Online Appendix

Collateralizing Liquidity

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1. Long-term contract

I show that focusing on short-term contracts is without loss of generality because the sequence of optimal short-term contracts in the baseline model implements the long-term contract that satisfies the participation constraints for the lender in periods 1 and 2.

Definition 1 (Long-term Contract). A long-term contract ψ consists of:

1. A loan amount in period 1, k.
2. Loan amounts in period 2 contingent on the report in period 1, k_L and k_H.
3. Repayments in terms of the consumption good
   in period 1, contingent on the report in period 1, r_L and r_H
   in period 2, contingent on the reports in periods 1 and 2, r_{LL}, r_{LH} , r_{HL}, and r_{HH}.
4. Repayments in terms of assets
   in period 1, contingent on the report in period 1, t_L and t_H
   in period 2, contingent on the report in periods 1 and 2, t_{LL}, t_{LH}, t_{HL}, and t_{HH}.

As in the case of the short-term contract, in equilibrium a long-term contract has to be feasible and incentive compatible. These conditions are expressed in the following borrower's problem as constraints $1-3$. Finally, I assume that the lender can walk away from the contract at any time and, therefore, his participation constraints have to be satisfied in periods 1 and 2. These are constraints 4 and 5 below. The borrower chooses a long-term contract to maximize his expected utility:

$$\max_{\psi} \mathbb{E}(\theta) k - \pi_L (r_L + dt_L) - \pi_H (r_H + dt_H) + (1+\beta) da \tag{1}$$

$$+ \beta \sum_{i=L,H} \pi_i (\mathbb{E}(\theta) k_i - \pi_L (r_{iL} + dt_{iL}) - \pi_H (r_{iH} + dt_{iH}) - dt_i), \tag{2}$$

subject to:

1. Feasibility constraints at $t=1$ and $t=2$.

2. Incentive compatibility constraints at $t=1$:

$$-r_L - dt_L + \beta (\mathbb{E}(\theta) k_L - dt_L - \pi_L (r_{LL} + dt_{LL}) - \pi_H (r_{LH} + d_{tH})) \tag{3}$$

$$= -r_H - dt_H + \beta (\mathbb{E}(\theta) k_H - dt_H - \pi_L (r_{HL} + dt_{HL}) - \pi_H (r_{HH} + dt_{HH})). \tag{4}$$

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3. Incentive compatibility constraints at $t = 2$[1]:

$$r_{iL} + dt_{iL} = r_{iH} + dt_{iH},\ i = L, H.$$

4. Participation constraint for the lender at $t = 1$,

$$\begin{aligned} kq + \beta\left(\pi_L k_L + \pi_H k_H\right) \leq \pi_L\left(r_L + dt_L\right) + \pi_H\left(r_H + dt_H\right) \qquad (5) \\ + \beta \sum_{i=L,H} \pi_i\left(dt_i + \pi_L\left(r_{iL} + dt_{iL}\right) + \pi_H\left(r_{iH} + dt_{iH}\right)\right). \qquad (6) \end{aligned}$$

5. Participation constraint for the lender at $t = 2$,

$$k_i \leq \pi_L\left(r_{iL} + dt_{iL}\right) + \pi_H\left(r_{iH} + dt_{iH}\right),\ i = L, H.$$

In contrast to the case in which only short-term contracts are allowed, using long-term contracts allows the borrower to directly contract on the loan amounts at $t = 2$.

Proposition 1. In the optimal long-term contract ϕ^* :

1. The loan amounts are

$$k^{**} = \left(1 + \beta + \beta\pi_H\left(\mathbb{E}\left(\theta\right) - 1\right)\right) da, \qquad (7)$$
$$k_H^{**} = a, \quad \text{and} \quad k_L^{**} = 0. \qquad (8)$$

2. Repayments in terms of consumption good are

$$r_L^{**} = 0, \quad r_H^{**} = r_L^{**} + da + \beta\left(\mathbb{E}\left(\theta\right) - 1\right) da, \qquad (9)$$
$$r_{2L}^{**} = 0, \quad \text{and} \quad r_{2H}^{**} = \theta_L k_H^{**}. \qquad (10)$$

3. Asset transfers are

$$t_L^{**} = a, \quad t_H^{**} = 0, \quad t_{2L}^{**} = 0, \quad \text{and} \quad t_{2H}^{**} = a.$$

Proof. Note that the participation constraints for the lender will hold with equality. Using the incentive compatibility constraints and the participation constraint in the first period, the objective function for the borrower becomes

$$\left(\mathbb{E}\left(\theta\right) - 1\right)\left(r_L + dt_L + \beta\left(dt_L + r_{2L} + dt_{2L}\right) + \beta\pi_H\mathbb{E}\left(\theta\right) d\left(t_L - t_H\right)\right) + \left(1 + \beta\right) da.$$

This implies

$$t_L^{**} = a, \quad t_H^{**} = 0, \quad t_{2L}^{**} = 0 \qquad (11)$$
$$r_{2L}^{**} = 0, \quad \text{and} \quad r_L^{**} = 0. \qquad (12)$$

Using these expressions in the participation and incentive compatibility constraints gives the contract in the proposition. □

Corollary 1. The optimal short-term contracts implement the optimal long-term contract.

[1] Risk neutrality implies that the incentive compatibility constraints at $t = 1$ and $t = 2$ can be expressed as one constraint, as stated in points 2 and 3.

The proof follows directly from Proposition 1.

Remark. *The optimal contract described here resembles the optimal contract in* **?**. *However, there are key differences between the assumptions and results in their paper and the ones presented here. In this paper the presence of a long-lived financial asset allows for lending to occur with short-term contracts. In fact, the existence of a long-lived financial assets implies that a sequence of short-term contracts can achieve the same allocation as the optimal long-term contract which, as in* **?**, *involves the firm not being operated if the low state is realized. Moreover, in* **?**, *the firm itself is pledged as collateral by allowing it to be liquidated in the event of a bad report. Therefore, the asset used as collateral is needed in order for the firm to operate and, thus, cannot be sold if it wants to continue operating. In my model, the project available to the borrower are inalienable and the financial asset that is used as collateral in equilibrium is unrelated to the operations of the firm. This implies that the borrower can choose to sell his assets and still be able to invest in his project, which is crucial to answer why assets are used as collateral instead of being sold to raise funds.*

2. Correlated dividends and returns

In the baseline model I assumed the financial asset was riskless. However, many risky assets are used as collateral in financial transactions. For example, mortgage back securities (MBS) were massively used as collateral to finance new mortgages or real estate related securities at the onset of the $2008-2009$ financial crisis. This extension allows for risky financial assets whose dividends may be correlated with the returns of the project operated by the borrower. I show that, consistent with the widely spread use of MBS as collateral, assets with payoffs highly correlated with the investment opportunity are better collateral.

Consider the baseline model presented in Section **??** with the only difference being that the asset's dividend is stochastic and it is potentially correlated with the return of the borrower's risky project. Although the dividends are paid at the end of each period, after the borrower invests in his risky project, they are known at the beginning of each period. This implies that there is only uncertainty about the dividend paid by the asset in the second period. The asset's dividend distribution is such that

$$\mathbb{E}\left(d_2|\theta_1=\theta_i\right)=d_i, \text{ for } i=L,H,$$

and

$$\mathbb{E}\left(d_2\right)=\pi_H d_H+\pi_L d_L=\bar{d}.$$

Given a dividend d_2 at $t=2$, the borrower's problem in the second period remains the same. The value of a borrower who holds a units of the asset at the beginning of the second period is

$$V_2^B\left(a,d_2\right)=\mathbb{E}\left(\theta\right)d_2 a.$$

Analogously, the value for the lender of holding a units of the asset at the beginning of the second period is

$$V_2^L\left(a,d_2\right)=d_2 a.$$

Given that the value functions for the borrower and the lender are linear in the dividend level, it is easy to see that all the results in the baseline model generalize to this extension. In particular, the asset's debt capacity becomes:

$$D=d_1+\beta\bar{d}+\beta\pi_H\left(\frac{V_2^B\left(a,d_H\right)}{a}-\frac{V_2^L\left(a,d_H\right)}{a}\right).$$

The following proposition shows that assets with dividends that are more highly positively correlated with the borrower's project are better collateral.

Proposition 2. Assets that have dividends that are more highly positively correlated with the risky project have a higher debt capacity, i.e.,

$$\frac{\partial D}{\partial d_H|_{\bar{d}}} > 0.$$

Proof. Since $\frac{V_2^L(a,d)}{a} = d$,

$$\frac{\partial D}{\partial d_H|_{\bar{d}}} = \beta \pi_H \left(\mathbb{E}\left(\theta\right) - 1 \right) > 0.$$

□

In the model, the financial asset partly resolves the non-contractibility of the return on the risky project by allowing the borrower to raise funds. Since in equilibrium the borrower values the asset more than the lender does, $V_2^B\left(a, d_H\right) - V_2^L\left(a, d_H\right)$ is the endogenous cost of defaulting on the promised amount, $r_H - r_L$, which allows the borrower to credibly commit to truthfully reporting the project's return. An asset that has dividends that are more highly positively correlated with the return of the risky project has a higher d_H, which implies a higher cost of default for borrowers. This higher cost of default allows the borrower to commit a larger amount of consumption goods in the high state and, thus, increases the debt capacity of the asset, which makes it better collateral.