frac{\delta}{1-\delta} \frac{\overline{q}-\underline{q}}{2} + 2(\Delta - \beta)$. The board is influential with respect to the first round if and only if $\tau(q^{**}) \leq p_1 < \tau(\overline{q})$ where

$$q_L^* = \underline{q} - 2\frac{\delta}{1-\delta} \frac{\overline{q}-\underline{q}}{2} + 2(\Delta - \beta) \in (\underline{q}, \overline{q}). \tag{120}$$

   Notice that q_L^* satisfies $\tau(q_L^*) = E[q|q < q_L^*] + \Delta$, and hence, if the bidder offers $p_1 < \tau(p_L^*)$ then the offer is rejected for sure, and bidder makes a zero profit in the next round. If the board is influential with respect to the initial offer, the bidder solves

$$\max_{\chi \in [q_L^*, \overline{q})} \Pr[q < \chi] \left(E[q|q < \chi] + \Delta - \tau(\chi)\right). \tag{121}$$

   The unconstrained optimum is

$$q^{**} = \underline{q} - \frac{\delta}{1-\delta} \frac{\overline{q}-\underline{q}}{2} + \Delta - \beta. \tag{122}$$

   Notice that $\Delta - \frac{\overline{q}-\underline{q}}{2} < \beta$ implies $q^{**} < \overline{q}$. Therefore, the bidder never makes an initial offer that is always accepted. Also notice that $\beta < \Delta - \frac{1}{2(1-\delta)} \frac{\overline{q}-\underline{q}}{2}$ implies $q^{**} < q_L^*$. Thus, if the bidder makes an initial offer with respect to which the board is influential, he offers $\tau(q_L^*)$, which generates a zero expected profit.

2. Suppose $\max\left\{\Delta - \frac{1}{\max\{2(1-\delta),1\}} \frac{\overline{q}-\underline{q}}{2}, \Delta - \frac{\overline{q}-\underline{q}}{2}\right\} < \beta < \Delta$. In this case, condition 115 becomes

$$\underline{q} + 2(\Delta - \beta) \max\{1 - 2\delta, 0\} \leq \chi < \overline{q} - 2(\Delta - \beta) \tag{123}$$

   and condition 116 becomes

$$\overline{q} - 2(\Delta - \beta) \leq \chi < \overline{q}. \tag{124}$$

   Let

$$\eta(x) = (1-\delta)(x + \beta) + \delta(x + 2\Delta - \beta) \tag{125}$$

Note that

$$p' \equiv \eta\left(\underline{q} + 2(\Delta - \beta)\max\{1-2\delta, 0\}\right) = \underline{q} + \beta + 2\max\{1-\delta, \delta\}(\Delta - \beta) \tag{126}$$

and

$$p' < \Delta + \frac{\overline{q} + \underline{q}}{2} \Leftrightarrow \Delta - \frac{1}{|1-2\delta|}\frac{\overline{q} - \underline{q}}{2} < \beta \tag{127}$$

However, $\Delta - \frac{1}{|1-2\delta|}\frac{\overline{q}-\underline{q}}{2} < \Delta - \frac{1}{\max\{2(1-\delta),1\}}\frac{\overline{q}-\underline{q}}{2}$, and hence, this condition always holds. Therefore, if $p_1 < p'$ the initial offer is rejected for sure, no information about q is revealed, and in the second round, based on Proposition 2, the bidder makes a zero profit. Therefore, the bidder maximizes the following

$$\Pi(p) = \begin{cases} 0 & \textit{if } p_1 < p' \\ \Pr[q < \chi]\left(E[q|q<\chi] + \Delta - \eta(\chi)\right) & \textit{if } p' \le p_1 < \tau(\overline{q} - 2(\Delta - \beta)) \\ \Pr[q < \chi]\left(E[q|q<\chi] + \Delta - \tau(\chi)\right) & \textit{if } \tau(\overline{q} - 2(\Delta - \beta)) \le p_1 \le \tau(\overline{q}) \\ E[q] + \Delta - \tau(\overline{q}) - (p_1 - \tau(\overline{q})) & \textit{if } \tau(\overline{q}) < p_1. \end{cases} \tag{128}$$

Clearly, bidding $\tau(\overline{q}) < p_1$ is sub-optimal. Notice that $q_L^* < \overline{q} - 2(\Delta - \beta)$ in this range, and since q_L^* satisfies $\tau(q_L^*) = E[q|q < q_L^*] + \Delta$, any initial offer $\tau(\overline{q} - 2(\Delta - \beta)) \le p_1 \le \tau(\overline{q})$ would generate a strictly negative profit for the bidder. Therefore, the bidder will not make such offer. Notice that

$$\arg\max_{\chi} \Pr[q < \chi]\left(E[q|q<\chi] + \Delta - \eta(\chi)\right) = \underline{q} + (\Delta - \beta)(1 - 2\delta) \tag{129}$$

and that $E[q|q<\chi] + \Delta - \eta(\chi) > 0$ if and only if $\chi > \underline{q} + 2(\Delta - \beta)(1-2\delta)$. Thus, if $\delta \ge \frac{1}{2}$, the bidder will make an initial offer that is rejected for sure, and move to the second round where his offer would be influential. If $\delta < \frac{1}{2}$ then the bidder would offer $p' = \eta\left(\underline{q} + 2(\Delta - \beta)(1-2\delta)\right)$ and make a zero profit.

Second, suppose $\Delta < \beta < \Delta + \frac{\overline{q}-\underline{q}}{2}$. Suppose also the board is influential with respect to the initial offer, and the initial offer p_1 is rejected if and only $\widetilde{q} \ge \chi \in (\underline{q}, \overline{q})$. The analysis of the second round is given by Proposition 2 where $\delta = 0$ and $\widetilde{q}$ is distributed uniformly on $[\chi, \overline{q}]$. Consider the following cases:

1. If $\overline{q} - 2(\beta - \Delta) \le \chi$ then $\Delta + \frac{\overline{q}-\chi}{2} \le \beta$, and based on Corollary 1, the board is not influential with respect to any revised offer. Therefore, the bidder offers shareholders $p_2 = \mathbb{E}[q|q > \chi] + \Delta$ and acquires the firm with probability one.

2. If $\overline{q}-2(\beta-\Delta)>\chi$ then $\Delta<\beta<\Delta+\frac{\overline{q}-\chi}{2}<\beta$, and based on Proposition 2 the bidder makes an offer that is always rejected by the shareholders and the board. The target remains independent.

Overall, given χ and q, the board's utility in the second round is

$$W_{board}(\chi,q)=\begin{cases} q+\beta & \textit{if } \chi<\overline{q}-2(\beta-\Delta) \\ \Delta+\frac{\overline{q}+\chi}{2} & \textit{if } \overline{q}-2(\beta-\Delta)\leq\chi. \end{cases} \tag{130}$$

As in the case where $\beta\leq\Delta$, $\chi(p_1)$ is determined such that

$$p_1 = \delta W_{board}(\chi,\chi)+(1-\delta)(\chi+\beta)\Leftrightarrow \tag{131}$$

$$p_1 = (1-\delta)(\chi+\beta)+\delta\times\begin{cases} \chi+\beta & \textit{if } \chi<\overline{q}-2(\beta-\Delta) \\ \Delta+\frac{\overline{q}+\chi}{2} & \textit{if } \overline{q}-2(\beta-\Delta)\leq\chi. \end{cases} \tag{132}$$

and note that $\overline{q}-2(\beta-\Delta)\in(\underline{q},\overline{q})$.

The board is influential with respect to the initial offer p_1 only if in addition

$$\underline{q}<\chi(p_1)<\overline{q} \text{ and } \mathbb{E}[\widetilde{q}|\widetilde{q}\leq\chi(p_1)]+\Delta\leq p_1<\mathbb{E}[\widetilde{q}|\widetilde{q}>\chi(p_1)]+\Delta. \tag{133}$$

The intersection of these conditions is

$$\underline{q} < \chi<\overline{q}-2(\beta-\Delta) \text{ and } \frac{\chi+\underline{q}}{2}+\Delta\leq\chi+\beta<\frac{\chi+\overline{q}}{2}+\Delta \tag{134}$$

or

$$\overline{q}-2(\beta-\Delta) \leq \chi<\overline{q} \text{ and } \frac{\chi+\underline{q}}{2}+\Delta\leq\tau(\chi)<\frac{\chi+\overline{q}}{2}+\Delta, \tag{135}$$

which is given by

$$\underline{q}<\chi<\overline{q}-2(\beta-\Delta) \tag{136}$$

and

$$\max\{\overline{q}-2(\beta-\Delta),q_L^*\}\leq\chi<\overline{q} \tag{137}$$

Note that $\beta > \Delta$ implies $\overline{q} - 2(\beta - \Delta) > q_L^*$. Therefore, the bidder maximizes the following

$$\Pi(p) = \begin{cases} 0 & \text{if } p_1 < \underline{q} + \beta \\ \Pr[q < \chi](E[q|q < \chi] + \Delta - \chi - \beta) & \text{if } \underline{q} + \beta \leq p_1 < \tau(\overline{q} - 2(\beta - \Delta)) \\ \Pr[q < \chi](E[q|q < \chi] + \Delta - \tau(\chi)) & \text{if } \tau(\overline{q} - 2(\beta - \Delta)) \leq p_1 \leq \tau(\overline{q}) \\ E[q] + \Delta - \tau(\overline{q}) - (p_1 - \tau(\overline{q})) & \text{if } \tau(\overline{q}) < p_1. \end{cases} \tag{138}$$

Clearly, bidding $\tau(\overline{q}) < p_1$ is sub-optimal. Since $q_L^* < \overline{q} - 2(\beta - \Delta)$ and q_L^* satisfies $\tau(q_L^*) = E[q|q < q_L^*] + \Delta$, any initial offer $\tau(\overline{q} - 2(\beta - \Delta)) \leq p_1 \leq \tau(\overline{q})$ would generate a strictly negative profit for the bidder. Therefore, the bidder will not make such offer. Notice that

$$E[q|q < \chi] + \Delta - \chi - \beta > 0 \Leftrightarrow \chi < \underline{q} - 2(\beta - \Delta). \tag{139}$$

Therefore, the bidder will make an initial and revised offers that are rejected for sure. ■

D. Veto power

The next result characterizes the conditions under which granting cooperate boards with veto power benefits target shareholders.

Proposition 9. *Suppose $\delta = 0$. The expected target shareholder value in equilibrium is lower when the board has veto power than when it does not, if and only if $\beta \leq \Delta$, or $\beta > \Delta$ and $b \leq b^{**}$, where*

$$b^{**} = \begin{cases} \left(\sqrt{\lambda(\overline{q} - \underline{q})} - \sqrt{\beta - \Delta}\right)^2 & \text{if } \Delta < \beta < \Delta + \lambda\frac{\overline{q}-\underline{q}}{4} \\ \sqrt{\beta^2 + 2\lambda(\overline{q} - \underline{q})(\lambda\frac{\overline{q}-\underline{q}}{2} + 2\Delta - \beta)} - \Delta & \text{if } \Delta + \lambda\frac{\overline{q}-\underline{q}}{4} \leq \beta < \Delta + \lambda\frac{\overline{q}-\underline{q}}{2} \\ \sqrt{\beta^2 + 2\lambda(\overline{q} - \underline{q})\Delta} - \Delta & \text{if } \Delta + \lambda\frac{\overline{q}-\underline{q}}{2} \leq \beta. \end{cases} \tag{140}$$

Proof. When the board has no veto power, the expected shareholder value in equilibrium is given by V, where $V = \mathbb{E}[\widetilde{q}] + \Delta$ if (14) is violated and V is given by (16) otherwise.

Suppose the board has veto power. The board exercises this power and blocks the takeover if and only $p < \tau(\widetilde{s})$. If $p \geq \tau(\widetilde{s})$ the board does not veto the deal, but target shareholders still must accept the offer for the deal to go forward. Conditional on $p \geq \tau(\widetilde{s})$, the board always prefers shareholders to accept the deal, and hence, will send a message that leads to the lowest beliefs about $\widetilde{q}$. Therefore, conditional on not rejecting the deal, the board cannot affect the decision of shareholders. In those cases, shareholders accept the offer if and only if

$\mathbb{E}[\widetilde{q}|\tau(\widetilde{s}) \leq p] + \Delta \leq p$. The takeover succeeds if and only if

$$\mathbb{E}[\widetilde{q}] + \Delta \leq p \textit{ and } \tau(\overline{q}) \leq p, \textit{ or } \mathbb{E}[\widetilde{q}|\tau(\widetilde{s}) \leq p] + \Delta \leq p \textit{ and } \tau(\widetilde{s})\} < p < \tau(\overline{q}), \tag{141}$$

which is equivalent to

$$\max\{\tau(\overline{q}), \mathbb{E}[\widetilde{q}] + \Delta\} \leq p \textit{ or } \max\{\tau(q_L), \tau(\widetilde{s})\} < p < \tau(\overline{q}). \tag{142}$$

Note that if $\delta = 0$ then (9) implies $\tau(x) = \lambda x + (1-\lambda)\mathbb{E}[\widetilde{q}] + \beta$ and (13) implies $q_L = \underline{q} + \frac{2}{\lambda}(\Delta - \beta)$. Let $\kappa = \lambda \frac{\overline{q}-\underline{q}}{2}$ and note that

$$\tau(\overline{q}) < \mathbb{E}[\widetilde{q}] + \Delta \Leftrightarrow \beta < \Delta - \kappa \tag{143}$$

$$q_L < \underline{q} \Leftrightarrow \Delta < \beta \tag{144}$$

$$q_L < \overline{q} \Leftrightarrow \Delta - \kappa < \beta \tag{145}$$

Overall, the takeover succeeds if and only if

$$\begin{cases} \mathbb{E}[\widetilde{q}] + \Delta & \textit{if } \beta < \Delta - \kappa \\ \tau(\max\{\widetilde{s}, q_L) & \textit{if } \Delta - \kappa < \beta \leq \Delta \leq p. \\ \tau(\widetilde{s}) & \textit{if } \Delta < \beta. \end{cases} \tag{146}$$

The comparison with Proposition 2 suggests that if $\beta \leq \Delta$ then the analysis with veto power is identical to the analysis without veto power. Let U be the expected shareholder value in equilibrium with veto power, then $U = V$ if $\beta \leq \Delta$.

If $\beta > \Delta$ then the takeover succeeds if and only if the board does not veto it. We make the following observations. First, the bidder has no reason to offer more than $\tau(\overline{q})$. Second, if $p_1 = \tau(x)$ for $\underline{q} \leq x \leq \overline{q}$ then the bidder's profit is $\pi(x)$ as defined by (25). Note that $\pi(\underline{q}) = 0$, $\pi(\overline{q}) = \mathbb{E}[\widetilde{q}] + \Delta + b - \tau(\overline{q})$ and $s^{**} = \arg\max \pi(x)$. Note that (15) implies $s^{**} = \underline{q} + \frac{1}{\lambda}(\Delta + b - \beta)$, if $s^{**} \in (\underline{q}, \overline{q})$ then $\pi(s^{**}) = \frac{(\Delta-\beta+b)^2}{4\kappa}$, and if $s^{**} \geq \overline{q}$ then $\pi(\overline{q}) > 0$. Third,

$$s^{**} < \underline{q} \Leftrightarrow b < \beta - \Delta \tag{147}$$

$$s^{**} < \overline{q} \Leftrightarrow b < \beta - \Delta + 2\kappa. \tag{148}$$

Combined, if $\beta > \Delta$ then the following holds: (i) If $b \leq \beta - \Delta$ then $p^* = \tau(\underline{q})$ the takeover fails with probability one. (ii) If $b \in [\beta - \Delta, \beta - \Delta + 2\kappa)$ then $p^* = \tau(s^{**})$ and the takeover succeeds if and only if $\widetilde{s} \leq s^{**}$. (iii) If $b \geq \beta - \Delta + 2\kappa$ then $p^* = \tau(\overline{q})$ and the takeover always succeeds. The analysis above implies that if $\beta > \Delta$ then

$$U = \begin{cases} \mathbb{E}[\widetilde{q}] & \text{if } b \in [0, \beta - \Delta) \\ \Pr[\widetilde{s} \leq s^{**}] \tau(s^{**}) + \Pr[\widetilde{s} > s^{**}] \mathbb{E}[\widetilde{q} | \widetilde{s} > s^{**}] & \text{if } b \in [\beta - \Delta, \beta - \Delta + 2\kappa) \\ \tau(\overline{q}) & \text{if } b \in [\beta - \Delta + 2\kappa, \infty), \end{cases} \tag{149}$$

$$= \mathbb{E}[\widetilde{q}] + \begin{cases} 0 & \text{if } b \in [0, \beta - \Delta) \\ \frac{(\Delta + b)^2 - \beta^2}{4\kappa} & \text{if } b \in [\beta - \Delta, \beta - \Delta + 2\kappa) \\ \kappa + \beta & \text{if } b \in [\beta - \Delta + 2\kappa, \infty). \end{cases} \tag{150}$$

There are three cases to consider. First, if $\beta \geq \kappa + \Delta$ then $V = \mathbb{E}[\widetilde{q}] + \Delta$, and hence, $U \leq V$ if and only if

$$b \in [0, \beta - \Delta) \text{ or, } b \in [\beta - \Delta, \beta - \Delta + 2\kappa) \text{ and } \frac{(\Delta + b)^2 - \beta^2}{4\kappa} \leq \Delta \Leftrightarrow$$
$$b \leq \sqrt{\beta^2 + 4\kappa\Delta} - \Delta. \tag{151}$$

Second, suppose $\Delta + \frac{\kappa}{2} \leq \beta < \Delta + \kappa$. Based on the proof of Proposition 2,

$$V = \mathbb{E}[\widetilde{q}] + \begin{cases} 0 & \text{if } b \in [0, \Delta - \beta + \kappa) \\ \kappa + 2\Delta - \beta & \text{if } b \in [\Delta - \beta + \kappa, \infty). \end{cases} \tag{152}$$

Note that $\Delta + \frac{\kappa}{2} \leq \beta \Rightarrow \Delta - \beta + \kappa \leq \beta - \Delta$. Therefore, $U \leq V$ if and only if

$$b \in [0, \beta - \Delta), \text{ or } b \in [\Delta - \beta, \beta - \Delta + 2\kappa) \text{ and } \frac{(\Delta + b)^2 - \beta^2}{4\kappa} \leq \kappa + 2\Delta - \beta \Leftrightarrow$$
$$b \leq \sqrt{\beta^2 + 4\kappa(\Delta + \kappa + \Delta - \beta)} - \Delta.$$

Third, suppose $\Delta < \beta < \Delta + \frac{\kappa}{2}$. Based on the proof of Proposition 2,

$$V = \mathbb{E}[\widetilde{q}] + \begin{cases} 0 & \text{if } b \in [0, \beta - \Delta) \\ \frac{(\Delta + b)^2 - \beta^2}{4\kappa} & \text{if } b \in \left[\beta - \Delta, \beta - \Delta + 2\kappa - 2\sqrt{\beta - \Delta}\sqrt{2\kappa}\right) \\ \kappa + 2\Delta - \beta & \text{if } b \in \left[\beta - \Delta + 2\kappa - 2\sqrt{\beta - \Delta}\sqrt{2\kappa}, \infty\right). \end{cases} \tag{153}$$

Therefore, $U \leq V$ if and only if

$$\begin{aligned}
b &\in [0, \beta - \Delta + 2\kappa) \text{ or,} \\
b &\in \left[\beta - \Delta, \beta - \Delta + 2\kappa - 2\sqrt{\beta - \Delta}\sqrt{2\kappa}\right) \text{ and } \frac{(\Delta + b)^2 - \beta^2}{4\kappa} \leq \kappa + 2\Delta - \beta \Leftrightarrow \\
b &\leq \beta - \Delta + 2\kappa - 2\sqrt{\beta - \Delta}\sqrt{2\kappa}. \qquad (154)
\end{aligned}$$

The combination of the above four cases provides the result. ■

The reasoning behind Proposition 9 is the following. With veto power, the board rejects the offer if $\tau(\widetilde{s}) > p$ and the transaction fails for sure. If $\tau(\widetilde{s}) \leq p$ then the board does not reject the offer and the approval of the takeover depends on shareholders' collective decision. Shareholders use the information from the board's decision and recommendation. It turns out that apart from the information that is conveyed by the board's decision not to reject the offer, no other information can be revealed by the board through communication. Overall, if the board does not reject the offer shareholders approve the takeover if and only if condition (11) holds.

If $\beta \leq \Delta$ then the board is less biased against the takeover than target shareholders, who suffer from the free-rider problem. In this case, the veto power of the board does not bind, and the analysis with veto power is identical to the analysis without veto power. If $\beta > \Delta$ then the board is more biased against the takeover than target shareholders. Therefore, whenever the board does not veto the deal, shareholders approve it. Here, granting the board with veto power benefits shareholders only if b is sufficiently high. Intuitively, if both β and b are high, granting a biased board a veto power is a mechanism by which shareholders can extract more surplus from the a highly motivated bidder. Interestingly, shareholders can be worse off by grating the board with veto power. This case is more likely when λ or $\overline{q} - \underline{q}$ are high, that is, when the board has more information. Intuitively, by granting the board veto power, the adverse selection problem is intensified since the board can more easily act based on this information, it does not need to convince shareholders to reject the deal. As a consequence, bidder is likely to shade the deal to avoid overpaying for the target. This effect can harm target shareholder value.