Internet Appendix to

"Can Financial Innovation Succeed by Catering to Behavioral Preferences? Evidence from a Callable Options Market"

(not to be included for publication)

Xindan Li, Avanidhar Subrahmanyam, Xuewei Yang[1]

March 12, 2017

In this appendix we first provide details of the derivations for the ex ante skewness and arbitrage-free pricing of CBBCs (Sections B.1 to B.4). We also provide some ancillary empirical results in Section B.5.

B.1 A Risk-neutral Pricing Formula for CBBCs

This section describes the basic pricing relations for CBBCs. By virtue of the risk-neutral valuation formula (e.g., Harrison and Pliska 1981), the price of a bull contract at $t \leqslant T_b$ is given by:

$$\begin{aligned} P_t^{\text{bull}}(T-t) &= e^{-r(T-t)}\mathbb{E}_t\left[(S_T - K)\,1_{\{T_b>T\}}\right] \\ &\quad +\mathbb{E}_t\left[e^{-r(T_b+T_0-t)}1_{\{T_b\leqslant T\}}\left(\min_{T_b\leqslant u\leqslant T_b+T_0} S_u - K\right)^+\right], \end{aligned} \tag{B.1}$$

where $r > 0$ is the constant risk-free rate, T is the maturity date, $S := (S_t)_{t\geqslant 0}$ is the price process of the underlying asset, K is the strike price, and $T_b := \inf\{t \geqslant 0; S_t \leqslant S_b\}$ is the first time that the price process S crosses the call level S_b. T_0 is the settlement period given the call level is hit. Here

[1]Corresponding author at: School of Management and Engineering, Nanjing University, 22 Hankou Road, Gulou District, Nanjing, Jiangsu 210093, China. E-mail address: xwyang@nju.edu.cn (Xuewei Yang), xwyang@aliyun.com (Xuewei Yang).

$(x)^+ := \max(x, 0)$, and $\mathbb{E}_t[\cdot]$ is the expectation under the risk-neutral measure given information known at time t. Similarly, the price of a bear contract can be expressed as:

$$\begin{aligned} P_t^{\text{bear}}(T-t) &= e^{-r(T-t)}\mathbb{E}_t\left[(K-S_T)\,1_{\{\widetilde{T}_b>T\}}\right] \\ &\quad +\mathbb{E}_t\left[e^{-r(\widetilde{T}_b+T_0-t)}1_{\{\widetilde{T}_b\leqslant T\}}\left(K-\max_{\widetilde{T}_b\leqslant u\leqslant \widetilde{T}_b+T_0} S_u\right)^+\right], \end{aligned} \tag{B.2}$$

with $\widetilde{T}_b := \inf\{t \geqslant 0; S_t \geqslant S_b\}$. Intuitively, if the asset price S hits the call level S_b before the maturity date T, the investor loses the value of the first expectation in (B.1) or (B.2), which is just a down-and-out option, and enters into an exotic option with a short maturity T_0. These represent the formulae for "exotic" CBBCs.

For a vanilla CBBC, the settlement price given MCE equals its call level, and the length of its settlement period is equal to zero. Accordingly, the values of vanilla bull/bear contracts are given by:

$$\mathbb{E}_t\left[e^{-r(T_b-t)}1_{\{T_b\leqslant T\}}\,(S_b-K)^+\right] \quad \text{and} \quad \mathbb{E}_t\left[e^{-r(\widetilde{T}_b-t)}1_{\{\widetilde{T}_b\leqslant T\}}\,(K-S_b)^+\right],$$

respectively. Here T_b and $\widetilde{T}_b$ are the same as those in (B.1) and (B.2).

B.2 Brownian Motion with Drift, First Passage Time, and its Running Minimum (Maximum)

In this section, we present some theoretical results related to Brownian motion with drift, its first passage time, and its running minimum (maximum). These results facilitate the derivation of closed-form formulae for the ex ante skewness and values of CBBC in the next two sections.

Assume W is a standard Brownian motion (Wiener process). For any $\sigma > 0$, $\mu \in \mathbb{R}$ and $b \in \mathbb{R}$, define $\tau_b := \inf\{t \geq 0 : \mu t + \sigma W_t = b\}$, then for any $a \in \mathbb{R}$ such that $b(b-a) \geq 0$, we have (e.g., Karatzas and Shreve 1991, Sections 2.8.A and 3.5.C):

$$
\begin{aligned}
&\mathbb{P}(\mu t+\sigma W_t \in da, \tau_b > t) \\
= \;& \frac{1}{\sqrt{2\pi t}\,\sigma} \exp\left(\frac{\mu a}{\sigma^2}-\frac{\mu^2 t}{2\sigma^2}\right)\left(\exp\left(-\frac{a^2}{2\sigma^2 t}\right)-\exp\left(-\frac{(2b-a)^2}{2\sigma^2 t}\right)\right) da,
\end{aligned}
$$

which yields:

$$
\begin{aligned}
& f(\lambda,\mu,\sigma,b,t) := \mathbb{E}\left[e^{\lambda(\mu t+\sigma W_t)} 1_{\{\tau_b>t\}}\right] \\
= & \begin{cases} \int_{-\infty}^{b} e^{\lambda a}\mathbb{P}(\mu t+\sigma W_t \in da, \tau_b > t), & b \geqslant 0, \\ \int_{b}^{\infty} e^{\lambda a}\mathbb{P}(\mu t+\sigma W_t \in da, \tau_b > t), & b \leqslant 0, \end{cases} \\
= & \begin{cases} e^{\frac{1}{2}\lambda t\left(\lambda\sigma^2+2\mu\right)}\left(N\left(d_1\right)-e^{2b\lambda+\frac{2b\mu}{\sigma^2}}N\left(-d_2\right)\right), & b \geqslant 0, \\ e^{\frac{1}{2}\lambda t\left(\lambda\sigma^2+2\mu\right)}\left(N\left(-d_1\right)-e^{2b\lambda+\frac{2b\mu}{\sigma^2}}N\left(d_2\right)\right), & b \leqslant 0, \end{cases}
\end{aligned}
\tag{B.3}
$$

with $N(\cdot)$ being the cumulative distribution function (cdf) of a standard normal distribution, and

$$
d_1 = \frac{b-\mu t-\lambda\sigma^2 t}{\sigma\sqrt{t}}, \quad d_2 = \frac{b+t\mu+\lambda\sigma^2 t}{\sigma\sqrt{t}}.
$$

Specifically, for $t \geq 0$, explicit formulae for the tail probability and the density of the first passage time τ_b can be expressed as:

$$
\begin{aligned}
\mathbb{P}\left(\tau_b > t\right) &= f(0,\mu,\sigma,b,t) = \begin{cases} N\left(\frac{b-\mu t}{\sigma\sqrt{t}}\right) - e^{\frac{2b\mu}{\sigma^2}}N\left(-\frac{b+t\mu}{\sigma\sqrt{t}}\right), & b \geqslant 0, \\ N\left(-\frac{b-\mu t}{\sigma\sqrt{t}}\right) - e^{\frac{2b\mu}{\sigma^2}}N\left(\frac{b+t\mu}{\sigma\sqrt{t}}\right), & b \leqslant 0, \end{cases} \\
\mathbb{P}(\tau_b \in dt) &= -\frac{\partial\mathbb{P}\left(\tau_b > t\right)}{\partial t}dt = \frac{|b|}{\sqrt{2\pi t^3}\sigma}\exp\left(-\frac{(b-\mu t)^2}{2t\sigma^2}\right) dt.
\end{aligned}
$$

We then obtain:

$$\begin{aligned}
\eta(\lambda,\mu,\sigma,b,T) &:= \mathbb{E}\left[e^{-\lambda\tau_b}1_{\{\tau_b\leqslant T\}}\right] = \int_0^T e^{-\lambda t}\mathbb{P}(\tau_b \in dt) \\
&= \begin{cases} e^{\frac{b(\mu+\mu_1)}{\sigma^2}} N\left(-d_3\right) + e^{\frac{b(\mu-\mu_1)}{\sigma^2}} N\left(-d_4\right), & b \geqslant 0, \\ e^{\frac{b(\mu+\mu_1)}{\sigma^2}} N\left(d_3\right) + e^{\frac{b(\mu-\mu_1)}{\sigma^2}} N\left(d_4\right), & b \leqslant 0, \end{cases}
\end{aligned} \tag{B.4}$$

where $d_3 = \frac{b+T\mu_1}{\sigma\sqrt{T}}$, $d_4 = \frac{b-T\mu_1}{\sigma\sqrt{T}}$ with $\mu_1 := \sqrt{2\lambda\sigma^2+\mu^2}$. As by products, we also have, for $\mu = 0$,

$$\theta(0,\sigma,b,T) = Tf(0,0,\sigma,b,T) + \frac{|b|}{\sigma^2}\left(\sqrt{\frac{2}{\pi}}\sigma\sqrt{T}e^{-\frac{b^2}{2\sigma^2 T}} - 2|b|N\left(-\frac{|b|}{\sigma\sqrt{T}}\right)\right),$$

and, for $\mu \neq 0$,

$$\begin{aligned}
&\theta(\mu,\sigma,b,T) := \mathbb{E}\left[\tau_b \wedge T\right] = T\mathbb{P}(\tau_b > T) + \mathbb{E}\left[\tau_b 1_{\{\tau_b\leqslant T\}}\right] \\
&= Tf(0,\mu,\sigma,b,T) + \begin{cases} \frac{b}{|\mu|}\left(e^{\frac{b(\mu-|\mu|)}{\sigma^2}} N\left(-\frac{b-|\mu|T}{\sigma\sqrt{T}}\right) - e^{\frac{b(\mu+|\mu|)}{\sigma^2}} N\left(-\frac{b+|\mu|T}{\sigma\sqrt{T}}\right)\right), & b \geqslant 0, \\ \frac{b}{|\mu|}\left(e^{\frac{b(\mu-|\mu|)}{\sigma^2}} N\left(\frac{b-|\mu|T}{\sigma\sqrt{T}}\right) - e^{\frac{b(\mu+|\mu|)}{\sigma^2}} N\left(\frac{b+|\mu|T}{\sigma\sqrt{T}}\right)\right), & b \leqslant 0. \end{cases}
\end{aligned}$$

To derive the analytic formula for ex ante skewness and values for CBBCs, we study the running minimum (maximum) of Brownian motion with drift. Define

$$\mathrm{Inf}(t) := \inf_{0\leqslant s\leqslant t}(\mu s + \sigma W_s), \quad \mathrm{Sup}(t) := \sup_{0\leqslant s\leqslant t}(\mu s + \sigma W_s),$$

then it is easy to see that:

$$\begin{aligned}
\mathbb{P}\left(\mathrm{Sup}(t) < b\right) &= \mathbb{P}(\tau_b > t), \quad b \geqslant 0, \\
\mathbb{P}\left(\mathrm{Inf}(t) > b\right) &= \mathbb{P}(\tau_b > t), \quad b \leqslant 0.
\end{aligned}$$

By virtue of the above formulas, we have:

$$
\begin{aligned}
& g(\lambda,\mu,\sigma,k,t) := \mathbb{E}\left[e^{\lambda \mathrm{Inf}(t)} 1_{\{\mathrm{Inf}(t)>k\}}\right] \\
= & \int_k^0 e^{\lambda b} \frac{-\partial \mathbb{P}\left(\mathrm{Inf}(t)>b\right)}{\partial b} \mathrm{d}b = \int_k^0 e^{\lambda b} \frac{-\partial \mathbb{P}\left(\tau_b>t\right)}{\partial b} \mathrm{d}b \\
= & \frac{2}{\mu+\mu_2}\left[-\mu_2 e^{\frac{1}{2}\lambda(\mu+\mu_2)t}\left(N\left(d_5\right)-N\left(d_6\right)\right)+\mu N\left(d_7\right)-\mu e^{k\lambda+\frac{2k\mu}{\sigma^2}} N\left(d_8\right)\right],
\end{aligned}
$$

for $k \leqslant 0$, and

$$
\begin{aligned}
& h(\lambda,\mu,\sigma,k,t) := \mathbb{E}\left[e^{\lambda \mathrm{Sup}(t)} 1_{\{\mathrm{Sup}(t)<k\}}\right] \\
= & \int_0^k e^{\lambda b} \frac{\partial \mathbb{P}\left(\mathrm{Sup}(t)<b\right)}{\partial b} \mathrm{d}b = \int_0^k e^{\lambda b} \frac{\partial \mathbb{P}\left(\tau_b>t\right)}{\partial b} \mathrm{d}b \\
= & \frac{2}{\mu+\mu_2}\left[\mu_2 e^{\frac{1}{2}\lambda(\mu+\mu_2)t}\left(N\left(d_5\right)-N\left(d_6\right)\right)+\mu N\left(-d_7\right)-\mu e^{k\lambda+\frac{2k\mu}{\sigma^2}} N\left(-d_8\right)\right],
\end{aligned}
$$

for $k \geqslant 0$, where $d_5 = \frac{k-\mu_2 t}{\sigma\sqrt{t}}$, $d_6 = -\frac{\mu_2 t}{\sigma\sqrt{t}}$, $d_7 = \frac{\mu t}{\sigma\sqrt{t}}$, $d_8 = \frac{k+\mu t}{\sigma\sqrt{t}}$ with $\mu_2 := \mu + \lambda\sigma^2$. The above functions $f(\cdots)$, $g(\cdots)$, $h(\cdots)$ and $\eta(\cdots)$ are key ingredients of the closed-form expressions for ex ante skewness and the values of CBBCs presented in the next two sections.

B.3 Explicit Formulae for Ex Ante Skewness

We now derive closed-form expressions for CBBCs' ex ante skewness. Following Boyer and Vorkink (2014), we define the measure of ex ante skewness for a CBBC over the horizon t to T as:

$$
\mathrm{SKEW}_t(\tau) := \frac{\mathbb{E}_t\left[R_t(\tau)-\mu_t(\tau)\right]^3}{\left[\sigma_t(\tau)\right]^3}, \qquad \tau := T-t, \tag{B.5}
$$

where $\mu_t(\tau) = \mathbb{E}_t[R_t(\tau)]$, $\sigma_t(\tau) = \left(\mathbb{E}_t\left[R_t^2(\tau)\right] - \mu_t^2(\tau)\right)^{1/2}$, and $R_t(\tau)$ denotes CBBC's return. In terms of the return's raw moments, (B.5) can be expressed as:

$$\mathrm{SKEW}_t(\tau) = \frac{\mathbb{E}_t\left[R_t^3(\tau)\right] - 3\mathbb{E}_t\left[R_t^2(\tau)\right]\mu_t(\tau) + 2\mu_t^3(\tau)}{\left(\mathbb{E}_t\left[R_t^2(\tau)\right] - \mu_t^2(\tau)\right)^{3/2}}. \tag{B.6}$$

Recalling the introduction of CBBCs presented in Section 2.1, the return from holding a bull contract to maturity, $R_t^{\mathrm{bull}}(\tau)$ is:

$$R_t^{\mathrm{bull}}(\tau) = \frac{(S_T - K)\,1_{\{T_b > T\}} + 1_{\{T_b \leqslant T\}}\left(\min\limits_{T_b \leqslant t \leqslant T_b + T_0} S_t - K\right)^+}{\hat{P}_t^{\mathrm{bull}}(\tau)}, \tag{B.7}$$

where T is the maturity date, $S := (S_t)_{t \geqslant 0}$ is the price process of HSI, K is the strike price, $\hat{P}_t^{\mathrm{bull}}(\tau)$ is the market price of the bull contract, and $T_b := \inf\{t \geqslant 0; S_t \leqslant S_b\}$ is the first time that the price process S crosses the call level S_b. Here $(x)^+ := \max(x, 0)$, and T_0 is the settlement period given the call level is hit. Define

$$M_{x,\theta} := \min_{0 \leqslant t \leqslant \theta} S_t, \text{ given } S_0 = x,$$

then from (B.7) we can rewrite the j-th raw moment of $R_t^{\mathrm{bull}}(\tau)$ as:

$$\begin{aligned} &\mathbb{E}_t\left[\left(R_t^{\mathrm{bull}}(\tau)\right)^j\right] \\ = \;& \frac{\mathbb{E}_t\left[(S_T - K)^j\,1_{\{T_b > T\}}\right] + \mathbb{E}\left[(M_{S_b,T_0} - K)^j\,1_{\{M_{S_b,T_0} > K\}}\right]\mathbb{P}_t(T_b \leqslant T)}{\left(\hat{P}_t^{\mathrm{bull}}(\tau)\right)^j}, \end{aligned} \tag{B.8}$$

where $\mathbb{P}_t$ is the probability given information as of time t. Noting that, at time t, $T_b > T$ is equivalent to $M_{S_t,T-t} > S_b$, Equation (B.8) shows that, in order to compute the raw moments for a bull contract, we need the joint distribution of the underlying asset price and its running minimum.

In the remaining part of this appendix, by virtue of the results presented in Section B.2, we derive explicit formulae for ex ante skewness defined by (B.5)-(B.6) under the log normal assumption. For ease of exposition, we introduce the following notation:

$$\Theta_1 := (r - d - \sigma^2/2, \sigma, s_b, T - t), \qquad \Theta_2 := (r - d - \sigma^2/2, \sigma, k_b, T_0), \tag{B.9}$$

where $s_b := \ln(S_b/S_t)$, $k_b := \ln(K/S_b)$, and d denotes the dividend yield of the HSI. To compute the ex ante skewness, we need (B.8) for $j = 1, 2, 3$, which consists of the following three components:

$$\mathbb{E}_t\left[(S_T - K)^j 1_{\{T_b > T\}}\right], \ \mathbb{E}_0\left[(M_{S_b,T_0} - K)^j 1_{\{M_{S_b,T_0} > K\}}\right], \ \mathbb{P}_t(T_b \leqslant T). \tag{B.10}$$

Under the log normal setting, the risk-neutral dynamics of the underlying asset is given by $S := (S_0 \exp((r - d - \sigma^2/2)t + \sigma W_t))_{t \geq 0}$ with $(W_t)_{t \geq 0}$ being a standard Brownian motion. The first hitting time of S on call level S_b is identical to the first hitting time of $(rt - dt - \sigma^2 t/2 + \sigma W_t)_{t \geq 0}$ on the level $\ln(S_b/S_0)$. Thus, by (B.3) and the definition of τ_b, we have:

$$\mathbb{P}_t(T_b \leqslant T) = 1 - \mathbb{P}_t(T_b > T) = 1 - f(0, \Theta_1).$$

We next concentrate on the computation of the first two components in (B.10). When $j = 1$, we have:

$$\begin{aligned}
& \mathbb{E}_t\left[(S_T - K) 1_{\{T_b > T\}}\right] \\
= \ & \mathbb{E}_t\left[S_T 1_{\{T_b > T\}}\right] - K\mathbb{P}_t(T_b > T) \\
= \ & \mathbb{E}_t\left[S_t \exp\left(\int_t^T (r - d - \sigma^2/2)dt + \int_t^T \sigma dW_t\right) 1_{\{T_b > T\}}\right] - K\mathbb{P}_t(T_b > T) \\
= \ & S_t \mathbb{E}_0\left[e^{(r-d-\sigma^2/2)(T-t) + \sigma W_{T-t}} 1_{\{\tau_1 > T - t\}}\right] - K\mathbb{P}(\tau_1 > T - t),
\end{aligned}$$

where $\tau_1 := \inf\{t \geq 0 : (r - d - \sigma^2/2)t + \sigma W_t = s_b\}$ with $s_b := \ln(S_b/S_t) < 0$. From (B.3), we have:

$$\mathbb{E}_t\left[(S_T - K)1_{\{T_b > T\}}\right] = S_t f(1, \Theta_1) - K f(0, \Theta_1). \tag{B.11}$$

Similarly,

$$\begin{aligned}
\mathbb{E}_t\left[(S_T - K)^j 1_{\{T_b > T\}}\right] &= \sum_{k=0}^{j} C_j^k (-K)^k S_t^{j-k} f(j-k, \Theta_1), \\
\mathbb{E}_0\left[\left(M_{S_b, T_0} - K\right)^j 1_{\{M_{S_b, T_0} > K\}}\right] &= \sum_{k=0}^{j} C_j^k (-K)^k S_b^{j-k} g(j-k, \Theta_2),
\end{aligned}$$

where $k_b := \ln(K/S_b) < 0$, and $C_j^k := \frac{j!}{k!(j-k)!}$ is the binomial coefficient. Recall that $\mathbb{P}_t(T_b \leqslant T) = 1 - f(0, \Theta_1)$. The raw moments are given by:

$$\begin{aligned}
&\mathbb{E}_t\left[\left(R_t^{\text{bull}}(\tau)\right)^j\right] \\
&= \frac{\sum_{k=0}^{j} C_j^k (-K)^k \left[S_t^{j-k} f(j-k, \Theta_1) + [1 - f(0, \Theta_1)] S_b^{j-k} g(j-k, \Theta_2)\right]}{\left(P_t^{\text{bull}}(\tau)\right)^j},
\end{aligned} \tag{B.12}$$

where $P_t^{\text{bear}}(\tau)$ is the market price of a bear contract. Similarly,

$$\begin{aligned}
\mathbb{E}_t\left[(K - S_T)^j 1_{\{T_b > T\}}\right] &= \sum_{k=0}^{j} C_j^k K^k (-S_t)^{j-k} f(j-k, \Theta_1), \\
\mathbb{E}_0\left[\left(K - \widetilde{M}_{S_b, T_0}\right)^j 1_{\{\widetilde{M}_{S_b, T_0} < K\}}\right] &= \sum_{k=0}^{j} C_j^k K^k (-S_b)^{j-k} h(j-k, \Theta_2),
\end{aligned}$$

where $\widetilde{M}_{x,\theta} := \left(\max_{0 \leqslant t \leqslant \theta} S_t \middle| S_0 = x\right)$, and Θ_1 and Θ_2 are given in (B.9) with $s_b := \ln(S_b/S_t) > 0$, $k_b := \ln(K/S_b) > 0$. The raw moments for bear contracts can be given by:

$$\mathbb{E}_t\left[\left(R_t^{\text{bear}}(\tau)\right)^j\right] \tag{B.13}$$

$$= \frac{\sum_{k=0}^{j} C_j^k K^k \left[(-S_t)^{j-k} f(j-k,\Theta_1) + (1-f(0,\Theta_1))(-S_b)^{j-k} h(j-k,\Theta_2)\right]}{\left(P_t^{\text{bear}}(\tau)\right)^j},$$

where $P_t^{\text{bear}}(\tau)$ is the market price of a bear contract. Substituting (B.12) and (B.13) into (B.6), we are able to obtain explicit formulae for ex ante skewness of CBBCs. Parenthetically, to the best of our knowledge, no work prior to ours provides explicit formulae for the ex ante skewness of CBBCs.

B.4 Closed-form Pricing Formulae for CBBCs

In this section, we provide explicit pricing formulae for CBBCs under the log normal assumption.[2] Recall (B.1). The time-t price of a bull contract with time-to-maturity $\tau = T - t$ can be written as:

$$P_t^{\text{bull}}(\tau) = C_1^{\text{bull}} + C_2^{\text{bull}}, \tag{B.14}$$

where:

$$\begin{aligned} C_1^{\text{bull}} &= \mathbb{E}_t\left[e^{-r(T-t)}\left(S_T - K\right) 1_{\{T_b > T\}}\right], \\ C_2^{\text{bull}} &= \mathbb{E}_t\left[e^{-r(T_b+T_0-t)}\left(M_{S_b,T_0} - K\right)^+ 1_{\{T_b \leqslant T\}}\right]. \end{aligned}$$

[2] There do exist explicit pricing formulae for CBBCs elsewhere in the literature; see, e.g., Eriksson (2006) and (Liu et al., 2011, Appendix A). In the latter paper, the authors determine explicit formulae by decomposing a bull contract into three parts: a down-and-out option, a standard floating strike lookback option, and a one-touch option. In this paper, we present formulae based on Black and Scholes 1973 and Merton 1974) as developed in Appendix B.3, that is, using the functions $f(\cdots)$, $g(\cdots)$, $h(\cdots)$ and $\eta(\cdots)$ defined in Appendix B.2.

Noting from Equation (B.11) that $\mathbb{E}_t\left[(S_T - K)\, 1_{\{T_b > T\}}\right] = S_t f(1,\Theta_1) - K f(0,\Theta_1)$, the explicit formula of C_1^{bull} is given by:

$$C_1^{\text{bull}} = e^{-r(T-t)}[S_t f(1,\Theta_1) - K f(0,\Theta_1)].$$

From the law of iterated expectations and the strong Markov property of the Black-Scholes model,

$$C_2^{\text{bull}} = \mathbb{E}\left[e^{-rT_0}\left(M_{S_b,T_0} - K\right) 1_{\{M_{S_b,T_0} > K\}}\right] \mathbb{E}_t\left[e^{-r\tau_1} 1_{\{\tau_1 \leqslant T-t\}}\right],$$

where $\tau_1 := \inf\{t \geq 0 : (r - d - \sigma^2/2)t + \sigma W_t = s_b\}$ with $s_b := \ln(S_b/S_t) < 0$. Assume the settlement period T_0 is known. Noting that $\mathbb{E}_t\left[e^{-r\tau_1} 1_{\{\tau_1 \leqslant T-t\}}\right] = \eta(r,\Theta_1)$, and $\mathbb{E}\left[\left(M_{S_b,T_0} - K\right) 1_{\{M_{S_b,T_0} > K\}}\right] = S_b g(1,\Theta_2) - K g(0,\Theta_2)$, we have:

$$C_2^{\text{bull}} = e^{-rT_0}\left[S_b g(1,\Theta_2) - K g(0,\Theta_2)\right] \eta(r,\Theta_1),$$

and where the function $\eta(\cdots)$ is given by (B.4). Substituting for C_1^{bull} and C_2^{bull} into (B.14), we obtain the explicit pricing formula for a bull contract. Similarly, the pricing formula for a bear contract can be expressed as:

$$P_t^{\text{bear}}(\tau) = C_1^{\text{bear}} + C_2^{\text{bear}},$$

where:

$$\begin{aligned} C_1^{\text{bear}} &= e^{-r(T-t)}[K f(0,\Theta_1) - S_t f(1,\Theta_1)], \\ C_2^{\text{bear}} &= e^{-rT_0}\left[K h(0,\Theta_2) - S_b h(1,\Theta_2)\right] \eta(r,\Theta_1). \end{aligned}$$

B.5 Additional Empirical Results

Table B.1: **Differences Between Contracts with Positive Trading Volume and Those with Zero Trading Volume**
Of the 12,400 CBBCs that are never bought or traded during our sample period, 11,369 contracts are called back on their listing days, and trading in them is immediately suspended. To show the differences between the 1,031 contracts with zero trading volume but no callback on listing day, and the 19,926 contracts with positive trading volume, this table reports the averages and standard deviations of Distance to Call Level (DtCL) on listing day, and Distance between Strike Price and Call Level (DbSC) for these two group of CBBCs. Differences between averages for the two groups as well as *p*-values for testing zero-difference are also reported. Statistical significance at the 10%, 5%, and 1% levels is indicated by *, **, and ***, respectively.

	Positive Volume		Zero Volume		Difference	
Variable	Mean	Std.	Mean	Std.	Diff	p-value
DtCL	853.7	895.8	3508.3	2764.5	-2654.6***	<0.001
DbSC	322.9	195.1	430.7	273.1	-107.8***	<0.001

Table B.2: **Average Weekly Returns for CBBC Strangles**

This table reports the average holding-period (measured in number of trading days) returns for CBBC strangles. All returns are scaled to weekly units (10-day and 20-day returns are divided by 2 and 4, respectively). On the first trading day of each month, CBBCs are first grouped by maturity, then in each group with the same maturity, we construct a strangle by choosing the bull (bear) contract whose DtCL is the closest to 500 among all bull (bear) contracts. In the column "Maturity Month," 1 means the strangle matures in the current month, 2 means the next month, and so on. NoS reports the number of strangles constructed (due to data availability, we are unable to construct strangles for each maturity month on every trading day). *p*-values for testing whether these averages are equal to zero are computed. Statistical significance at the 10%, 5%, and 1% levels is indicated by *, **, and ***, respectively.

Maturity		Holding Periods		
Month	NoS	5	10	20
1	41	0.493	-0.003	-6.516
2	55	-1.486	-2.963**	-3.398***
3	67	-2.110	-4.130*	-6.530***
4	67	-4.481	-3.946*	-6.518***
5	65	-5.806**	-7.744***	-5.584***
6	68	-8.185***	-9.449***	-6.339***
7	65	-10.198***	-5.952***	-5.814***
8	55	-9.513***	-6.320**	-3.305*
9	36	-8.040***	-7.438***	-3.533**
10	20	-1.760	-1.821	-2.596
11	9	-8.543*	-9.298***	-5.466
12	4	0.160	-6.952	-7.789

Table B.3: **OLS Regression of Net Buy Volume, and Net Buy Volume Scaled by Issue Size on Skewness, Gamma, Vega, Leverage, Price, and Volatility**

This table reports the OLS regression of the average difference between buy and sell volume (net buy volume), and net buy volume scaled by issue size on (hold-to-maturity) ex ante skewness, gamma, vega, leverage, closing price, and (11-day) return volatility of CBBCs. The sample period is from January 2009 through December 2014. Ex ante skewness, gamma, vega, and leverage are computed under the log-normal assumption, where leverage (elasticity) is defined as per Figure 1 in the paper. Since these variables are highly skewed, we use the respective variable rank instead of the variable itself. Specifically, for each variable, an observation is ranked as i if the observation lies in the ith percentile. In the column labeled "Net Buy Volume," we first sort all observations into net buy volume percentiles. Then for each percentile, we assign a net buy volume rank (1 to 100), and compute the averages of skewness, gamma, vega, leverage, price, and volatility ranks. Reported in Panel A is the OLS regression of the averaged rank on the averages of the ranks for the other six variables. The column labeled "Net Buy Volume/Issue Size" reports results analogous to the second column but with net buy volume replaced by net buy volume scaled by issue size. t-statistics are reported in parenthesis. Statistical significance at the 10%, 5%, and 1% levels is indicated by *, **, and ***, respectively.

	Net Buy Volume	Net Buy Volume/Issue Size
Skewness	32.82***	33.73***
	(9.64)	(10.51)
Gamma	-15.43***	-11.05***
	(-3.06)	(-2.64)
Vega	13.44***	12.22***
	(3.43)	(3.31)
Leverage	-3.21	-2.58
	(-0.78)	(-0.74)
Price	5.15*	6.87***
	(1.86)	(2.96)
Volatility	-21.64***	-21.22***
	(-5.56)	(-5.62)

Table B.4: **Gross Profits and the Difference between Market Closing and BS-Merton Value**
This table reports descriptive statistics of issuers' daily gross profits (million HKD) in different quintiles of the difference between market closing and BS-Merton value. The column Diff(1) reports the difference between the sample mean of each upper quintile and that of the lowest quintile. The column Diff(2) reports the difference between two successive quintiles. p-values for testing the null hypothesis of no difference between sample means are also reported.

Difference Quintile	Mean	Std	Skew	Diff(1)	p-value	Diff(2)	p-value
Low	-0.033	0.986	-2.893	—	—	—	—
2	-0.040	1.039	-4.323	-0.007	0.142	—	—
3	0.001	1.081	0.242	0.034	<0.001	0.041	<0.001
4	0.091	1.170	3.017	0.124	<0.001	0.090	<0.001
High	0.196	1.194	2.887	0.229	<0.001	0.106	<0.001

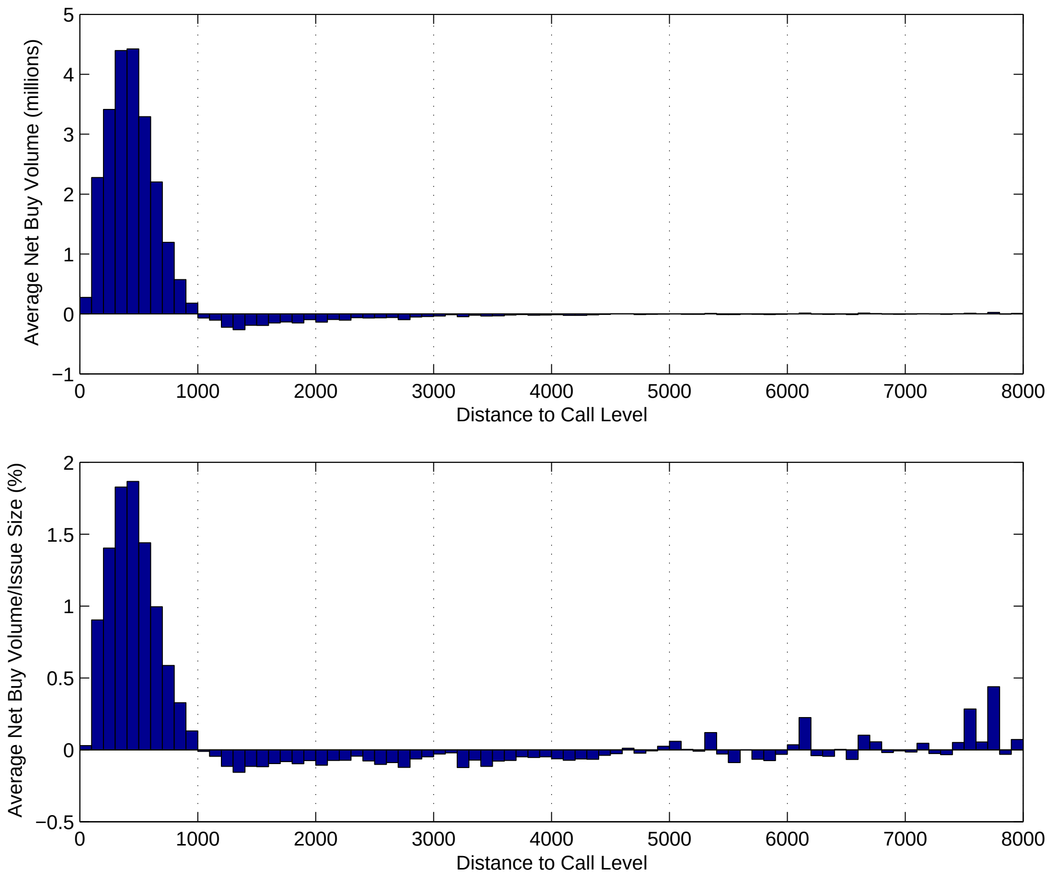

Figure B.1: **Average Net Buy Volume, and Average Net Buy Volume Scaled by Issue Size in Different Bins of Distance to Call Level**
This figure reports the average net buy volume, and the average net buy volume scaled by issue size, for different ranges of the distance to call level (DtCL) across all contracts on all trading days. The bin size is 100. Reported are averaged values for daily trading records falling within each bin. On each day, the distance to call level is defined as the absolute difference between contract's call level and the closing price of HSI on that day.

References

Black, F., Scholes, M., 1973. The pricing of options and corporate liabilities. The Journal of Political Economy 81 (3), 637–654.

Boyer, B., Vorkink, K., 2014. Stock options as lotteries. Journal of Finance 69 (4), 1485–1527.

Eriksson, J., 2006. Explicit pricing formulas for turbo warrants. RISK magazine ISSN, 1082–6394.

Harrison, J., Pliska, S., 1981. Martingales and stochastic integrals in the theory of continuous trading. Stochastic Processes and their Applications 11 (3), 215–260.

Karatzas, I., Shreve, S., 1991. Brownian Motion and Stochastic Calculus. Springer.

Liu, X., Luo, X., Zhang, J., 2011. The mechanism of callable bull/bear contracts. Working paper, The University of Hong Kong, 1–28.

Merton, R., 1974. On the pricing of corporate debt: the risk structure of interest rates. The Journal of Finance 29 (2), 449–470.