Online Appendix to "Liquidity standards and the value of an informed lender of last resort"[1]

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Abstract

We consider a dynamic model in which receiving support from the lender of last resort (LLR) may help banks to weather investor runs. We show the need for regulatory liquidity standards when the underlying social trade-offs make the uninformed LLR inclined to support troubled banks during a run. Liquidity standards increase the time available before the LLR must decide on supporting the bank. This facilitates the arrival of information on the bank's financial condition and improves the efficiency of the decision taken by the LLR, a role that can be modified but not replaced with the use of capital regulation.

Keywords: Liquidity standards, lender of last resort, bank runs

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A The late run equilibrium

We denote a *late run equilibrium* (or LR equilibrium) the subgame perfect Nash equilibrium that begins after the shock arrives at $t = 0$ in which debtholders only start exercising their puts if further news confirm that the bank's illiquid assets are bad. In this equilibrium, the arrival of good news allows the bank to end the crisis with its liquidity untouched.

In the LR equilibrium the situation of the bank only changes when the news arrive. So, if it is not a profitable deviation for an individual debtholder to exercise her put at $t = 0$, then it will not be a profitable deviation either at any other point before news arrive. But news, on the other hand, arrive in finite time with probability one, revealing the bank to be good with probability μ and bad with probability $1 - \mu$. So debtholders' value of not exercising their put in the late run equilibrium can be written as

$$V_0^{LR}(C) = \mu B + (1 - \mu) [C + q_b(1 - C)], \tag{38}$$

reflecting that debtholders are eventually paid B if the bank is good (recall that there is no discounting) and receive an expected payoff $C + q_b(1 - C)$ if the bank is bad (recall Eq. (10)).

Sustaining an equilibrium with late runs requires having $V_0^{LR}(C) \geq D$, so the following proposition can be proven by direct inspection of the relevant expressions.

Proposition 8 *A LR equilibrium exists if and only if $V_0^{LR}(C) \geq D$. That condition holds when the bank is sufficiently likely to be good. When $V_0^{LR}(0) \geq D$, the LR equilibrium exists even with $C = 0$. When $V_0^{LR}(0) < D \leq V_0^{LR}(\bar{C})$, there is a minimum liquidity standard*

$$\hat{C} = \frac{D - \mu B - (1 - \mu)q_b}{(1 - \mu)(1 - q_b)} \in (0, \bar{C}] \tag{39}$$

such that the LR equilibrium exists if and only if $C \in [\hat{C}, \bar{C}]$.

Proof Evident from the arguments provided above.■

The case with $\hat{C} \in (0, \bar{C}]$ shows that when banks hold (moderate amounts of) liquidity this facilitates the sustainability of the late run equilibrium. It does so by enhancing the value of the bank when its illiquid assets are bad, which in turn increases debtholders' payoffs from waiting for news. In other words, cash reassures debtholders about the value of their stake at the bank and makes them willing to delay the exercise of their option to run.

From an allocational perspective, making the debtholders effectively more patient during a crisis contributes to "buying" the time needed for the arrival of news that, eventually, facilitate an efficient resolution of the crisis, in that the bank with good assets continues and the bank with bad assets is liquidated.[37]

What is the connection between liquidity and LLR support in the LR equilibrium? On the one hand, by facilitating the sustainability of the LR equilibrium, liquidity may contribute to actually make LLR support unneeded on the equilibrium path. On the other, the LLR's willingness to support the bank when its assets are known to be good rules out the possibility of self-fulfilling prophecies that might precipitate the start of a run at date $t = 0$ and lead it not to stop even after news indicating that assets are good.

A.1 Welfare and firm value in the late run equilibrium

Let us first consider the case in which $D \leq V_0^{LR}(\bar{C})$, which means that the LR equilibrium can be sustained by choosing a suitable value of C. And suppose that C is set at a value that indeed sustains the LR equilibrium. How large is the welfare generated by the bank in these circumstances?

We measure the ex ante welfare associated with this equilibrium, $W_{-1}^{LR}(C)$, as the expected value of the overall payoffs generated by the bank from $t = -1$ onwards, that is, the returns produced by its initial assets across possible states. Using the fact that, in the LR equilibrium, good illiquid assets continue up to termination, while bad assets are liquidated early, we obtain

$$W_{-1}^{LR}(C) = C + \{[(1-\varepsilon) + \varepsilon\mu]a_g + \varepsilon(1-\mu)q_b\}(1-C) \tag{40}$$

or, in terms of the notation introduced in Eq. (13),

$$W_{-1}^{LR}(C) = C + A_H(1-C), \tag{41}$$

where $A_H - 1$ was referred to as the fundamental net present value potentially associated with the bank's investment in illiquid assets. Importantly, $W_{-1}^{LR}(C)$ is linear in C, and strictly decreasing in C if an only if $A_H - 1 > 0$. Therefore:

[37] Strictly speaking, the bank with bad assets continues up to the exhaustion of its cash. Alternatively, we could assume that a resolution authority forces the bank into liquidation as soon as it learns the bank is bad. Given the absence of discounting, none of our equations and results would change in such an alternative scenario.

Proposition 9 *If the illiquid assets have strictly positive fundamental net present value at* $t = -1$*, and the LR equilibrium is (rightly) anticipated to prevail, it is not socially optimal to set* C *strictly larger than* $\max\{\hat{C}, 0\}$*, where* $\hat{C}$ *is given by Eq. (39).*

Proof Proposition 8 implies that sustaining the LR equilibrium requires a minimal C of either 0, if $V_0^{LR}(0) \geq D$, or some $\hat{C} \in (0, \bar{C}]$, if $V_0^{LR}(0) < D \leq V_0^{LR}(\bar{C})$. Assume that the LR equilibrium is (rightly) anticipated to prevail whenever $C \geq \max\{\hat{C}, 0\}$, where $\hat{C}$ is given by Eq. (39). However, under $A_H - 1 > 0$, $W_{-1}^{LR}(C)$ is decreasing in C. So setting C strictly larger than $\max\{\hat{C}, 0\}$ would be detrimental to welfare.■

In the LR equilibrium, the LLR never supports the bank, so the full value reflected in $W_{-1}^{LR}(C)$ also constitutes the ex ante *total market value* of the bank in this equilibrium, $TMV_{-1}^{LR}(C)$. This is the object that bank owners aim to maximize when choosing C and selling the bank's debt and equity to investors. Therefore:

Proposition 10 *If the illiquid assets have positive fundamental net present value at* $t = -1$ *and the LR equilibrium is (rightly) anticipated to prevail, it is not privately optimal for bank owners to set* C *strictly larger than* $\max\{\hat{C}, 0\}$*, where* $\hat{C}$ *is given by Eq. (39).*

Proof Given the absence of subsidies associated with LLR support, we have $TMV_{-1}^{LR}(C) = W_{-1}^{LR}(C)$ and the result follows trivially from the arguments provided in the proof of Proposition 9.■

So, conditional on inducing a LR equilibrium, there appears to be no discrepancy between the private and the social incentives for the choice of C and, hence, no clear rationale for regulatory liquidity standards. Bank owners and the social planner agree that setting $C = \hat{C}$ is the most efficient way to guarantee the existence of the LR equilibrium. However, there might be situations where, even if a LR equilibrium can be sustained with $C = \hat{C} > 0$, bank owners find it privately optimal to set $C < \hat{C}$ and induce the emergence of a different equilibrium (e.g. the ER equilibrium) where the total market value of the bank is larger than $TMV_{-1}^{LR}(\hat{C})$. To discuss this in greater detail, one would need to analyze systematically the possible coexistence of ER and LR equilibria in our model, as we do in the next subsection.

A.2 Early run vs. late run equilibria

To analyze the possible coexistence of the ER and LR equilibria, it is useful to start comparing $V_t^{ER}(C)$ with $V_0^{LR}(C)$. To this effect, it is convenient to rewrite Eq. (11) as

$$
\begin{aligned}
V_t^{ER}(C) &= \mu\{D + [1 - \exp(-(\delta+\lambda)(\tau - t))]\frac{\lambda}{\delta+\lambda}(B - D)\} \\
&\quad +(1-\mu)\{D - \exp(\delta t)[D - C - q_b(1-C)]\} \\
&\quad -\mu \exp(-\lambda(\tau - t))\exp(\delta t)[D - C - q_g(1-C)] \\
&\quad +\xi \exp(-\lambda(\tau - t))\exp(\delta t)[D - C - \bar{q}(1-C)]. \qquad (42)
\end{aligned}
$$

1. The first term can be compared to the first term in Eq. (38): it is smaller. The term multiplied by μ is smaller than B because all of the factors that multiply the term $B - D > 0$ within the curly brackets are smaller than one.

2. The second term can be compared to the second term in Eq. (38): it is weakly smaller. Specifically, it is identical for $t = 0$ and decreasing in t, so it is strictly smaller for $t \in (0, \tau]$.

3. The third term is negative, while there are no further terms in Eq. (38).

4. The fourth term is zero in the weak bank case ($\xi = 0$) and positive (and equal to the expected subsidy associated with LLR support) in the strong bank case ($\xi = 1$).

Therefore:

1. In the weak bank case ($\xi = 0$), we necessarily have $V_t^{ER}(C) < V_0^{LR}(C)$ for all $t \in [0, \tau]$, and hence $V_t^{ER}(C) < D$ for all $t \in [0, \tau]$ whenever $V_0^{LR}(C) < D$. Hence either the ER equilibrium or the LR equilibrium always exist. In fact, in situations with $V_0^{ER}(C) \leq D \leq V_0^{LR}(C)$, the LR and the ER equilibria coexist, due to the self-fulfilling potential of the prophecies (on likelihood that the bank ends up liquidated) attached to the ER equilibrium.

2. In the strong bank case ($\xi = 1$), the fourth term in Eq. (42) is a source of ambiguity for the comparison between $V_t^{ER}(C)$ and $V_0^{LR}(C)$. In fact, in this case, the third and fourth terms in Eq. (42) can be consolidated into a net positive term:

$$+(1-\mu)\exp(-\lambda(\tau - t))\exp(\delta t)[D - C - q_b(1-C)], \qquad (43)$$

whose comparison with the positive gap between $V_0^{LR}(C)$ and the first two terms of $V_t^{ER}(C)$ is generally ambiguous. In this case, analytical conditions guaranteeing $V_t^{ER}(C) \leq D$ for all $t \in [0, \tau]$ whenever $V_0^{LR}(C) < D$ are convoluted. Yet, numerical examples show that there are parameter values under for this property is preserved, as well as cases in which it is not.

A.3 Taxonomy of equilibria in the strong bank case

To further understand the taxonomy of situations that we may find in the strong bank case, Fig. A1 depicts the values of D, $V_0^{LR}(C)$, and $V_t^{ER}(C)$ for all $t \in [0, \tau]$ for a number of examples. The time passed since the possible start of the early run, t, appears on the horizontal axes, while the values of D, $V_0^{LR}(C)$, and $V_t^{ER}(C)$ appear on the horizontal ones. The examples rely on a variation of the strong bank baseline example described in Table 1 of the paper. Specifically, the following parameters are kept fixed throughout the examples that appear in the various panels of Fig. A1:

Table A1
Parameter values behind Fig. A1

ε	a_g	a_b	q_g	q_b	λ	δ	b	D
0.2	1.2	0.5	0.85	0.75	2.5	0.167	0.0476	1.05

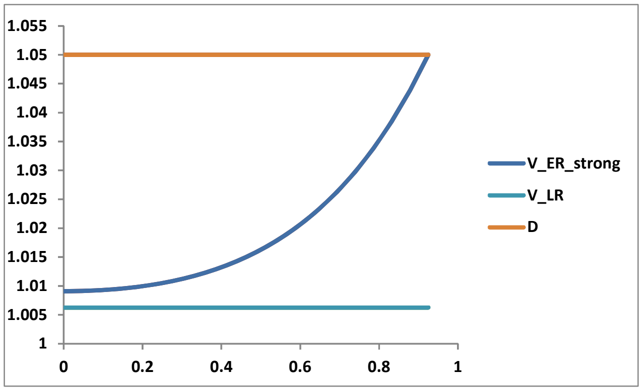
Under all combinations of parameters explored in Fig. A1, the bank is strong, i.e. the expected value of its illiquid assets is higher, unconditionally, if continued than if early liquidated, so the previously described complexity regarding the potential taxonomy of equilibria arises. We generate the various panels of Fig. A1 by varying the probability of the illiquid assets being good, μ, by rows, and the bank's cash holdings, C, by columns, as indicated under each panel.

Panel 1 describes a case (with $\mu = 0.7$ and $C = 0.15$) in which the ER equilibrium is sustainable while the LR equilibrium is not. Interestingly, in this case the subsidy linked to LLR support makes $V_t^{ER}(C) > V_0^{LR}(C)$ for all t. Panel 2 shows that holding more liquidity ($C = 0.3$) lengthens the potential duration of the run and modifies the time-profile of $V_t^{ER}(C)$, which now starts below $V_0^{LR}(C)$ but eventually becomes larger than it, but never larger than D. So the ER equilibrium is sustainable, while the LR equilibrium is not.

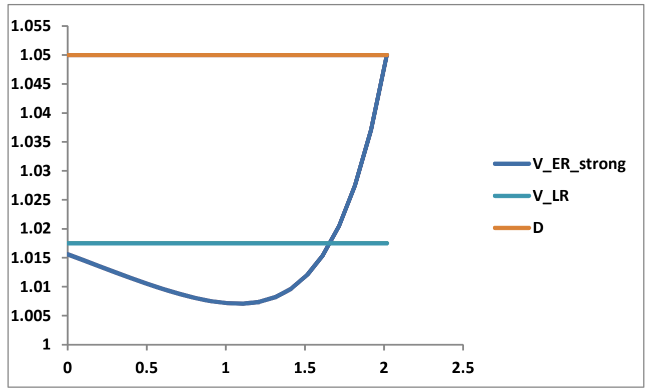
Panels 3 and 4 illustrate what happens when μ is larger, very close to (but still below) the bound above which the LR equilibrium would become sustainable even with $C = 0.15$.

In Panel 3, the ER equilibrium is sustainable while the LR equilibrium is not. In this case, increasing C to 0.3 makes the LR equilibrium sustainable (because $V_0^{LR}(C) > D$), while it turns the ER equilibrium unsustainable (because $V_t^{ER}(C)$ is larger than D at low values of t).

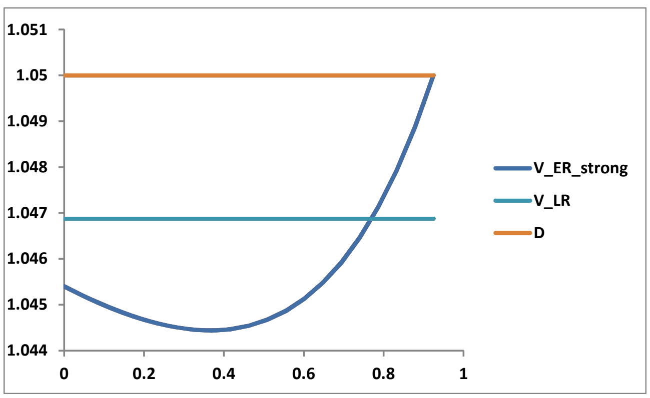
In panels in the bottom row, μ is large enough ($\mu = 0.85$) for the LR equilibrium to be sustainable even with $C = 0.1$ (Panel 5) but with those liquidity holdings the ER equilibrium is also sustainable. In this case, increasing C to 0.2 (Panel 6) makes the ER equilibrium unsustainable, while the LR equilibrium remains sustainable. The reason why the ER ceases to exist is that the additional liquidity holdings reduce the effective net subsidy associated with LLR support in a way that makes $V_t^{ER}(C)$ larger than D at some (low) values of t. This means that a debtholder's best response to anticipating that subsequently debtholders will exercise their put options is no longer to exercise her own option, so the logic sustaining the ER equilibrium unwinds.



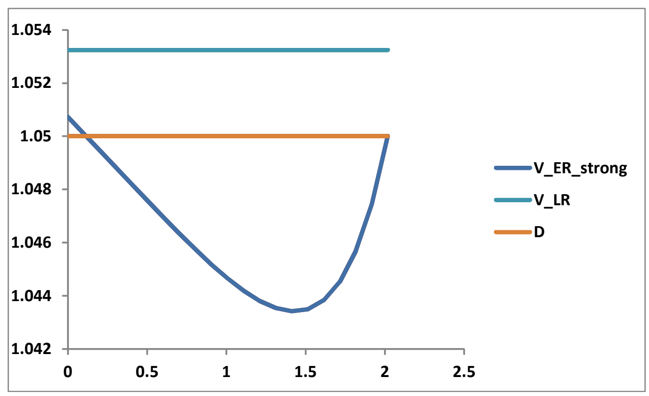
1. (μ, C)=(0.70,0.15). Only ER is an equilibrium



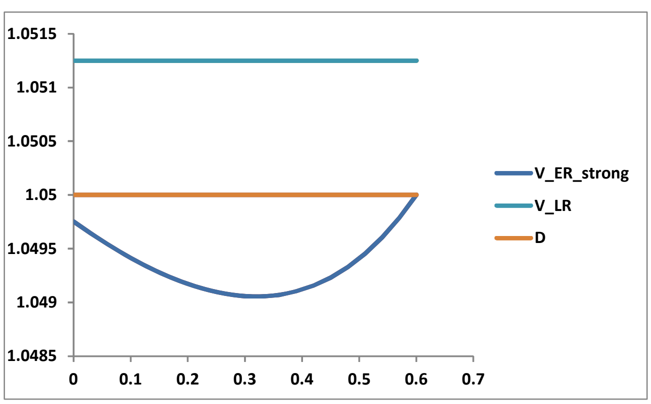
2. (μ, C)=(0.70,0.30). Only ER is an equilibrium



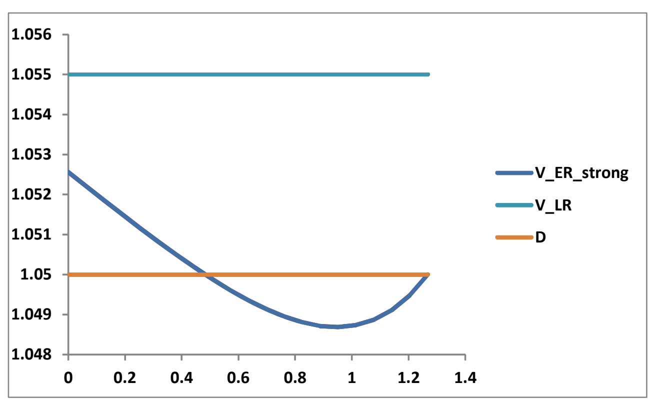
3. (μ, C)=(0.83,0.15). Only ER is an equilibrium



4. (μ, C)=(0.83,0.30). Only LR is an equilibrium



5. (μ, C)=(0.85,0.10). Both LR and ER are equilibria



6. (μ, C)=(0.85,0.20). Only LR is an equilibrium

Figure A1. Early vs. late run equilibria in the strong bank case

B Determinants of optimal liquidity holdings

In principle, the analysis of the determinants of the optimal liquidity standards C^* could be undertaken by performing a standard comparative statics exercise on the first order condition that characterizes C^* when it is interior, $dW_{-1}^{ER}(C)/dC = 0$, where an expression for $dW_{-1}^{ER}(C)/dC$ appears in Eq. (33) in the paper. However, analytically it is only possible to obtain non-ambiguous signs for the corresponding dC^*/dz for some of the z parameters. The signs (or, in case of ambiguity, interrogation signs) in the following table are obtained by standard use of the Implicit Function Theorem on $dW_{-1}^{ER}(C)/dC = 0$. Detailed derivations are omitted for brevity.

Table B1
Dependence of optimal liquidity holdings on key parameters*

	μ	ε	a_g	a_b	q_g	q_b	b
Strong bank case	−	+	+	−	0	?	0
Weak bank case	?	+	?	0	−	+	0

*For z=δ, λ, D the signs of dC^*/dz are ambiguous.

In the rest of this appendix we explore the numerical examples introduced in Section 5.2 of the paper. We analyze and discuss how the socially optimal liquidity holdings C^* in those examples vary with some of the key parameters of the model. The examples seek to understand qualitatively the key forces influencing C^*. The two baseline cases describe in Table 1 of the paper only differ in the probability μ with which illiquid assets remain good if the bank gets hit by the shock at $t = 0$. In the strong bank baseline ($\mu = 0.5$), conditional on being hit by the shock at $t = 0$, illiquid assets have an expected continuation value of 0.85, which is larger than their expected liquidation value of 0.75. In the weak bank baseline ($\mu = 0.25$), these values are 0.675 and 0.725, respectively, so their rank switches. In both cases, the value of δ implies a relatively conservative maturity structure, with an average debt maturity of 6 months, while the value of λ implies a relatively rapid revelation of information, with an expected span of 0.4 months (about 12 days).[38] The probability that the bank is hit by a shock is set intentionally high (20% per month) to magnify the importance of the trade-offs and make the qualitative effects more visible.

All panels in Fig. B1 (strong bank baseline) and Fig. B2 (weak bank baseline) depict,

[38] The corresponding expected time spans can be computed as $1/\delta$ and $1/\lambda$, respectively.

as functions of a different varying parameter, the socially optimal liquidity holdings C^*. The value of such a parameter in the baseline parameterization is indicated by a dashed vertical line. The value (if any) for which moving the parameter further up or down switches from the strong bank case ($\xi = 1$) to the weak bank case ($\xi = 0$) is indicated with a vertical dotted line. As one can see, the baseline parameterization involves an interior value of C^* but changing the parameters often leads to the corner solution with $C^* = 0$.

For a number of parameters, the response of C^* is qualitatively identical across the strong and weak bank cases. Specifically, C^* is decreasing (until it reaches the lower bound of zero) in the continuation value of good assets a_g, while is increasing (once it abandons its lower bound of zero) in the probability ε of the shock hitting the bank at $t = 0$.

The response of C^* to the Poisson rates δ and λ is also similar across the strong and weak bank cases but interestingly non-monotonic. Increasing the rate δ at which debt can be put by investors during a run (which can be interpreted as the result of shortening the average debt maturity) or the rate λ at which information about the quality of the illiquid assets arrives during a run first increases but eventually decreases the optimal liquidity holdings. This is because first liquidity becomes more needed (as δ increases) or more effective (as λ increases) as a means to buy time for information to arrive during the run. However, once δ is large enough (relative to λ), the run happens too quickly and liquidity becomes a too expensive mechanism to buy information—it requires a too large sacrifice of ex ante profitable investment in the illiquid asset. Similarly, once λ is large enough (relative to δ), liquidity holdings are so effective in providing time to obtain the relevant information that C^* declines in response to an increase in λ. For the same logic, the effect of D on C^* is (very mildly) non-monotonic in the weak bank example, while it is negative over the depicted range of values of D in the strong bank example.

The remaining parameters affect C^* differently across the strong and weak bank cases. Specifically, the optimal liquidity holding are decreasing in the continuation value of bad assets a_b in the strong bank case, since larger a_b implies a lower type II error when a bad bank receives LLR support, but C^* does not depend on a_b in the weak bank case, since a bank with bad assets never continues in such a scenario (and hence a_b does not enter the relevant welfare calculations). By the symmetric logic, C^* does not depend on the liquidation value of good assets q_g in the strong bank case, but declines with q_g in the weak bank case.

Finally, the sign of the response of C^* to variations in parameters q_b and μ switches depending on whether the bank is supported or not by the LLR in the absence of news. In the strong bank regime, where the uninformed LLR chooses $\xi = 1$, the optimal liquidity holdings are increasing in the liquidation value of bad assets q_b (reflecting the higher value of discovering that the bank is bad on time not to support it) and decreasing in the probability μ that the illiquid assets are good (by exactly the reverse logic). Instead, in the weak bank regime, C^* decreases mildly with q_b (reflecting the larger return overall associated with the investment in illiquid assets and, hence, the large opportunity cost of cash holdings) and increases with μ (reflecting the greater value of preventing the mistaken liquidation of good assets that occurs when the uninformed LLR decides $\xi = 0$).

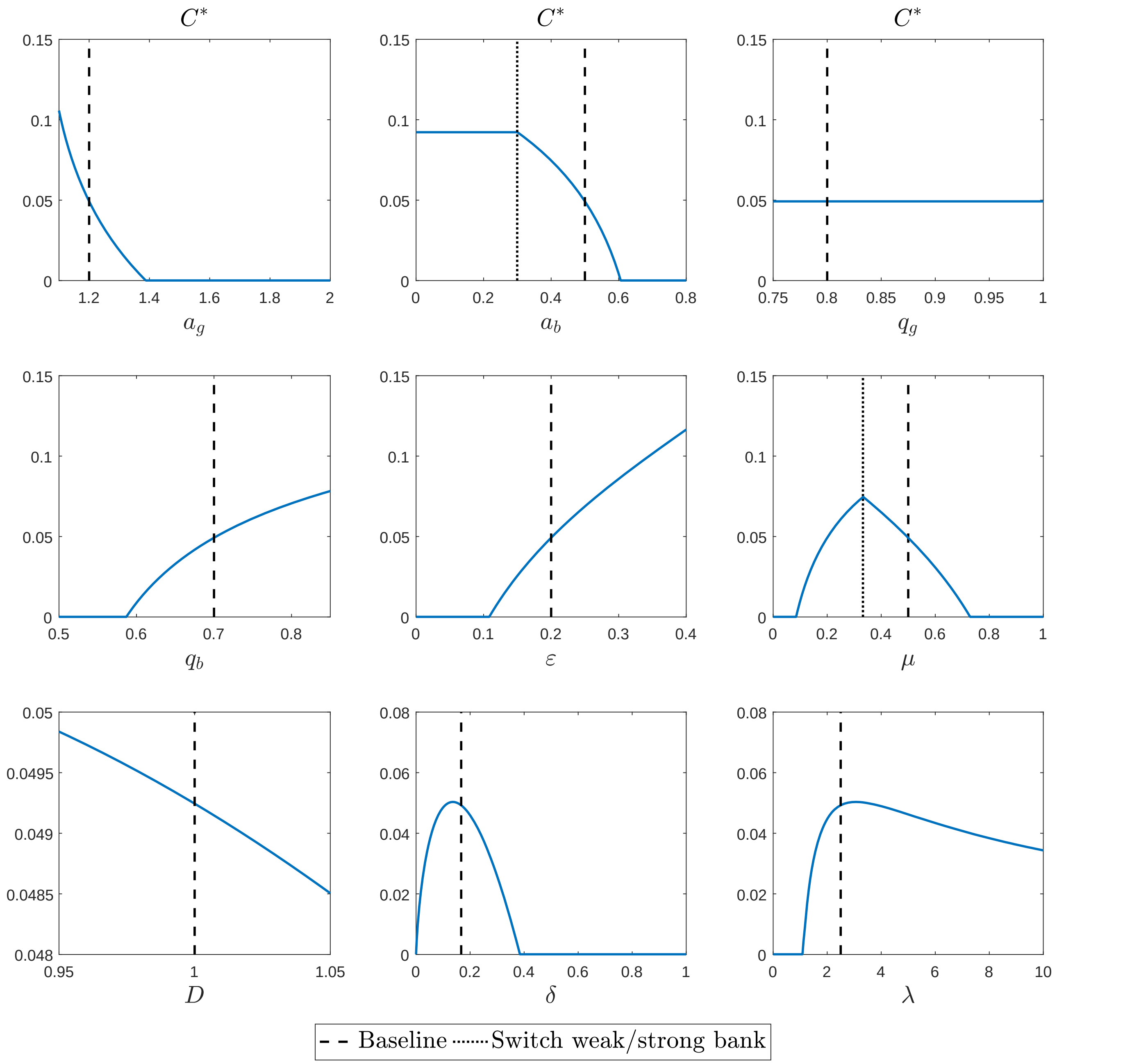


Figure B1 Comparative statics of C^* in the strong bank baseline case

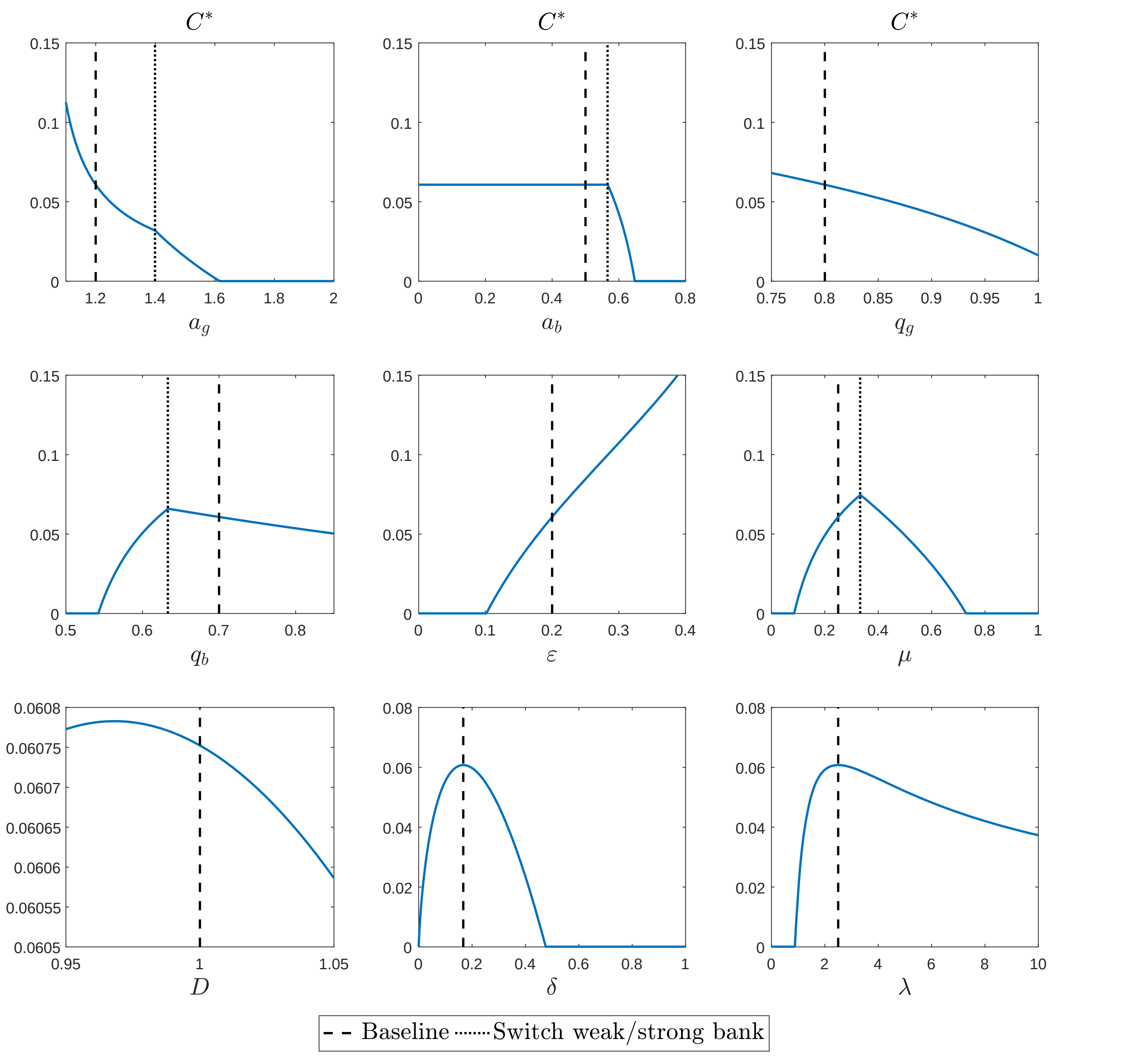


Figure B2 Comparative statics of C^* in the weak bank baseline case

C Additional extensions to our model

In this section, we discuss two additional issues: bankers' incentives to produce information on their financial condition, and a variant of our baseline model in which what needs to be discovered about the bank is not the quality of its assets but the potential systemic importance of its failure.

C.1 Banks' incentives to produce information

In our model, we assume that the arrival of information on the bank's financial condition follows an exogenous Poisson process. As we discussed, the rate λ at which information comes out and the nature of it (whether it is good or bad news) have important implications. For example, the arrival of good news at any time before the bank's cash gets exhausted during an early run eliminates debtholders' incentives to continue exercising their puts, leading the run to an end. In contrast, when the news is bad, debtholders continue exercising their puts and it becomes clear that the LLR will not support the bank once it exhausts its cash. In this context, bank owners are not going to be generally indifferent about whether information gets disclosed, or the rate at which it is disclosed. And this may have implications for the need for supervision to play an active role in the discovery of the relevant information.

To see this, suppose bank owners have the ability to affect the speed at which information gets disclosed during an early run. In the weak bank case, since by default LLR support will not be blindly granted, bank owners will find it advantageous to disclose information about the bank's assets. If the bank is bad, things will not be worse than without the information. But if the bank is good, some extra value can be generated. Social interest in this case is aligned with bank incentives. By contrast, in the strong bank case, bank owners will not be interested in accelerating the production of information. In fact, they will try to delay it, since keeping the LLR "blind" is a way to guarantee its support, and appropriate the corresponding implicit subsidy. So, in this case, involving the LLR in bank supervision or entrusting another agency with the responsibility to produce information about the bank's financial condition (and to share it with the LLR) may be crucial.

C.2 Systemically important banks

One key feature of systemically important banks (SIBs), especially in the absence of a fully effective regime for the recovery and resolution of too-big-to-fail institutions, is the possibility that their early (and disorderly) liquidation causes significant damage to the rest of the financial system or the wider economy (e.g. in the form of fire sale externalities, contagion, etc.). This suggests that for a LLR dealing with a SIB, the trade-offs relevant for deciding whether to grant liquidity support or not might be driven by considerations beyond the fundamental solvency of the bank (or, in model terms, the intrinsic quality of its illiquid assets). One important consideration is the size of the *systemic externalities* that might be avoided by supporting the bank. These externalities increase the social value of allowing the bank to continue in operation after it exhausts its cash, as opposed to pushing it into liquidation.

From the perspective of the LLR, a change in the size of these systemic externalities can play the same role as change in the quality of the illiquid assets in our model. And, from this viewpoint, we could also assimilate non-SIBs to our weak banks (i.e. banks that, in the absence of further information, would not be supported by the LLR) and SIBs to our strong banks (i.e. banks that, in the absence of further information, would be supported). Hence, it is natural to establish a parallel between our model and a model in which banks in trouble (say, to simplify, with bad illiquid assets) can generate small or large systemic externalities if they fail. In such a setup, liquidity standards would give the LLR time to receive information on the size of the externalities.

A full formal analysis of this alternative framework would require more than a pure relabeling of the objects present in our model. Parallel to our model, the size of the systemic externalities is relevant for the LLR decision and, through it, for debtholders' expectations on whether the bank will be supported or not. But one important difference is that systemic externalities do not directly affect debtholders' payoffs contingent on continuation so their size being large or small cannot be fully assimilated to the value of illiquid assets being high or low in our model. Hence, the details of several equations would change.

Yet, it is safe to conjecture that non-SIBs will have greater incentives than SIBs to choose liquidity holdings close to those that maximize social welfare, since, by a logic similar to the one explored in our model, the subsidies that they will expect to obtain through the support granted by a blind LLR are lower (if any) than the subsidies that SIBs will expect to obtain. As in our analysis, the socially optimal liquidity standards would have to trade-off gains from increasing the likelihood that the LLR gets informed about the true systemic importance of the bank and the losses from forcing banks to ex ante forgo potentially more profitable uses of funds.