Online Appendix for "Securitized Markets, International Capital Flows, and Global Welfare"

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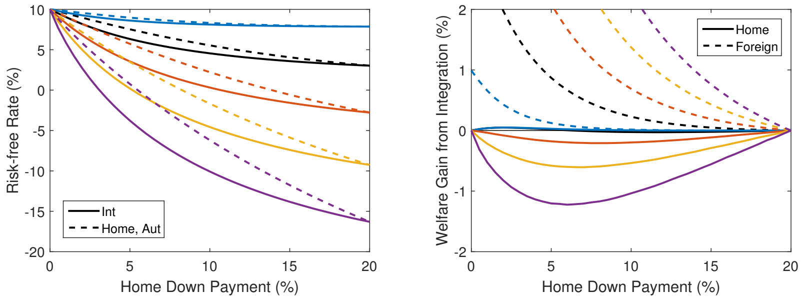
B Sensitivity analysis over parameter values

In this appendix we solve the two country model in Section 2 with different parameter values. We consider changing the standard deviation of technology σ and the relative risk aversion coefficient γ to show how the welfare consequences for Home, driven by risk sharing, depend on underlying risk.

Figure B.1 shows the risk-free rate and the welfare gain (relative to autarky) when σ ranges over $\sigma = 0.1, 0.2, \dots, 0.5$. The baseline case ($\sigma = 0.2$) is shown in black. The larger σ, the lower the risk-free rate and the larger the welfare gain for Foreign, which is natural because Foreign gains from more risk sharing the riskier the technology is. As before, the welfare implication for Home is ambiguous. Home loses more from financial integration with higher σ because the interest rate drops by more, which reduces risk sharing.



(a) Risk-free rate. (b) Welfare gain.

Figure B.1: The effect of changing standard deviation σ. Black: baseline case ($\sigma = 0.2$); Blue: low σ; Purple: high σ.

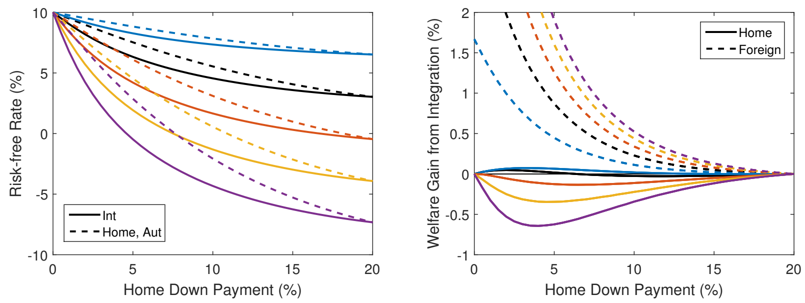
Figure B.2 shows the risk-free rate and the welfare gain when γ ranges over $\gamma = 1, 2, \dots, 5$. The baseline case ($\gamma = 2$) is shown in black. The result is quantitatively similar to changing σ. However, with low risk aversion ($\gamma = 1$), Home also uniformly gains from financial integration for all collateral levels.

C Default costs

In this appendix we discuss the effects of default costs, which we model as a constant depreciation of the collateral after default. Thus the ABS return in (2.3) is replaced with

$$\mathrm{E}\left[(1-\delta)A^i c_j D + R_b^j (1-D)\right],$$

where $\delta \in [0,1]$ is the depreciation rate and $D = 1(A^i c_j < R_b^j)$ is a dummy variable for default. Given the nature of the model, the cost δ is best interpreted



(a) Risk-free rate. (b) Welfare gain.

Figure B.2: The effect of changing relative risk aversion γ. Black: baseline case ($\gamma = 2$); Blue: low γ; Purple: high γ.

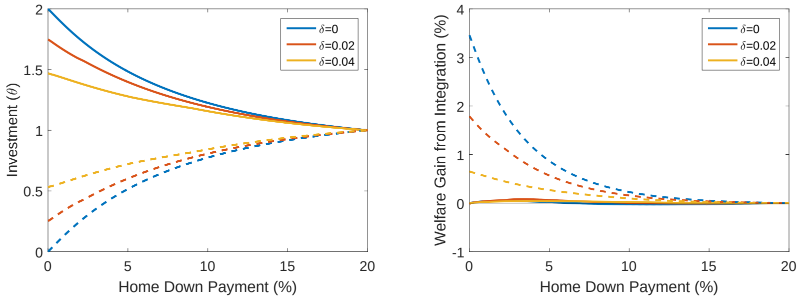
as financial frictions and intermediation costs from creating safe assets out of risky investments (*i.e.*, through pooling). Thus, the reader should not merely have in mind literally costs from resolving default. Accordingly, we loosely calibrate default costs in the following way. Note that δ is the intermediation cost of making all risky claims in the economy into safe assets (this is literally what happens when the collateral constraint is $c = 1$). With risk aversion of $\gamma = 2$, the maximum intermediation costs that agents in the model will tolerate is roughly 5%, at which point investors do not borrow but prefer autarky. (This strikes us as a reasonable number for tolerable intermediation costs.) We thus consider intermediation costs of about 2–4%. We solve the model without aggregate risk, supposing default costs are given by $\delta = 0, 2\%, 4\%$.[13]

In autarky, when default costs are sufficiently high and collateral constraints are sufficiently low, then in equilibrium investors do not borrow with maximum leverage (default costs increase borrowing rates R_b, discouraging borrowing). Because of default costs, the risk-free rate is lower in equilibrium (borrowing rates contain an additional credit spread), and as a result welfare is lower because of the deadweight loss from default costs.

Figure C.1 plots investments (essentially, capital flows) and welfare gains from intermediation when countries are symmetric. As is clearly evident, flows are smaller when intermediation costs are high (since borrowing decreases), and Foreign welfare gains are smaller as well. On the other hand, Home welfare is unaffected. Figure C.2 plots the same, but when the Foreign bloc is large ($W^F = 5$, the savings glut). Again, our results on flows and welfare gains are robust, except that now Home welfare gains are dampened when intermediation costs are large.

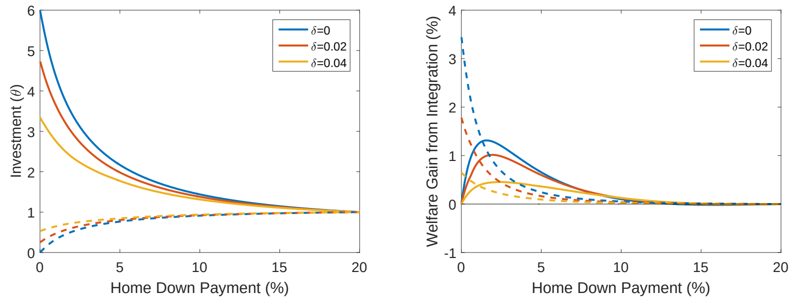
In summary, introducing default costs have two quantitative effects on equilibrium with financial integration. First, with default costs capital flows are quantitatively smaller but continue to flow from Foreign to Home. Capital

[13] Bris et al. (2006) report that the default costs are in the range of 0–10%. Our model can sustain higher level of default costs when risk aversion is higher, in which case the benefit from risk sharing outweighs the loss from default. For example, when risk aversion is 10, even default costs of 25% do not completely shut down borrowing, and investors continue to use maximum leverage for collateral levels above $c = 1.08$. Thus, calibrating default costs to match evidence from the data requires carefully specifying the risk aversion of agents, and a careful exercise would likely require heterogeneity in risk aversion.



(a) Capital flows. (b) Welfare gains.

Figure C.1: Default costs (δ). Solid: Home, dashed: Foreign.



(a) Capital flows. (b) Welfare gains.

Figure C.2: Country size ($W^F = 5$) and default costs (δ). Solid: Home, dashed: Foreign.

flows are smaller when default costs are higher for two reasons: (i) lower Home risk-free rates provide less incentive for Foreign investors to buy Home ABS, (ii) Home investors use less leverage. Interestingly, capital continues to flow from Foreign to Home even if in autarky Home investors would use *less* leverage than Foreign investors would. This is because Home investors get better risk sharing due to the lower collateral constraint, even if they use less leverage, and so they borrow from Foreign to finance investment.

Second, default costs do not qualitatively change the welfare gains from financial integration, but they have quantitative implications. When default is costly, the welfare gains for Foreign are smaller; however, the effect of default costs on Home investors is subtle. If default costs are not too large, welfare gains from financial integration can be slightly larger than when there are no default costs. In general, though, the welfare gains to Home are slightly muted, which is consistent with our baseline results that welfare gains for Home are modest and depend on parameters.

D Tranching

One of the most important recent financial innovations has been the "tranching" of assets or collateral. In tranched securitization, the dividend payments are divided into state-contingent payoffs, which better caters the needs of different investors.[14]

It is worth understanding why tranching would be meaningful in this situation, and why it would change results. The result of our paper so far has been that because collateral rates differ across countries, agents in each country have different exposures to risk, and international trade transpires precisely because agents face different levels of risk. Trade allows agents to hold portfolios that share risk more effectively than the autarkic portfolios. With ex ante identical agents, tranching has no effect on the autarky equilibrium because investors hold identical portfolios and thus have identical exposure to aggregate risks. However, when agents in each country are subject to different collateral rates, investors in different countries hold different portfolios and are thus subject to different aggregate risk exposure. Because collateral levels differ across countries, the payoffs to ABS are linearly independent. However, investors cannot sell short ABS, and thus the ABS need not span the aggregate states. Tranching, by design, allows agents to isolate aggregate risks, and thus tranching will lead to greater risk sharing.

D.1 Model

We model tranching as the ability of financial intermediaries to divide the payments from a pool of collateral into different bonds that pay in different states. To simplify, we consider perfectly correlated ("world shocks") so that there are only two aggregate states. To allow for the cleanest form of tranching, we consider splitting the Home pool of collateral into Arrow securities that pay in each state: the s-th tranche pays the value of the Home collateral in state s, and zero otherwise.[15] Furthermore, we suppose that only the Home financial sector has the ability to tranche assets. We call these tranches "Home-s", denoted by Hs. The payoff of this tranche in state s' per unit of capital investment is

$$D^{Hs}(s') = \begin{cases} R^H_{\text{ABS}}(s), & (s' = s) \\ 0. & (s' \neq s) \end{cases}$$

Let q_s be the price of one share of the Hs tranche. Since holding 1 share of all tranches is the same as holding the entire ABS, which has price 1, we have

$$\sum_{s=1}^{S} q_s = 1.$$

The budget constraint is modified as follows. Let $\phi_{Hs}, \phi_F \geq 0$ be the fraction of wealth invested in the ABS tranches of each country, and let $\psi \geq 0$ be the fraction borrowed. By accounting, the budget constraint of country j with tranching becomes

$$\theta^j + \sum_{s=1}^{S} \phi^j_{Hs} + \phi^j_F - \psi^j = 1.$$

[14] See Fostel and Geanakoplos (2012a) for details about tranching.

[15] While considering Arrow securities is an extreme case, the equilibrium asset holdings with two states are not so far from "balanced" so that we could instead consider two tranches, one safer and one slightly riskier, and still get similar quantitative results.

The collateral constraint remains the same. Since the return on tranche s is D^{Hs}/q_s, the return on portfolio $\pi = (\theta, (\phi_{Hs}), \phi_F, \psi)$ of agent i in country j is

$$R^i(\pi) = A^i\theta + \sum_{s=1}^{S} \frac{D^{Hs}}{q_s}\phi_{Hs} + R^F_{\text{ABS}}\phi_F - \min\left\{A^i c_j, R^j_b\right\}\psi.$$

Equilibrium is modified to include market clearing in tranches. Financial intermediaries create as many tranches as are backed by collateral, so

$$W^H\phi^H_{Hs} + W^F\phi^F_{Hs} = q_s W^H \psi^H$$

for all $s = 1, \dots, S$. The left-hand side is the world investment in tranche Hs. The right-hand side is the total value of tranche Hs, which equals price q_s times the number of shares $W^H\psi^H$. The market clearing condition for Foreign ABS is unchanged. As before, borrowing rates and tranche prices are determined such that agents optimize and asset markets clear.

One caveat with tranching is that the Foreign ABS is a redundant asset since there are as many tranches as aggregate states, which implies that equilibrium portfolios are indeterminate. To overcome this issue, we assume that investors are home-biased. Thus Foreign investors hold Foreign ABS as much as possible.

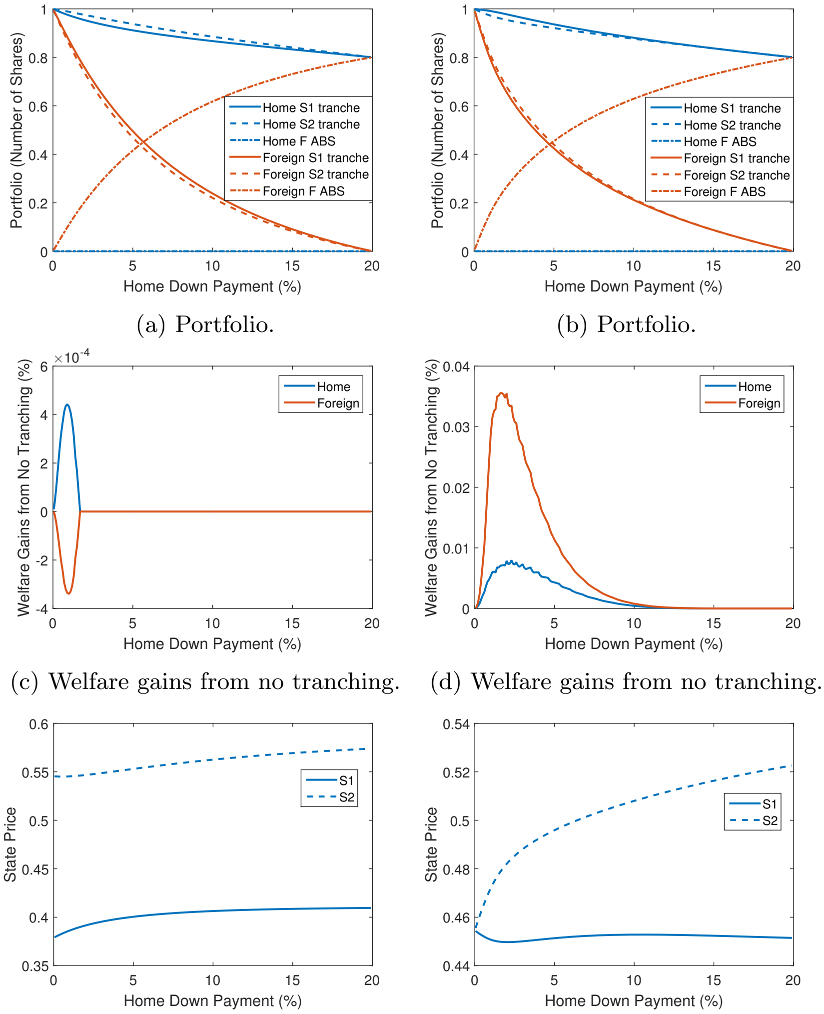
D.2 Numerical results with tranching

Figure D.1 shows the results with tranching. The figures in the left are for first-moment shocks, whereas those in the right are for second-moment shocks. Since the interest rates, investment, and welfare hardly change from the case without tranching, we omit the figures.

Tranching has a large impact on international capital flows. Without tranching, gross flows arise because countries try to insure against aggregate risk using the two ABS (Figures 5.1e and 5.3c). With tranching, since Home tranches are essentially Arrow securities, Foreign ABS becomes a redundant asset. Since Foreign borrows less after financial integration and therefore the market capitalization of Foreign ABS is small even compared to Foreign capital, Foreign can absorb all Foreign ABS. Thus capital flows only from Foreign to Home through the purchase of tranches (Figures D.1a and D.1b), just as in the case without aggregate risk (Theorem 3.5).

According to Figures D.1a and D.1b, each country holds roughly "balanced" portfolios of tranches. However, while with first-moment shocks Home holds more shares of the $H2$ tranche (corresponding to low investment returns), Home holds more shares of the $H1$ tranche (corresponding to low variance) with second-moment shocks. This result may appear surprising, because state $s = 1$ corresponds to either high return or low variance, which is the "good" state. Since Home is relatively insured against idiosyncratic risks, we would expect that Home will demand larger shares of the asset that pays in the "bad" aggregate state, which is $s = 2$. The reason why Home holds more shares of the $H1$ tranche with second-moment shocks is that the high variance state is indeed "good" for Home because the high idiosyncratic variance makes default more likely and enables more risk sharing.

Since there are two aggregate states and two ABS, tranching would change welfare only when the two ABS do not fully span the aggregate states, *i.e.*,



(a) Portfolio. (b) Portfolio.

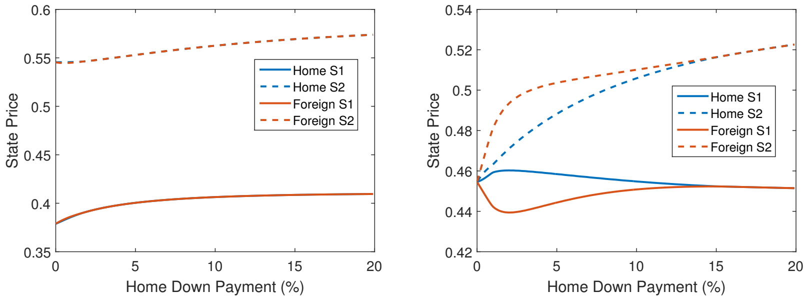
(c) Welfare gains from no tranching. (d) Welfare gains from no tranching.

(e) State prices. (f) State prices.

Figure D.1: Effect of tranching with first-moment shocks (left) and second-moment shocks (right).

when the no-shortselling constraint binds for at least one country. This is the case when Home down payments are less than 5% with first-moment shocks or less than 15% with second-moment shocks. In those cases, tranching affects welfare but only slightly: with first-moment shocks, Foreign loses after tranching (Figure D.1c); with second-moment shocks, both countries gain (Figure D.1d).

Since tranching completes the market with respect to aggregate states, the state prices become the same across countries. Figure D.2 plots state prices without tranching. With first-moment shocks (Figure D.1e), the state prices hardly change from the case without tranching because the prices were nearly identical across countries (Figure D.2a). With second-moment shocks, the state prices without tranching are quite different across countries (Figure D.2b), and the state prices after tranching lie in between (Figure D.1f).



(a) First-moment Shocks. (b) Second-moment shocks.

Figure D.2: State prices without tranching.

E Solution Algorithm

E.1 Model without aggregate risk

We solve for the equilibrium by reducing the equilibrium conditions to one equation in one unknown, as follows. Given the risk-free rate R_f, the borrowing rates in each country are determined by $R_f = \mathrm{E}\left[\min\left\{A^i c_j, R_b^j\right\}\right]$. These borrowing rates completely determine the portfolio return. Since $\theta^j + \phi^j - \psi^j = 1$ by the budget constraint and $\theta^j = c_j\psi^j$ by the maximum leverage property, we have $\phi^j = 1-(c_j-1)\psi^j$. Therefore we can express the portfolio return as a function of ψ^j alone, and numerically solve the portfolio problem. Thus all allocations are functions of R_f, and we can invoke the market clearing condition for capital to pin down the equilibrium risk-free rate R_f.

E.2 Model with aggregate risk but no tranching

It is numerically more efficient to start from allocations and then to solve for allocations and borrowing rates jointly. Given (ϕ_H^j, ϕ_F^j), we can compute θ^j, ψ^j by the budget constraint and the maximum leverage property. There are thus 6 unknowns, $(\phi_H^j, \phi_F^j, R_b^j)$ for $j = H, F$. We compute them by solving 6 nonlinear equations, the two first-order conditions (Home and Foreign ABS) for each country, and the 2 market clearing conditions for ABS.

Finding a solution requires a fairly accurate initial guess of the solution. When country shocks are perfectly correlated ($\rho = 1$), we use the solution of the case with no aggregate shocks as an initial guess. When $\rho < 1$, we start from $\rho = 1$ and iteratively solve for the case with slightly smaller ρ using the previous solution as an initial guess until we reach the desired ρ.

E.3 Model with tranching

Since Home tranches are essentially Arrow securities, Foreign ABS is a redundant asset. Therefore we solve for the equilibrium with Arrow securities first and then replicate them by Home tranches and Foreign ABS. Let n_s^H be the number of shares of state s Arrow security (which pays dividend 1 in state s and 0 otherwise) held by Home per unit of wealth, and let $n^H = \left\{n_s^H\right\}_{s=1}^S$.

There are $S+3$ unknown variables, $n^H, \psi^H, R_b^H, R_b^F$. Given ψ^H, we obtain $\theta^H = c_H \psi^H$ by maximum leverage. Letting $\pi^H = (\theta^H, n^H, \psi^H)$ be the Home portfolio, we obtain the Home portfolio return

$$R(\pi^H) = A\theta^H + \sum_{s=1}^{S} D_s n_s^H - \min\left\{Ac_H, R_b^H\right\}\psi^H,$$

where D_s is the dividend of state s Arrow security. By the first-order condition, the state s price is

$$\tilde{q}_s = \frac{\mathrm{E}[R(\pi^H)^{-\gamma} D_s]}{\mathrm{E}[R(\pi^H)^{1-\gamma}]}.$$

We obtain θ^F by capital market clearing, and also $\psi^F = \theta^F / c_F$ by maximum leverage. We can then compute the Foreign holdings of Arrow securities, n^F, using the market clearing condition

$$W^H n_s^H + W^F n_s^F = R_{\mathrm{ABS}}^H(s) W^H \psi^H + R_{\mathrm{ABS}}^F(s) W^F \psi^F,$$

where $s = 1, \ldots, S$. Since the Foreign portfolio is determined, we obtain the Foreign portfolio return $R(\pi^F)$. Finally, we compute the $S+3$ unknown variables $n^H, \psi^H, R_b^H, R_b^F$ by solving $S+3$ nonlinear equations. The first S equations are the Foreign first-order conditions

$$\tilde{q}_s = \frac{\mathrm{E}[R(\pi^F)^{-\gamma} D_s]}{\mathrm{E}[R(\pi^F)^{1-\gamma}]},$$

where $s = 1, \ldots, S$. The other three are Foreign budget constraint

$$\theta^F + \sum_{s=1}^{S} \tilde{q}_s n_s^F - \psi^F = 1$$

and the no-arbitrage pricing of Home and Foreign ABS,

$$\sum_{s=1}^{S} \tilde{q}_s R_{\mathrm{ABS}}^j(s) = 1,$$

where $j = H, F$.