

# Index Option Returns and Generalized Entropy Bounds Online Appendix

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# 1 Summary

The on-line appendix is organized as follows.

Appendix A discusses bound frontiers generated by alternative option strategies. Appendix B shows test results for alternative option portfolios and disaster model calibrations. Appendix C reports test results based on alternative option portfolios that explore information in the option cross section. Appendix D provides results on testing the habit model (Campbell and Cochrane, 1999) and the long-run risks model (Bansal and Yaron, 2004).

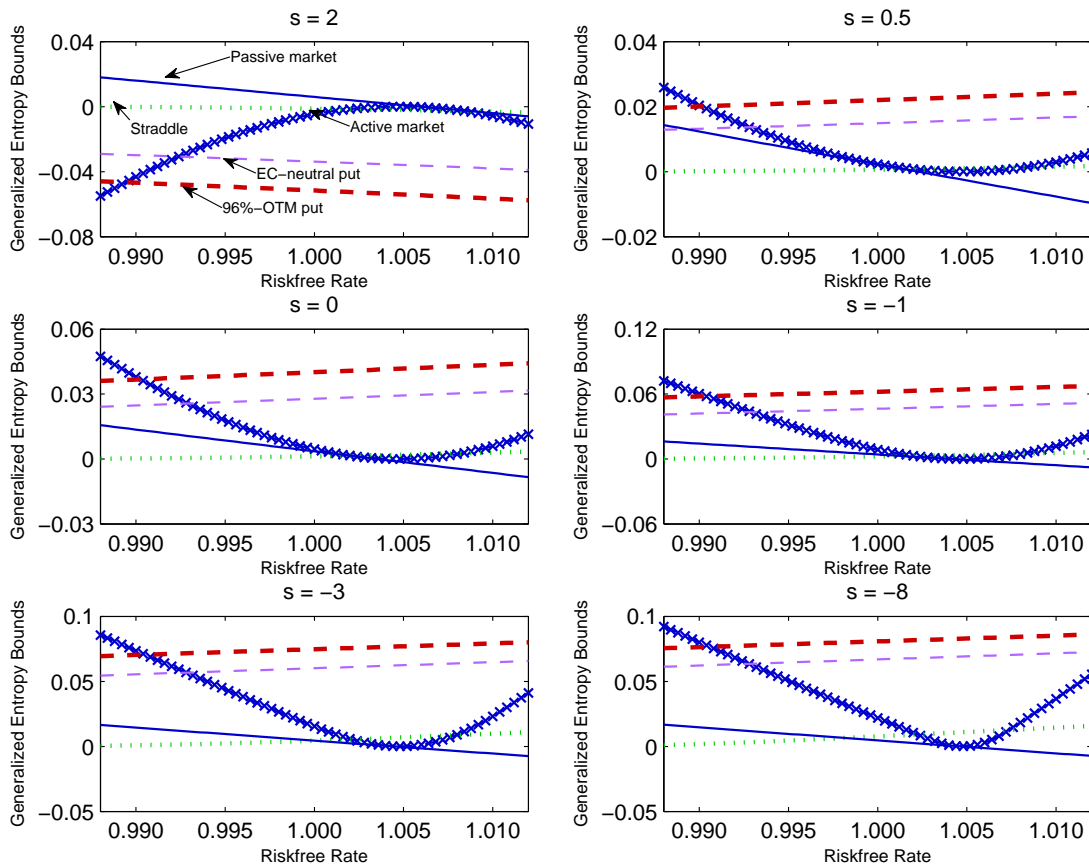
# A Bound Frontiers Generated by Alternative Option Strategies

Instead of focusing on the exact crash-neutral puts, we examine alternative option strategies by comparing their bound frontiers. We do not study these option strategies in the main text because they may imply a negative gross return out of the sample. Our goal is to have a sense of the relative performance of option strategies in restricting moments of the pricing kernel.

Following Hansen and Jagannathan (1991) and Snow (1990), we plot the bound frontier (i.e., the right-hand sides of the inequalities in (10) and (11)) at different levels of the risk-free rate. In particular, for each strategy and at a hypothetical risk-free rate, we search for the portfolio weight  $\alpha_g$  that optimizes the right-hand side of (10) and (11). We plot the optimized values of the right-hand side of (10) and (11) in Figure A.1. We also report the optimal portfolio weights in Table A.1.<sup>1</sup> Notice that when the power  $s$  equals 2, the permissible region for the pricing kernel is below the depicted bound frontier, whereas at other powers the permissible region is above the bound frontier. We plot the bound frontiers corresponding to five strategies: three option trading strategies (i.e., 96%-OTM put, ATM straddle, and the exact crash-neutral put) and, for comparison purposes, two strategies that involve the market index.

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<sup>1</sup>The mean and standard deviation of the real risk-free rate for the short sample (1996-2011) is 4bp and 40bp per month, respectively. We therefore center the monthly risk-free rate at zero and extend to  $\pm 3$  standard deviations away from the center in Figure A.1.



**Figure A.1: Bound Frontiers Implied by Option Strategies and the Market Index.** We plot the right-hand side of Eq. (10) and (11) for a variety of option strategies and the market index. In particular, at a hypothetical level of monthly risk-free rate (x-axis), we search for the portfolio weight that optimizes the right-hand side of Eq. (10) and (11). We plot the optimized values of the right-hand side of Eq. (10) and (11) against the monthly risk-free rate. When  $s = 2$  ( $s < 1$ ), the permissible region for the pricing kernel is below (above) the depicted bound frontier. The strategies we plot are: a passive market strategy for which we allocate 100% of wealth to the market index (solid line), an active market strategy for which the weight on the market index varies with the assumed level of the risk-free rate (line with 'x'), ATM straddle (dotted line), EC-neutral put (thin dashed line), and 96%-OTM put (thick dashed line).

Table A.1: **Optimal Portfolio Weights on Option Strategies and Market Index.**

For a given power  $s$  and at a hypothetical risk-free rate of zero, we search for the portfolio weight  $\alpha_g$  that optimizes the right-hand side of (10) and (11) in-sample for different trading strategies. Panel A shows the optimal weights. Panel B shows the in-sample constraints on weights (such that the in-sample portfolio gross returns are always positive), as given by  $\alpha_g(\min) = 1/(1 - \max_{1 \leq t \leq T} \{R_t\})$  and  $\alpha_g(\max) = 1/(1 - \min_{1 \leq t \leq T} \{R_t\})$ , where  $R_t$  is the strategy return realized between time  $t - 1$  and  $t$ .

| Power (s)                       | Market | 96%-OTM put | ATM straddle | EC-neutral put |
|---------------------------------|--------|-------------|--------------|----------------|
| Panel A: Optimal Weights        |        |             |              |                |
| 2                               | -1.954 | 0.488       | 0.149        | 0.410          |
| 0.5                             | 0.941  | -0.199      | -0.070       | -0.173         |
| 0                               | 1.840  | -0.341      | -0.137       | -0.310         |
| -1                              | 3.450  | -0.437      | -0.260       | -0.449         |
| -3                              | 5.428  | -0.442      | -0.461       | -0.467         |
| -8                              | 5.667  | -0.442      | -0.663       | -0.467         |
| Panel B: Constraints on Weights |        |             |              |                |
| $\alpha_g(\min)$                | -8.508 | -0.442      | -0.667       | -0.467         |
| $\alpha_g(\max)$                | 5.667  | 1.178       | 1.286        | 0.702          |

Several patterns emerge from Figure A.1 and Table A.1.

First, strategies involving the OTM put option dominate other strategies across all powers and a wide range of the risk-free rate. At  $s = 2$ , which corresponds to the HJ bound, the Sharpe ratio of a strategy essentially determines the strength of the bound that the strategy imposes on the pricing kernel. As a result, consistent with the rankings of the absolute Sharpe ratios in Table 1, the 96%-OTM put implies the strongest constraint, and is followed by the market index and lastly by the ATM straddle.

At other powers, the bounds fall into the realm of the generalized entropy bounds and hence have utility-based interpretations, as discussed previously. In particular, a bound at power  $s$  corresponds to the transformed optimized utility of a CRRA investor with a risk-aversion of  $1/(1 - s)$ .<sup>2</sup> This interpretation allows us to relate our results to existing research. For example, Driessen and Maenhout (2005) study the asset allocation problem for an investor who has access to index options. Their results lend support to our results. For instance, at  $s = 0.5$ , both their paper and our results show that an agent with a risk-aversion of 2 ( $= 1/(1 - 0.5)$ ) will have a substantial short position in the 96%-OTM put. While their paper allows the market

<sup>2</sup>Strictly speaking, while the second component (i.e.,  $\frac{1}{s} \log E(M^s)$ ) in the definition of the generalized entropy bounds corresponds to the transformed optimized utility of a CRRA investor, the first component (i.e.,  $\log E(M)$ ) adjusts this utility by the risk-free rate.

index to be in investors' choice sets and has an estimate of about  $-10\%$  for  $\alpha_g$ , our estimate is around  $-20\%$  by excluding the market index. As another example, at  $s = 0$ , which corresponds to the logarithmic utility case as in the original entropy bound, Driessen and Maenhout's estimate on the 96%-OTM put is around  $-15\%$ . Our estimate again roughly doubles their estimate. Aside from the differences in the choice set and the sample period, both Driessen and Maenhout and our study show the economic benefits by allowing investors to trade index options.

As the power  $s$  becomes negative, investors are less risk-averse than the case with a logarithmic utility so the relative gain in expected return by shorting put options further outweighs the loss in having a large variance (as well as a negative skewness and a large kurtosis as shown in Table 1).<sup>3</sup> Consequently, the optimal bounds require even larger positions (in magnitude) in the OTM put. On the other hand, since the ATM straddle has a much smaller average return (in magnitude) than the 96%-OTM put and a much larger variance than the market index, strategies involving the ATM straddle imply inferior bounds compared to those involving either the OTM put or the market index.

To gain more insights into our findings and relate to our theoretical framework, it is worthwhile to repeat our previous discussion about the interpretation of bounds. Although the marginal investor determines the market prices of jump and volatility risks,<sup>4</sup> investors with different risk attitudes all reveal valuable information about these prices through their asset allocation decisions. For example, the market price of jump risk is a key determinant for the price of an OTM put.<sup>5</sup> In equilibrium, its price is determined by the marginal investor. However, for an investor who is less risk averse than the marginal investor and hence does not value the hedging benefits offered by the OTM puts as much as the marginal investor, she will have incentives to take short positions in the OTM puts and thereby increase her expected utility. In our framework, this expected utility is reflected in the generalized entropy bounds, which impose a set of thresholds for a valid pricing kernel (as determined by the marginal investor) to satisfy. Hence, by studying bounds imposed by investors with different risk attitudes, we indirectly infer about properties of the pricing kernel.

Our findings (i.e., strategies involving OTM put options imply sharper bounds than alternative strategies, in particular strategies that are based on ATM straddles) also highlight the more important role played by jump risks than volatility risks in making inference on disaster models in our application. Notice that this does not imply that volatility risks are not important to model diagnosis under all circumstances,

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<sup>3</sup>Table 1 shows that strategies that take a long position in the 96%-OTM put have a large variance, a positive skewness, and a large kurtosis. As a result, strategies that take a short position will have a large variance, a negative skewness, and a large kurtosis.

<sup>4</sup>The compensation for jump and volatility risks in the market index is well documented by the option pricing literature. Buraschi and Jackwerth (2001), Coval and Shumway (2001), and Bakshi and Kapadia (2003) show the presence of volatility risk premiums. Bates (2002), Pan (2002), and Ait-Sahalia, Wang, and Yared (2001) show the presence of jump risk premiums.

<sup>5</sup>See, e.g., Bates (2002), Pan (2002) and Ait-Sahalia, Wang and Yared (2001).

as we are only considering static option strategies. To better identify the impact of volatility risks, we likely need to construct dynamic option trading strategies given the strong predictability of market volatilities.<sup>6</sup> We leave this to future research.

The bound frontiers generated above can be used to confront candidate pricing kernels. However, they may appear too stringent for several reasons. First, in-sample asset allocation often generates noisy estimates for portfolio weights (Brandt, 2000, Driessen and Maenhout, 2005). More importantly, the permissible region for portfolio weights that generates strictly positive gross returns depends on the particular sample that we examine. In-sample optimized portfolio weights may not work out of sample. Indeed, as Table A.1 shows, the optimal weights for several strategies are close to the boundaries of their corresponding permissible ranges. Second, transaction costs and margin requirements for real-world option trading strategies will likely limit the sizes of the short positions one can take. Although transaction costs are low for the index option market (Bakshi, Cao, and Chen, 1997) and our long positions in the risk-free asset can serve as margins, microstructure and liquidity issues may become non-negligible if we have excessive short positions in index options. Lastly, the weights for the optimal portfolios depend on the assumed level of the risk-free rate. This is cumbersome for our application since we have to rebalance our portfolios based on the prevailing risk-free rate. Given these concerns, we focus on option portfolios that are based on the exact crash-neutral puts in our main text.

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<sup>6</sup>Liu (2017) uses the generalized entropy bounds to study the implications of dynamic strategies on representative agent models.

## **B Disaster Model: Alternative Option Portfolios and Model Calibrations**



Table B.1: **Testing Disaster Models with Option Return Bounds: Baseline Disaster Model + Exact Crash-Neutral Puts ( $ECNput(92\%, 85\%)$ )**

The baseline disaster model is characterized by  $\omega_B = 0.017$  and  $\theta_B = -0.38$  (Table C.1). Other model parameters are given in Table C.1, except for  $\gamma$  (risk aversion) and  $R_f$  (annualized risk-free rate). For a given  $\gamma$  and  $R_f$ , we solve for the implied mean consumption growth rate ( $\mu + \omega\theta$ ) through Eq. (26). We use the exact crash-neutral put portfolio specified in section 4.2 (i.e.,  $ECNput$ ), which takes a long position in the riskless bond, a short position in the 92%-OTM put, and an offsetting long position in the 85%-OTM put. The put strategy return is given by  $R_{g,t+1} = (p_{t+1}^H - p_{t+1}^L)/(p_t^H - p_t^L)$ , where  $p_{t+1}^H(p_t^H)$  is the future (current) price of the 92%-OTM put and  $p_{t+1}^L(p_t^L)$  is the future (current) price of the 85%-OTM put. The combined portfolio return is given by  $R_{p,t+1} = \alpha_{g,t}R_{g,t+1} + (1 - \alpha_{g,t})R_{f,t}$ , where  $R_{f,t}$  is the risk-free rate and  $\alpha_{g,t}$  is the weight on the crash-neutral put as given by  $\alpha_{g,t} = \max\{-0.15, -R_{f,t}/(\bar{R}_t^{ECN} - R_{f,t})\}$ , where  $\bar{R}_t^{ECN}$  is the theoretical upper bound for the return of the crash-neutral put as given by  $\bar{R}_t^{ECN} = ((0.92 - 0.85)p_t^M)/(p_t^H - p_t^L)$ , where  $p_t^M$  is the spot price for the market index. For a given asset return  $R$  with monthly observations, we estimate the right-hand side of (10) by its sample counterpart (denoted as  $\widehat{RHS}(R)$ ). We also obtain the monthly  $GEF$  for the model under consideration (denoted as  $GEF(M)$ ). ‘‘RHS’’ reports the annualized  $\widehat{RHS}(R)$  (i.e.,  $12 \times \widehat{RHS}(R)$ ). ‘‘Distance’’ reports the annualized distance between  $\widehat{RHS}(R)$  and  $GEF(M)$ , and is given by  $12 \times (GEF(M) - \widehat{RHS}(R))$  if  $s \leq 1$  and  $12 \times (\widehat{RHS}(R) - GEF(M))$  if  $s \geq 1$ .  $P$ -values are obtained through bootstrap.

|              |  |                 | Generalized Entropies (s) |        |        |        |
|--------------|--|-----------------|---------------------------|--------|--------|--------|
|              |  |                 | s = 0.5                   | s = 0  | s = -1 | s = -2 |
| $\gamma = 2$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = -0.025) | <i>RHS</i>      | 0.212                     | 0.293  | 0.325  | 0.335  |
|              |  | <i>Distance</i> | -0.205                    | -0.281 | -0.307 | -0.314 |
|              |  | <i>p-value</i>  | (0.00)                    | (0.00) | (0.00) | (0.00) |
|              | $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.001)  | <i>RHS</i>      | 0.151                     | 0.241  | 0.274  | 0.284  |
|              |  | <i>Distance</i> | -0.143                    | -0.229 | -0.256 | -0.262 |
|              |  | <i>p-value</i>  | (0.01)                    | (0.01) | (0.00) | (0.00) |
|              | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.025)  | <i>RHS</i>      | 0.092                     | 0.193  | 0.225  | 0.235  |
|              |  | <i>Distance</i> | -0.085                    | -0.181 | -0.207 | -0.214 |
|              |  | <i>p-value</i>  | (0.07)                    | (0.03) | (0.01) | (0.01) |
| $\gamma = 4$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = 0.007)  | <i>RHS</i>      | 0.212                     | 0.293  | 0.325  | 0.335  |
|              |  | <i>Distance</i> | -0.148                    | -0.204 | -0.218 | -0.220 |
|              |  | <i>p-value</i>  | (0.01)                    | (0.02) | (0.01) | (0.01) |
|              | $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.020)  | <i>RHS</i>      | 0.151                     | 0.241  | 0.274  | 0.284  |
|              |  | <i>Distance</i> | -0.086                    | -0.153 | -0.167 | -0.168 |
|              |  | <i>p-value</i>  | (0.06)                    | (0.06) | (0.03) | (0.03) |
|              | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.032)  | <i>RHS</i>      | 0.092                     | 0.193  | 0.225  | 0.235  |
|              |  | <i>Distance</i> | -0.028                    | -0.104 | -0.118 | -0.119 |
|              |  | <i>p-value</i>  | (0.30)                    | (0.13) | (0.09) | (0.08) |
| $\gamma = 5$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = 0.029)  | <i>RHS</i>      | 0.212                     | 0.293  | 0.325  | 0.335  |
|              |  | <i>Distance</i> | -0.051                    | -0.089 | -0.095 | -0.093 |
|              |  | <i>p-value</i>  | (0.17)                    | (0.17) | (0.13) | (0.13) |
|              | $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.039)  | <i>RHS</i>      | 0.151                     | 0.241  | 0.274  | 0.284  |
|              |  | <i>Distance</i> | 0.010                     | -0.038 | -0.044 | -0.041 |
|              |  | <i>p-value</i>  | (0.56)                    | (0.33) | (0.30) | (0.30) |
|              | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.049)  | <i>RHS</i>      | 0.092                     | 0.193  | 0.225  | 0.235  |
|              |  | <i>Distance</i> | 0.069                     | 0.011  | 0.005  | 0.007  |
|              |  | <i>p-value</i>  | (0.90)                    | (0.53) | (0.51) | (0.52) |

Table B.2: **Testing Disaster Models with Option Return Bounds: Baseline Disaster Model + Exact Crash-Neutral Puts ( $ECNput(96\%, 92\%)$ )**

The baseline disaster model is characterized by  $\omega_B = 0.017$  and  $\theta_B = -0.38$  (Table C.1). Other model parameters are given in Table C.1, except for  $\gamma$  (risk aversion) and  $R_f$  (annualized risk-free rate). For a given  $\gamma$  and  $R_f$ , we solve for the implied mean consumption growth rate ( $\mu + \omega\theta$ ) through Eq. (26). We use the exact crash-neutral put portfolio specified in section 4.2 (i.e.,  $ECNput$ ), which takes a long position in the riskless bond, a short position in the 96%-OTM put, and an offsetting long position in the 92%-OTM put. The put strategy return is given by  $R_{g,t+1} = (p_{t+1}^H - p_{t+1}^L)/(p_t^H - p_t^L)$ , where  $p_{t+1}^H(p_t^H)$  is the future (current) price of the 96%-OTM put and  $p_{t+1}^L(p_t^L)$  is the future (current) price of the 92%-OTM put. The combined portfolio return is given by  $R_{p,t+1} = \alpha_{g,t}R_{g,t+1} + (1 - \alpha_{g,t})R_{f,t}$ , where  $R_{f,t}$  is the risk-free rate and  $\alpha_{g,t}$  is the weight on the crash-neutral put as given by  $\alpha_{g,t} = \max\{-0.15, -R_{f,t}/(\bar{R}_t^{ECN} - R_{f,t})\}$ , where  $\bar{R}_t^{ECN}$  is the theoretical upper bound for the return of the crash-neutral put as given by  $\bar{R}_t^{ECN} = ((0.96 - 0.92)p_t^M)/(p_t^H - p_t^L)$ , where  $p_t^M$  is the spot price for the market index. For a given asset return  $R$  with monthly observations, we estimate the right-hand side of (10) by its sample counterpart (denoted as  $\widehat{RHS}(R)$ ). We also obtain the monthly  $GEF$  for the model under consideration (denoted as  $\widehat{GEF}(M)$ ). ‘‘RHS’’ reports the annualized  $\widehat{RHS}(R)$  (i.e.,  $12 \times \widehat{RHS}(R)$ ). ‘‘Distance’’ reports the annualized distance between  $\widehat{RHS}(R)$  and  $\widehat{GEF}(M)$ , and is given by  $12 \times (\widehat{GEF}(M) - \widehat{RHS}(R))$  if  $s \leq 1$  and  $12 \times (\widehat{RHS}(R) - \widehat{GEF}(M))$  if  $s \geq 1$ .  $P$ -values are obtained through bootstrap.

|              |  |                 | Generalized Entropies (s) |         |          |          |
|--------------|--|-----------------|---------------------------|---------|----------|----------|
|              |  |                 | $s = 0.5$                 | $s = 0$ | $s = -1$ | $s = -2$ |
| $\gamma = 2$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = -0.025) | <i>RHS</i>      | 0.162                     | 0.213   | 0.237    | 0.245    |
|              |  | <i>Distance</i> | -0.154                    | -0.201  | -0.219   | -0.224   |
|              |  | <i>p-value</i>  | (0.00)                    | (0.01)  | (0.01)   | (0.00)   |
|              | $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.001)  | <i>RHS</i>      | 0.100                     | 0.162   | 0.186    | 0.194    |
|              |  | <i>Distance</i> | -0.093                    | -0.150  | -0.168   | -0.173   |
|              |  | <i>p-value</i>  | (0.04)                    | (0.05)  | (0.03)   | (0.02)   |
|              | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.025)  | <i>RHS</i>      | 0.041                     | 0.113   | 0.137    | 0.145    |
|              |  | <i>Distance</i> | -0.034                    | -0.101  | -0.119   | -0.124   |
|              |  | <i>p-value</i>  | (0.25)                    | (0.12)  | (0.08)   | (0.07)   |
| $\gamma = 4$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = 0.007)  | <i>RHS</i>      | 0.162                     | 0.213   | 0.237    | 0.245    |
|              |  | <i>Distance</i> | -0.097                    | -0.125  | -0.131   | -0.130   |
|              |  | <i>p-value</i>  | (0.04)                    | (0.08)  | (0.06)   | (0.06)   |
|              | $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.020)  | <i>RHS</i>      | 0.100                     | 0.162   | 0.186    | 0.194    |
|              |  | <i>Distance</i> | -0.036                    | -0.073  | -0.079   | -0.078   |
|              |  | <i>p-value</i>  | (0.25)                    | (0.20)  | (0.17)   | (0.17)   |
|              | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.032)  | <i>RHS</i>      | 0.041                     | 0.113   | 0.137    | 0.145    |
|              |  | <i>Distance</i> | 0.023                     | -0.025  | -0.030   | -0.030   |
|              |  | <i>p-value</i>  | (0.66)                    | (0.38)  | (0.35)   | (0.35)   |
| $\gamma = 5$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = 0.029)  | <i>RHS</i>      | 0.162                     | 0.213   | 0.237    | 0.245    |
|              |  | <i>Distance</i> | -0.001                    | -0.009  | -0.007   | -0.003   |
|              |  | <i>p-value</i>  | (0.48)                    | (0.45)  | (0.45)   | (0.48)   |
|              | $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.039)  | <i>RHS</i>      | 0.100                     | 0.162   | 0.186    | 0.194    |
|              |  | <i>Distance</i> | 0.061                     | 0.042   | 0.044    | 0.048    |
|              |  | <i>p-value</i>  | (0.88)                    | (0.68)  | (0.69)   | (0.72)   |
|              | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.049)  | <i>RHS</i>      | 0.041                     | 0.113   | 0.137    | 0.145    |
|              |  | <i>Distance</i> | 0.119                     | 0.091   | 0.093    | 0.097    |
|              |  | <i>p-value</i>  | (0.99)                    | (0.85)  | (0.87)   | (0.88)   |

Table B.3: **Testing Disaster Models with Option Return Bounds: Mild Disaster Model + Exact Crash-Neutral Puts ( $ECNput(96\%, 85\%)$ )**

The mild disaster model is characterized by  $\omega_B = 0.04$  and  $\theta_B = -0.15$  (Table C.1). Other model parameters are given in Table C.1, except for  $\gamma$  (risk aversion) and  $R_f$  (annualized risk-free rate). For a given  $\gamma$  and  $R_f$ , we solve for the implied mean consumption growth rate ( $\mu + \omega\theta$ ) through Eq. (26). We use the exact crash-neutral put portfolio specified in section 4.2 (i.e.,  $ECNput$ ), which takes a long position in the riskless bond, a short position in the 96%-OTM put, and an offsetting long position in the 85%-OTM put. The put strategy return is given by  $R_{g,t+1} = (p_{t+1}^H - p_{t+1}^L)/(p_t^H - p_t^L)$ , where  $p_{t+1}^H(p_t^H)$  is the future (current) price of the 96%-OTM put and  $p_{t+1}^L(p_t^L)$  is the future (current) price of the 85%-OTM put. The combined portfolio return is given by  $R_{p,t+1} = \alpha_{g,t}R_{g,t+1} + (1 - \alpha_{g,t})R_{f,t}$ , where  $R_{f,t}$  is the risk-free rate and  $\alpha_{g,t}$  is the weight on the crash-neutral put as given by  $\alpha_{g,t} = \max\{-0.15, -R_{f,t}/(\bar{R}_t^{ECN} - R_{f,t})\}$ , where  $\bar{R}_t^{ECN}$  is the theoretical upper bound for the return of the crash-neutral put as given by  $\bar{R}_t^{ECN} = ((0.96 - 0.85)p_t^M)/(p_t^H - p_t^L)$ , where  $p_t^M$  is the spot price for the market index. For a given asset return  $R$  with monthly observations, we estimate the right-hand side of (10) by its sample counterpart (denoted as  $\widehat{RHS}(R)$ ). We also obtain the monthly  $GEF$  for the model under consideration (denoted as  $\widehat{GEF}(M)$ ). “RHS” reports the annualized  $\widehat{RHS}(R)$  (i.e.,  $12 \times \widehat{RHS}(R)$ ). “Distance” reports the annualized distance between  $\widehat{RHS}(R)$  and  $\widehat{GEF}(M)$ , and is given by  $12 \times (\widehat{GEF}(M) - \widehat{RHS}(R))$  if  $s \leq 1$  and  $12 \times (\widehat{RHS}(R) - \widehat{GEF}(M))$  if  $s \geq 1$ .  $P$ -values are obtained through bootstrap.

|              |  |                 | Generalized Entropies (s) |        |        |        |
|--------------|--|-----------------|---------------------------|--------|--------|--------|
|              |  |                 | s = 0.5                   | s = 0  | s = -1 | s = -2 |
| $\gamma = 2$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = -0.026) | <i>RHS</i>      | 0.185                     | 0.244  | 0.301  | 0.318  |
|              |  | <i>Distance</i> | -0.179                    | -0.234 | -0.285 | -0.297 |
|              |  | <i>p-value</i>  | (0.01)                    | (0.04) | (0.01) | (0.01) |
|              | $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.000)  | <i>RHS</i>      | 0.123                     | 0.192  | 0.250  | 0.267  |
|              |  | <i>Distance</i> | -0.118                    | -0.182 | -0.233 | -0.246 |
|              |  | <i>p-value</i>  | (0.06)                    | (0.08) | (0.02) | (0.02) |
|              | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.024)  | <i>RHS</i>      | 0.065                     | 0.144  | 0.201  | 0.218  |
|              |  | <i>Distance</i> | -0.059                    | -0.134 | -0.185 | -0.197 |
|              |  | <i>p-value</i>  | (0.21)                    | (0.14) | (0.06) | (0.04) |
| $\gamma = 4$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = -0.005) | <i>RHS</i>      | 0.185                     | 0.244  | 0.301  | 0.318  |
|              |  | <i>Distance</i> | -0.146                    | -0.184 | -0.218 | -0.204 |
|              |  | <i>p-value</i>  | (0.03)                    | (0.07) | (0.03) | (0.04) |
|              | $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.012)  | <i>RHS</i>      | 0.123                     | 0.192  | 0.250  | 0.267  |
|              |  | <i>Distance</i> | -0.084                    | -0.133 | -0.167 | -0.153 |
|              |  | <i>p-value</i>  | (0.13)                    | (0.14) | (0.07) | (0.08) |
|              | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.025)  | <i>RHS</i>      | 0.065                     | 0.144  | 0.201  | 0.218  |
|              |  | <i>Distance</i> | -0.025                    | -0.084 | -0.118 | -0.104 |
|              |  | <i>p-value</i>  | (0.36)                    | (0.24) | (0.15) | (0.17) |
| $\gamma = 5$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = 0.012)  | <i>RHS</i>      | 0.185                     | 0.244  | 0.301  | 0.318  |
|              |  | <i>Distance</i> | -0.099                    | -0.124 | -0.145 | -0.077 |
|              |  | <i>p-value</i>  | (0.09)                    | (0.16) | (0.10) | (0.24) |
|              | $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.022)  | <i>RHS</i>      | 0.123                     | 0.192  | 0.250  | 0.267  |
|              |  | <i>Distance</i> | -0.037                    | -0.072 | -0.093 | -0.026 |
|              |  | <i>p-value</i>  | (0.30)                    | (0.27) | (0.20) | (0.40) |
|              | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.032)  | <i>RHS</i>      | 0.065                     | 0.144  | 0.201  | 0.218  |
|              |  | <i>Distance</i> | 0.021                     | -0.024 | -0.045 | 0.023  |
|              |  | <i>p-value</i>  | (0.60)                    | (0.41) | (0.34) | (0.57) |

Table B.4: **Testing Disaster Models with Option Return Bounds: Severe Disaster Model + Exact Crash-Neutral Puts ( $ECNput(96\%, 85\%)$ )**

The severe disaster model is characterized by  $\omega_B = 0.01$  and  $\theta_B = -0.60$  (Table C.1). Other model parameters are given in Table C.1, except for  $\gamma$  (risk aversion) and  $R_f$  (annualized risk-free rate). For a given  $\gamma$  and  $R_f$ , we solve for the implied mean consumption growth rate ( $\mu + \omega\theta$ ) through Eq. (26). We use the exact crash-neutral put portfolio specified in section 4.2 (i.e.,  $ECNput$ ), which takes a long position in the riskless bond, a short position in the 96%-OTM put, and an offsetting long position in the 85%-OTM put. The put strategy return is given by  $R_{g,t+1} = (p_{t+1}^H - p_{t+1}^L)/(p_t^H - p_t^L)$ , where  $p_{t+1}^H(p_t^H)$  is the future (current) price of the 96%-OTM put and  $p_{t+1}^L(p_t^L)$  is the future (current) price of the 85%-OTM put. The combined portfolio return is given by  $R_{p,t+1} = \alpha_{g,t}R_{g,t+1} + (1 - \alpha_{g,t})R_{f,t}$ , where  $R_{f,t}$  is the risk-free rate and  $\alpha_{g,t}$  is the weight on the crash-neutral put as given by  $\alpha_{g,t} = \max\{-0.15, -R_{f,t}/(\bar{R}_t^{ECN} - R_{f,t})\}$ , where  $\bar{R}_t^{ECN}$  is the theoretical upper bound for the return of the crash-neutral put as given by  $\bar{R}_t^{ECN} = ((0.96 - 0.85)p_t^M)/(p_t^H - p_t^L)$ , where  $p_t^M$  is the spot price for the market index. For a given asset return  $R$  with monthly observations, we estimate the right-hand side of (10) by its sample counterpart (denoted as  $\widehat{RHS}(R)$ ). We also obtain the monthly  $GEF$  for the model under consideration (denoted as  $\widehat{GEF}(M)$ ). “RHS” reports the annualized  $\widehat{RHS}(R)$  (i.e.,  $12 \times \widehat{RHS}(R)$ ). “Distance” reports the annualized distance between  $\widehat{RHS}(R)$  and  $\widehat{GEF}(M)$ , and is given by  $12 \times (\widehat{GEF}(M) - \widehat{RHS}(R))$  if  $s \leq 1$  and  $12 \times (\widehat{RHS}(R) - \widehat{GEF}(M))$  if  $s \geq 1$ .  $P$ -values are obtained through bootstrap.

|              |  |                 | Generalized Entropies (s) |        |        |        |
|--------------|--|-----------------|---------------------------|--------|--------|--------|
|              |  |                 | s = 0.5                   | s = 0  | s = -1 | s = -2 |
| $\gamma = 2$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = -0.023) | <i>RHS</i>      | 0.185                     | 0.244  | 0.301  | 0.318  |
|              |  | <i>Distance</i> | -0.175                    | -0.227 | -0.278 | -0.293 |
|              |  | <i>p-value</i>  | (0.01)                    | (0.04) | (0.01) | (0.01) |
|              | $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.003)  | <i>RHS</i>      | 0.123                     | 0.192  | 0.250  | 0.267  |
|              |  | <i>Distance</i> | -0.113                    | -0.176 | -0.227 | -0.241 |
|              |  | <i>p-value</i>  | (0.07)                    | (0.08) | (0.03) | (0.02) |
|              | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.028)  | <i>RHS</i>      | 0.065                     | 0.144  | 0.201  | 0.218  |
|              |  | <i>Distance</i> | -0.054                    | -0.127 | -0.178 | -0.193 |
|              |  | <i>p-value</i>  | (0.22)                    | (0.15) | (0.06) | (0.04) |
| $\gamma = 4$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = 0.022)  | <i>RHS</i>      | 0.185                     | 0.244  | 0.301  | 0.318  |
|              |  | <i>Distance</i> | -0.067                    | -0.093 | -0.131 | -0.142 |
|              |  | <i>p-value</i>  | (0.18)                    | (0.22) | (0.12) | (0.10) |
|              | $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.035)  | <i>RHS</i>      | 0.123                     | 0.192  | 0.250  | 0.267  |
|              |  | <i>Distance</i> | -0.005                    | -0.041 | -0.080 | -0.091 |
|              |  | <i>p-value</i>  | (0.46)                    | (0.36) | (0.23) | (0.20) |
|              | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.047)  | <i>RHS</i>      | 0.065                     | 0.144  | 0.201  | 0.218  |
|              |  | <i>Distance</i> | 0.053                     | 0.007  | -0.031 | -0.042 |
|              |  | <i>p-value</i>  | (0.76)                    | (0.51) | (0.38) | (0.34) |
| $\gamma = 5$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = 0.069)  | <i>RHS</i>      | 0.185                     | 0.244  | 0.301  | 0.318  |
|              |  | <i>Distance</i> | 0.157                     | 0.160  | 0.129  | 0.121  |
|              |  | <i>p-value</i>  | (0.99)                    | (0.91) | (0.88) | (0.87) |
|              | $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.079)  | <i>RHS</i>      | 0.123                     | 0.192  | 0.250  | 0.267  |
|              |  | <i>Distance</i> | 0.219                     | 0.211  | 0.180  | 0.172  |
|              |  | <i>p-value</i>  | (1.00)                    | (0.96) | (0.95) | (0.95) |
|              | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.089)  | <i>RHS</i>      | 0.065                     | 0.144  | 0.201  | 0.218  |
|              |  | <i>Distance</i> | 0.277                     | 0.260  | 0.229  | 0.221  |
|              |  | <i>p-value</i>  | (1.00)                    | (0.99) | (0.99) | (0.98) |

Table B.5: **Testing Disaster Models with Option Return Bounds: BCM + Exact Crash-Neutral Puts ( $ECNput(96\%, 85\%)$ )**

The BCM calibration is given by Appendix C. For a given  $\gamma$  and  $R_f$ , we solve for the implied mean consumption growth rate ( $\mu + \omega\theta$ ) through Eq. (26). We use the exact crash-neutral put portfolio specified in section 4.2 (i.e.,  $ECNput$ ), which takes a long position in the riskless bond, a short position in the 96%-OTM put, and an offsetting long position in the 85%-OTM put. The put strategy return is given by  $R_{g,t+1} = (p_{t+1}^H - p_{t+1}^L)/(p_t^H - p_t^L)$ , where  $p_{t+1}^H$  ( $p_t^H$ ) is the future (current) price of the 96%-OTM put and  $p_{t+1}^L$  ( $p_t^L$ ) is the future (current) price of the 85%-OTM put. The combined portfolio return is given by  $R_{p,t+1} = \alpha_{g,t}R_{g,t+1} + (1 - \alpha_{g,t})R_{f,t}$ , where  $R_{f,t}$  is the risk-free rate and  $\alpha_{g,t}$  is the weight on the crash-neutral put as given by  $\alpha_{g,t} = \max\{-0.15, -R_{f,t}/(\bar{R}_t^{ECN} - R_{f,t})\}$ , where  $\bar{R}_t^{ECN}$  is the theoretical upper bound for the return of the crash-neutral put as given by  $\bar{R}_t^{ECN} = ((0.96 - 0.85)p_t^M)/(p_t^H - p_t^L)$ , where  $p_t^M$  is the spot price for the market index. For a given asset return  $R$  with monthly observations, we estimate the right-hand side of (??) by its sample counterpart (denoted as  $\widehat{RHS}(R)$ ). We also obtain the monthly  $GEF$  for the model under consideration (denoted as  $GEF(M)$ ). “RHS” reports the annualized  $\widehat{RHS}(R)$  (i.e.,  $12 \times \widehat{RHS}(R)$ ). “Distance” reports the annualized distance between  $\widehat{RHS}(R)$  and  $GEF(M)$ , and is given by  $12 \times (GEF(M) - \widehat{RHS}(R))$  if  $s \leq 1$  and  $12 \times (\widehat{RHS}(R) - GEF(M))$  if  $s \geq 1$ .  $P$ -values are obtained through bootstrap.

|              |  |                 | Generalized Entropies (s) |        |        |        |
|--------------|--|-----------------|---------------------------|--------|--------|--------|
|              |  |                 | s = 0.5                   | s = 0  | s = -1 | s = -2 |
| $\gamma = 2$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = -0.029) | <i>RHS</i>      | 0.185                     | 0.244  | 0.301  | 0.318  |
|              |  | <i>Distance</i> | -0.183                    | -0.240 | -0.296 | -0.312 |
|              |  | <i>p-value</i>  | (0.01)                    | (0.03) | (0.01) | (0.00) |
|              | $R_f = 1.00$<br>(Imp. Consp. Gr. = -0.003) | <i>RHS</i>      | 0.123                     | 0.192  | 0.250  | 0.267  |
|              |  | <i>Distance</i> | -0.122                    | -0.189 | -0.245 | -0.261 |
|              |  | <i>p-value</i>  | (0.05)                    | (0.07) | (0.02) | (0.01) |
|              | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.021)  | <i>RHS</i>      | 0.065                     | 0.144  | 0.201  | 0.218  |
|              |  | <i>Distance</i> | -0.063                    | -0.140 | -0.196 | -0.212 |
|              |  | <i>p-value</i>  | (0.19)                    | (0.13) | (0.05) | (0.03) |
| $\gamma = 4$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = -0.011) | <i>RHS</i>      | 0.185                     | 0.244  | 0.301  | 0.318  |
|              |  | <i>Distance</i> | -0.173                    | -0.225 | -0.276 | -0.290 |
|              |  | <i>p-value</i>  | (0.01)                    | (0.04) | (0.01) | (0.01) |
|              | $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.021)  | <i>RHS</i>      | 0.123                     | 0.192  | 0.250  | 0.267  |
|              |  | <i>Distance</i> | -0.111                    | -0.174 | -0.225 | -0.239 |
|              |  | <i>p-value</i>  | (0.07)                    | (0.09) | (0.03) | (0.02) |
|              | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.014)  | <i>RHS</i>      | 0.065                     | 0.144  | 0.201  | 0.218  |
|              |  | <i>Distance</i> | -0.053                    | -0.125 | -0.176 | -0.190 |
|              |  | <i>p-value</i>  | (0.23)                    | (0.16) | (0.06) | (0.05) |
| $\gamma = 5$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = -0.005) | <i>RHS</i>      | 0.185                     | 0.244  | 0.301  | 0.318  |
|              |  | <i>Distance</i> | -0.160                    | -0.208 | -0.256 | -0.270 |
|              |  | <i>p-value</i>  | (0.02)                    | (0.05) | (0.02) | (0.01) |
|              | $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.005)  | <i>RHS</i>      | 0.123                     | 0.192  | 0.250  | 0.267  |
|              |  | <i>Distance</i> | -0.099                    | -0.157 | -0.205 | -0.218 |
|              |  | <i>p-value</i>  | (0.09)                    | (0.11) | (0.04) | (0.03) |
|              | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.015)  | <i>RHS</i>      | 0.065                     | 0.144  | 0.201  | 0.218  |
|              |  | <i>Distance</i> | -0.040                    | -0.108 | -0.156 | -0.170 |
|              |  | <i>p-value</i>  | (0.29)                    | (0.19) | (0.09) | (0.06) |

## C Exploring the Option Cross-Section: Alternative Option Portfolios

Table C.1: **Testing Disaster Models with Option Return Bounds: Baseline Disaster Model + Crash-Neutral Puts Based on the Option Cross Section**  
 $(CNput^{EW}(100\%, 85\%) + CNput^{VIXW}(100\%, 85\%))$

The baseline disaster model is characterized by  $\omega_B = 0.017$  and  $\theta_B = -0.38$  (Table C.1). Other model parameters are given in Table C.1, except for  $\gamma$  (risk aversion) and  $R_f$  (annualized risk-free rate). For a given  $\gamma$  and  $R_f$ , we solve for the implied mean consumption growth rate ( $\mu + \omega\theta$ ) through Eq. (26). We use the crash-neutral put portfolios that take a long position in the riskless bond, short positions in all puts with a strike-to-price ratio that falls into (85%, 100%), and an offsetting long position in the 85%-OTM put. The strategy return is given by  $R_{g,t+1} = (\bar{p}_{t+1}^H - p_{t+1}^L)/(\bar{p}_t^H - p_t^L)$ , where  $\bar{p}_{t+1}^H$  ( $\bar{p}_t^H$ ) is the weighted average future (current) price of all puts with a strike-to-price ratio between 85% and 100%, and  $p_{t+1}^L$  ( $p_t^L$ ) is the future (current) price of the 85%-OTM put. The portfolio return is given by  $R_{p,t+1} = \alpha_{g,t}R_{g,t+1} + (1 - \alpha_{g,t})R_{f,t}$ , where  $R_{f,t}$  is the risk-free rate and  $\alpha_{g,t}$  is the weight on the crash-neutral put as given by  $\alpha_{g,t} = \max\{-0.15, -R_{f,t}/(\bar{R}_t^{ECN} - R_{f,t})\}$ , where  $\bar{R}_t^{ECN}$  is the theoretical upper bound for the return of the crash-neutral put as given by  $\bar{R}_t^{ECN} = ((\bar{K} - 0.85)p_t^M)/(\bar{p}_t^H - p_t^L)$ , where  $\bar{K}$  is the weighted average strike-to-price ratio among long positions in puts and  $p_t^M$  is the spot price for the market index. Put prices for short positions are equally-weighted for  $CNput^{EW}(100\%, 85\%)$  and VIX-weighted for  $CNput^{VIXW}(100\%, 85\%)$ . For a given asset return  $R$  with monthly observations, we estimate the right-hand side of (10) by its sample counterpart (denoted as  $\widehat{RHS}(R)$ ). We also obtain the monthly  $GEF$  for the model under consideration (denoted as  $GEF(M)$ ). ‘‘RHS’’ reports the annualized  $\widehat{RHS}(R)$  (i.e.,  $12 \times \widehat{RHS}(R)$ ). ‘‘Distance’’ reports the annualized distance between  $\widehat{RHS}(R)$  and  $GEF(M)$ , and is given by  $12 \times (GEF(M) - \widehat{RHS}(R))$  if  $s \leq 1$  and  $12 \times (\widehat{RHS}(R) - GEF(M))$  if  $s \geq 1$ .  $P$ -values are obtained through bootstrap.

|  |                 | Generalized Entropies (s)          |         |          |          |                                |         |          |          |
|--|-----------------|------------------------------------|---------|----------|----------|--------------------------------|---------|----------|----------|
|  |                 | Equally-Weighted Option Portfolios |         |          |          | VIX-Weighted Option Portfolios |         |          |          |
|  |                 | $s = 0.5$                          | $s = 0$ | $s = -1$ | $s = -2$ | $s = 0.5$                      | $s = 0$ | $s = -1$ | $s = -2$ |
| <b><math>\gamma = 2</math></b>             |                 |                                    |         |          |          |                                |         |          |          |
| $R_f = 0.95$<br>(Imp. Consp. Gr. = -0.025) | <i>RHS</i>      | 0.197                              | 0.266   | 0.316    | 0.332    | 0.208                          | 0.285   | 0.330    | 0.344    |
|  | <i>Distance</i> | -0.190                             | -0.253  | -0.298   | -0.311   | -0.201                         | -0.273  | -0.312   | -0.323   |
|  | <i>p-value</i>  | (0.01)                             | (0.02)  | (0.00)   | (0.00)   | (0.00)                         | (0.01)  | (0.00)   | (0.00)   |
| $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.001)  | <i>RHS</i>      | 0.135                              | 0.214   | 0.265    | 0.280    | 0.147                          | 0.234   | 0.279    | 0.293    |
|  | <i>Distance</i> | -0.128                             | -0.202  | -0.247   | -0.259   | -0.139                         | -0.222  | -0.261   | -0.272   |
|  | <i>p-value</i>  | (0.04)                             | (0.05)  | (0.01)   | (0.01)   | (0.02)                         | (0.03)  | (0.01)   | (0.00)   |
| $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.025)  | <i>RHS</i>      | 0.077                              | 0.165   | 0.216    | 0.232    | 0.088                          | 0.185   | 0.230    | 0.244    |
|  | <i>Distance</i> | -0.070                             | -0.153  | -0.198   | -0.211   | -0.081                         | -0.173  | -0.212   | -0.223   |
|  | <i>p-value</i>  | (0.16)                             | (0.09)  | (0.04)   | (0.03)   | (0.11)                         | (0.06)  | (0.02)   | (0.01)   |
| <b><math>\gamma = 4</math></b>             |                 |                                    |         |          |          |                                |         |          |          |
| $R_f = 0.95$<br>(Imp. Consp. Gr. = 0.007)  | <i>RHS</i>      | 0.197                              | 0.266   | 0.316    | 0.332    | 0.208                          | 0.285   | 0.330    | 0.344    |
|  | <i>Distance</i> | -0.133                             | -0.177  | -0.209   | -0.217   | -0.144                         | -0.197  | -0.224   | -0.229   |
|  | <i>p-value</i>  | (0.03)                             | (0.07)  | (0.03)   | (0.02)   | (0.02)                         | (0.04)  | (0.02)   | (0.01)   |
| $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.020)  | <i>RHS</i>      | 0.135                              | 0.214   | 0.265    | 0.280    | 0.147                          | 0.234   | 0.279    | 0.293    |
|  | <i>Distance</i> | -0.071                             | -0.126  | -0.158   | -0.165   | -0.082                         | -0.145  | -0.172   | -0.178   |
|  | <i>p-value</i>  | (0.15)                             | (0.14)  | (0.07)   | (0.06)   | (0.11)                         | (0.10)  | (0.05)   | (0.04)   |
| $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.032)  | <i>RHS</i>      | 0.077                              | 0.165   | 0.216    | 0.232    | 0.088                          | 0.185   | 0.230    | 0.244    |
|  | <i>Distance</i> | -0.012                             | -0.077  | -0.109   | -0.116   | -0.024                         | -0.097  | -0.124   | -0.129   |
|  | <i>p-value</i>  | (0.42)                             | (0.25)  | (0.15)   | (0.13)   | (0.35)                         | (0.19)  | (0.11)   | (0.10)   |
| <b><math>\gamma = 5</math></b>             |                 |                                    |         |          |          |                                |         |          |          |
| $R_f = 0.95$<br>(Imp. Consp. Gr. = 0.029)  | <i>RHS</i>      | 0.197                              | 0.266   | 0.316    | 0.332    | 0.208                          | 0.285   | 0.330    | 0.344    |
|  | <i>Distance</i> | -0.036                             | -0.062  | -0.086   | -0.090   | -0.047                         | -0.081  | -0.100   | -0.102   |
|  | <i>p-value</i>  | (0.29)                             | (0.29)  | (0.21)   | (0.19)   | (0.23)                         | (0.22)  | (0.16)   | (0.15)   |
| $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.039)  | <i>RHS</i>      | 0.135                              | 0.214   | 0.265    | 0.280    | 0.147                          | 0.234   | 0.279    | 0.293    |
|  | <i>Distance</i> | 0.026                              | -0.010  | -0.035   | -0.038   | 0.014                          | -0.030  | -0.049   | -0.051   |
|  | <i>p-value</i>  | (0.63)                             | (0.45)  | (0.36)   | (0.35)   | (0.58)                         | (0.38)  | (0.31)   | (0.30)   |
| $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.049)  | <i>RHS</i>      | 0.077                              | 0.165   | 0.216    | 0.232    | 0.088                          | 0.185   | 0.230    | 0.244    |
|  | <i>Distance</i> | 0.084                              | 0.039   | 0.014    | 0.011    | 0.073                          | 0.019   | -0.000   | -0.002   |
|  | <i>p-value</i>  | (0.89)                             | (0.62)  | (0.54)   | (0.53)   | (0.87)                         | (0.56)  | (0.49)   | (0.48)   |

Table C.2: Testing Disaster Models with Option Return Bounds: Baseline Disaster Model + Crash-Neutral Puts Based on the Option Cross Section ( $CNput^{EW}(92\%, 85\%) + CNput^{VIXW}(92\%, 85\%)$ )

The baseline disaster model is characterized by  $\omega_B = 0.017$  and  $\theta_B = -0.38$  (Table C.1). Other model parameters are given in Table C.1, except for  $\gamma$  (risk aversion) and  $R_f$  (annualized risk-free rate). For a given  $\gamma$  and  $R_f$ , we solve for the implied mean consumption growth rate ( $\mu + \omega\theta$ ) through (26). We use the crash-neutral put portfolios that take a long position in the riskless bond, short positions in all puts with a strike-to-price ratio that falls into (85%, 92%), and an offsetting long position in the 85%-OTM put. The strategy return is given by  $R_{g,t+1} = (\bar{p}_{t+1}^H - p_{t+1}^L) / (\bar{p}_t^H - p_t^L)$ , where  $\bar{p}_{t+1}^H$  ( $\bar{p}_t^H$ ) is the weighted average future (current) price of all puts with a strike-to-price ratio between 85% and 92%, and  $p_{t+1}^L$  ( $p_t^L$ ) is the future (current) price of the 85%-OTM put. The portfolio return is given by  $R_{p,t+1} = \alpha_{g,t}R_{g,t+1} + (1 - \alpha_{g,t})R_{f,t}$ , where  $R_{f,t}$  is the risk-free rate and  $\alpha_{g,t}$  is the weight on the crash-neutral put as given by  $\alpha_{g,t} = \max\{-0.15, -R_{f,t} / (\bar{R}_t^{ECN} - R_{f,t})\}$ , where  $\bar{R}_t^{ECN}$  is the theoretical upper bound for the return of the crash-neutral put as given by  $\bar{R}_t^{ECN} = ((K - 0.85)p_t^M) / (\bar{p}_t^H - p_t^L)$ , where  $K$  is the weighted average strike-to-price ratio among long positions in puts and  $p_t^M$  is the spot price for the market index. Put prices for short positions are equally-weighted for  $CNput^{EW}(92\%, 85\%)$  and VIX-weighted for  $CNput^{VIXW}(92\%, 85\%)$ . For a given asset return  $R$  with monthly observations, we estimate the right-hand side of (10) by its sample counterpart (denoted as  $\widehat{RHS}(R)$ ). We also obtain the monthly  $GEF$  for the model under consideration (denoted as  $GEF(M)$ ). ‘‘RHS’’ reports the annualized  $\widehat{RHS}(R)$  (i.e.,  $12 \times \widehat{RHS}(R)$ ). ‘‘Distance’’ reports the annualized distance between  $\widehat{RHS}(R)$  and  $GEF(M)$ , and is given by  $12 \times (GEF(M) - \widehat{RHS}(R))$  if  $s \leq 1$  and  $12 \times (\widehat{RHS}(R) - GEF(M))$  if  $s \geq 1$ .  $P$ -values are obtained through bootstrap.

|                                |                 | Generalized Entropies (s)          |        |        |        |                                |        |        |        |
|--------------------------------|-----------------|------------------------------------|--------|--------|--------|--------------------------------|--------|--------|--------|
|                                |                 | Equally-Weighted Option Portfolios |        |        |        | VIX-Weighted Option Portfolios |        |        |        |
|                                |                 | s = 0.5                            | s = 0  | s = -1 | s = -2 | s = 0.5                        | s = 0  | s = -1 | s = -2 |
| <b><math>\gamma = 2</math></b> |                 |                                    |        |        |        |                                |        |        |        |
| $R_f = 0.95$                   | <i>RHS</i>      | 0.217                              | 0.301  | 0.324  | 0.332  | 0.217                          | 0.302  | 0.325  | 0.332  |
| (Imp. Consp. Gr. = -0.025)     | <i>Distance</i> | -0.209                             | -0.289 | -0.307 | -0.310 | -0.210                         | -0.290 | -0.307 | -0.311 |
|                                | <i>p-value</i>  | (0.00)                             | (0.00) | (0.00) | (0.00) | (0.00)                         | (0.00) | (0.00) | (0.00) |
| $R_f = 1.00$                   | <i>RHS</i>      | 0.155                              | 0.250  | 0.273  | 0.280  | 0.155                          | 0.251  | 0.273  | 0.281  |
| (Imp. Consp. Gr. = 0.001)      | <i>Distance</i> | -0.148                             | -0.238 | -0.255 | -0.259 | -0.148                         | -0.239 | -0.256 | -0.259 |
|                                | <i>p-value</i>  | (0.00)                             | (0.00) | (0.00) | (0.00) | (0.00)                         | (0.00) | (0.00) | (0.00) |
| $R_f = 1.05$                   | <i>RHS</i>      | 0.096                              | 0.201  | 0.224  | 0.232  | 0.097                          | 0.202  | 0.225  | 0.232  |
| (Imp. Consp. Gr. = 0.025)      | <i>Distance</i> | -0.089                             | -0.189 | -0.207 | -0.210 | -0.089                         | -0.190 | -0.207 | -0.211 |
|                                | <i>p-value</i>  | (0.04)                             | (0.01) | (0.01) | (0.00) | (0.03)                         | (0.01) | (0.00) | (0.00) |
| <b><math>\gamma = 4</math></b> |                 |                                    |        |        |        |                                |        |        |        |
| $R_f = 0.95$                   | <i>RHS</i>      | 0.217                              | 0.301  | 0.324  | 0.332  | 0.217                          | 0.302  | 0.325  | 0.332  |
| (Imp. Consp. Gr. = 0.007)      | <i>Distance</i> | -0.152                             | -0.213 | -0.218 | -0.216 | -0.153                         | -0.213 | -0.218 | -0.217 |
|                                | <i>p-value</i>  | (0.00)                             | (0.01) | (0.00) | (0.00) | (0.00)                         | (0.01) | (0.00) | (0.00) |
| $R_f = 1.00$                   | <i>RHS</i>      | 0.155                              | 0.250  | 0.273  | 0.280  | 0.155                          | 0.251  | 0.273  | 0.281  |
| (Imp. Consp. Gr. = 0.020)      | <i>Distance</i> | -0.091                             | -0.162 | -0.166 | -0.165 | -0.091                         | -0.162 | -0.167 | -0.165 |
|                                | <i>p-value</i>  | (0.03)                             | (0.03) | (0.02) | (0.02) | (0.03)                         | (0.02) | (0.02) | (0.01) |
| $R_f = 1.05$                   | <i>RHS</i>      | 0.096                              | 0.201  | 0.224  | 0.232  | 0.097                          | 0.202  | 0.225  | 0.232  |
| (Imp. Consp. Gr. = 0.032)      | <i>Distance</i> | -0.032                             | -0.113 | -0.118 | -0.116 | -0.032                         | -0.113 | -0.118 | -0.117 |
|                                | <i>p-value</i>  | (0.24)                             | (0.08) | (0.06) | (0.06) | (0.24)                         | (0.08) | (0.06) | (0.06) |
| <b><math>\gamma = 5</math></b> |                 |                                    |        |        |        |                                |        |        |        |
| $R_f = 0.95$                   | <i>RHS</i>      | 0.217                              | 0.301  | 0.324  | 0.332  | 0.217                          | 0.302  | 0.325  | 0.332  |
| (Imp. Consp. Gr. = 0.029)      | <i>Distance</i> | -0.056                             | -0.097 | -0.095 | -0.090 | -0.056                         | -0.098 | -0.095 | -0.090 |
|                                | <i>p-value</i>  | (0.12)                             | (0.11) | (0.10) | (0.11) | (0.12)                         | (0.11) | (0.10) | (0.11) |
| $R_f = 1.00$                   | <i>RHS</i>      | 0.155                              | 0.250  | 0.273  | 0.280  | 0.155                          | 0.251  | 0.273  | 0.281  |
| (Imp. Consp. Gr. = 0.039)      | <i>Distance</i> | 0.006                              | -0.046 | -0.043 | -0.038 | 0.006                          | -0.047 | -0.044 | -0.038 |
|                                | <i>p-value</i>  | (0.54)                             | (0.27) | (0.27) | (0.29) | (0.53)                         | (0.26) | (0.27) | (0.29) |
| $R_f = 1.05$                   | <i>RHS</i>      | 0.096                              | 0.201  | 0.224  | 0.232  | 0.097                          | 0.202  | 0.225  | 0.232  |
| (Imp. Consp. Gr. = 0.049)      | <i>Distance</i> | 0.065                              | 0.003  | 0.006  | 0.011  | 0.064                          | 0.002  | 0.005  | 0.010  |
|                                | <i>p-value</i>  | (0.92)                             | (0.50) | (0.52) | (0.54) | (0.92)                         | (0.50) | (0.52) | (0.55) |



## D Alternative Asset Pricing Models

Table D.1: **Testing the Habit Model with Option Return Bounds: Campbell and Cochrane (1999) Model + Exact Crash-Neutral Puts ( $ECNput(96\%, 85\%)$ )**

Our model calibration for the habit model follows Table I in Campbell and Cochrane (1999). For a given  $\gamma$ , we calculate the model implied  $R_f$ . We use the exact crash-neutral put portfolio specified in section 4.2 (i.e.,  $ECNput$ ), which takes a long position in the riskless bond, a short position in the 96%-OTM put, and an offsetting long position in the 85%-OTM put. The put strategy return is given by  $R_{g,t+1} = (p_{t+1}^H - p_{t+1}^L)/(p_t^H - p_t^L)$ , where  $p_{t+1}^H(p_t^H)$  is the future (current) price of the 96%-OTM put and  $p_{t+1}^L(p_t^L)$  is the future (current) price of the 85%-OTM put. The combined portfolio return is given by  $R_{p,t+1} = \alpha_{g,t}R_{g,t+1} + (1 - \alpha_{g,t})R_{f,t}$ , where  $R_{f,t}$  is the risk-free rate and  $\alpha_{g,t}$  is the weight on the crash-neutral put as given by  $\alpha_{g,t} = \max\{-0.15, -R_{f,t}/(\bar{R}_t^{ECN} - R_{f,t})\}$ , where  $\bar{R}_t^{ECN}$  is the theoretical upper bound for the return of the crash-neutral put as given by  $\bar{R}_t^{ECN} = ((0.96 - 0.85)p_t^M)/(p_t^H - p_t^L)$ , where  $p_t^M$  is the spot price for the market index. For a given asset return  $R$  with monthly observations, we estimate the right-hand side of (10) by its sample counterpart (denoted as  $\widehat{RHS}(R)$ ). We also obtain the monthly  $GEF$  for the model under consideration (denoted as  $GEF(M)$ ). “RHS” reports the annualized  $\widehat{RHS}(R)$  (i.e.,  $12 \times \widehat{RHS}(R)$ ). “Distance” reports the annualized distance between  $\widehat{RHS}(R)$  and  $GEF(M)$ , and is given by  $12 \times (GEF(M) - \widehat{RHS}(R))$  if  $s \leq 1$  and  $12 \times (\widehat{RHS}(R) - GEF(M))$  if  $s \geq 1$ .  $P$ -values are obtained through bootstrap.

|              |  |                 | Generalized Entropies (s) |         |          |          |
|--------------|--|-----------------|---------------------------|---------|----------|----------|
|              |  |                 | $s = 0.5$                 | $s = 0$ | $s = -1$ | $s = -2$ |
| $\gamma = 2$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = -0.015) | <i>RHS</i>      | 0.185                     | 0.244   | 0.301    | 0.318    |
|              |  | <i>Distance</i> | -0.117                    | -0.114  | -0.015   | 0.078    |
|              |  | <i>p-value</i>  | (0.06)                    | (0.18)  | (0.44)   | (0.76)   |
|              | $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.011)  | <i>RHS</i>      | 0.123                     | 0.192   | 0.250    | 0.267    |
|              |  | <i>Distance</i> | -0.048                    | -0.055  | 0.059    | 0.162    |
|              |  | <i>p-value</i>  | (0.25)                    | (0.32)  | (0.69)   | (0.94)   |
|              | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.035)  | <i>RHS</i>      | 0.065                     | 0.144   | 0.201    | 0.218    |
|              |  | <i>Distance</i> | 0.006                     | -0.007  | 0.067    | 0.181    |
|              |  | <i>p-value</i>  | (0.52)                    | (0.47)  | (0.72)   | (0.96)   |
| $\gamma = 4$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = 0.027)  | <i>RHS</i>      | 0.185                     | 0.244   | 0.301    | 0.318    |
|              |  | <i>Distance</i> | -0.042                    | -0.034  | 0.264    | 0.548    |
|              |  | <i>p-value</i>  | (0.28)                    | (0.60)  | (0.99)   | (1.00)   |
|              | $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.040)  | <i>RHS</i>      | 0.123                     | 0.192   | 0.250    | 0.267    |
|              |  | <i>Distance</i> | 0.013                     | 0.078   | 0.302    | 0.594    |
|              |  | <i>p-value</i>  | (0.55)                    | (0.73)  | (1.00)   | (1.00)   |
|              | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.052)  | <i>RHS</i>      | 0.065                     | 0.144   | 0.201    | 0.218    |
|              |  | <i>Distance</i> | 0.080                     | 0.131   | 0.342    | 0.621    |
|              |  | <i>p-value</i>  | (0.86)                    | (0.86)  | (1.00)   | (1.00)   |

Table D.2: Testing the Long-run Risks Model with Option Return Bounds: Bansal, Kiku and Yaron (2012) Model + Exact Crash-Neutral Puts ( $ECNput(96\%, 85\%)$ )

Our model calibration for the long-run risks model follows Table I in Bansal, Kiku, and Yaron (2012). For a given  $\gamma$ , we calculate the model implied  $R_f$ . We use the exact crash-neutral put portfolio specified in section 4.2 (i.e.,  $ECNput$ ), which takes a long position in the riskless bond, a short position in the 96%-OTM put, and an offsetting long position in the 85%-OTM put. The put strategy return is given by  $R_{g,t+1} = (p_{t+1}^H - p_{t+1}^L)/(p_t^H - p_t^L)$ , where  $p_{t+1}^H(p_t^H)$  is the future (current) price of the 96%-OTM put and  $p_{t+1}^L(p_t^L)$  is the future (current) price of the 85%-OTM put. The combined portfolio return is given by  $R_{p,t+1} = \alpha_{g,t}R_{g,t+1} + (1 - \alpha_{g,t})R_{f,t}$ , where  $R_{f,t}$  is the risk-free rate and  $\alpha_{g,t}$  is the weight on the crash-neutral put as given by  $\alpha_{g,t} = \max\{-0.15, -R_{f,t}/(\bar{R}_t^{ECN} - R_{f,t})\}$ , where  $\bar{R}_t^{ECN}$  is the theoretical upper bound for the return of the crash-neutral put as given by  $\bar{R}_t^{ECN} = ((0.96 - 0.85)p_t^M)/(p_t^H - p_t^L)$ , where  $p_t^M$  is the spot price for the market index. For a given asset return  $R$  with monthly observations, we estimate the right-hand side of (10) by its sample counterpart (denoted as  $\widehat{RHS}(R)$ ). We also obtain the monthly  $GEF$  for the model under consideration (denoted as  $GEF(M)$ ). “RHS” reports the annualized  $\widehat{RHS}(R)$  (i.e.,  $12 \times \widehat{RHS}(R)$ ). “Distance” reports the annualized distance between  $\widehat{RHS}(R)$  and  $GEF(M)$ , and is given by  $12 \times (GEF(M) - \widehat{RHS}(R))$  if  $s \leq 1$  and  $12 \times (\widehat{RHS}(R) - GEF(M))$  if  $s \geq 1$ .  $P$ -values are obtained through bootstrap.

|               |  |                 | Generalized Entropies (s) |        |        |        |
|---------------|--|-----------------|---------------------------|--------|--------|--------|
|               |  |                 | s = 0.5                   | s = 0  | s = -1 | s = -2 |
| $\gamma = 10$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = -0.085) | <i>RHS</i>      | 0.185                     | 0.244  | 0.301  | 0.318  |
|               |  | <i>Distance</i> | -0.132                    | -0.138 | -0.088 | 0.006  |
|               |  | <i>p-value</i>  | (0.04)                    | (0.13) | (0.21) | (0.51) |
|               | $R_f = 1.00$<br>(Imp. Consp. Gr. = -0.004) | <i>RHS</i>      | 0.123                     | 0.192  | 0.250  | 0.267  |
|               |  | <i>Distance</i> | -0.030                    | -0.006 | 0.125  | 0.300  |
|               |  | <i>p-value</i>  | (0.33)                    | (0.47) | (0.87) | (1.00) |
|               | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.086)  | <i>RHS</i>      | 0.065                     | 0.144  | 0.201  | 0.218  |
|               |  | <i>Distance</i> | 0.180                     | 0.347  | 0.782  | 1.262  |
|               |  | <i>p-value</i>  | (1.00)                    | (0.99) | (1.00) | (1.00) |
| $\gamma = 12$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = -0.082) | <i>RHS</i>      | 0.185                     | 0.244  | 0.301  | 0.318  |
|               |  | <i>Distance</i> | -0.098                    | -0.069 | 0.051  | 0.218  |
|               |  | <i>p-value</i>  | (0.09)                    | (0.28) | (0.67) | (0.99) |
|               | $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.002)  | <i>RHS</i>      | 0.123                     | 0.192  | 0.250  | 0.267  |
|               |  | <i>Distance</i> | 0.042                     | 0.139  | 0.417  | 0.744  |
|               |  | <i>p-value</i>  | (0.71)                    | (0.87) | (1.00) | (1.00) |
|               | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.118)  | <i>RHS</i>      | 0.065                     | 0.144  | 0.201  | 0.218  |
|               |  | <i>Distance</i> | 0.585                     | 1.157  | 2.404  | 3.703  |
|               |  | <i>p-value</i>  | (1.00)                    | (1.00) | (1.00) | (1.00) |
| $\gamma = 15$ | $R_f = 0.95$<br>(Imp. Consp. Gr. = -0.076) | <i>RHS</i>      | 0.185                     | 0.244  | 0.301  | 0.318  |
|               |  | <i>Distance</i> | -0.019                    | 0.088  | 0.370  | 0.712  |
|               |  | <i>p-value</i>  | (0.39)                    | (0.76) | (1.00) | (1.00) |
|               | $R_f = 1.00$<br>(Imp. Consp. Gr. = 0.012)  | <i>RHS</i>      | 0.123                     | 0.192  | 0.250  | 0.267  |
|               |  | <i>Distance</i> | 0.208                     | 0.470  | 1.084  | 1.761  |
|               |  | <i>p-value</i>  | (1.00)                    | (1.00) | (1.00) | (1.00) |
|               | $R_f = 1.05$<br>(Imp. Consp. Gr. = 0.076)  | <i>RHS</i>      | 0.065                     | 0.144  | 0.201  | 0.218  |
|               |  | <i>Distance</i> | 0.131                     | 0.249  | 0.592  | 0.995  |
|               |  | <i>p-value</i>  | (0.97)                    | (0.98) | (1.00) | (1.00) |