# Should Long-Term Investors Time Volatility?
# Internet Appendix

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**Abstract**

This appendix contains additional results.

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## A. Estimation of parameters

We estimate the model using Simulated Method of Moments (Duffie and Singleton, 1990) and use the estimated parameters in Table 1 to discuss the model implications for portfolio choice.

We first calibrate the real riskless rate ($r = 1\%$) which reflect the U.S. experience in the post-war sample. We also calibrate the volatility lower bound to ($\underline{\sigma} = 7\%$) based on the data.[1]

We estimate the remaining eight parameters. Let $\theta = (\overline{\sigma}^2, \nu_\sigma, \kappa_\sigma, \alpha_\mu, \kappa_\mu, \rho_{\sigma,\mu}, \overline{\mu})$ be the vector of parameters to be estimated. Our SMM estimator is given by

$$\hat{\theta} = \arg\min_{\theta}(g(\theta) - g_T)'\widehat{\Omega}^{-1}(g(\theta) - g_T), \tag{1}$$

where $g_T$ is a set of target moments in the data and $g(\theta)$ is the vector of moments in the model for parameters $\theta$. We use the optimal weighting matrix $\widehat{\Omega}^{-1}$ where $\widehat{\Omega}$ is the variance-covariance matrix of the targeted moments.

We choose the vector of target moments $g_T$ to be informative about the parameters $\theta$. Our target moments are: (1) average excess return on the risky asset (equity premium), (2) average realized monthly variance, (3) the auto-correlation coefficient of the logarithm of realized monthly variance, (4) the variance of log realized variance, (5-6) the R-squared of a predictability regression of one year and five year-ahead returns on the price-dividend

[1] Here we use that the minimum of the VIX from 1990-2015 is 10%, so our 7% for the longer 90 year sample is reasonable. Note we use VIX to calibrate this number rather than realized volatility, because realized volatility is noisy and hence would not properly measure a lower bound for true volatility.

ratio, (7) the regression coefficient of realized returns on changes in realized variance (contemporaneous), (8-9) the alpha of the volatility managed market portfolio on the market portfolio (see (Moreira and Muir, 2017)) as well as the risk-return tradeoff estimated from regressions of future returns on realized volatility. The the alpha of the volatility managed portfolio is defined by the regression $\frac{c}{RV_t^2} R_{t+1} = \alpha + \beta R_{t+1} + \varepsilon_{t+1}$ where the alpha measures whether one can increase Sharpe ratios through volatility timing. Moreira and Muir (2017) show this alpha measures the strength of the risk-return tradeoff over time, but is a sharper measure than standard forecasting regressions.

We follow the influence function technique from Erickson and Whited (2002) to estimate variance-covariance matrix of target moments $\widehat{\Omega}$. We use the inverse of this variance-covariance matrix as the SMM weighting matrix. The parameter standard errors are estimated by sampling a vector of targeted moments from the distribution implied by the influence function approach, i.e. $\widetilde{g}_T^i \sim N(g_T, \widehat{\Omega})$. For each $i$ targeted moment realization we re-estimate the model obtaining new parameter estimates. This procedure recovers the distribution of parameters implied by the data. The implied standard errors are reported in Panel B of Table 1.

## B. Numerical solution

Our solution method follows Judd (1998, Chapter 11). We first conjecture the value function $g(\mu, f(\sigma^2))$ expressed as bivariate Chebyshev polynomials of order N . We use N = 6. For our baseline parameter estimates results do not change with N up to 10, but take increasingly more time to solve. We calculate the derivatives of these functions as well as the optimal portfolio. We then plug these quantities into the HJB (**??**) and project the resulting residuals onto the complete set of Chebyshev polynomials up to order N. We use the built-in Matlab routine fsolve to find the coefficients of the asset price polynomials that make the projected residuals equal to zero.

## C. Certainty equivalent fee calculation

We use $J(\{w = w'\})$ to denote the value function of an investor with investment policy $w'$. Given that the investor life-time utility can be expressed as $V = W^{1-\gamma}/(1-\gamma)exp((1-\gamma)J(\{w = w'\}))$, then the unconditional expected life-time utility of an investor with investment policy $w = \omega$ is $V(\{w = w'\}) = W^{1-\gamma}/(1-\gamma)E[exp((1-\gamma)J(\{w = w'\}))]$. To compare gains across investors it is useful to compute the utility for an investor that invest all her wealth in the riskless asset.

The HJB (equation (**??**)) for an investor that only invests in risk-free asset is given by the solution of

$$0 = \beta\left(log(\beta) - J(\{w = 0\}) - 1\right) + (r + f), \tag{2}$$

with $f = 0$. This yields $J(\{w = 0\}|f) = (log(\beta) + (r + f - \beta)/\beta))$ and expected life-time utility equal to

$$V(\{w=0\}|f) = W^{1-\gamma}/(1-\gamma)\beta exp((r+f-\beta)/\beta), \tag{3}$$

where $V(\{w=0\}|f=0)$ is the life-time utility of the investor that invests all his wealth in the risk-less asset. For a given portfolio policy $w'$ consider the "fee" $f$ that makes the investor indifferent between the risk-free strategy and the strategy $w'$,i.e. $V(\{w=0\}|f) = V(\{w=w'\})$. This fee can be written as,

$$f(\{w=w'\}) = \beta(1+log(E[\exp((1-\gamma)J(\{w=w'\}))]/\beta)) - r \tag{4}$$

A positive fee $f$ means that switching from the risk-free strategy to to $w'$ gives the same utility gain as a wealth flow of $Wf$ per period. That can be thought as the fee an investor that does not have access to the risk-asset would be willing to pay per period to invest with policy $w'$.

We compare the utility gain across portfolio strategies by comparing these fees. For example, let $\Delta = f(\{w=w'\}) - f(\{w=w''\})$ , then $\Delta$ is the fee that an investor would be willing to pay to switch from strategy $w''$ to $w'$.

# D. Multi-frequency variation in expected returns

In the interest of parsimony we have modeled expected returns as a uni-variate process and use the conditional correlation between innovation to volatility and expected returns to match the relation between expected returns and volatility. Here we show that this choice does not materially affect our results. Here we extend our framework as follows

$$\mu_t = x_t + \delta(\sigma_t^2 - E[\sigma_t^2]), \tag{5}$$

where $x_t$ is the pure expected return state variable, which follows dynamics given by (**??**), and the expected return $\mu_t$ is allowed to depend on the level of stock market variance. We re-estimate the model with this additional parameter $\delta$. The table below compare our baseline results where we restrict $\delta = 0$ with the unrestricted case. The estimation recovers a negative $\delta = -0.55$ resulting in even more aggressive volatility timing. Utility gains of volatility timing are also larger. As a share of total gains from timing is grows from about 70% to more than 85%. If, anything, we find that allowing for a direct relation between the level of volatility and expected returns further increase the gains of volatility timing.

| | Constrained | Unconstrained |
|---|---|---|
| $\delta$ | 0.00 | -0.23 |
| $\zeta$ | 0.85 | 0.91 |
| $a^*$ | 0.14 | 0.14 |
| $b^*$ | 0.99 | 1.06 |
| $\Delta U$ | 2.84 | 3.70 |
| $a^\sigma$ | 0.14 | 0.07 |
| $b^\sigma$ | 0.89 | 1.01 |
| $\Delta U^\sigma$ | 2.36 | 3.23 |
| $\Delta U^\sigma / \Delta U$ | 82.92 | 87.29 |

## E. Additional references on conventional wisdom

We include links (click for hyperlink) to additional advice on how to respond to volatility. These sources are meant to convey the conventional view given by practitioners and academics that investors should not respond to increases in volatility, and that long horizon investors may in fact want to buy stocks during periods of high volatility.

Fidelity: "A natural reaction to that fear might be to reduce or eliminate any exposure to stocks, thinking it will stem further losses and calm your fears, but that may not make sense in the long run." "Do not try to time the market." "Invest regularly despite volatility."

Charles Schwab: "Understandably, investors often become nervous when markets are volatile, and we are hearing many questions from clients." "They should also resist the urge to buy and sell based solely on recent market movements, as it could hobble their performance over time."

Forbes: "5 Tips To Survive Stock Market Volatility In Retirement."
"Stay The Course: While staying the course might sound boring to you, it is likely the absolute best thing to do right now. In fact, market volatility is the main catalyst behind a lot of bad financial behaviors – most specifically – buying high and selling low. Despite this widespread knowledge, retirees often overreact when the market drops and divest some of their equities. One way to minimize this harmful financial behavior is to hire a financial advisor. One of the great benefits of having a financial advisor is to steady your emotions during volatile markets. In fact, the retirement income certified professionals surveyed by The American College of Financial Services stated that keeping their clients from overreacting during volatile markets was a crucial aspect of protecting their clients? retirement security.
Reduce Your Withdrawals: As a retiree, you often need to sell your investments in order to generate the income needed to meet your retirement expenses. Selling investments and taking portfolio distributions during a volatile market highlights a unique retirement risk called sequence of returns risk. Sequence of returns risk is unique because until you start

withdrawing money from your investments it has no impact on your portfolio. However, when you sell stocks right after a significant market downturn, you lock in lower returns which can negatively impact the longevity of your investment portfolio. As such, if you can be flexible when markets are volatile and avoid selling as much stock, you can vastly improve how long your retirement portfolio will last. For many people, reducing market withdrawals also involves reducing expenses, even if it is just for a short period of time."

CNBC: "Investing for dummies – and smart guys – in a whipsaw market." "What may be good advice for a 21-year-old may not be the best course of action for a 65-year-old."

New York Times: "Stocks are most useful for long-term goals. So unless those goals have changed in the last few days, it probably doesn't make much sense to overhaul an investment strategy based on a blip of market activity." "Plenty of research shows that if you miss just a few days of the market's biggest gains, your long-term portfolio will suffer badly."

US News: "Volatility can also provide opportunities for investors looking for bargains. Amid an uncertain outlook for the market and key influencers such as interest rates and China, it might be a good idea to take a page from "Dr. Strangelove" – or learn to stop worrying and love the volatility."

USA Today: "Don't attempt a strategy of bailing out temporarily until things 'calm down'."

# References


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