Internet Appendix for "Liquidity, Innovation, and Endogenous Growth"

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A The model with learning about an entrant's type

In this section, we study an environment characterized by incomplete information and financiers' learning about an entrant's ability to achieve technological breakthroughs. This environment is consistent with entrepreneurial finance schemes, according to which financiers stage their investments, learn about the firm's potential, and often deny further financing if no breakthroughs (in the entrepreneurial finance jargon, "milestones") are attained (see Gompers, 1995; Lerner, Leamon, and Hardymon, 2012). This financing scheme usually applies to startups (as we interpret our entrants) rather than to mature, large firms that have R&D laboratories (as we interpret our incumbents). In this model extension, therefore, we only focus on entrants.

We assume that there are two types of entrants, good and bad. Good types eventually attain a breakthrough, whereas bad types never attain one. We assume that neither an entrant itself nor financiers know the entrant's type. The only source of information—which is common to all agents—is the random variable that equals one if a technological breakthrough is attained (and equals zero otherwise). We denote by π_t the time$-t$ posterior probability that a given entrant is of good type. Standard filtering results imply that the posterior likelihood $\ell_t = \log \frac{\pi_t}{1-\pi_t}$ of an entrant being of good type evolves as:

$$d\ell_t \;=\; -\phi z_E(\ell_t, c_{Et})dt. \tag{1}$$

This likelihood is monotonically decreasing over time when no breakthrough is attained, and jumps up if a technological breakthrough is attained. By the definition of ℓ_t, we have $\pi(\ell) = e^{\ell}/(1+e^{\ell})$. In a Markov perfect equilibrium, the entrant's firm value, $v_E(\ell_t, c_{Et})$, is a function of ℓ_t and c_{Et}. Standard results imply that entrant value satisfies the following

HJB equation:

$$
\begin{aligned}
(r-g)v_E(\ell, c_E) = \max_{z_E} \Bigg\{ & (\delta - g)c_E v_{Ec}(\ell, c_E) - \frac{\zeta_E}{2} z_E^2(\ell, c_E) v_{Ec}(\ell, c_E) \\
& - \phi z_E(\ell, c_E) v_{E\ell}(\ell, c_E) + \ \pi(\ell)\phi z_E(\ell, c_E)[\Lambda w^* + c_E - v_E(\ell, c_E) - \Lambda \kappa_T] \Bigg\}
\end{aligned}
\tag{2}
$$

subject to

$$
v_E(\ell, 0) = \max\{\psi_E \kappa_E, v_E(\ell, C_{E*}(\ell)) - (1+\epsilon_E)C_{E*}(\ell) - \omega_E\}. \tag{3}
$$

This boundary condition implies that, when cash reserves are depleted, the firm shuts down operations if the liquidation value of R&D assets (the first term in the max operator) is greater than the firm continuation value when raising funds (the second term in the max operator). $C_{E*}(\ell)$ is the issue amount that is pinned down by the first-order condition $v_{Ec}(\ell, C_{E*}(\ell)) = 1 + \epsilon_E$, which warrants that the post-issuance level of cash reserves equalizes the benefit (the left-hand side) and the cost (the right-hand side) of an equity issuance. Maximizing (2) gives the optimal innovation rate

$$
z_E(\ell, c) \ = \ \frac{\frac{e^\ell}{1+e^\ell}\phi[\Lambda w^* + c_E - v_E(\ell, c_E) - \Lambda\kappa_T] - \phi v_{E\ell}(\ell, c_E)}{v_{Ec}(\ell, c_E)\zeta_E}. \tag{4}
$$

Because v_E is decreasing in ℓ, so is

$$
v_E(\ell, C_{E*}(\ell)) - (1+\epsilon_E)C_{E*}(\ell) \ = \ \max_{c_E}(v_E(\ell, c_E) - (1+\epsilon_E)c_E). \tag{5}
$$

It is straightforward to show that there exists a critical level ℓ^* such that

$$
v_E(\ell^*, C_{E*}(\ell^*)) - (1+\epsilon_E)C_{E*}(\ell^*) - \omega_E - \psi_E\kappa_E = 0, \tag{6}
$$

and the firm is shut down when ℓ hits ℓ^* and cash reserves are depleted. Because the dynamics of ℓ_t for a bad-type firm are deterministic, ℓ^* is hit at a deterministic time t^*. If ℓ_0 is sufficiently close to ℓ^*, then this t^* is small enough to ensure that only one round of financing is optimal. When only one financing round is optimal, the entrants' boundary condition at zero is $v_E(0) = \psi_E \kappa_E$.

Thus, contrary to the case considered in the main text (in which entrants have access to successive financing rounds and face the same issuance costs at each round), learning implies that only one financing round may be an equilibrium outcome. In a previous version of the paper, we have analyzed the implications of this setting, and our main results are unchanged. The (less tractable) case in which entrants have access to a finite number of financing rounds stands in between these corner cases.

B Incumbents' optimal production decisions

Whereas we focus on innovation in the main text, in this paragraph we analyze the properties of the incumbents' optimal production rate and the associated selling price.

When incumbents hold their target level of cash reserves, financial constraints are relaxed and effective risk aversion is zero by Eq. (17). Then, the equality $X(C^*) = X^*$ holds by Eq. (35)—i.e., the optimal production rate coincides with the one associated with the UE. When incumbents hold *less* than their target cash level, they are effectively risk averse and seek to limit cash flow risk by scaling down production (recall that cash flow volatility is given by $\sigma X(c)$). As shown in Lemma 5, F is decreasing in $A(c)$. Thus, if effective risk aversion $A(c)$ is decreasing in cash reserves c, then the optimal production rate $X(c)$ is increasing in cash reserves. That is, in the presence of financing frictions, incumbents respond to adverse operating shocks by decreasing their production rate.

Through the demand schedule of the final good sector, the selling price associated with this optimal production rate is given by:

$$p(c) = X(c)^{-\beta} \geq (X^*)^{-\beta} = (1-\beta)^{-1} = p^*. \tag{7}$$

This equation implies that $p(c)$ decreases if the production rate increases. Because the optimal production rate increases with cash reserves, the price $p(c)$ associated with the CE (and the related markup, $p(c) - 1$) is greater than the price p^* associated with the UE (see Section 3) for any $c < C^*$, and is equal to p^* only at $c = C^*$.

Fig. IA.1 shows the optimal production rate and the associated selling price in the three environments UE, CE0, and CE1 that we study in Section 5. The figure shows that incumbents produce less (and charge larger markups) in the presence of financing costs but when they hold their target cash level $c = C^*$. That is, after a cash outflow, constrained firms are more effectively risk averse, decrease production, and increase markups. Our result is consistent with Gilchrist, Schoenle, Sim, and Zakrajšek (2017), who show that firms with limited internal liquidity *increased* prices in the face of the financial constraints associated with the 2007-2009 financial crisis. Our result also implies that in the presence of financing frictions, markups are *countercyclical* to firm operating shocks.

C Robustness

C.1 Additional comparative statics

In this section, we provide additional numerical results that, for the sake of brevity, we omitted from the main text.[1]

[1]The parameters used in the numerical analyses are as in Table 1 unless otherwise stated.

Proposition 1(a) illustrates that the sensitivity of the incumbents' innovation rate to cash reserves depends on cash flow volatility. Consistently, Fig. IA.2 illustrates that $z(c)$ is increasing with cash reserves when β is large or σ is small, because cash flow volatility is highly sensitive to the parameters σ and β. Fig. IA.3 complements this analysis by showing the optimal innovation rate of incumbents in the CE *and* in the UE, for different values of β and σ. For large values of β or small values of σ—i.e., when cash flows are not very volatile—$z(c)$ can be lower than z^* for levels of cash reserves close to $c = 0$. Nevertheless, such large values of β deliver unreasonable markup ranges—in fact, the markup range shifts upward when β increases. For instance, if we set the model parameters as in Table 1 and only change $\beta = 0.19$, we get a markup equal to 23.5% in the UE and between 23.5% (at $c = C^*$) and 54.5% (at $c = 0$) in the CE (we discuss how we set β at the beginning of Section 5). We then rule out this or larger values of β. Similarly, $z(c)$ can go below z^* for cash reserves close to $c = 0$ whenever σ is sufficiently small, but we constrain our choice of σ so that operating volatility swings within a realistic range. For $\sigma = 0.25$, average cash flow volatility is 7.75% (cash flow volatility ranges in $[2.8\%, 8.7\%]$ depending on the level of cash reserves). Hence, we do not consider values of σ below this level. It is worth noting that because the mass of incumbents holding low levels of cash reserves is negligible (recall that $\eta(0) = 0$), the equilibrium impact of incumbents who set $z(c) < z^*$ when c is close to zero is minor.

In Fig. 2 and Fig. 3, we analyze the innovation rates of incumbents and entrants as well as the stationary distributions of cash reserves as functions of κ. We did so to preserve a reasonable proportion of R&D assets to total assets. In Fig. IA.4 of this Appendix, we spell out the impact of κ_E and κ_T on innovation rates and on the stationary distributions of cash reserves. First, the figure shows that κ_E and κ_T have similar effects on $z(c)$: By reducing the rate of creative destruction, an increase in κ_E or κ_T shifts the incumbents' innovation rates up. In turn, the incumbent distribution of cash is quite insensitive to κ, κ_E, and κ_T. Turning to entrants, $z_E(c_E)$ increases with κ_E (as it does with κ, see Fig. 2) but is quite insensitive to κ_T. An increase in either κ_E and κ_T is associated with a decrease in the mass of active entrants, which decreases competition from new entrants and boosts the innovation rate of active entrants. This is the reason why $z_E(c_E)$ shifts upward as κ_E increases. On top of this effect, an increase in κ_T also reduces the gain from attaining a breakthrough (see Eq. (19)), which reduces the entrants' incentives to invest in innovation. Because of these offsetting strengths, $z_E(c_E)$ is almost insensitive to κ_T. Similarly, most of the variation of $\eta_E(c_E)$ in κ is driven by κ_E rather than by κ_T.

Finally, to complement the analysis in Fig. 6, Fig. IA.5 shows the growth rate and the incumbents' and the entrants' contributions to growth as a function of the proportional cost ϵ_E, for two levels of κ. It shows that the sensitivity of these endogenous quantities to ϵ_E is qualitatively similar to the sensitivity to ω_E.

C.2 Shrinking the gap between incumbents and entrants

In this section, we consider an environment in which incumbents and entrants are characterized by identical technologies and financing costs. Specifically, we assume that incumbents and entrants have the same innovation technology (i.e., $\lambda = \Lambda = 1.05$, $\zeta = \zeta_E = 0.65$) and that they face the same financing frictions (i.e., $\epsilon = \epsilon_E = 0.06$ and $\omega = \omega_E = 0.01$) and liquidation costs ($\psi = \psi_E = 0.9$). We also assume that the entry costs faced to become entrant and to become incumbent are the same (i.e., $\kappa_E = \kappa_T = 0.65/2$ in the baseline case). Our goal is to show that the different response of incumbents' and entrants' innovation rates to financing frictions is not driven by the specific parameters of our baseline parameterization (Table 1), but rather by the structural difference between these two types of firms. Incumbents are established firms that earn monopoly rents from selling inputs to the consumption good sector and, at the same time, invest in innovation. Differently, entrants are startups that seek to gain lead over some existing inputs, which they do not currently produce. Hence, entrants do not have stable cash flows that can be used to finance their innovation expenditures.

Fig. IA.6 compares this environment (labeled as CE) with an identical, but unconstrained one (in which financing costs are zero, labeled as UE). The top panel shows that $z(c)$ and $z_E(c_E)$ display patterns similar to those in Fig. 1. In particular, $z(c)$ is larger than z^* for any level of cash reserves, whereas $z_E(c_E)$ exceeds z_E^* if cash reserves are sufficiently large. Turning to the distribution of cash, the bottom panel shows that $\eta(0)$ is zero whereas $\eta_E(0)$ is positive. Finally, Fig. IA.7 shows that the mass of active entrants is smaller in the CE than in the UE, for any κ. As a result, the entrant contribution to growth is lower in the CE than in the UE, and so is the rate of creative destruction. Consistent with Proposition 3, the lower rate of creative destruction boost the incumbents' contribution to growth, which is larger in the CE than in the UE. Again, g is not monotonic in entry costs and the gap between g and g^* shrinks as κ increases.

References

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Gompers, P., 1995. Optimal investment, monitoring, and the staging of venture capital. Journal of Finance 50, 1461–1489.

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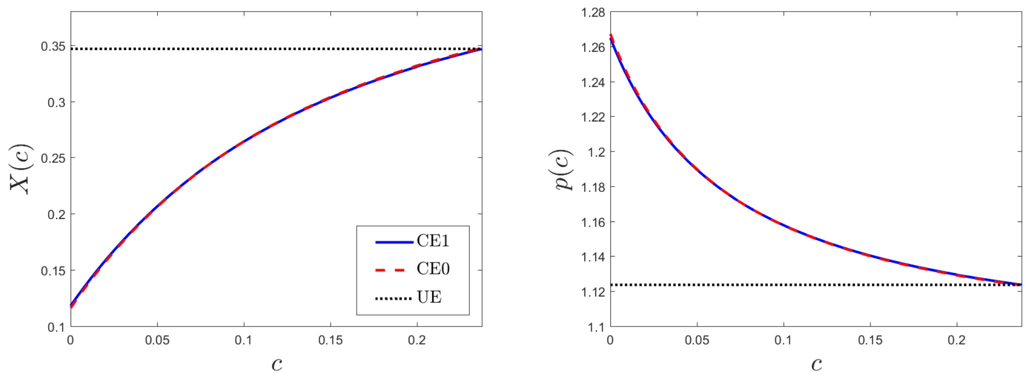

Figure IA.1: Incumbents' production and pricing policies.

The figure shows the incumbents' production rate $X(c)$ and the associated selling price $p(c)$ as functions of cash reserves $c \in [0, C^*]$. The dotted line illustrates the UE (i.e., the environment with no financing costs), the dashed line illustrates the CE0 (i.e., the environment in which entrants and incumbents face the same financing costs), and the solid line illustrates the CE1 (i.e., the environment in which entrants face greater financing costs than incumbents).

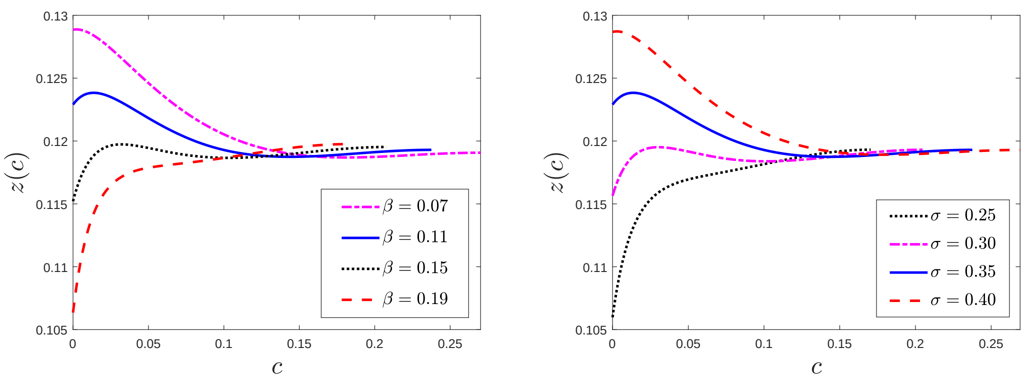

Figure IA.2: Cash flows characteristics and incumbents' innovation (I).
The figure shows the innovation rate of incumbents $z(c)$ as a function of cash reserves $c \in [0, C^*]$ for different values of the the elasticity β (left panel) and the coefficient of cash flow volatility σ (right panel).

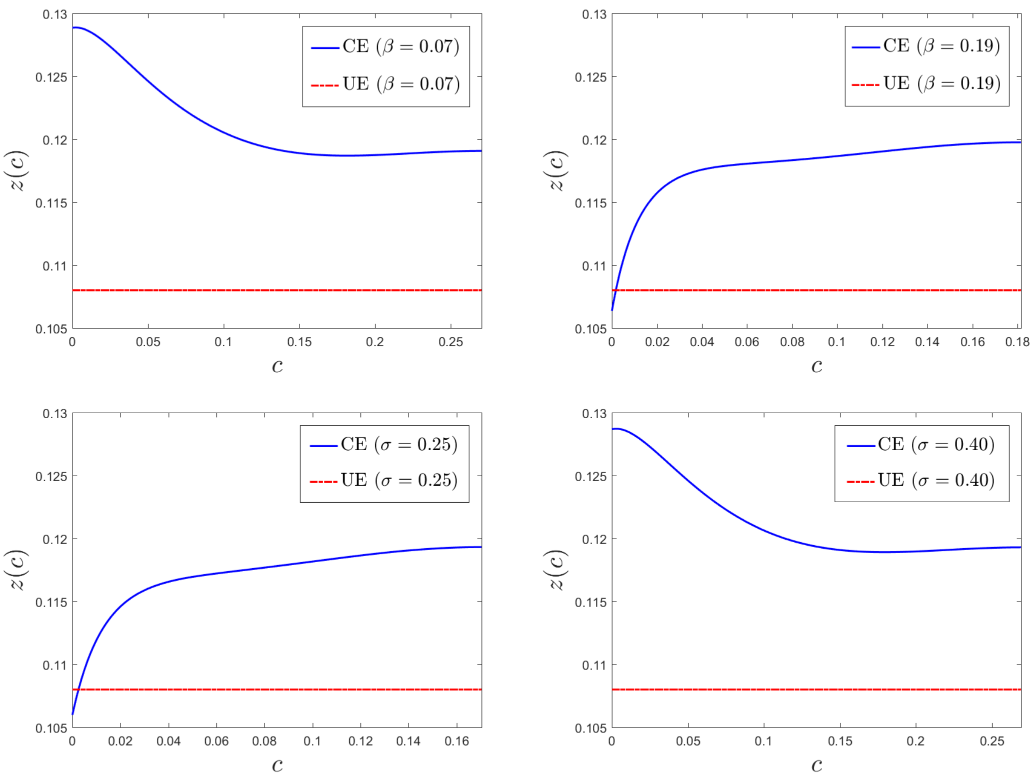

Figure IA.3: Cash flow characteristics and incumbents' innovation (II). The figure shows the innovation rate of incumbents $z(c)$ as a function of cash reserves $c \in [0, C^*]$, for high and low values of the elasticity β (top panel) and the cash flow volatility coefficient σ (bottom panel). The solid blue line represents the constrained economy described in Table 1, whereas the dashed red line represents the unconstrained economy (in which financing is costless).

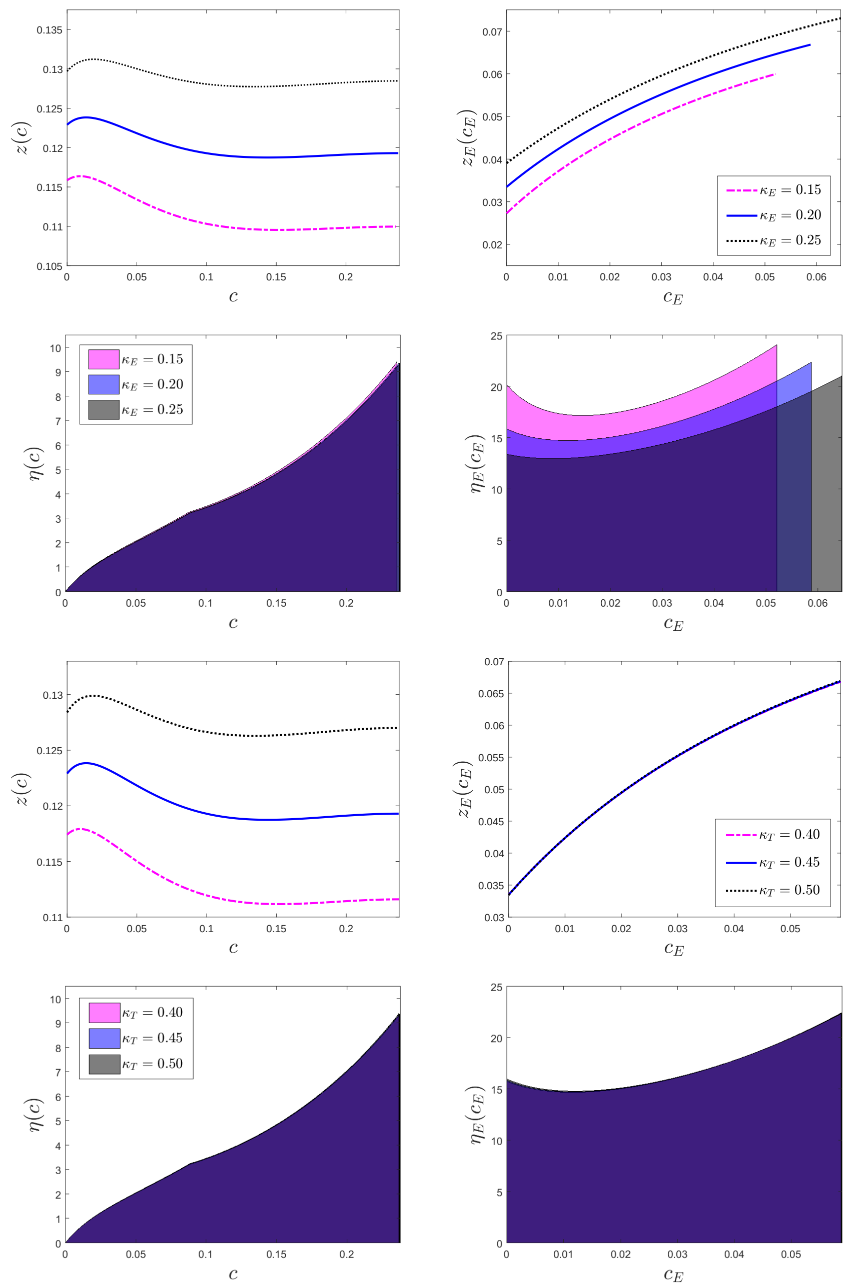

Figure IA.4: Entry costs, innovation, and the distribution of cash.
The figure shows the optimal innovation rates of incumbents and entrants ($z(c)$ and $z_E(c_E)$) and their stationary distributions of cash reserves ($\eta(c)$ and $\eta_E(c_E)$) as functions of cash reserves ($c \in [0, C^*]$ for incumbents; $c_E \in [0, C_{E*}]$ for entrants) for different values of the entry costs κ_E (first and second panel) and κ_T (third and fourth panel).

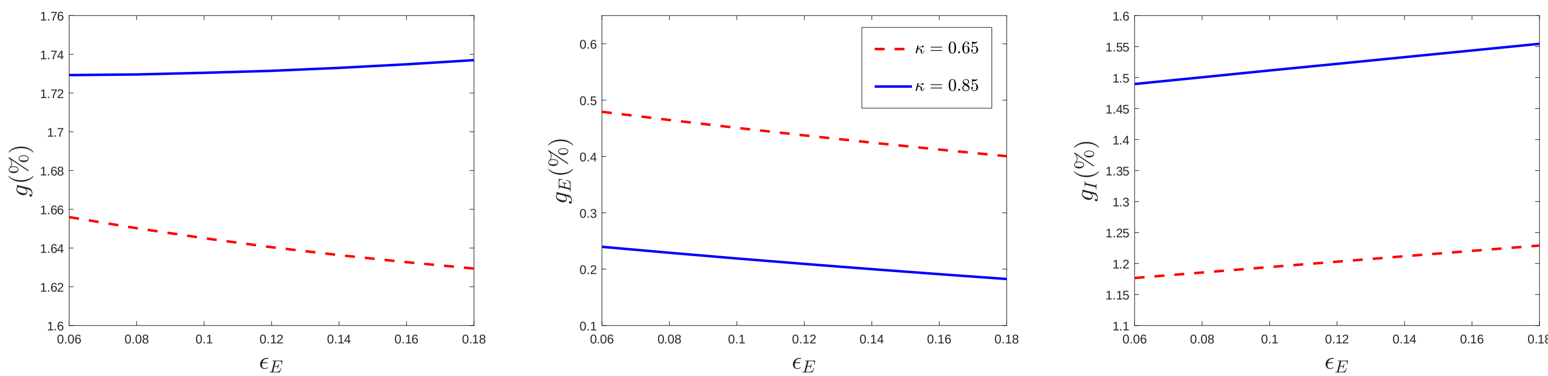

Figure IA.5: Financing frictions and growth.

The figure shows the rate of economic growth (g) and the entrants' and the incumbents' contributions to growth (respectively, g_E and g_I) as functions of the financing cost ϵ_E. The solid line refers to an environment in which the entry cost κ is equal to 0.85, whereas the dashed line refers to an environment in which the entry cost κ is equal to 0.65 (as in the baseline parameterization reported in Section 1).

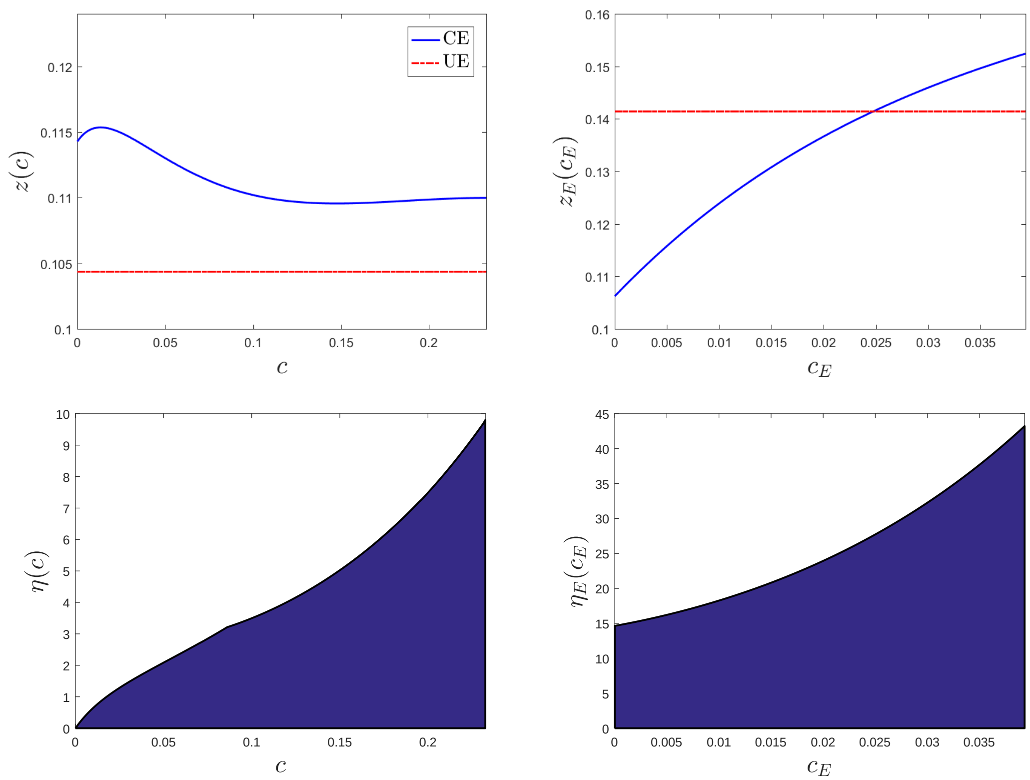

Figure IA.6: Shrinking the gap between incumbents and entrants (I).
The figure shows the optimal innovation rates of incumbents and entrants ($z(c)$ and $z_E(c_E)$) and their stationary distribution of cash ($\eta(c)$ and $\eta_E(c_E)$) as functions of cash reserves ($c \in [0, C^*]$ for incumbents; $c_E \in [0, C_{E*}]$ for entrants), in an environment in which incumbents and entrants are characterized by the same structural parameters. The solid blue line represents the constrained economy, whereas the dashed red line represents the unconstrained economy.

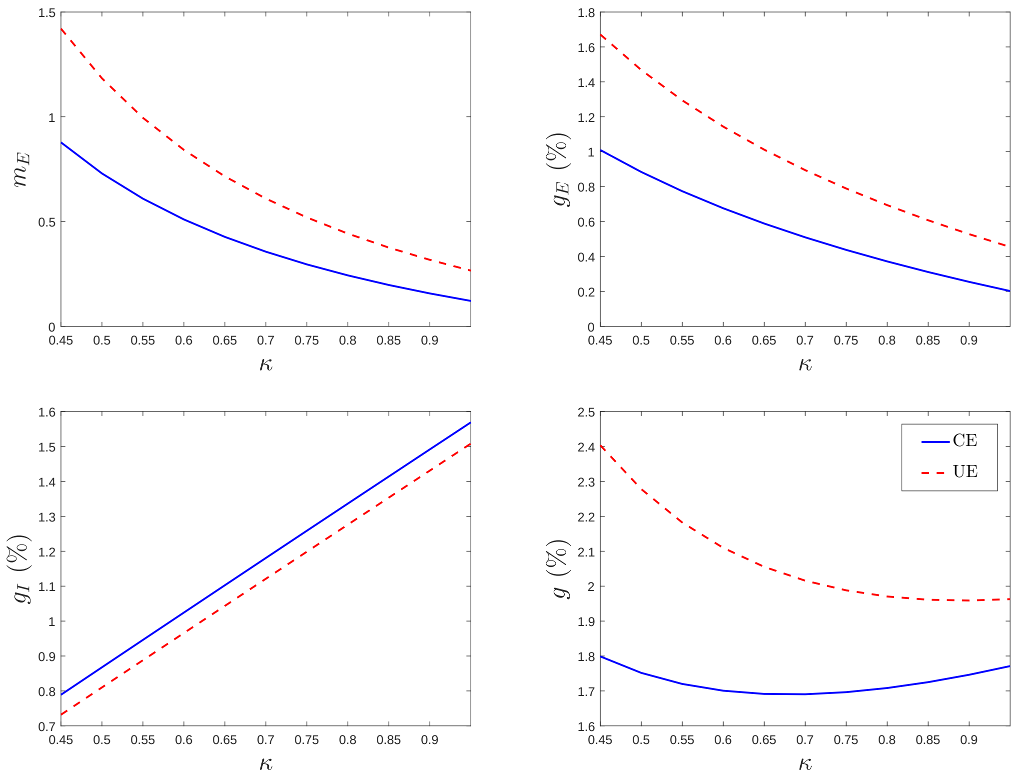

Figure IA.7: Shrinking the gap between incumbents and entrants (II).
The figure shows the measure of active entrants (m_E), the contribution to growth of entrants and incumbents (g_E and g_I), and the growth rate (g) as functions of the entry cost κ, in an environment in which incumbents and entrants are characterized by the same structural parameters. The solid blue line represents the constrained economy, whereas the dashed red line represents the unconstrained economy.