Supplementary Appendix to

Composition of Wealth, Conditioning Information, and the Cross-Section of Stock Returns

Not for publication

Appendix E. Comparison with parametric approaches

Could the conclusions reached above be obtained using more standard econometric approaches? Assume that the conditional means of consumption growth and excess returns, as well as their conditional covariance - $E_t\left(\frac{\Delta C_{t+1}}{C_t}\right)$, $E_t R^{ei}_{t+1}$, and $Cov_t(R^{ei}_{t+1}, \frac{\Delta C_{t+1}}{C_t})$ - are all linear in the vector of conditioning variables z_t (which includes the constant). Then we can estimate (e.g. as in Duffee (2005)) the following system:

$$\frac{\Delta C_{t+1}}{C_t} = \kappa' z_t + u^c_{t+1}, \tag{E.1}$$

$$R^{ei}_{t+1} = \mu_i' z_t + u^i_{t+1}, \tag{E.2}$$

$$\widetilde{Cov}^{\,i}_{t+1} = \delta_i' z_t + u^{ci}_{t+1} \tag{E.3}$$

where $\widetilde{Cov}^{\,i}_{t+1} = \left(\frac{\Delta C_{t+1}}{C_t} - E_t\frac{\Delta C_{t+1}}{C_t}\right)\left(R^{ei}_{t+1} - E_t R^{ei}_{t+1}\right) = u^c_{t+1}u^i_{t+1}$ is the 'ex-post' covariance of consumption growth and excess returns on asset i, so that the ex ante conditional covariance is given by its projection on the vector of conditioning variables:

$$Cov_t(R^{ei}_{t+1}, \frac{\Delta C_{t+1}}{C_t}) = E_t\widetilde{Cov}^{\,i}_{t+1} = \delta_i' z_t. \tag{E.4}$$

Table G.8 shows the coefficients from the regressions of returns and the ex-post consumption covariances on z_t for several choices of the conditioning variable. The assets used are three portfolios formed from the 6 benchmark portfolios sorted on market capitalization on book/market equity ratios used by Fama and French (1992). The growth portfolio is the equal-weighted average of the small and large growth portfolios, the value and neutral portfolios are, similarly, equal-weighted averages across value and neutral portfolios, respectively.

If high values of z_t are associated with "bad times" and, consequently, a high price of consumption risk, the assets whose covariances with consumption growth are increasing in

z_t are riskier. If the CCAPM holds, their expected excess returns should also increase in z_t. Duffee (2005) finds that an increase in the ratio of stock market wealth to consumption is associated with a rise in the covariance of the aggregate stock market return and consumption growth. However, it is also associated with *low* expected stock returns. The top panel illustrates that the same is true for each of the book/market-sorted portfolios. In fact, their does not appear to be much difference in the sensitivities of either conditional expected returns or conditional covariances to this variable, despite the fact that it appears to be a useful scaling variable as shown in section 4.

The two middle panels of table G.8 display the sensitivities of first and second moments of returns to *cay*. It does appear that *cay* plays a similar role at quarterly frequency to the role played by *ac* at monthly frequency: rising *cay* not only predicts higher expected returns, but also lower covariances of consumption with returns, presumably due to the declining share of financial assets in total wealth. The expected return sensitivities exhibit the pattern familiar from section 4.1: value returns are not quite as predictable as growth returns (in terms of the slope coefficient). There is virtually no difference in covariances if the entire sample is used for the estimation. However, using a shorter subsample ending in the second quarter of 2003, which is closer to the sample used by Lettau and Ludvigson (2001b), I find that the covariance of value returns with consumption growth actually increases when *cay* goes up, while growth returns' covariance declines. This is consistent with the argument of Lettau and Ludvigson (2001b) that value is riskier in "bad times," but inconsistent with the fact that value's expected returns are not more but less sensitive than growth's expected returns. Further, the coefficients for the conditional covariances are not statistically significantly different from zero, as their standard errors are very large. This might be in part due to the fact that the linear model is misspecified. Finally, using the labor-to-consumption ratio as the predictive variable (bottom panel) leads to similar conclusions: covariances and expected returns appear to move in the opposite directions for all portfolios, and while there is some heterogeneity across covariance sensitivities, there is much less difference in expected return sensitivities.

In principle, one could go further and impose conditional moment restrictions on the asset returns jointly. This entails making parametric assumptions on the functional form of risk prices. For example, one could follow Duffee (2005) and assume that $\gamma_t = \gamma_0 + \gamma_1 x_t$. Then the model could be estimated using the instrumental variables GMM approach of Campbell (1987) and Harvey (1989). However, such a model would be misspecified by construction, since expected returns, covariances, and prices of risk cannot be all linear. Thus even if the true conditional model holds, it could produce non-trivial pricing errors. Brandt and Chapman (2007) emphasize that the nonlinearity need not be large to produce a spurious rejection. Alternatively, one could avoid imposing parametric structure on the prices of risk and only make assumptions about the dynamics of conditional second moments, as done, for example, by Ferson and Harvey (1999), among others. I discuss this approach in Appendix E and show that, indeed, one can reject the conditional CCAPM using *cay*. Still, the conditional restrictions imposed using this method rely crucially on the linear specification of conditional betas. Therefore, if the linear model for conditional betas is misspecified, it is possible that the conditional tests will reject even the true conditional model. Ghysels (1998) argues that this problem is potentially quite severe, to the extent that the conditional beta models can perform even worse empirically than the unconditional models. Given the substantial difference in the estimated sensitivities of consumption covariances to the conditioning variable between the samples the concern over misspecification should make it hard to argue in favor of using the parametric approaches for imposing conditional moment restrictions.

Appendix F. Testing conditional factor models using beta representation

Consider the setup of Lettau and Ludvigson (2001b), who specify a conditional consumption CAPM with a single conditioning variable, *cay* - the cointegrating residual of consumption, financial wealth and labor income, so that $\tilde{\mathbf{f}}_{\mathbf{t+1}} = \left[\frac{\Delta C_{t+1}}{C_t}, \frac{\Delta C_{t+1}}{C_t} \times cay_t\right]$ in (8) above. Their tests concentrate on the beta representation

$$E(R^{ei}_{t+1}) = \eta_0 + \eta_1 \beta^i_{cay_t} + \lambda_0 \beta^i_{\Delta C_{t+1}} + \lambda_1 \beta^i_{\Delta C_{t+1} \times cay_t}, \qquad \text{(F.1)}$$

which is equivalent to (8) except that they allow a non-zero (and time-varying) cross-sectional intercept ($\eta_0 + \eta_1 cay_t$), which implies that the conditional zero-beta rate is not necessarily equal to the risk-free interest rate. The estimate and test this specification using the standard cross-sectional regression methodology of Fama and MacBeth (1973), first estimating the betas (loadings) of returns on the scaled factors $\left[cay_t, \frac{\Delta C_{t+1}}{C_t}, \frac{\Delta C_{t+1}}{C_t} \times cay_t\right]$ by time-series regression and then regressing the cross-section of returns on the cross-section of betas to obtain the risk premium estimates λ (and η).

An alternative approach would be to test the conditional implications of the consumption CAPM using *cay* as the conditioning variable. The conditional beta representation is given[16] by

$$E_t(R^{ei}_{t+1}) = \eta_t + \lambda_t \beta^i_t, \tag{F.4}$$

where η_t, λ_t, and β^i_t are all functions of *cay*. Conditioning down obtains

$$E(R^{ei}_{t+1}) = E\left(\eta_t + \lambda_t \beta^i_t\right). \tag{F.5}$$

Assuming, as Lettau and Ludvigson (2001b) do, that conditional betas (and risk premia) are linear, i.e. $\beta^i_t = \beta^i_0 + \beta^i_1 cay_t$, these pricing implications can also be tested using the Fama-Macbeth methodology (e.g. Ferson and Harvey (1999)). Specifically, the parameters

[16]Lettau and Ludvigson (2001b) start with the stochastic discount factor model $E_t[M_{t+1}R^i_{t+1}] = 1$, where $M_{t+1} = a_t + b_t \frac{\Delta C_{t+1}}{C_t}$. Taking the unconditional expectation and assuming the SDF coefficients are linear functions of the conditioning variable yields

$$E[(a_0 + a_1 cay_t + (b_0 + b_1 cay_t)\frac{\Delta C_{t+1}}{C_t})R^i_{t+1}] = 1 \tag{F.2}$$

and standard manipulations produce the expected return-beta representation (F.1). Alternatively, working with the conditional expectation directly, the conditional expected returns are given by

$$E_t(R^i_{t+1}) = \frac{1}{a_t} - \frac{b_t}{a_t} E_t[\frac{\Delta C_{t+1}}{C_t} R^i_{t+1}], \tag{F.3}$$

which leads to the beta representation for excess returns (F.4).

β_0^i and β_1^i can be estimated as factor loadings in the time series regressions

$$R_{t+1}^{ei} = \alpha_0 + \alpha_1 cay_t + \beta_0^i \frac{\Delta C_{t+1}}{C_t} + \beta_1^i \frac{\Delta C_{t+1}}{C_t} cay_t \tag{F.6}$$

Then the fitted conditional betas $\hat{\beta}_t^i = \hat{\beta}_0^i + \hat{\beta}_1^i cay_t$ can be used in the cross-sectional regressions (at each date t) to estimate η_t and λ_t. The latter can be used to obtain either the unconditional averages of the risk premium and the zero-beta rate, or can be projected on the conditioning information set. Average of the conditional pricing errors for each asset are then given straightforwardly as

$$u^i = E(R_{t+1}^{ei}) - E\left(\hat{\eta}_t + \hat{\lambda}_t \hat{\beta}_t^i\right). \tag{F.7}$$

Both of these are valid approaches to testing a conditional factor model. However, the latter approach has more power, since it imposes additional restrictions on the dynamics of conditional betas and expected returns. A simple way to illustrate the dramatic differences between the two approaches is to compare the average pricing errors. Figure G.9 plots the average returns on the 25 portfolios formed on size and book-to-market (see Appendix for data description) against the average returns predicted by four empirical models: the unconditional consumption CAPM, the unconditional scaled-factor specification of conditional CCAPM in (F.1), the three-factor model of Fama and French (1993), and the conditional specification of conditional CCAPM in (F.4). The unconditional consumption CAPM (top left panel) is well-known to have virtually no explanatory power for the average returns of the Fama-French portfolios. In contrast, the scaled CCAPM of Lettau and Ludvigson (2001b) does a relatively good job at lining up the predicted mean returns against the actual ones (top right panel), reducing the square root of the average (squared) pricing errors (alphas) by a third compared to the unconditional CCAPM (from 0.6% to 0.4% for quarterly returns). This performance is comparable to the well-known ability of the Fama-French portfolio-based model to explain the cross-section of value and size-sorted portfolios (bottom left panel). However, imposing the conditional restrictions (F.4) eliminates virtually all of the advantage of the conditional model over the unconditional one. The conditional model generates very little dispersion in the predicted average returns (bottom right panel), thus failing to explain any of the variation in the observed mean portfolio returns.

Appendix G. Consumption of stockholders

The fact that not all households participate in the equity market suggests an alternative interpretation of the composition effect, i.e. the tendency of the conditional covariances of stock returns with aggregate consumption growth to decline as a the contribution of financial wealth to consumption decreases. Since equity, which represents a large fraction of total financial wealth, is concentrated in the hands of stockholders, their consumption is likely to be disproportionately effected by stock market fluctuations, relative to the consumption of non-stockholders. Thus, a decrease in the value of equity would reduce the stockholders' relative share of aggregate consumption, and therefore reduce the sensitivity of aggregate consumption to the fluctuations in stock market wealth. Indeed, consistent with this interpretation, Malloy, Moskowitz, and Vissing-Jørgensen (2005) use household-level data from the Consumer Expenditure Survey (CEX) to show that the consumption-wealth residual *cay* is highly negatively correlated with the time-varying share of stockholders' consumption in the aggregate consumption.

The direct implication of this interpretation of the composition effect is that the canonical asset pricing relation 2 is misspecified as long as the measure of aggregate consumption includes all households rather than just those that are marginal in the asset market of interests (i.e., stockholders in the case where stock returns are the test assets). In order to verify whether my conclusions are robust to this type of misspecification I use the data from Malloy, Moskowitz, and Vissing-Jørgensen (2005) to test the conditional CCAPM. Their measure of quarterly stockholder consumption growth is available at a monthly frequency (i.e., for overlapping quarterly growth rates), but for a shorter time period (03.1983 - 11.2004) than the aggregate data used elsewhere in the paper. As a benchmark comparison, I also use the monthly series of quarterly aggregate consumption growth based on the NIPA data constructed by Malloy, Moskowitz, and Vissing-Jørgensen (2005) for the same time period.

I construct the monthly analog of the *cay* variable as a cointegrating residual of monthly series for aggregate consumption, stock market wealth, and labor income; the resulting series has very similar properties to the *cay* variable of Lettau and Ludvigson (2001b).

As before, I estimate conditional expected returns and conditional covariances of returns with consumption growth jointly, by selecting kernel bandwidth so as to minimize the conditional pricing errors for the cross-section of portfolio returns. The evidence in table G.9 shows that if differences between "good" and "bad" states in conditional covariances of returns and consumption growth are measured the same way as above, the composition effect is statistically detectable for stockholder consumption, at least for the large growth portfolio, while the differences are not statistically significant for the NIPA aggregate consumption growth measure over the same sample period (however, in both cases statistical significance is somewhat sensitive to the choice of "high" and "low" states. Moreover, the magnitudes of differences in covariances between high and low states are greater for stockholder consumption than for aggregate consumption, which is likely due to the fact that levels of covariances are proportionally higher for latter than for the former. For the Value minus Growth portfolio returns, in both cases the difference is positive and statistically significant for the small portfolios, consistent with the conditional CCAPM of the value effect, but not for the large portfolios. As before, however, the differences in expected returns on these portfolios are negative, albeit not statistically significantly.

In terms of the average pricing errors, the consumption CCAPM, both unconditional and conditional, that uses stockholder consumption does appear to perform somewhat better than the model with aggregate consumption estimated over the same sample period. Table G.10 displays the average pricing errors for the two sets of models, using either *cay* or the stock market wealth-consumption ratio *ac*. While all of the versions of the CCAPM that uses NIPA aggregate consumption growth have large and highly statistically significant pricing errors on the Small Value minus Small Growth and Small Growth minus Large Growth portfolios, for the stockholder consumption CAPM these pricing errors are smaller (although still substantial) and not statistically different from zero, with the exception of the conditional CCAPM using *ac* where it is significant. However, for the stockholder consumption CAPM the Small Value minus Large Value portfolio has a large (2 % per quarter) and statistically significant pricing error, either unconditionally or when *cay* is used as the conditioning variable. Moreover, the lack of statistical significance might be in part attributed to the short sample, which makes estimated pricing errors highly imprecise, especially in the nonparametric setting. Overall, there is evidence that using stockholder consumption to measure risk in asset returns improves the performance of a canonical consumption-based asset pricing model, but does not fully explain the cross section of equity returns. This conclusion is consistent with the evidence documented above that high average return portfolios (e.g. small value) do not seem to have higher conditional expected returns than low average return portfolios at times their risk measured by conditional covariance with consumption growth is higher.

Table G.8: **Sensitivity of conditional moments to conditioning variables**
Regression slope coefficients of portfolio excess returns and their ex-post covariances with consumption growth on the lagged conditioning variable. Standard errors are given in the parentheses.

ac **- monthly data**

	$E(R^i)$	R^2	Cov^i	R^2
Growth	-0.77	0.01	0.51	0.00
	(0.45)		(2.32)	
Neutral	-0.57	0.01	0.68	0.00
	(0.34)		(1.47)	
Value	-0.64	0.01	0.64	0.00
	(0.34)		(1.44)	

cay **- quarterly data**

	$E(R^i)$	R^2	Cov^i	R^2
Growth	1.35	0.03	-4.47	0.01
	(0.42)		(3.74)	
Neutral	1.11	0.03	-4.29	0.02
	(0.35)		(2.99)	
Value	1.03	0.03	-4.60	0.02
	(0.38)		(3.31)	

cay **- quarterly data up to 2003**

	$E(R^i)$	R^2	Cov^i	R^2
Growth	2.35	0.07	-1.29	0.00
	(0.57)		(9.54)	
Neutral	1.87	0.07	1.22	0.00
	(0.47)		(8.12)	
Value	1.79	0.05	2.46	0.00
	(0.50)		(8.32)	

yc **- quarterly data**

	$E(R^i)$	R^2	Cov^i	R^2
Growth	-0.25	0.01	0.11	0.00
	(0.19)		(19.97)	
Neutral	-0.15	0.00	0.44	0.00
	(0.17)		(17.61)	
Value	-0.18	0.00	0.70	0.00
	(0.21)		(20.72)	

Table G.9: **Differences in conditional moments of portfolio returns - stockholders**

Bootstrap tests of differences in conditional covariances of returns on the benchmark portfolios with stockholder consumption growth and differences in conditional mean excess returns, estimated jointly using $z = cay$ as the conditioning variable, where $z^L = -0.0174$ and $z^H = 0.02$ correspond to the 10th and 90th percentiles of the distribution of cay (in the entire sample IV.1952 - IV.2008), respectively. The test statistics are differences in point estimates of conditional moments evaluated at these two states for each test portfolio. The p-values for the one-sided tests reported in the parentheses are computed using the bootstrap distributions of the corresponding test statistics centered at zero. Conditional means and covariances are estimated jointly using monthly observations of quarterly consumption growth measures based on, alternatively, the NIPA aggregate data, or the stockholder consumption data from the CEX, both for the period 03.1983 - 11.2004 (see Malloy, Moskowitz, and Vissing-Jørgensen (2005) for detailed description).

Panel A: NIPA

	$E(R\|z^H) - E(R\|z^L)$	$100 \times (cov(R, \Delta c\|z^H) - cov(R, \Delta c\|z^L))$
Small Growth	1.75	-1.82
	(0.25)	(0.06)
Small Value	0.76	-0.12
	(0.37)	(0.45)
Large Growth	2.64	-1.13
	(0.06)	(0.09)
Large Value	1.14	-0.35
	(0.25)	(0.30)
Small Value minus Growth	-0.99	1.69
	(0.33)	(0.04)
Large Value minus Growth	-1.50	0.79
	(0.17)	(0.08)

Panel B: CEX stockholders

	$E(R\|z^H) - E(R\|z^L)$	$100 \times (cov(R, \Delta c\|z^H) - cov(R, \Delta c\|z^L))$
Small Growth	2.14	-9.73
	(0.16)	(0.06)
Small Value	0.33	-3.92
	(0.41)	(0.21)
Large Growth	2.83	-7.93
	(0.03)	(0.05)
Large Value	0.88	-5.35
	(0.25)	(0.07)
Small Value minus Growth	-1.81	5.82
	(0.16)	(0.05)
Large Value minus Growth	-1.95	2.58
	(0.07)	(0.16)

Table G.10: **Average pricing errors: stockholder consumption**

CCAPM estimated using monthly observations of quarterly consumption growth measures based on, alternatively, the NIPA aggregate data, or the stockholder consumption data from the CEX, both for the period 03.1983 - 11.2004 (see Malloy, Moskowitz, and Vissing-Jørgensen (2005) for detailed description).
P-values for the test that individual pricing errors are equal to zero given in the parentheses are computed using (semi)parametric stationary bootstrap with 10000 replications.

Model	SV-SG	SG-LG	SV-LV	LV-LG
unconditional CCAPM (NIPA)	3.43	-3.16	-0.13	0.40
	(0.00)	(0.00)	(0.35)	(0.31)
unconditional CCAPM (stockholders)	1.84	1.11	2.26	0.69
	(0.10)	(0.16)	(0.01)	(0.22)
CCAPM (NIPA) scaled with *cay*	3.08	-3.20	-0.30	0.18
	(0.00)	(0.00)	(0.17)	(0.52)
CCAPM (stockholders) scaled with *cay*	1.23	-1.15	1.41	-1.33
	(0.15)	(0.05)	(0.06)	(0.00)
CCAPM (NIPA) scaled with *ac*	-0.33	-1.53	-0.83	-1.03
	(0.03)	(0.27)	(0.10)	(0.09)
CCAPM (stockholders) scaled with *ac*	0.62	-0.39	-0.27	0.51
	(0.62)	(0.45)	(0.07)	(0.23)
conditional CCAPM (NIPA) with *cay*	3.45	-3.15	-0.12	0.42
	(0.00)	(0.00)	(0.44)	(0.36)
conditional CCAPM (stockholders) with *cay*	1.88	1.01	2.15	0.74
	(0.13)	(0.24)	(0.05)	(0.28)
conditional CCAPM (NIPA) with *ac*	3.29	-2.96	-0.10	0.43
	(0.00)	(0.00)	(0.46)	(0.21)
conditional CCAPM (stockholders) with *ac*	2.19	0.41	1.74	0.86
	(0.05)	(0.60)	(0.21)	(0.14)
average returns	2.28	-0.79	1.16	0.34

Figure G.9: **Fama-MacBeth regressions**

Each panel plots the average excess returns on the 25 portfolios sorted on size (S, 1 = low, 5 = high) and book-to-market (B, 1 = low, 5 = high)), against the average returns predicted by one of the four models:
unconditional consumption CAPM, $E(R^{ei}_{t+1}) = \eta + \lambda \beta^i_{\Delta C_{t+1}}$;
Fama-French three-factor model, $E(R^{ei}_{t+1}) = \eta + \lambda_M \beta^i_{RMRF} + \lambda_S \beta^i_{SMB} + \lambda_H \beta^i_{HML}$;
unconditional version of the conditional consumption CAPM scaled with *cay*,

$$E(R^{ei}_{t+1}) = \eta_0 + \eta_1 \, cay_t + \lambda_0 \beta^i_{\Delta C_{t+1}} + \lambda_1 \beta^i_{\Delta C_{t+1} \times cay_t}; \tag{G.1}$$

conditional consumption CAPM using *cay* as the conditioning variable:

$$E(R^{ei}_{t+1}) = E\left(\eta_t + \lambda_t \beta^i_t\right), \text{ where } \beta^i_t = b^i_0 + b^i_1 \, cay_t. \tag{G.2}$$