Internet Appendix for "Off-Balance Sheet Funding, Voluntary Support and Investment Efficiency"

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Access to storage technology

In the baseline model, we have assumed that the bank has no access to a storage technology. In this Internet Appendix, we relax this restriction, and show that our main results on the signaling value of voluntary support and the optimal funding mode continue to hold.

Allowing the bank access to a storage technology enriches the mechanisms in the baseline model along two dimensions. First, the bank may store any remaining funds at $t = 1$ after obligatory or voluntary repayments to debtholders, and invest some of them into the second project at $t = 2$. In presence of asymmetric information, such (partial) self-financing constitutes an alternative means to signal strength for the bank, potentially reducing the reliance on voluntary support. Second, the bank may want to raise more funds at $t = 0$ than what is necessary for investment, as the additional funds can be stored until $t = 1$ and enhance its capability to signal strength when the second investment is undertaken under asymmetric information.[1]

For technical reasons we introduce an additional ingredient in this extension. Following the systematic failure of the first project at $t = 1$, which creates information asymmetry at

[1] In practice, this is achieved by setting the price at which the off-balance sheet vehicle purchases the first project from the bank at $d_0 \geq 1$. The off-balance sheet vehicle then raises d_0 by issuing debt backed by the project. Therefore at $t = 0$, the vehicle receives zero net cash flow, while the bank receives a net cash flow equal to $d_0 - 1$, where 1 is the initial investment in the first project and d_0 is the proceeds from selling the project to the off-balance sheet vehicle. The net cash flow $d_0 - 1$ can be interpreted as a set-up fee charged by the bank to the vehicle.

that date, there is a small probability $\phi > 0$ that a transparency shock that exogenously reveals the bank type $j \in \{g, b\}$ is realized at $t = 2$ just before investment takes place. This small but positive probability that information is exogenously revealed allows us to ensure uniqueness of equilibrium in presence of the alternative signaling device considered in this extension. For the rest of this extension, we consider the limit case $\phi \to 0$ so that asymmetric information persists with probability tending to one following the systematic shock (as in the baseline model) and the parameter ϕ will play no further role.[2]

In the rest of this Internet Appendix we show how the main formal results in the equilibrium analysis of the baseline model extend to this set-up.

IA.1. Second investment and voluntary support

We first consider the bank's investment in the second project following the systematic failure of the first project ($\sigma = S$). If the first project was financed on-balance sheet, the access to a storage technology does not affect the investment efficiency in the second project because the bank is contractually obliged to exhaust all its funds at $t = 1$ to repay the debt.

Suppose that the bank has financed the first project off-balance sheet raising $d_0 \geq 1$ units of funds from competitive investors. Following the systematic failure of the first project, even though the bank is not contractually obliged to use its own funds $d_0 - 1 + Y_S$ at $t = 1$ to repay the off-balance sheet debt holders, it can voluntarily provide them support $s \leq d_0 - 1 + Y_S$. The remaining funds $d_0 - 1 + Y_S - s$ are stored until $t = 2$ and at that date the bank decides whether or not to invest and, if so, how much of its own funds $i \leq \min\{I, d_0 - 1 + Y_S - s\}$ to use for investment. After observing the history of the bank's decisions (s and i) and updating their belief $\hat{\alpha}$ on the bank's type, competitive investors set a promised repayment $D_3(i, \hat{\alpha})$ to provide the $I - i$ units of funds necessary for investment.

It can be shown that the use of self-financing i satisfies an analogous single-crossing condition to that for voluntary support in Eq. (11) of the paper so that this action also

[2] More generally, if the probability $\phi \in (0, 1)$ of the transparency shock does not tend to zero, this parameter could be economically interpreted as a measure of transparency as it captures how easy it is for investors to obtain information about the quality of the new investment opportunities of the bank once uncertainty about them arises. All the formal results in this section, which are presented for the case $\phi \to 0$, can be extended to the case of a fixed $\phi > 0$ as is shown in the proofs of these results at the end of this Internet Appendix.

constitutes a signal of quality. The bank thus faces a bi-dimensional signaling problem: Each of the bank types decides how to use its limited internal funds to signal quality with a combination of voluntary support (at $t = 1$) and self-financing (at $t = 2$). The following proposition characterizes the equilibrium of this bi-dimensional signaling game.

Proposition IA.1. *Suppose the bank has financed the first project off-balance sheet with an initial funding amount d_0 and the project fails due to systematic reasons at $t = 1$. There exists a unique equilibrium of the subsequent support and investment game, characterized as follows:*

- *The good bank voluntarily provides support $\widetilde{s}^{Off}$ at $t = 1$ and invests with as much remaining own funds as possible at $t = 2$. Moreover, $\widetilde{s}^{Off} = d_0 - 1 + Y_S$ if and only if $d_0 - 1 + Y_S \leq \overline{Y}_S(I)$, where $\overline{Y}_S(I)$ is given by Proposition 1.*
- *The bad bank mimics the good bank in the support decision at $t = 1$ with probability $\widetilde{\pi}^{Off}$. The bad bank invests in the second project if and only if it mimics the good bank at $t = 1$. Moreover, $\widetilde{\pi}^{Off} = \pi^{Off}(d_0 - 1 + Y_S, I)$, where $\pi^{Off}(\cdot)$, is given by Proposition 1. In particular, $\widetilde{\pi}^{Off}$ is decreasing in d_0, and $\widetilde{\pi}^{Off} = 0$ if and only if $d_0 - 1 + Y_S \geq \overline{Y}_S(I)$.*

The proposition shows that the main results on the signaling value of voluntary support in Proposition 1 in the paper remain valid. The g bank provides voluntary support in equilibrium to signal strength even in the presence of self-financing as an alternative signaling opportunity and the b bank mimics with a probability that coincides with that in the baseline model for the same amount of available funds at $t = 1$.[3] Moreover, the g bank exhausts all its available funds to provide support ($\widetilde{s}^{Off} = d_0 - 1 + Y_S$) when its available funds at $t = 1$ are not sufficient to achieve full separation.

A final important result in Proposition IA.1 is that separation across bank types increases in the initial funding amount d_0, as the access to storage allows the bank to carry those funds until $t = 1$ and use them for signaling purposes at that date. This suggests that if the bank uses off-balance sheet funding it might decide to set $d_0 > 1$.

[3]Notice that following the first project failure, in the baseline model the bank funds at $t = 1$ amount to Y_S while in this extension they are increased to $d_0 - 1 + Y_S$.

IA.2. First investment and optimal funding mode

The bank's optimal choice between on- and off-balance sheet funding then results from a similar trade-off between the efficiency gains and costs in the two investments as in the baseline model in the paper. The characterization of the optimal funding mode is given by the following proposition.

Proposition IA.2. *There exists a threshold* $\widetilde{I} \in \mathbf{R}^+ \cup \{\infty\}$*, such that off-balance sheet funding is strictly optimal if and only if* $I > \widetilde{I}$*. Moreover, there exists* $\widetilde{m} > 0$ *such that* $\widetilde{I} \neq \infty$ *for all* $m < \widetilde{m}$*. Finally, the threshold* $\widetilde{I}$ *satisfies* $\widetilde{I} < \overline{I}$ *if and only if* $Y_S < \overline{Y}_S(I)$*, where* $\overline{I}$ *is given by Proposition 2 and* $\overline{Y}_S(\cdot)$ *is given by Proposition 1.*

The first part of the proposition shows that the main results of the baseline model in Proposition 2 hold in this extension: Off-balance sheet funding is optimal when the scale of the second project I is sufficiently large and/or the marginal value of effort m is low.

Proposition IA.2 also shows that access to a storage technology may increase or decrease the threshold scale of the second project $\overline{I}$ below which off-balance sheet funding is optimal, depending on the pay-off of the asset-in-place Y_S and the scale of the second investment. When Y_S is low such that $Y_S < \overline{Y}_S(I)$, in the baseline model the bank is unable to achieve full separation even when it exhausts all pay-offs from the asset-in-place to provide voluntary support. Access to a storage technology enables the bank to raise additional funds $d_0 > 1$ at $t = 0$ and carry $d_0 - 1$ until $t = 1$ for signaling purposes. This increases the degree of separation of bank types and improves the bank's second investment efficiency. The increased amount of voluntary support also constitutes additional skin-in-the-game and enhances the bank's effort incentives for the first project. Overall, access to a storage technology in this case increases the bank's investment efficiency in both projects and thus overall expected profit under off-balance sheet funding. When Y_S is high such that $Y_S > \overline{Y}_S(I)$, however, the bank is able to achieve full separation with pay-offs from the asset-in-place through voluntary support. Access to a storage technology enables the bank to reduce (ex post) costly voluntary support, while still achieving full separation across bank types by signaling with a combination of voluntary support and self-financing. While this does not affect the investment efficiency of the second project, reduced voluntary support decreases the bank's skin-in-the-game and thus effort in the first project. Therefore in this case, access to a storage technology decreases the bank's investment efficiency and thus expected profits under off-balance sheet funding.

Proofs

Proof of Proposition IA.1 While the main text in this Internet Appendix focuses on the limit as $\phi \to 0$, in this proof we derive the results for any given $\phi \in (0, 1)$. The results given by the proposition are then obtained by taking the limit as $\phi \to 0$.

Suppose the bank has financed the first project off-balance sheet with an initial funding amount d_0 and the project fails due to systematic reasons at $t = 1$. For the ease of notation, let $w_1 \equiv d_0 - 1 + Y_S$ denote the amount of the bank's own funds at $t = 1$. Each bank chooses among the set of actions $\widetilde{A}_1 \equiv \{(s, i) \in [0, w_1] \times \widetilde{A}_2 : s + i \leq w_1\}$, where $i \in \widetilde{A}_2 \equiv [0, w_1] \cup \{i_0\}$ denotes the bank's action at $t = 2$ conditional on information asymmetry persists (with probability $1 - \phi$). In particular, $i \in [0, w_1]$ means that the bank invests in this contingency with i amount of self-financing, raising the remaining $I - i$ from outside investors, while i_0 means that the bank does not invest in this contingency. We extend the natural order in the interval $[0, w_1]$ to the set $\widetilde{A}_2$ by assuming that for any $i \in [0, w_1]$ we have $i > i_0$. Notice that the bank's set of actions $\widetilde{A}_1$ does not include the bank's investment decision conditional on the transparency shock realizing at $t = 2$ (with probability ϕ). This is because, in this contingency, the bank invests if and only if it is of type g, irrespective of how much self-financing it contributes, since externally financing is always fairly priced. The formal equilibrium definition is analogous to that in the proof of Proposition 1.

The expected profits as of $t = 1$ for a bank of type j that provides support s at $t = 1$ and chooses i when information asymmetry persists at $t = 2$, given the investors' belief $\hat{\alpha}$, are given by

$$\widetilde{\Pi}_{j,1}(s, i, \hat{\alpha}) = \begin{cases} \phi \left[w_1 - s + \max\{(p_j R - 1)I, 0\}\right] & \\ \quad (1 - \phi)\left[w_1 - s - i + p_j(RI - \widetilde{D}_3(i, \hat{\alpha}))\right], & \text{if } i \in [0, w_1] \text{ and } \widetilde{D}_3(i, \hat{\alpha})) \leq RI, \\ w_1 - s + \phi \max\{(p_j R - 1)I, 0\}, & \text{otherwise,} \end{cases} \tag{IA.1}$$

where $\widetilde{D}_3(i, \hat{\alpha}))$ for $i \in [0, w_1]$ is given by

$$\widetilde{D}_3(i, \hat{\alpha})) = \frac{I - i}{\hat{\alpha} p_g + (1 - \hat{\alpha}) p_b}. \tag{IA.2}$$

We first establish the following claims.

Claim IA.1. *In equilibrium, if a g bank plays action (s, i) with positive probability, then $i = \min\{w_1 - s, I\}$.*

Proof. We prove this claim by contradiction. Suppose there exists $i \in [0, \min\{w_1 - s, I\}) \cup \{i_0\}$ such that $\pi_g(s,i) > 0$. We thus have that the equilibrium profits $\widetilde{\Pi}^*_{j,1}$ must satisfy

$$\widetilde{\Pi}^*_{g,1} = \widetilde{\Pi}_{g,1}(s,i,\alpha(s,i)) \text{ and } \widetilde{\Pi}^*_{b,1} \geq \max\{\widetilde{\Pi}_{b,1}(s,i,\alpha(s,i)), \widetilde{\Pi}_{j,1}(s,i_0,\alpha(s,i_0))\}, \tag{IA.3}$$

where the inequality follows because the b bank may play another action in equilibrium which gives it a strictly higher pay-off.

We now prune the supposed equilibrium by constructing a profitable deviation for the g bank under the D1 refinement. Consider an off-equilibrium deviation by the bank to contribute some $i' \in (i, \min\{w_1 - s, I\}]$. The set of beliefs that make deviation strictly optimal for the j bank is defined as:

$$\widetilde{\Lambda}_j(s,i') = \left\{\hat{\alpha} \in [0,1] : \widetilde{\Pi}_{j,1}(s,i',\hat{\alpha}) > \widetilde{\Pi}^*_{j,1}\right\}. \tag{IA.4}$$

Using Eq. (IA.4) and the fact that $\frac{\partial \widetilde{\Pi}_{j,1}(s,i,\hat{\alpha})}{\partial \hat{\alpha}} > 0$, the deviation set $\widetilde{\Lambda}_b(s,i')$ satisfies

$$\widetilde{\Lambda}_b(s,i') \subset \hat{\widetilde{\Lambda}}_b(s,i') \equiv \left\{\hat{\alpha} \in [0,1] : \widetilde{\Pi}_{b,1}(s,i',\hat{\alpha}) > \max\{\widetilde{\Pi}_{b,1}(s,i,\alpha(s,i)), w_1 - s\}\right\}. \tag{IA.5}$$

We now show that $\hat{\widetilde{\Lambda}}_b(s,i') \subsetneq \widetilde{\Lambda}_g(s,i')$, which then implies that $\widetilde{\Lambda}_b(s,i') \subsetneq \widetilde{\Lambda}_g(s,i')$. First, if $\hat{\widetilde{\Lambda}}_b(s,i') \neq \emptyset$, then $\widetilde{\Pi}^*_{b,2} \geq w_1 - s$ implies that $\widetilde{D}_3(i',\hat{\alpha}) \leq R$ for all $\hat{\alpha} \in \hat{\widetilde{\Lambda}}_b(s,i')$, so that we have the following property for all $\hat{\alpha}$ in $\hat{\widetilde{\Lambda}}_b(s,i')$:

$$\widetilde{\Pi}_{b,1}(s,i',\hat{\alpha}') \geq \widetilde{\Pi}_{b,1}(s,i,\hat{\alpha}) \Rightarrow \widetilde{\Pi}_{g,1}(s,i',\hat{\alpha}') \geq \widetilde{\Pi}_{g,1}(s,i,\hat{\alpha}). \tag{IA.6}$$

This implies that $\hat{\widetilde{\Lambda}}_b(s,i') \subseteq \widetilde{\Lambda}_g(s,i')$. Further, $p_b R < 1$ implies that $\hat{\widetilde{\Lambda}}_b(s,i') \subsetneq (0,1]$. This and the fact that $\frac{\widetilde{\Pi}_{j,1}(s,i,\hat{\alpha})}{\partial \hat{\alpha}} > 0$ then imply that $\hat{\widetilde{\Lambda}}_b(s,i') \subsetneq \widetilde{\Lambda}_g(s,i')$. Second, if $\hat{\widetilde{\Lambda}}_b(s,i') = \emptyset$, then we also have $\hat{\widetilde{\Lambda}}_b(s,i') \subsetneq \widetilde{\Lambda}_g(s,i')$, because $p_g R > 1$ and $1 \in \widetilde{\Lambda}_g(s,i') \neq \emptyset$.

The D1 refinement then implies that $\alpha(s,i') = 1$, but then using that $\alpha(s,i') \geq \alpha(s,i)$, $\frac{\partial \widetilde{\Pi}_{b,1}(s,i,\hat{\alpha})}{\partial \hat{\alpha}} > 0$ and $\frac{\partial \widetilde{\Pi}_{b,1}(s,i,\hat{\alpha})}{\partial i} > 0$ we have that

$$\widetilde{\Pi}_{g,1}(s,i',\alpha(s,i')) \geq \widetilde{\Pi}_{g,1}(s,i',\alpha(s,i)) > \widetilde{\Pi}_{g,1}(s,i,\alpha(s,i)) = \widetilde{\Pi}^*_{g,1}, \tag{IA.7}$$

a contradiction. □

Claim IA.2. *In equilibrium, the b bank plays with positive probability only actions that are either played with positive probability by the g bank, or $(s,i) = (0,i)$ and receives $\widetilde{\Pi}_{b,1}(0,i,\alpha(0,i)) = w_1$.*

Proof. Proof analogous to that of Claim 2. □

Claim IA.3. *If in equilibrium* $\int_{\widetilde{A}_2} \pi_b(0,i)di < 1$ *then the* g *bank plays with probability one the action* $(s,i) = (w_1, 0)$.

Proof. The proof of this claim is analogous to that of Claim 3. The logic is that, if this claim is not true, then we can construct a profitable deviation for the g bank (s', i') where $s'^{*} + \epsilon$ and $i' = i - \epsilon \geq 0$ for some sufficiently small ϵ. □

Claim IA.4. *If in equilibrium* $\int_{\widetilde{A}_2} \pi_b(0,i)di = 1$ *then the* g *bank plays with probability one the action* (s,i)*, where* $i = \min\{w_1 - s, I\}$ *and* s *is the minimum value such that* $\widetilde{\Pi}_{b,1}(s,i,1) \leq w_1$.

Proof. Proof analogous to that of Claim 4. □

Analogous to the baseline model, Claims IA.1–IA.4 imply that equilibria are characterized by the probability that the b bank mimics the pure strategy action played by the g bank. Let us denote by $\widetilde{s}^{Off}$ the support given by the g bank in equilibrium and define $\widetilde{i}^{Off} = w_1 - \widetilde{s}^{Off}$ and $\widetilde{\pi}^{Off} = \pi_b(\widetilde{s}^{Off}, \widetilde{i}^{Off})$. We use the results in the previous claims without explicit reference in the rest of the proof.

Let us first characterize the existence of a separating equilibrium, i.e. an equilibrium with $\widetilde{\pi}^{Off} = 0$ (or equivalent, $\int_{\widetilde{A}_2} \pi_b(0,i)di = 1$). In a separating equilibrium, we have $\alpha(\widetilde{s}^{Off}, \widetilde{i}^{Off}) = 1$. Separation is sustained in equilibrium if and only if the incentive compatibility constraint for the b bank not to mimic at $t = 1$ is satisfied, i.e. $\widetilde{\Pi}_{b,1}(\widetilde{s}^{Off}, \widetilde{i}^{Off}, 1) \leq w_1$, which is the case if and only if $\widetilde{s}^{Off} \geq \overline{s}(w_1, \phi, I)$, where $\overline{s}(w_1, \phi, I)$ is defined by

$$\widetilde{\Pi}_{b,1}(\overline{s}(w_1,\phi,I), \min\{w_1 - \overline{s}(w_1,\phi,I), I\}, 1) = w_1 \quad \Leftrightarrow$$
$$\phi\left[w_1 - \overline{s}(w_1,\phi,I)\right] + (1-\phi)p_b\left[RI - \frac{I - w_1 + \overline{s}(w_1,\phi,I)}{p_g}\right] = w_1. \tag{IA.8}$$

Notice that $\overline{s}(w_1, \phi, I)$ is continuous and decreasing in w_1. $\widetilde{s}^{Off} < w_1$ thus implies that a separating equilibrium exists if and only if $w_1 \geq \widetilde{Y}_S(\phi, I)$, where $\widetilde{Y}_S(\phi, I)$ is defined by

$$\widetilde{\Pi}_{b,1}(\widetilde{Y}_S(\phi,I), 0, 1) = \widetilde{Y}_S(\phi,I) \qquad \Leftrightarrow \qquad \widetilde{Y}_S(\phi,I) = (1-\phi)\frac{p_b}{p_g}(p_g R - 1)I. \tag{IA.9}$$

Notice that there exists $\overline{w}_1(\phi, I) > \widetilde{Y}_S(\phi, I)$, such that $\overline{s}(w_1, \phi, I) > 0$ if and only if $w_1 < \overline{w}_1(\phi, I)$, where $\overline{w}_1(\phi, I)$ is given by $\overline{s}(\overline{w}_1(\phi,I), \phi, I) > 0$, or

$$p_b\left[RI - \frac{I - \overline{w}_1(\phi,I)}{p_g}\right] = \overline{w}_1(\phi,I). \tag{IA.10}$$

By construction, for $w_1 \geq \widetilde{Y}_S(\phi, I)$, there is a unique separating equilibrium such that $\widetilde{s}^{Off} = \max\{\overline{s}(w_1, \phi, I), 0\}$. Conversely, if a separating equilibrium exists then $w_1 \geq \widetilde{Y}_S(\phi, I)$.

Next, we characterize the semi-pooling equilibrium, i.e., equilibria with $\widetilde{\pi}^{Off} \in (0, 1)$. In such an equilibrium, we have $\widetilde{s}^{Off} = w_1$, $\widetilde{i}^{Off} = 0$ and $\alpha(\widetilde{s}^{Off}, \widetilde{i}^{Off}) = \frac{\alpha}{\alpha + (1-\alpha)\widetilde{\pi}^{Off}}$. This is an equilibrium if and only if the b bank is indifferent between $(\widetilde{s}^{Off}, \widetilde{i}^{Off})$ and $(0, i)$, i.e. $\widetilde{\Pi}_{b,1}(\widetilde{s}^{Off}, \widetilde{i}^{Off}, \alpha(\widetilde{s}^{Off}, \widetilde{i}^{Off})) = w_1$, which implies that $\widetilde{\pi}^{Off}(w_1, \phi, I)$ is defined by

$$(1-\phi)p_b \left[R - \frac{1}{\alpha^{Off} p_g + (1-\alpha^{Off})p_b}\right] I = Y_S, \quad \text{where } \alpha^{Off} = \frac{\alpha}{\alpha + (1-\alpha)\pi^{Off}}. \tag{IA.11}$$

To summarize, there exists a unique equilibrium, in which the g bank plays $(\widetilde{s}^{Off}, \widetilde{i}^{Off})$, where

$$\widetilde{s}^{Off}(w_1, \phi, I) = \begin{cases} w_1, & \text{if } w_1 \leq \widetilde{Y}_S(\phi, I), \\ \overline{s}(w_1, \phi, I), & \text{if } w_1 \in [\widetilde{Y}_S(\phi, I), \overline{w}_1(\phi, I)], \\ 0, & \text{if } w_1 \geq \overline{w}_1(\phi, I), \end{cases} \quad \text{and} \quad \widetilde{i}^{Off} = \min\{w_1 - \widetilde{s}^{Off}, I\}. \tag{IA.12}$$

The b bank mimics with probability $\widetilde{\pi}^{Off}(w_1, \phi, I)$, where $\widetilde{\pi}^{Off} > 0$ if and only if $w_1 < \widetilde{Y}_S(\phi, I)$.

Focusing on the case of $\phi \to 0$, we have the result stated in this proposition. First, notice that $\lim_{\phi \to 0} \widetilde{Y}(\phi, I) = \overline{Y}(I)$, where $\overline{Y}(\phi, I)$ is defined by Proposition 1. We thus have that, for $\phi \to 0$, $\widetilde{s}^{Off} = w_1 = d_0 - 1 + Y_S$ if and only if $w_1 = d_0 - 1 + Y_S \leq \overline{Y}_S(I)$. Second, for $\phi \to 0$, $\widetilde{\pi}^{Off} = \pi^{Off}(d_0 - 1 + Y_S, I)$, where $\pi^{Off}(\cdot)$ is defined by Proposition 1. This implies that $\widetilde{\pi}^{Off}$ is decreasing in d_0. □

Proof of Proposition IA.2 As in the proof of Proposition IA.1, we derive the results for any given $\phi \in (0, 1)$ in this proof. The results given by this proposition are then obtained by taking the limit as $\phi \to 0$.

We first characterize the bank's effort choice for the first investment e and investment decision at $t = 2$ under on-balance sheet funding in the following claim.

Claim IA.5. *Suppose the bank invests in the first project on-balance sheet and chooses the amount of funds $d_0 \geq 1$ to raise from investors at $t = 0$. The bank's optimal effort choice is given by $e^{On}(E[Y_\sigma])$, where $e^{On}(E[Y_\sigma])$ is defined in Lemma 2. If the project fails due to systematic reasons at $t = 1$ and information asymmetry persists at $t = 2$, the bank's investment decision is as described in Lemma 1.*

Proof. Suppose the first project is funded on-balance sheet and the bank raises $d_0 \geq 1$ units of funds from investors at $t = 0$. Let $D_1 \leq R$ denote the competitive promised repayment required by the investors. Taking into account that the bank's asset-in-place pays off Y_σ at $t = 1$ and that the bank is contractually obliged to use it if necessary to satisfy debt repayments, the promise D_1 satisfies the following break-even condition:

$$(p_g + me)D_1 + q\min\{d_0 - 1 + Y_S, D_1\} + (1 - q - p_g - me)\min\{d_0 - 1 + Y_{\overline{S}}, D_1\} = d_0. \tag{IA.13}$$

Notice that Eq. (IA.13) and $Y_\sigma < 1$ imply $D_1 > d_0 - 1 + Y_\sigma$. This implies that if $R_1 = 0$, the bank exhausts its own funds to repay the on-balance sheet debt holders. As a result, for a given D_1, the bank chooses effort e to maximize

$$(p_g + me)(d_0 - 1 + Y_{\overline{S}} + R - D_1) - c(e). \tag{IA.14}$$

Notice that this expression is analogous to Eq. (6) in the baseline model. The bank's optimal effort choice thus satisfies the following first order condition

$$c'(e) = m(d_0 - 1 + Y_{\overline{S}} + R - D_1). \tag{IA.15}$$

Eqs. (IA.13) and (IA.15) jointly determine D_1 and e. After some algebraic manipulation, the solution is identical to that described in Lemma 2 and does not depend on d_0.

Finally, since the equilibrium repayment D_1 satisfies $D_1 > d_0 - 1 + Y_\sigma$, the bank has no funds after repaying the debt holders if the first project fails at $t = 1$. If the failure is due to systematic reasons and information asymmetry persists at $t = 2$, it follows that the bank's investment decision is as described in Lemma 1. □

Next, we consider off-balance sheet funding. In the following claims, we characterize first the bank's effort choice for the first investment e for a given initial funding amount d_0, then the bank's optimal choice of initial funding amount.

Claim IA.6. *Suppose the bank invests in the first project off-balance sheet. For a given initial funding amount $d_0 \geq 1$, the bank's optimal effort choice is given by the largest solution to the equation*

$$c'(e) = m\left(R - \frac{1 - q\left[\alpha + (1-\alpha)\widetilde{\pi}^{Off}\right]\widetilde{s}^{Off}}{p_g + me}\right),$$

where $w_1 = d_0 - 1 + Y_S$ is the bank's own funds at $t = 1$ following the systematic failure of the first project. Moreover, the optimal effort, which we denote by $\widetilde{e}^{Off}(w_1, \phi, I)$, is increasing in I and strictly so if and only if $w_1 < \widetilde{Y}_S(\phi, I)$. Finally, $\widetilde{e}^{Off}(w_1, \phi, I)$ is strictly increasing in w_1 if and only if $w_1 < \widetilde{Y}_S(\phi, I)$, where $\widetilde{Y}_S(\phi, I)$ is defined in the proof of Proposition IA.1.

Proof. We first derive the bank's optimal effort choice. Taking into account that the bank provides voluntary support following the systematic failure of the project as described in Proposition IA.1, the promised repayment D_1 satisfies the following break-even condition:

$$(p_g + me)D_1 + q\left[\alpha + (1-\alpha)\widetilde{\pi}^{Off}\right]\widetilde{s}^{Off} = d_0, \tag{IA.16}$$

where $\widetilde{\pi}^{Off}$ and $\widetilde{s}^{Off}$ are defined in the proof of Proposition IA.1. Moreover, for a given D_1, the bank chooses effort e to maximize

$$(p_g + me)(d_0 - 1 + Y_{\overline{S}} + R - D_1) + (1 - q - p_g - me)(d_0 - 1 + Y_{\overline{S}}) - c(e). \tag{IA.17}$$

This expression is analogous to Eq. (14) in the baseline model. The bank's optimal effort choice thus satisfies the same first order condition as in the baseline model, given by Eq. (15). Eqs. (IA.16) and (15) jointly determine D_1 and e. After some algebraic manipulation, the solution is as described in this claim.

We now consider the properties of $\widetilde{e}^{Off}(w_1, \phi, I)$, using the properties of $\widetilde{s}^{Off}(w_1, \phi, I)$ and $\widetilde{\pi}^{Off}(w_1, \phi, I)$ given in the proof of Proposition IA.1. Consider the following two cases. If $w_1 < \widetilde{Y}_S(\phi, I)$, $\widetilde{s}^{Off}(w_1, \phi, I) = w_1$ and $\widetilde{\pi}^{Off}(w_1, \phi, I)$ is strictly decreasing in w_1 and increasing in I. Therefore $\widetilde{e}^{Off}(w_1, \phi, I)$ is strictly increasing in w_1 in I. If $w_1 \geq \widetilde{Y}_S(\phi, I)$, $\widetilde{s}^{Off}(w_1, \phi, I) = \max\{\overline{s}(w_1, \phi, I), 0\}$ and $\widetilde{\pi}^{Off}(w_1, \phi, I) = 0$. Notice that $\overline{s}(w_1, \phi, I)$ given by Eq. (IA.8) is decreasing in w_1. Therefore $\widetilde{e}^{Off}(w_1, \phi, I)$ is decreasing in w_1. To summarize, $\widetilde{e}^{Off}(w_1, \phi, I)$ is increasing in I and strictly so if and only if $w_1 < \widetilde{Y}_S(\phi, I)$; $\widetilde{e}^{Off}(w_1, \phi, I)$ is also strictly increasing in w_1 if and only if $w_1 < \widetilde{Y}_S(\phi, I)$. □

Claim IA.7. *Suppose the bank invests in the first project off-balance sheet. The bank optimally chooses an initial funding amount* $d_0^{Off}(Y_S, \phi, I) = \widetilde{w}_1(Y_S, \phi, I) + 1 - Y_S$, *where* $\widetilde{w}_1(Y_S, \phi, I) \geq Y_S$, *with equality if and only if* $Y_S \geq \widetilde{Y}_S(\phi, I)$.

Proof. The bank chooses the initial funding amount d_0 to maximize the expected profits as of $t = 0$, given by

$$\widetilde{\Pi}_0(\widetilde{e}^{Off}, \widetilde{\pi}^{Off}) \equiv V^{FB} - \int_{\widetilde{e}^{Off}}^{e^{FB}} (mR - c'(e))de - (1-\phi)q(1-\alpha)\widetilde{\pi}^{Off}(1 - p_b R)I. \tag{IA.18}$$

$\widetilde{\Pi}_0(\cdot)$ is defined analogous to $\Pi_0(\cdot)$ given by Eq. (9) in the baseline model, with the only difference being that second project inefficiency occurs only when the information asymmetry persists until $t = 2$ (with probability $1 - \phi$).

Since $\widetilde{e}^{Off}(w_1, \phi, I)$ and $\widetilde{\pi}^{Off}(w_1, \phi.I)$ only indirectly depend on d_0 through w_1, choosing the optimal initial funding amount d_0^{Off} is equivalent to choosing the optimal $\widetilde{w}_1^{Off} \geq Y_S$. The optimal funding can then be residually derived as $d_0^{Off} = \widetilde{w}_1^{Off} + 1 - Y_S$.

Using the properties of $\widetilde{e}^{Off}(w_1, \phi, I)$ derived in the proof of Proposition IA.1 and the properties of $\widetilde{e}^{Off}(w_1, \phi, I)$ given by Claim IA.6, we have that $\Pi_0(\widetilde{e}^{Off}, \widetilde{\pi}^{Off})$ is increasing in w_1 if and only if $w_1 \leq \widetilde{Y}_S(\phi, I)$. This implies that $\widetilde{w}_1^{Off} = Y_S$ for all $Y_S \geq \widetilde{Y}_S(\phi, I)$.

If $Y_S < \widetilde{Y}_S(\phi, I)$, then $w_1 = \widetilde{Y}_S(\phi, I)$ is feasible if and only if there exists $D_1 < R$ that satisfies Eq. (IA.16), or equivalently,

$$\left[p_g + me^{Off}(\widetilde{Y}_S(\phi, I), \phi, I)\right] R + q\alpha\widetilde{Y}_S(\phi, I) \geq \widetilde{Y}_S(\phi, I) + 1 - Y_S. \tag{IA.19}$$

Notice that the right hand side of the above expression is decreasing in Y_S. Further, Eq. (IA.19) is satisfied with strict inequality for $Y_S = \widetilde{Y}_S(\phi, I)$. Therefore there exists $\underline{Y}_S(\phi, I) < \widetilde{Y}_S(\phi, I)$, such that $w_1 = \widetilde{Y}_S(\phi, I)$ is feasible if and only if $Y_S \in [\underline{Y}_S(\phi, I), \widetilde{Y}_S(\phi, I)]$. In this case, the bank optimally chooses $\widetilde{w}_1^{Off} = \widetilde{Y}_S(\phi, I)$.

If $Y_S < \underline{Y}_S(\phi, I)$, $\Pi_0(\widetilde{e}^{Off}, \widetilde{\pi}^{Off})$ is increasing in w_1 for all feasible w_1, while all feasible w_1 satisfy $w_1 < \widetilde{Y}_S(\phi, I)$ as argued above. Therefore the bank chooses the maximum feasible w_1, denoted by $\overline{w}_1^{Off}(Y_S, \phi, I)$, which is the largest solution to

$$\left[p_g + m\widetilde{e}^{Off}(w_1, \phi, I)\right] R + q\left[\alpha + (1-\alpha)\widetilde{\pi}^{Off}(w_1, \phi, I)\right] w_1 = w_1 + 1 - Y_S. \tag{IA.20}$$

To summarize, the bank's optimal initial funding amount is given by $d_0^{Off}(Y_S, \phi, I) = \widetilde{w}_1^{Off}(Y_S, \phi, I) + 1 - Y_S$, where

$$\widetilde{w}_1^{Off}(Y_S, \phi, I) = \begin{cases} \overline{w}_1^{Off}(Y_S, \phi, I), & \text{if } Y_S \leq \underline{Y}_S(\phi, I), \\ \widetilde{Y}_S(\phi, I), & \text{if } Y_S \in [\underline{Y}_S(\phi, I), \widetilde{Y}_S(\phi, I)], \\ Y_S, & \text{if } Y_S \geq \widetilde{Y}_S(\phi, I). \end{cases} \tag{IA.21}$$

□

We can now characterize the bank's choice of optimal funding mode, which is determined by the difference in the bank's expected profits between off- and on-balance sheet funding. Using the results of Claims IA.5–IA.7, this is given by

$$\begin{aligned} \Delta\widetilde{\Pi}_0(E[Y_\sigma], Y_S, \phi, I) \equiv\ & \widetilde{\Pi}_0(\widetilde{e}^{Off}(\widetilde{w}_1^{Off}(Y_S, \phi, I), \phi, I), \widetilde{\pi}^{Off}(\widetilde{w}_1^{Off}(Y_S, \phi, I), \phi, I)) \\ & - \Pi_0(e^{On}(E[Y_\sigma]), \pi^{On}) \\ =\ & q(1-\phi)(1-\alpha)\left(\pi^{On} - \widetilde{\pi}^{Off}(\widetilde{w}_1^{Off}(Y_S, \phi, I), \phi, I)\right)(1 - p_b R)I \\ & - \int_{\widetilde{e}^{Off}(\widetilde{w}_1^{Off}(Y, \phi, I), \phi, I)}^{e^{On}(Y)} (mR - c'(e))de. \end{aligned} \tag{IA.22}$$

Notice that $\Delta\widetilde{\Pi}_0(E[Y_\sigma], Y_S, \phi, I)$ is strictly increasing in I. To see this, consider the following three cases. If $Y_S \leq \underline{Y}_S(\phi, I)$, then Eq. (IA.20) implies that $\widetilde{w}_1^{Off}(Y_S, \phi, I) = \overline{w}_1^{Off}(Y_S, \phi, I)$

is increasing in I. Using the properties of $\widetilde{e}^{Off}(w_1,\phi,I)$ stated in Claim IA.6, we have that

$$\begin{aligned}
&\frac{\partial\Delta\widetilde{\Pi}_0(E[Y_\sigma],Y_S,\phi,I)}{\partial I}\\
&= q\alpha\frac{(1-\phi)\frac{p_g-p_b}{p_b}\left[\overline{w}_1^{Off}(Y_S,\phi,I)\right]^2}{\left[(1-\phi)(1-p_bR)I+\overline{w}_1^{Off}(Y_S,\phi,I)\right]^2}\\
&\quad - q(1-\phi)(1-\alpha)\frac{\partial\widetilde{\pi}^{Off}(\overline{w}_1^{Off}(Y_S,\phi,I),\phi,I)}{\partial w_1}\frac{\partial\overline{w}_1^{Off}(Y_S,\phi,I)}{\partial I}(1-p_bR)I\\
&\quad + \left[\frac{\widetilde{e}^{Off}(\overline{w}_1^{Off}(Y_S,\phi,I),\phi,I)}{\partial I}+\frac{\widetilde{e}^{Off}(\overline{w}_1^{Off}(Y_S,\phi,I),\phi,I)}{\partial w_1}\frac{\partial\overline{w}_1^{Off}(Y_S,\phi,I)}{\partial I}\right]\\
&\qquad \times(mR-c'(\widetilde{e}^{Off}))>0,
\end{aligned} \tag{IA.23}$$

If $Y_S\in[\underline{Y}_S(\phi,I),\widetilde{Y}_S(\phi,I)]$, we have

$$\Delta\widetilde{\Pi}_0(E[Y_\sigma],Y_S,\phi,I)=q(1-\phi)(1-\alpha)\pi^{On}(1-p_bR)I-\int_{\widetilde{e}^{Off}(\overline{Y}(\phi,I),\phi,I)}^{e^{On}(Y)}(mR-c'(e))de, \tag{IA.24}$$

$$\begin{aligned}
\frac{\partial\Delta\widetilde{\Pi}_0(E[Y_\sigma],Y_S,\phi,I)}{\partial I}&=q(1-\phi)(1-\alpha)\pi^{On}(1-p_bR)\\
&\quad+\left[\frac{\widetilde{e}^{Off}(\widetilde{Y}_S(\phi,I),\phi,I)}{\partial I}+\frac{\widetilde{e}^{Off}(\widetilde{Y}_S(\phi,I),\phi,I)}{\partial w_1}\frac{\partial\widetilde{Y}_S(\phi,I)}{\partial I}\right]\\
&\qquad\times(mR-c'(\widetilde{e}^{Off}))>0
\end{aligned} \tag{IA.25}$$

If $Y_S\geq\widetilde{Y}_S(\phi,I)$, we have

$$\Delta\widetilde{\Pi}_0(E[Y_\sigma],Y_S,\phi,I)=q(1-\phi)(1-\alpha)\pi^{On}(1-p_bR)I-\int_{\widetilde{e}^{Off}(Y_S,\phi,I)}^{e^{On}(Y)}(mR-c'(e))de, \tag{IA.26}$$

$$\frac{\partial\Delta\widetilde{\Pi}_0(E[Y_\sigma],Y_S,\phi,I)}{\partial I}=q(1-\phi)(1-\alpha)\pi^{On}(1-p_bR)+\frac{\widetilde{e}^{Off}(Y_S,\phi,I)}{\partial I}(mR-c'(\widetilde{e}^{Off}))>0. \tag{IA.27}$$

It then follows that there exists a threshold $\widetilde{I}\in\mathbf{R}^+\cup\{\infty\}$, such that $\Delta\widetilde{\Pi}_0(E[Y_\sigma],Y_S,\phi,I)>0$ if and only if $I>\widetilde{I}$. Notice that $\widetilde{I}>0$ because $\Delta\widetilde{\Pi}_0(E[Y_\sigma],Y_S,\phi,0)<0$.

Next, we consider how $\widetilde{I}$ depends on m. As $m\to 0$, $\Delta\widetilde{\Pi}_0(E[Y_\sigma],Y_S,\phi,I)>0$ for all I. This implies that there exists $\widetilde{m}>0$, such that $\widetilde{I}\neq\infty$ for all $m<\widetilde{m}$.

Finally, we consider, as $\phi \to 0$, how $\widetilde{I}$ compares to $\overline{I}$ given by Proposition 1. Comparing the results Claims IA.5–IA.7 to those of Proposition 1 and Lemma 3, we have that

$$\lim_{\phi \to 0} \widetilde{\pi}^{\mathit{Off}}(\widetilde{w}_1^{\mathit{Off}}(Y_S, \phi, I), \phi, I) \begin{cases} < \pi^{\mathit{Off}}(Y_S, I), & \text{if } Y_S < \overline{Y}_S(I), \\ = \pi^{\mathit{Off}}(Y_S, I), & \text{if } Y_S \geq \overline{Y}_S(I), \end{cases} \tag{IA.28}$$

and that

$$\lim_{\phi \to 0} \widetilde{e}^{\mathit{Off}}(\widetilde{w}_1^{\mathit{Off}}(Y_S, \phi, I), \phi, I) \begin{cases} > e^{\mathit{Off}}(Y_S, I), & \text{if } Y_S < \overline{Y}_S(I), \\ = e^{\mathit{Off}}(Y_S, I), & \text{if } Y_S = \overline{Y}_S(I), \\ < e^{\mathit{Off}}(Y_S, I), & \text{if } Y_S > \overline{Y}_S(I). \end{cases} \tag{IA.29}$$

This implies that $\lim_{\phi \to 0} \Delta\widetilde{\Pi}_0(E[Y_\sigma], Y_S, \phi, I) > \Delta\Pi_0(E[Y_\sigma], Y_S, I)$ if and only if $Y_S < \overline{Y}_S(I)$. Equivalently, $\widetilde{I} < \overline{I}$ if and only if $Y_S < \overline{Y}_S(I)$. $\square$