Bank Net Worth and Frustrated Monetary Policy Online Appendix

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B Extensions

This online appendix has extensions of the model and is supplemental.

B.1 Variable investment scale

A shortcoming of the model with unit firm investment is that aggregate lending and investment are unrealistically fixed. Fluctuations in bank net worth only affect the *price* of loans. There is no means to study how the *quantity* of credit fluctuates. One way to allow variable loan quantity is to insert a downward sloping demand curve for firm investment. That way, aggregate investment, lending, and output in the economy can adjust with the state of bank net worth. Another advantage of inserting variable investment is that total lending of a bank can be nicely decomposed into two parts: the *scale* (the quantity lent at each location) and the *scope* (the space of locations). Bank equity will affect each part differently.

B.1.1 Demand curve for loans

Isoelastic investment demand and linear production generate tractable results. So, let $\iota_{s(i)}$ denote the investment demand of entrepreneur s who borrows from bank i. The quantity $\iota_{s(i)}$ is the scale of investment in a firm. Because firms are financed entirely using bank credit, $\iota_{s(i)}$ is the amount lent to entrepreneur s. Let $y_{s(i)}$ denote the production output of the firm. The investment demand and production functions are

$$\iota_{s(i)} = \frac{b}{r_{L,i}}, \tag{18}$$

$$y_{s(i)} = \kappa \iota_{s(i)}, \tag{19}$$

where $b > 0$ is a constant, $\kappa = \overline{\kappa}$ if the firm is successful, and $\kappa = \underline{\kappa}$ if the firm fails. Let σ be the probability of success, just as in the case with unit investment. The value of a firm to an entrepreneur is the expected output from production, net of the principal and interest payments on a loan. Again, the entrepreneur expects to receive nothing should his or her firm fail.

The expected utility of entrepreneur s is

$$U_{s(i)} = \sigma \left(\overline{\kappa} - r_{L,i}\right) \iota_{s(i)} - \tau |s - i|,$$

and he or she can also choose an outside option worth w. To conserve notation, let

$$v_{s(i)} \equiv \sigma \left(\overline{\kappa} - r_{L,i}\right) \iota_{s(i)}$$

be the expected net consumption of an entrepreneur who starts a firm. Using exactly the same procedure as before, one can construct monopoly and competitive demand curves for bank i. They are

$$\Delta_{i,m} = \frac{v_{s(i)} - w}{\tau/2},$$

$$\Delta_{i,c} = \frac{v_{s(i)} - v_s}{\tau} + \frac{1}{n},$$

where $v_s \equiv \sigma\left(\overline{\kappa} - r_L\right)\iota_s$ is the expected net consumption for the entrepreneur if he or she borrowed from a neighboring bank that charged loan rate r_L. These two demand curves are analogous to their counterparts in (4) and (5) when investment was fixed.

B.1.2 Bank problem

A bank's profit function consists of loan repayments from successful firms, recovery values from failed firms, and financial payments to depositors and equity holders. For simplicity, suppose $\underline{\kappa} = 0$, which makes recovery valueless. Banks still issue loans along arcs Δ_i centered at their home locations, but now, each loan has amount ι_i. To distinguish these two dimensions of lending, I refer to Δ_i as the *scope* of bank i's loan portfolio and ι_i as the *scale* of the portfolio. Bank i's total lending is $\Delta_i\iota_i$.

Like before, a bank faces a minimum equity capital requirement: $e_{0,i} + e_i \geq \lambda\left(\Delta_i\iota_i + f\right)$, which now accounts for the variable scale of lending. The constraint again binds. With this, the profit function of a bank is

$$\pi_i = \sigma r_{L,i}\Delta_i\iota_i - r_\lambda\left(\Delta_i\iota_i + f\right), \tag{20}$$

where again $r_\lambda = (1-\lambda)\,r + \lambda r_e$ is the weighted average cost of bank funding.

B.1.3 Equilibrium

The definition of an equilibrium is identical to Definition 2.8. Like before, the cost of bank equity r_e is pinned down by clearing in the equity market. Rather than (6), however, the equity market clearing condition is now

$$\xi = \lambda\left(\iota + nf\right) - ne_0. \tag{21}$$

Also, instead of a circle, the equilibrium is represented by a cylinder, as depicted in Figure 6. In equilibrium, each bank has loan portfolio scope $\Delta = \frac{1}{n^*}$ and scale $\iota = \frac{b}{r_L^*}$, where the number of banks is n^* and lending rate is r_L^*.

I focus again on kinked and competitive equilibria. Like before, the kinked lending rate is taken off the monopoly portion of a bank's demand curve, whereas the competitive lending rate comes from the first order condition of a bank's problem. Proposition 7 presents the two loan rates.

Proposition 7. *(Loan rates, variable investment scale)* When the scale of firm investment can vary, the bank loan rate in a competitive equilibrium is

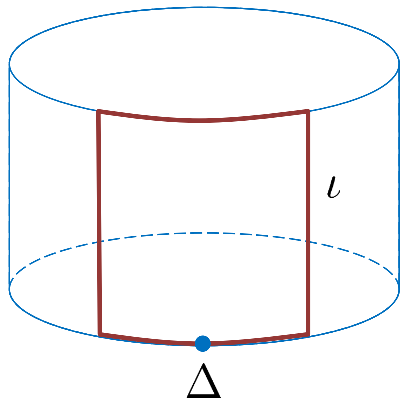
$$r_{L,c} = \frac{1}{\frac{\sigma}{r_{\lambda,c}} - \frac{\tau}{b\sigma\overline{\kappa}n_c}}. \tag{22}$$

The kinked equilibrium loan rate is

$$r_{L,k} = \frac{\overline{\kappa}}{1 + \frac{1}{b\sigma}\left(w + \frac{\tau}{2n_k}\right)}. \tag{23}$$

The comparative statics of both lending rates are the same as in the unit investment case. Less risky firms (higher σ) are charged a lower competitive loan rate but a higher kinked

Figure 6: Equilibrium Loan Market, Variable Investment Scale



Notes: Firms are uniformly distributed around the base of the cylinder. A typical bank is located at the dot. Each bank's loan portfolio in equilibrium has scope $\Delta = \frac{1}{n^*}$ and scale $\iota = \frac{b}{r_{L^*}}$,.

rate. A larger distance cost τ increases the competitive rate but decreases the kinked. More competition (higher n) decreases the competitive rate but perversely increases the kinked rate for the same reasons given in the unit investment case. Again, in a kinked equilibrium the interest rate channel is closed—with $r_{\lambda,k}$ not entering the price of firm credit—whereas in a competitive equilibrium, the channel is open.

What is mainly different now, compared to the unit scale case, is the presence of the investment demand parameter b. When b is larger, demand for loans is higher, and locally monopolist banks in a kinked equilibrium exploit the higher demand by charging a higher loan rate. In contrast, competitive banks charge a lower rate when b is higher. The lower rate is a consequence of the unitary elasticity for investment demand. Aggregate loan repayments (principal plus interest) $r_L \iota = b$ is constant. A positive increase in investment demand compels competitive banks to lower rates or lose customers. Each bank expects to issue a larger loan per borrower, so it charges a lower rate on the amount lent.

Using the equity market clearing condition in (21), one can solve for the weighted average cost of bank funding $r_{\lambda,c}$ in a competitive equilibrium when the number of banks is held fixed at n_c. In a kinked equilibrium, one cannot perform that exercise because the cost of bank funding $r_{\lambda,k}$ does not directly enter the equity market clearing condition. It only enters indirectly through n_k.

The cost of bank funding in a competitive equilibrium with variable investment is

$$r_{\lambda,c} = \frac{\lambda b \sigma}{\xi - n_c (f\lambda - e_0) + \frac{\lambda \tau}{\bar{\kappa} \sigma n_c}}.$$

Expansions in aggregate external equity ξ or retained earnings e_0 lower the cost of equity $r_{e,c}$ and by extension the cost of overall bank funding $r_{\lambda,c}$. A tighter equity capital constraint share λ raises the cost of equity. A larger demand for investment funds (higher b) also raises bank funding costs. Greater loan demand increases the amount of external funding banks require to supply the loans, which puts upward pressure on $r_{e,c}$.

Substituting $r_{\lambda,c}$ into the price of firm credit in (22) expresses both the competitive loan rate and aggregate investment as functions of bank net worth:

$$r_{L,c} = \frac{b\lambda}{\xi - n_c\left(f\lambda - e_0\right)}, \tag{24}$$

$$\iota_c = \frac{\xi - n_c\left(f\lambda - e_0\right)}{\lambda}. \tag{25}$$

From (24) and (25), one can see that better capitalized banks (higher ξ or e_0) reduce the price of firm credit, which encourages firm borrowing and enlarges investment and output. Here too one can see that the scale ι_c of a typical bank's loan portfolio also expands with its net worth.

B.1.4 Proof of Proposition 7

For the kinked lending rate, substitute the entrepreneur's expected net consumption from running the firm

$$v_{s(i)} = \sigma b\left(\frac{\overline{\kappa}}{r_{L,i}} - 1\right)$$

into the monopoly demand curve $\Delta_{i,m} = \frac{v_{s(i)}-w}{\tau/2}$. Solving for the lending rate and using the equilibrium condition $\Delta_{i,m} = \frac{1}{n_k}$ in a kinked equilibrium delivers (23).

For the competitive lending rate, start with the first derivative of the profit function with respect to the lending rate r_L:

$$\pi' = \sigma r_L\left(\Delta\iota' + \Delta'\iota\right) + \sigma\Delta\iota - r_\lambda\left(\Delta\iota' + \Delta'\iota\right).$$

Re-arranging gives the first order condition in the competitive case:

$$\left(\sigma r_{L,c} - r_\lambda\right)\left(\Delta\iota' + \Delta'\iota\right) = -\sigma\Delta\iota.$$

In the competitive equilibrium, $\Delta_{i,c} = \frac{1}{n_c}$.Furthermore, $\iota' = -\frac{b}{r_{L,c}^2}$ and $\Delta'_{i,c} = -\frac{\sigma\overline{\kappa}b}{\tau r_{L,c}^2}$. Making these substitutions and solving for $r_{L,c}$ gives (22).

B.2 Bank diversification

Here, I provide an extension in which banks weigh the benefits of diversifying into new locations against the consequences of other banks' competitive responses. The key distinguishing feature from the model in the main text is that firm outputs are now pairwise correlated. The size of a bank's loan portfolio Δ_i directly influences its level of diversification and the amount of external equity issued. To simplify, I set the probability of success $\sigma = \frac{1}{2}$ and the starting retained earnings $e_{0,i} = 0$. In this new environment, I show that the results on monetary transmission from the main model are unchanged.

The economy lasts for one period. Firms produce one of two possible outputs $\overline{\kappa}$ or $\underline{\kappa}$ at the end of the period. However, the *probability* that a firm produces the high output (succeeds) takes a special form. This form allows all firms to have the same *expected* probability of success prior to financing, but different *actual* probabilities of success after initiation. At the beginning of the period, the probability that the firm located at point j on the circle succeeds is *random*. This random probability takes the form

$$\widetilde{\Pr}\left(H|j,\tilde{u}\right)=\frac{1}{2}\left(1+\cos\left(2\pi\left(j+\tilde{u}\right)\right)\right), \tag{26}$$

where $\tilde{u}\sim U\left[0,1\right]$. The object in (26) is a random measure that maps a realization of the uniform shock $\tilde{u}$ to a probability distribution over the high and low outputs at location j. I call (26) the *success probability* of a firm.

The shock $\tilde{u}$ is realized in the middle of the period. The periodicity of the cosine function guarantees $\widetilde{\Pr}:[0,1]\mapsto[0,1]$. Other properties of a firm's success probability are presented in Lemma 1.

Lemma 1. The *success probability* in (26) satisfies the following properties:

1. (*Distributional symmetry*) The probability density function of a firm's success probability is the same at all locations.
2. (*Mean and variance*) Each firm is expected to succeed half the time with variance $\frac{1}{8}$.
3. (*Distance-dependent covariance*) The covariance between the success probabilities of firms j and k on the circle is $\frac{1}{8}\cos\left(2\pi\left(j-k\right)\right)$.

The form in (26) makes a firm's probability of reaching the high state invariant to its location. Prior to the realization of $\tilde{u}$, the probability distribution of a firm's outcome is the same as that of its two neighbors. As a result, firms at every location share the same expected probability of generating the high output $\overline{\kappa}$—namely, $\frac{1}{2}$.

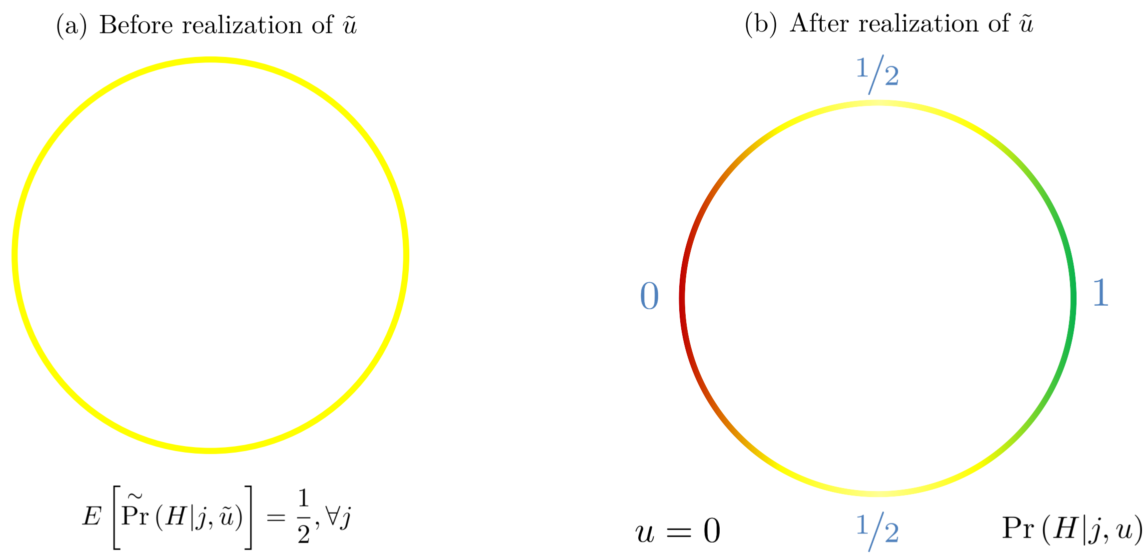
Another important feature of production is that the covariance of success probabilities between firms depends exclusively on the distance between those firms rather than on their locations. From the lemma, the correlation between the success probabilities of firms located at positions j and k on the circle is

$$\operatorname{corr}\left(\widetilde{\Pr}\left(H|j,\tilde{u}\right),\widetilde{\Pr}\left(H|k,\tilde{u}\right)\right)=\cos\left(2\pi\left(j-k\right)\right).$$

The above expression implies firms located near one another on the circle have more positively correlated probabilities of success than those located farther apart. Firms positioned opposite one another on the circle have the lowest correlated probability. This correlation structure is meant to capture the notion of integrated industries (e.g., metals and automobiles) or nearby geographic areas (e.g., neighboring cities) sharing more correlated production outcomes than more "distant" ones.

Figures 7(a)-7(b) present an illustration of firm uncertainty. At the start of the period, each firm around the circle bears the same uncertainty of firm success, having one-half chance of producing the high output. In Figure 7(a), that symmetry is marked by a common color yellow throughout the circle. Once the shock $\tilde{u}$ is drawn in the middle of the period, firms bear different probabilities of success according to their locations. Firms near one another on the circle share similar likelihoods of generating the high output. In Figure 7(b), that asymmetry is marked by the different colors of red (low probability of success) and green (high probability of success).

Figure 7: Example of Success Probabilities across Firms



Notes: At the beginning of the period, all firms share the same expected success probability of one-half. This common probability of success in expectation is represented in Figure 7(a) by the color yellow along the entire circle. In the middle of the period, the shock $\tilde{u}$ is realized. The example in Figure 7(b) has a realized value of $u = 0$. At that moment, firms differ in their success probabilities according to (26). In Figure 7(b), arcs of the circle with firms having high success probability are colored green. Arcs with firms having low success probability are colored red. The four numbers positioned around the circle are the success probabilities of the firms located at those positions.

A firm's life follows this sequence: At the beginning of the period, the firm is financed and the entrepreneur's investment is made. In the middle of the period, $\tilde{u}$ is realized, which determines the firm's actual probability of high production, denoted $\Pr\left(H|j,u\right)$. No action related to firm financing or firm investment can be made at that time. Finally, at the end of the period, the firm produces either the high or low output.

B.2.1 Bank diversification

The new form of firm uncertainty prevents banks from perfectly diversifying their portfolios with infinitesimal lending arcs. Facing correlated firm productions, banks adjust the size of their loan portfolios to influence their degree of diversification. Consider a typical bank i on the circle. For a given lending arc Δ_i, a fraction of a bank's borrowers will repay their loans. Prior to the realization of $\tilde{u}$, this fraction is unknown. I call this fraction the (random) *repayment rate* of bank $i's$ loan portfolio. It is defined as

$$\begin{aligned}\widetilde{\Pr}\left(H|j,\tilde{u}\right) &= \frac{1}{\Delta_i}\int_{-(\Delta_i)/2}^{(\Delta_i)/2}\frac{1}{2}\left(1+\cos\left(2\pi\left(i+j+\tilde{u}\right)\right)\right)dj \\ &= \frac{1}{2}+\underbrace{\frac{\sin\left(\pi\Delta_i\right)}{\Delta_i}}_{\text{diversification}}\underbrace{\frac{\cos\left(2\pi\left(i+\tilde{u}\right)\right)}{2\pi}}_{\substack{\text{residual}\\ \text{uncertainty}}}. \end{aligned} \tag{27}$$

Two components comprise the repayment rate of a portfolio: diversification and residual uncertainty. The diversification component captures the reduction in the uncertainty of a loan portfolio's payoff when a bank chooses a larger arc length around the circle. The residual uncertainty component reflects the risk that remains in an imperfectly diversified loan portfolio. Important properties of the repayment rate are presented in Lemma 2.

Lemma 2. The *repayment rate* of bank $i's$ portfolio satisfies the following properties:

1. (*Common mean*) The expected repayment rate is always $\frac{1}{2}$, no matter the choice of Δ_i.
2. (*No diversification*) As $\Delta_i \downarrow 0$, the bank's repayment rate approaches the same probability of a single firm succeeding.
3. (*Declining variance*) As Δ_i increases, the variance of the repayment rate declines.
4. (*Perfect diversification*) As $\Delta_i \uparrow 1$, the repayment rate approaches $\frac{1}{2}$, no matter the realization of $\tilde{u}$.

The repayment rate of an imperfectly diversified bank is a random variable prior to the realization of $\tilde{u}$. Nevertheless, the *expected* repayment rate on a portfolio is always $\frac{1}{2}$, no matter the size of Δ_i. A bank expects half its loan portfolio to repay and half to default.

As a bank lends to more and more entrepreneurs around the circle, it reduces the variability of its repayment rate by diversifying its loan portfolio. Eventually, if a bank lends the circumference of the circle, its portfolio becomes risk-free and immune to the random realization of $\tilde{u}$. In this case, half the portfolio *will* succeed and half *will* fail.

B.2.2 Bank capital structure

A bank's capital structure consists of equity and deposits. Given the uncertainty of firm outcomes, bank equity capital is risky and equity holders are willing to bear that risk. Depositors, on the other hand, have a strict preference for risk-free, safe assets. Because there is no federal deposit insurance, depositors impose there own kind of guarantee that deposits are fully repaid. They do so by capping the quantity lent to a bank by the maximum amount they can recover if the bank defaulted. This way, bank deposits are fully collateralized.

The maximum amount recoverable is the lowest possible profit on a bank's loan portfolio. Part of the profits from lending is the amount collected from borrower repayment. Because a bank always earns more on loans repaid than loans liquidated, the lowest possible loan profits is associated with the *minimum* repayment rate.[3] Taking the minimum of (27) over

[3] Bank i suffers its minimum repayment rate if the shock lands at location $|\frac{1}{2}-i|$ if $i \in \left[0,\frac{1}{2}\right]$ or location $1-|\frac{1}{2}-i|$ if $i \in \left(\frac{1}{2},1\right)$. If the shock lands at some other location for all $i \in \mathcal{N}$, where $\mathcal{N}$ is the set of bank locations, every bank's realized repayment rate exceeds the minimum.

the shock $\tilde{u}$ gives

$$\Pr_{\min}\left(r_{L,i}|\Delta_i\right) = \frac{1}{2}\left(1 - \frac{\sin\left(\pi\Delta_i\right)}{\pi\Delta_i}\right).$$

Denote the minimum loan profits for bank i as $\pi_{i,\min}$. At the beginning of the period, depositors are willing to lend an amount up to the discounted face value of $\pi_{i,\min}$. Because deposits are safe, the deposit rate is the risk-free interest rate r. Let d_i be the amount of deposits a bank chooses. Banks can raise deposits up to the amount

$$d_i \leq \frac{\pi_{i,\min}}{r}. \tag{28}$$

I call the maximum amount a bank can raise in deposits its *debt capacity.* Because constraint (28) ensures that bank deposits are risk-free, it can also be considered a kind of deposit insurance that is established by the market instead of the government. Additional financial capital is obtained from the equity market at the required expected equity return r_e. Like in the main model, equity will be at least as expensive as debt ($r_e \geq r$) in equilibrium.

The minimum repayment rate of a bank determines its debt capacity, which influences its leverage. If a bank contemplates increasing Δ_i and expanding its lending operations to more and more industries or geographic areas across the circle, it would diversify its loan portfolio. In that case, depositors would be willing to lend more to the bank. The minimum repayment rate of a bank is increasing in Δ_i, and so too will its debt capacity and leverage. Hence, the model produces a positive correlation between bank diversification and leverage, which Liang and Rhoades (1991), McAllister and McManus (1993) and Demsetz and Strahan (1997) document empirically.

The debt capacity constraint in (28) can equivalently be considered a minimum equity capital requirement that is imposed by the market rather than an outside rule. Denote the total assets of the bank by $a_i = \Delta_i + f$. Substituting the balance sheet identity $a_i \equiv d_i + e_i$, one can write the constraint as $e_i \geq \Delta_i + f - \frac{\pi_{i,\min}}{r}$. So, rather than choosing an amount d_i in deposits, a bank instead chooses an amount e_i in equity, provided its choice satisfies a minimum amount. Because Δ_i and $\pi_{i,\min}$ are endogenous, so too is the equity capital requirement.

B.2.3 Bank decision

With this set-up, expected profits of a typical bank are

$$\pi_i = \frac{1}{2} r_{L,i}\Delta_i + \frac{1}{2}\underline{\kappa}\Delta_i - FC\left(\Delta_i\right). \tag{29}$$

The financing cost function $FC\left(\Delta_i\right)$ consists of the payments to depositors and equity holders. The function is

$$FC\left(\Delta_i\right) = rd_i + r_e\left(\Delta_i + f - d_i\right). \tag{30}$$

The minimum loan profits that determine the constraint of (28) are

$$\begin{aligned}\pi_{i,\min} &= \Pr_{\min}\left(r_{L,i}|\Delta_i\right) r_{L,i}\Delta_i \\ &\quad + \left(1 - \Pr_{\min}\left(r_{L,i}|\Delta_i\right)\right)\underline{\kappa}\Delta_i.\end{aligned}$$

In its decision problem, a bank maximizes (29) subject to (28).

B.2.4 Monetary transmission

Like the main model, monetary transmission depends on the kind of equilibrium. Entrepreneur preferences are identical to those in the main text, which makes the demand curve for bank credit the same as well. Because $r_e \geq r$ in equilibrium, (28) will bind. Solving a bank's problem gives the competitive and kinked lending rates. Proposition 8 presents those rates.

Proposition 8. *(Lending rates)* The bank lending rate in a competitive equilibrium that features an endogenous equity capital requirement is

$$r_{L,c} = \frac{r_{e,c} + \frac{1}{2}\left(\frac{2\tau}{n_c} - \underline{\kappa}\right) + \left(\frac{r_{e,c}}{r} - 1\right)\phi\left(\frac{1}{n_c}\right)}{\frac{1}{2} + \left(\frac{r_{e,c}}{r} - 1\right)\psi\left(\frac{1}{n_c}\right)}, \tag{31}$$

where the functions ϕ and ψ are defined in Online Appendix B.2.10 by equations (37) and (38), respectively. The kinked equilibrium lending rate is

$$r_{L,k} = \overline{\kappa} - 2w - \frac{\tau}{n_k}. \tag{32}$$

B.2.5 Pass-through when the equity constraint is slack

Consider first the competitive lending rate. Suppose the supply of equity were so large that the equity market cleared at the lower-bound price $r_e = r$. In this situation, deposits and equity are perfect substitutes, so a bank faces a single cost of financial capital r. Because the bank could finance itself entirely with equity, the constraint (28) would be slack. The functions ϕ and ψ in (31) reflect a bank's debt capacity. They enter the lending rate if the constraint binds. They are set to zero for now.

The lending rate in such a competitive equilibrium would be

$$r_{L,c} = 2r - \underline{\kappa} + \frac{2\tau}{n_c}. \tag{33}$$

The competitive lending rate is analogous to the one in the main text given in (7). It consists of the marginal cost of financing plus a markup. The interest rate enters the bank lending rate linearly, making monetary transmission one-for-one after the adjustment for the riskiness of the loan via the leading coefficient 2.

B.2.6 Pass-through when the equity constraint binds

Staying in a competitive equilibrium, now suppose equity were scarce, so that the equity market cleared at price $r_e > r$. A bank would have strict preference for cheaper deposit financing and raise the maximum amount of deposits. The equity requirement binds, and the debt capacity of the bank now becomes important when the bank chooses its competitive loan rate.

That loan rate is now (31). The rate reflects the blend of debt and equity in the bank's capital structure. The functions ϕ and ψ capture this blend, and they adjust the marginal cost of financing after an incremental growth in the loan portfolio. This fact can be seen clearly from Lemma 3 below, which presents a typical bank's marginal financing cost function $FC'(\Delta_i)$.

Lemma 3. (*Marginal financing cost*) The marginal financing cost function of a typical bank i is

$$\frac{dFC(\Delta_i)}{d\Delta_i} = r_e + \left(\frac{r_e}{r} - 1\right)\left(\phi(\Delta_i) - \psi(\Delta_i)\, r_{L,i}\right). \tag{34}$$

A marginal increase in a bank's loan portfolio has two effects on its cost of funding. The first effect is a higher financing cost from the need for more equity at price r_e to fund the portfolio. The second effect is a decrease in the cost of funding as the bank tilts its capital structure to cheaper deposit financing because of greater diversification.

The second term in (34) is the cost savings from diversification. It reflects the change in the minimum possible profit $\pi_{i,\min}$, which influences a bank's debt capacity. For a fixed lending rate $r_{L,i}$, an expansion in Δ_i increases the debt capacity of the bank at rate ψ. The higher debt capacity generates financing cost savings at rate $\left(\frac{r_e}{r} - 1\right)$, which are passed onto entrepreneurs via a lower lending rate.

Greater diversification decreases a bank's minimum failure rate on its portfolio $(1 - \mathrm{Pr}_{\min})$. While a lower failure rate means the bank will receive payment $r_{L,i}$ on more of its loans, it also means the bank will retrieve the low output $\underline{\kappa}$ on less of its loans, as fewer will default. Less recovery values from fewer defaults reduces the minimum loan profit of the bank and its debt capacity. The function ϕ is the rate at which debt capacity decreases with Δ_i. Lower debt capacity increases marginal financing costs, which raises the competitive lending rate.

When the market's constraint on bank financing binds, the degree of pass-through now relies on bank capital structure. The relation is non-linear, and interest rate pass-through depends on the functions ϕ and ψ, which reflect a bank's debt capacity and its diversification. The pass-through also depends on the cost of equity capital r_e, which will be a function of aggregate bank net worth.

B.2.7 Negative pass-through

In a kinked equilibrium, the interest rate channel is closed completely in the "short run" when the number of banks n_k is held fixed for the same reasons given in the main text. One can see the absence of pass-through from (32). A lower interest rate will only have an indirect effect on bank lending rates through adjustments in the number of banks n_k over the "long run."

A lower interest rate will reduce the average cost of operating a bank, increase profits, and encourage entry into the lending market. An important perverse feature of the kinked lending rate in (8), however, is that more banks in the credit market leads all of them to *raise* their loan rates.

In a kinked credit market, banks are local monopolists. More banks on the circle means that an entrepreneur can find one that specializes in an industry or area "closer" to the entrepreneur's preferred location. A bank takes advantage of its greater local monopoly power by charging a higher lending rate.

Conversely, fewer banks lead all of them to *reduce* their loan rates. When a bank exits the lending market, the average entrepreneur needs to "travel" a longer distance on the circle and contract with a bank that is less specialized in his or her particular industry or location than before. Undertaking the firm becomes less attractive to the entrepreneur relative to the outside option w. Because a typical bank in a kinked market is competing against its borrowers' outside options, it needs to lower the lending rate to encourage an entrepreneur to borrow instead.

By stimulating bank entry, an accommodative monetary policy has the unintended effect of increasing the cost of bank credit to firms and worsening the commercial loan spread. I call a decrease to the policy rate that leads to an increase in the bank loan rate (and vice versa) "negative pass-through."

To study all the effects of a policy rate cut, consider again the expected profit function of a bank:

$$\pi_i = \frac{1}{2}\left(r_{L,i} + \underline{\kappa}\right)\Delta_i - r_e\left(\Delta_i + f\right) + \left(r_e - r\right)\frac{\pi_{i,\min}}{r}.$$

A direct effect of a decline in r is to encourage bank entry from higher profits because of lower funding costs. Another first order effect works in the same direction: A lower policy rate increases the debt capacity $\frac{\pi_{i,\min}}{r}$ of a bank, allowing it to raise cheaper debt financing over equity capital. This effect can be considered an "asset pricing" channel of monetary policy in this extension because the rate cut increases the value of the collateral banks post to raise cheaper debt, which increases expected bank profits. These first order effects lead to more entry and negative pass-through.

One second order equilibrium effect works in the same direction: Higher debt capacities reduce demand for equity capital, which lowers the cost of equity r_e and further increases profits and entry. Another second order effect comes from the reaction of the equity market and works in the opposing direction: More banks in the lending market increases the demand for equity capital, which puts upward pressure on r_e and dampens entry. A final second order effect also works in the opposite direction: More banks on the circle reduces the arc length of each one and the minimum possible profits $\pi_{i,\min}$, which limits the capacity to raise cheaper debt and reduces entry.

Provided the positive forces for entry dominate, a kinked equilibrium features negative pass-through once accounting for entry. Figure 8 illustrates the effect of a lower interest rate on the average cost curve that encourages bank entry and negative pass-through. The average cost curve shifts inward from the lower funding costs, which raises profits of existing banks and gives reason for other banks to enter the lending market.

B.2.8 Proof of Lemma 1

The success probability of a firm at location j from (26) is

$$\widetilde{\Pr}\left(H|j,\tilde{u}\right) = \frac{1}{2}\left(1 + \cos\left(2\pi\left(j + \tilde{u}\right)\right)\right).$$

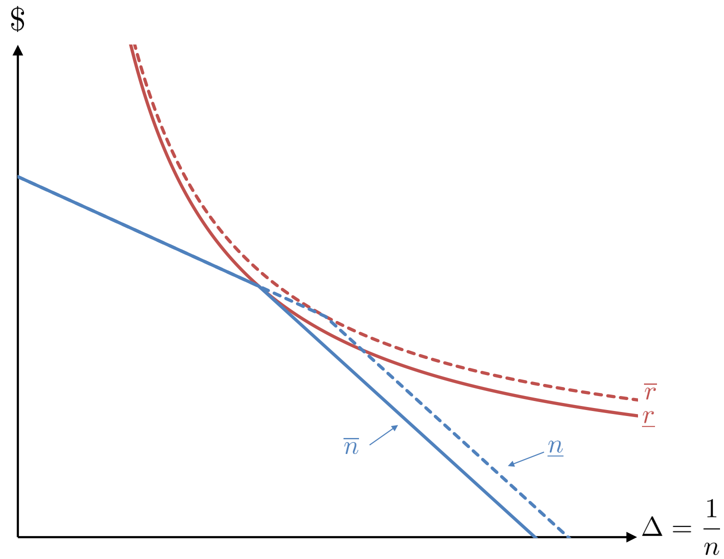
The success probability can be treated as a transformation of a uniform random variable. Let $x_j = 2\pi\left(j + \tilde{u}\right)$ be that transformation. The random variable x_j has support $\left[2\pi j, 2\pi\left(j+1\right)\right]$. For ease of notation, define

$$Y_j \equiv \frac{1}{2}\left(1 + \cos\left(x_j\right)\right).$$

For any $y \in [0,1]$, the equation

$$y = \frac{1}{2}\left(1 + \cos\left(x_j\right)\right)$$

Figure 8: Bank Entry from Lower Policy Rate, Kinked Equilibrium



Notes: The policy rate declines from $\overline{r}$ to $\underline{r}$, which shifts the red average cost curve inward from the dotted to solid line. This change leads the number of banks to increase from $\underline{n}$ to $\overline{n}$, which pushes the blue average revenue curve inward from the dotted to solid line. In the example, ex post profits $v = 0$.

has two solutions in $x_j \in [2\pi j, 2\pi(j+1)]$. Therefore, the transformed density is

$$\begin{aligned} f_{Y_j}(y) &= 2 \times \frac{2}{\sqrt{1-y^2}} \times \frac{1}{2\pi} \\ &= \frac{2}{\pi\sqrt{1-y^2}}, \end{aligned}$$

for $y \in [0,1]$ and zero otherwise for all j. The leading factor of 2 accounts for the two solutions in the support. Thus, the density of the success probability is the same at all locations.

The expected probability of success for a single firm is $\frac{1}{2}$. Because $\tilde{u} \sim U[0,1]$, integrate the success probability over the unit interval to get

$$\begin{aligned} E\left[\widetilde{\Pr}(H|j,\tilde{u})\right] &= \int_0^1 \frac{1}{2}\left(1 + \cos\left(2\pi(j+\tilde{u})\right)\right) d\tilde{u} \\ &= \frac{1}{2} + \frac{1}{2\pi}\left[\sin(2\pi(j+1)) - \sin(2\pi j)\right] \\ &= \frac{1}{2} + \frac{1}{2\pi}\left[\sin(2\pi j)\cos(2\pi) + \cos(2\pi j)\sin(2\pi) - \sin(2\pi j)\right] \\ &= \frac{1}{2} + \frac{1}{2\pi}\left[\sin(2\pi j) - \sin(2\pi j)\right] \\ &= \frac{1}{2}, \end{aligned}$$

where the third equality follows from the sum-difference formula for sine.

The variance of the success probability is

$$\begin{aligned}\sigma^2\left[\widetilde{\Pr}\left(H|j,\tilde{u}\right)\right] &= E\left[\widetilde{\Pr}\left(H|j,\tilde{u}\right)^2\right] - E\left[\widetilde{\Pr}\left(H|j,\tilde{u}\right)\right]^2 \\ &= \frac{1}{4}E\left[\left(1+\cos\Big(2\pi\left(j+\tilde{u}\right)\Big)\right)^2\right] - \frac{1}{4} \\ &= \frac{1}{4}\int_0^1 \cos^2\Big(2\pi\left(j+\tilde{u}\right)\Big)\,d\tilde{u}.\end{aligned}$$

Using the half-angle trigonometric formula $\cos^2 u = \frac{1+\cos(2u)}{2}$, the variance can be written as

$$\begin{aligned}\sigma^2\left[\widetilde{\Pr}\left(H|j,\tilde{u}\right)\right] &= \frac{1}{8}\int_0^1 \left[1+\cos\left(4\pi\left(j+\tilde{u}\right)\right)\right]d\tilde{u} \\ &= \frac{1}{8}\left[1+\frac{1}{4\pi}\left[\sin\left(4\pi\left(j+1\right)\right)-\sin\left(4\pi j\right)\right]\right] \\ &= \frac{1}{8}.\end{aligned}$$

Finally, the covariance of success probabilities between firms located at j and k on the circle is

$$\begin{aligned}\operatorname{cov}\left(\widetilde{\Pr}\left(H|j,\tilde{u}\right),\widetilde{\Pr}\left(H|k,\tilde{u}\right)\right) &= E\left[\widetilde{\Pr}\left(H|j,\tilde{u}\right)\widetilde{\Pr}\left(H|k,\tilde{u}\right)\right] - \frac{1}{4} \\ &= \frac{1}{4}E\left[\cos\Big(2\pi\left(j+\tilde{u}\right)\cos\Big(2\pi\left(k+\tilde{u}\right)\Big)\right].\end{aligned}$$

Using the cosine product formula $\cos\left(a\right)\cos\left(b\right) = \frac{1}{2}\left[\cos\left(a+b\right)+\cos\left(a-b\right)\right]$ gives

$$\begin{aligned}\operatorname{cov}\left(\widetilde{\Pr}\left(H|j,\tilde{u}\right),\widetilde{\Pr}\left(H|k,\tilde{u}\right)\right) &= \frac{1}{8}\cos\left(2\pi\left(j-k\right)\right)+\frac{1}{8}E\left[\cos\left(2\pi\left(j+k+2\tilde{u}\right)\right)\right] \\ &= \frac{1}{8}\cos\left(2\pi\left(j-k\right)\right)+\frac{1}{8}\int_0^1\cos\left(2\pi\left(j+k+2\tilde{u}\right)\right)d\tilde{u} \\ &= \frac{1}{8}\cos\left(2\pi\left(j-k\right)\right).\end{aligned}$$

B.2.9 Proof of Lemma 2

The repayment rate of bank i, conditional on its chosen arc length Δ_i, is

$$\widetilde{\Pr}\left(r_{L,i}|\Delta_i,\tilde{u}\right) = \frac{1}{2}+\frac{\sin\left(\pi\Delta_i\right)}{\Delta_i}\frac{\cos\Big(2\pi\left(i+\tilde{u}\right)\Big)}{2\pi}.$$

As $\Delta_i \downarrow 0$, the bank's lending arc reduces to its home location. To see this, apply L′Hôpital′s rule:

$$\begin{aligned}
\lim_{\Delta_i \downarrow 0} \widetilde{\Pr}\left(r_{L,i}|\Delta_i, \tilde{u}\right) &= \lim_{\Delta_i \downarrow 0} \left\{ \frac{1}{2} + \pi \cos\left(\pi \Delta_i\right) \frac{\cos\Big(2\pi\left(i+\tilde{u}\right)\Big)}{2\pi} \right\} \\
&= \frac{1}{2}\Big(1+\cos\Big(2\pi\left(i+\tilde{u}\right)\Big)\Big) \\
&= \widetilde{\Pr}\left(H|j,\tilde{u}\right).
\end{aligned}$$

Next, to see that the expected repayment rate of a bank's portfolio is always $\frac{1}{2}$, no matter its arc length Δ_i, integrate the repayment rate over the unit interval:

$$\begin{aligned}
E\left[\widetilde{\Pr}\left(r_{L,i}|\Delta_i,\tilde{u}\right)\right] &= \int_0^1 \left[\frac{1}{2} + \frac{\sin\left(\pi\Delta_i\right)}{\Delta_i}\frac{\cos\Big(2\pi\left(i+\tilde{u}\right)\Big)}{2\pi}\right] d\tilde{u} \\
&= \frac{1}{2} + \frac{\sin\left(\pi\Delta_i\right)}{\left(2\pi\right)^2\Delta_i}\left[\sin\left(2\pi\left(i+1\right)\right) - \sin\left(2\pi i\right)\right] \\
&= \frac{1}{2}.
\end{aligned}$$

The variance of the repayment rate is

$$\begin{aligned}
\sigma^2\left[\widetilde{\Pr}\left(r_{L,i}|\Delta_i,\tilde{u}\right)\right] &= \int_0^1 \left(\widetilde{\Pr}\left(r_{L,i}|\Delta_i,\tilde{u}\right) - E\left[\widetilde{\Pr}\left(r_{L,i}|\Delta_i,\tilde{u}\right)\right]\right)^2 d\tilde{u} \\
&= \int_0^1 \left(\widetilde{\Pr}\left(r_{L,i}|\Delta_i,\tilde{u}\right) - \frac{1}{2}\right)^2 d\tilde{u} \\
&= \int_0^1 \left(\frac{\sin\left(\pi\Delta^i\right)}{\Delta^i}\frac{\cos\Big(2\pi\left(i+\tilde{u}\right)\Big)}{2\pi}\right)^2 d\tilde{u} \\
&= \left[\frac{\sin\left(\pi\Delta_i\right)}{2\pi\Delta_i}\right]^2 \int_0^1 \cos^2\Big(2\pi\left(i+\tilde{u}\right)\Big) d\tilde{u}.
\end{aligned}$$

Using the half-angle formula again, the variance can be written as

$$\begin{aligned}
\sigma^2\left[\widetilde{\Pr}\left(r_{L,i}|\Delta_i,\tilde{u}\right)\right] &= \frac{1}{2}\left[\frac{\sin\left(\pi\Delta_i\right)}{2\pi\Delta_i}\right]^2 \int_0^1 \left[1+\cos\left(4\pi\left(i+\tilde{u}\right)\right)\right] d\tilde{u} \\
&= \frac{1}{2}\left[\frac{\sin\left(\pi\Delta_i\right)}{2\pi\Delta_i}\right]^2 + \frac{1}{8\pi}\left[\frac{\sin\left(\pi\Delta_i\right)}{2\pi\Delta_i}\right]^2 \left[\sin\left(4\pi\left(i+1\right)\right) - \sin\left(4\pi i\right)\right] \\
&= \frac{1}{2}\left[\frac{\sin\left(\pi\Delta_i\right)}{2\pi\Delta_i}\right]^2. \qquad (35)
\end{aligned}$$

The variance of the repayment rate is strictly decreasing for $\Delta_i \in (0,1)$. To see why, take the first derivative of (35) with respect to Δ_i:

$$\begin{aligned} \frac{\partial \sigma^2\left[\widetilde{\Pr}\left(r_{L,i}|\Delta_i,\tilde{u}\right)\right]}{\partial \Delta_i} &= \left[\frac{\sin\left(\pi\Delta_i\right)}{2\pi\Delta_i}\right]\frac{\partial\frac{\sin(\pi\Delta_i)}{2\pi\Delta_i}}{\partial\Delta_i} \\ &= \left[\frac{\sin\left(\pi\Delta_i\right)}{2\pi\Delta_i}\right]\left[\frac{\pi\Delta_i\cos\left(\pi\Delta_i\right)-\sin\left(\pi\Delta_i\right)}{2\pi\left(\Delta_i\right)^2}\right] \\ &= \frac{\pi\Delta_i\cos\left(\pi\Delta_i\right)\sin\left(\pi\Delta_i\right)-\sin^2\left(\pi\Delta_i\right)}{4\pi^2\left(\Delta_i\right)^3}. \end{aligned} \tag{36}$$

The sign of $\frac{\partial \sigma^2\left[\widetilde{\Pr}\left(r_{L,i}|\Delta_i,\tilde{u}\right)\right]}{\partial \Delta_i}$ is determined by the numerator of (36), as the denominator is always positive. The variance is non-increasing in Δ_i if the numerator

$$\pi\Delta_i\cos\left(\pi\Delta_i\right)\sin\left(\pi\Delta_i\right)-\sin^2\left(\pi\Delta_i\right)\leq 0.$$

For $\Delta_i = 0$ and $\Delta_i = 1$, the numerator is zero, making the above inequality hold. For $\Delta_i \in (0,1)$, $\sin\left(\pi\Delta_i\right) \neq 0$, so the expression can be written as

$$\pi\Delta_i\cos\left(\pi\Delta_i\right)\leq\sin\left(\pi\Delta_i\right).$$

Perform a change of variable $\theta = \pi\Delta_i$. The aim is to show

$$\theta\cos\theta < \sin\theta$$

over the domain $\theta \in (0,\pi)$. Because $\cos\theta \leq 0$ and $\sin\theta > 0$ over the interval $\theta \in \left[\frac{\pi}{2},\pi\right)$, the relation holds there. Now define the function

$$f\left(\theta\right) \equiv \sin\theta - \theta\cos\theta.$$

Note that $\lim_{\theta\downarrow 0} f\left(\theta\right) = 0$ and $f'\left(\theta\right) = \theta\sin\theta > 0$ for $\theta \in \left(0,\frac{\pi}{2}\right)$. Therefore, $f\left(\theta\right) > 0$ over the lower half of the interval. This proves the numerator of (36) is negative for $\Delta_i \in (0,1)$ and that the variance of the repayment rate is non-increasing for $\Delta_i \in [0,1]$ and strictly decreasing over $\Delta_i \in (0,1)$.

Finally, as $\Delta_i \uparrow 1$, the repayment rate has the following limit:

$$\begin{aligned} \lim_{\Delta_i\uparrow 1}\widetilde{\Pr}\left(r_{L,i}|\Delta_i,\tilde{u}\right) &= \frac{1}{2}+\sin\left(\pi\right)\frac{\cos\Big(2\pi\left(i+\tilde{u}\right)\Big)}{2\pi}, \\ &= \frac{1}{2}. \end{aligned}$$

Thus, the repayment rate becomes a constant $\frac{1}{2}$, no matter the realization of the random variable $\tilde{u}$.

B.2.10 Proof of Proposition 8 and Lemma 3

In a competitive equilibrium, the first order condition for optimality is

$$\frac{1}{2}\left(r_{L,i}+\underline{\kappa}\right)+\frac{1}{2}\Delta_i\left(\frac{dr_{L,i}}{d\Delta_i}\right)=\frac{dFC\left(\Delta_i\right)}{d\Delta_i}.$$

From (30) and a binding constraint (28), the marginal financing cost function is

$$\frac{dFC\left(\Delta_i\right)}{d\Delta_i}=r_{e,c}+\left(1-\frac{r_{e,c}}{r}\right)\frac{d\pi_{i,\min}}{d\Delta_i}.$$

Computing $\frac{d\pi_{i,\min}}{d\Delta_i}$ gives for the marginal financing cost function:

$$\begin{aligned}\frac{dFC\left(\Delta_i\right)}{d\Delta_i} &= \left(1-\frac{r_{e,c}}{r}\right)\left(1-\Pr_{\min}\left(\Delta_i\right)\right)\underline{\kappa}\\ &\quad+\left(1-\frac{r_{e,c}}{r}\right)\left(r_{L,i}+\Delta^i\left(\frac{dr_{L,i}}{d\Delta_i}\right)\right)\Pr_{\min}\left(\Delta_i\right)\\ &\quad+\left(1-\frac{r_{e,c}}{r}\right)\left(r_{L,i}-\underline{\kappa}\right)\Delta_i\frac{d\Pr_{\min}\left(\Delta_i\right)}{d\Delta_i}\\ &\quad+r_{e,c}.\end{aligned}$$

Substituting the slope of the competitive demand curve $\frac{dr_{L,i}}{d\Delta_i}=-2\tau$ and re-arranging terms gives

$$\begin{aligned}\frac{dFC\left(\Delta_i\right)}{d\Delta_i} &= \left(1-\frac{r_{e,c}}{r}\right)\left[\Pr_{\min}\left(\Delta_i\right)\left(r_{L,i}-2\tau\Delta_i\right)+\left(1-\Pr_{\min}\left(\Delta_i\right)\right)\underline{\kappa}\right]\\ &\quad+\left(1-\frac{r_{e,c}}{r}\right)\left(r_{L,i}-\underline{\kappa}\right)\Delta_i\frac{d\Pr_{\min}\left(\Delta_i\right)}{d\Delta_i}+r_{e,c}.\end{aligned}$$

The first line in the above expression is the marginal financing cost savings from greater diversification and larger debt capacity, holding fixed the minimum repayment rate. The first term of the second line is the cost savings from a higher minimum repayment rate after a marginal increase in the loan portfolio size Δ_i. The bank gets repaid on more of its loans and recovers less of the failed firm production. The final term $r_{e,c}$ of the second line is the marginal cost of equity financing.

The marginal financing cost function can be conveniently represented by separating terms involving $r_{L,i}$. Doing so gives

$$\begin{aligned}
\frac{dFC(\Delta_i)}{d\Delta_i} &= -\left(\frac{r_{e,c}}{r}-1\right)\left[\Pr_{\min}(\Delta_i)+\frac{d\Pr_{\min}(\Delta_i)}{d\Delta_i}\Delta_i\right]r_{L,i} \\
&\quad +\left(\frac{r_{e,c}}{r}-1\right)\left[2\tau\Delta_i\Pr_{\min}(\Delta_i)-\underline{\kappa}\left(1-\Pr_{\min}(\Delta_i)\right)\right] \\
&\quad +\left(\frac{r_{e,c}}{r}-1\right)\underline{\kappa}\Delta_i\frac{d\Pr_{\min}(\Delta_i)}{d\Delta_i} \\
&\quad +r_{e,c}.
\end{aligned}$$

Next, define the functions

$$\begin{aligned}
\phi(\Delta_i) &\equiv 2\tau\Delta_i\Pr_{\min}(\Delta_i)-\underline{\kappa}\left(1-\Pr_{\min}\left(\Delta^i\right)\right) \\
&\quad +\underline{\kappa}\Delta_i\frac{d\Pr_{\min}(\Delta_i)}{d\Delta_i}, \qquad (37) \\
\psi(\Delta_i) &\equiv \Pr_{\min}(\Delta_i)+\frac{d\Pr_{\min}(\Delta_i)}{d\Delta_i}\Delta_i, \qquad (38)
\end{aligned}$$

and re-write the marginal financing cost function as

$$\frac{dFC(\Delta_i)}{d\Delta_i}=r_{e,c}+\left(\frac{r_{e,c}}{r}-1\right)\left(\phi(\Delta_i)-\psi(\Delta_i)\,r_{L,i}\right).$$

The function $\psi > 0$. Provided $\underline{\kappa}$ is not too large, then $\phi > 0$. The terms in ϕ are those that increase the marginal cost of financing for the bank, whereas those in ψ decrease the marginal cost of financing.

Using the representation of $\frac{dFC(\Delta_i)}{d\Delta_i}$ in the last expression above, the optimality condition becomes

$$\frac{1}{2}\left(r_{L,i}+\underline{\kappa}-2\tau\Delta_i\right)=r_{e,c}+\left(\frac{r_{e,c}}{r}-1\right)\left(\phi(\Delta_i)-\psi(\Delta_i)\,r_{L,i}\right).$$

Solving for $r_{L,i}$ and using the equilibrium condition $\Delta_i=\frac{1}{n_c}$ gives the competitive equilibrium lending rate:

$$r_{L,c}=\frac{r_{e,c}+\frac{1}{2}\left(\frac{2\tau}{n_c}-\underline{\kappa}\right)+\left(\frac{r_{e,c}}{r}-1\right)\phi\left(\frac{1}{n_c}\right)}{\frac{1}{2}+\left(\frac{r_{e,c}}{r}-1\right)\psi\left(\frac{1}{n_c}\right)}.$$

In a kinked equilibrium, the first order condition for optimality does not hold with equality, so the lending rate is instead the monopoly demand curve. Just as in the main text, solving (4) for the lending rate and setting $\Delta_i=\frac{1}{n_k}$ gives (32).

B.3 Welfare analysis

In this section, I analyze the welfare implications of the model and explore the differences between the decentralized equilibrium and the social optimum. The (consumer) surplus of an entrepreneur that is located a distance x from his or her bank is

$$s_e=\overline{\kappa}-\frac{w}{\sigma}-r_L-\tau x.$$

The (producer) surplus of a bank obtained on a loan to a borrower that is located a distance x from the bank is

$$s_b = r_L + (1-\sigma)\,\underline{\kappa} - r_\lambda,$$

where $r_\lambda = (1-\lambda)\,r + \lambda r_e$ is the marginal cost of bank financing. With n banks operating in the loan market, the marginal borrower travels a distance $1/2n$. Aggregating across borrowers and banks, total welfare is

$$\begin{aligned} W &= 2n \int_0^{1/2n} (s_b + s_e)\,dx - nfr_\lambda \\ &= 2n \int_0^{1/2n} \left[\overline{\kappa} - \frac{w}{\sigma} - (r_\lambda - (1-\sigma)\,\underline{\kappa}) - \tau x\right] dx - nfr_\lambda. \end{aligned}$$

Integrating gives the welfare function

$$W = \overline{\kappa} - \frac{w}{\sigma} - (r_\lambda - (1-\sigma)\,\underline{\kappa}) - \frac{\tau}{4n} - nfr_\lambda. \tag{39}$$

The number of banks that maximizes welfare is

$$n^* = \frac{1}{2}\sqrt{\frac{\tau}{r_\lambda f}}. \tag{40}$$

Comparing n^* to the competitive and kinked number of banks in (10) and (11), one can observe that if ex post profits $v = 0$,

$$n^* < n_k < n_c$$

for fixed bank cost of funding r_λ. This relation implies that the decentralized equilibria feature excess variety: too many banks serve the loan market and concentration is too low relative to the social optimum. Nevertheless, when ex post profits are strictly positive, the central bank can set them so that concentration increases to match the social optimum. For example, in the case of the competitive equilibrium, if

$$v = 3fr_\lambda,$$

then $n^* = n_c$. The larger the financed cost of entry, the larger the gap between the decentralized and welfare-maximizing concentration. A central bank can close this gap with higher bank rents that restricts entry.

Another notable feature about welfare in the kinked equilibrium is that certain comparative statics are perverse. For example, consider total entrepreneur welfare:

$$\begin{aligned} W_e &= 2n \int_0^{1/2n} s_e(x)\,dx \\ &= \overline{\kappa} - \frac{w}{\sigma} - r_L - \frac{\tau}{4n}. \end{aligned}$$

Substituting the kinked lending rate from (8) gives

$$W_e = \frac{\tau(2-\sigma)}{4\sigma n}.$$

One can see that if the distance cost τ increases, entrepreneur welfare *increases*. Recall that the kinked loan rate declines if τ increases, both directly and indirectly via a decline in the number of banks. So, although the average disutility from travel has increased with a rise in τ, the drop in the loan rate is more than enough to compensate borrowers so that entrepreneur welfare rises.

Finally, we can examine welfare with an endogenous cost of bank funding r_λ upon equity market clearing. Doing so reveals another tool the central bank can use to reduce entry and induce the decentralized equilibrium to match the social optimum. That tool is the capital share requirement λ. Indeed, using the competitive cost of bank funding $r_{\lambda,c}$ from (12), one can see that the competitive equilibrium is socially optimal when

$$\lambda = \frac{e_0 + \xi\sqrt{\frac{4v}{3\tau}}}{f - \sqrt{\frac{4v}{3\tau}}}.$$

The capital requirement is increasing in both internal equity e_0 and ξ, which suggests a procyclical capital requirement can maximize welfare. At times when the supply of bank equity is flush and the cost of equity is low, excessive entry occurs. A higher capital requirement raises demand for external equity and puts upward pressure on the cost of equity, which curtails entry and makes the social optimum achievable.

B.4 Price discriminating banks

In this extension, I assume that banks can identify the location of any prospective borrower. They are free to price discriminate by offering a loan rate that depends on the borrower's location. For simplicity let a firm's recovery value $\underline{\kappa} = 0$. Despite the presence of price discrimination, the results on monetary transmission from the main model are the same.

I consider first-degree price discrimination in that a bank can capture the entire consumer surplus. A simple way to insert price discrimination is to allow banks to charge a personalized fixed premium to each entrepreneur for taking out a loan. The fixed premium could be a loan application or closing fee.

The premium would need to depend on the borrower's distance from the bank. It would be highest for those closest to the bank because these borrowers retain the largest surplus under uniform pricing, which the main model uses. The personalized fixed premium is a two-part tariff or affine pricing schedule. It is equivalent to a system of personalized prices in which each borrower pays a sum equal to his or her willingness to pay.

Under both the kinked and competitive cases, let $S_i(x)$ be the net surplus for an entrepreneur located a distance x from bank i. The bank is charging lending rate $r_{L,i}$. The total amount of money $T_i(x)$ the entrepreneur pays for a loan from bank i is

$$T_i(x) = S_i(x) + r_{L,i}.$$

B.4.1 Kinked case

In the kinked case, the indifference condition for the entrepreneur located a distance x from bank i is

$$\sigma\left(\overline{\kappa}-r_{L,i}\right)-\tau x=w.$$

Without price discrimination, the equilibrium kinked lending rate is $r_{L,k}=\overline{\kappa}-\frac{w}{\sigma}-\frac{\tau}{2\sigma n_k}$. For an entrepreneur to have surplus, it must mean that

$$\sigma\left(\overline{\kappa}-r_{L,i}\right)-\tau x-w\geq 0.$$

The surplus is the left-hand-side of this inequality. Substituting the kinked lending rate for $r_{L,i}$ gives

$$S_i\left(x\right)=\tau\left(\frac{1}{2n_k}-x\right).$$

The surplus is positive for $x\leq\frac{1}{2n_k}$. This upper bound on x is the edge of bank $i's$ local monopoly market. The personalized premium is decreasing in the borrower's distance from bank i. The bank has the most market power over those borrowers nearest to it. The nearest entrepreneurs can be charged the largest premium and yet still take out the loan.

Because the kinked interest rate with price discrimination does not depend on the interest rate (holding fixed the number of banks), the absence of interest rate pass-through in a kinked equilibrium holds.

B.4.2 Competitive case

For the competitive case, it is easiest to assume that the supply of equity is so large that $r_{e,c}=r$, which makes the equity capital requirement slack. The profit function is

$$\pi=\sigma r_{L,i}\Delta_i-r\left(\Delta_i+f\right),$$

and the equilibrium competitive lending rate is

$$r_{L,c}=\frac{1}{\sigma}\left(r+\frac{\tau}{n_c}\right).$$

The indifference condition for a borrower located a distance x from bank i is

$$\sigma r_{L,i}+\tau x=\sigma r_L+\tau\left(\frac{1}{n_c}-x\right).$$

The entrepreneur has surplus when borrowing from bank i so long as

$$\sigma r_L+\tau\left(\frac{1}{n_c}-x\right)-\left(\sigma r_{L,i}+\tau x\right)\geq 0.$$

Simplifying gives the surplus when borrowing from bank i:

$$S_i\left(x\right)=\sigma\left(r_L-r_{L,i}\right)+\tau\left(\frac{1}{n_c}-2x\right).$$

From above, the lower that bank i sets the lending rate, the more surplus goes to the entrepreneur; the closer the entrepreneur is to bank i, the more surplus he or she receives.

In the symmetric equilibrium, bank lending rates match, so the surplus is

$$S(x) = \tau \left(\frac{1}{n_c} - 2x \right).$$

This surplus is positive for $x \leq \frac{1}{2n_c}$. An entrepreneur located at distance $x = \frac{1}{2n_c}$ is in between the two banks. That entrepreneur is marginal, so he or she will not be charged a personalized premium. Everyone else will be charged the fixed premium according to their distances from the bank. Compared to the kinked case, the surplus in the competitive case declines twice as fast as x increases due to the entrepreneur's credible alternative of borrowing from a competing bank. Like in the main text, the competitive lending rate features no interrupted pass-through.

B.5 Smoothing the kink

In this extension, I provide one way to "smooth" the kink in the demand curve for bank credit (i.e., make the region differentiable) in order to study the effects on monetary transmission. For simplicity, I assume a bank cannot recover any value from a loan in default ($\underline{\kappa} = 0$) and that the probability of a loan's repayment $\sigma = \frac{1}{2}$. I also assume the bank is flush with equity so that its cost of financial capital is the interest rate r. Because smoothing the kink still preserves the concavity in the demand curve for bank loans, the results on monetary transmission from the main model are unchanged.

B.5.1 General Case

Generally, a firm with market power passes through less of its marginal costs to prices at points on a downward-sloping demand curve that feature greater concavity. A kink is a sharp way of creating concavity in the demand curve for loans when consumer preferences would otherwise imply a linear demand curve (and perfect pass-through). As long as the smoothing procedure preserves the highest concavity at the smoothed kink, the pass-through will be lowest there, just as when the kink is sharp.

One way to smooth the kink is to assume that banks are unsure about which demand curve they face when setting a loan price. Because banks set prices at the margin, they may equivalently be unsure about the slope of the demand curve at a given price.

The two possible slopes a bank faces is either $-\tau$ or -2τ. I assume the bank assigns probability h that the slope is -2τ, which is to say that it believes with probability h that it is competing with a neighboring bank. The probability with which the bank believes it is a local monopolist is $1 - h$.

I assume that h is an increasing function of a bank's market share Δ_i, is continuous, and is three times differentiable over the domain I specify below. This kind of uncertainty can be rationalized by a bank not knowing the precise boundary of its neighbor's market. The bank knows, however, that as it expands market share, it has more and more likely penetrated that boundary. I assume the bank knows the number of banks n operating in the lending market. Under these assumptions, the profit function of a bank is

$$\pi = \frac{1}{2} r_L \Delta - r \left(\Delta + f\right).$$

The bank will chose a market share Δ that satisfies the first order condition

$$\Omega \equiv r_L\left(\Delta\right) + \Delta r_L'\left(\Delta\right) - 2r = 0.$$

By the implicit function theorem, the quantity pass-through $\frac{\partial \Delta}{\partial r}$ is

$$\begin{aligned}
\frac{\partial \Delta}{\partial r} &= -\frac{\frac{\partial \Omega}{\partial r}}{\frac{\partial \Omega}{\partial \Delta}} \\
&= \frac{2}{r_L' + \Delta r_L''\left(\Delta\right) + r_L'} \\
&= \frac{2}{2r_L' + \Delta r_L''\left(\Delta\right)}.
\end{aligned}$$

By the chain rule, the interest rate pass-through is

$$\frac{\partial r_L}{\partial r} = r_L' \frac{\partial \Delta}{\partial r}.$$

Substituting the quantity pass-through gives

$$\frac{\partial r_L}{\partial r} = r_L' \left(\frac{2}{2r_L' + \Delta r_L''}\right).$$

By symmetry in market shares and a completely covered circle, $\Delta = \frac{1}{n}$. Substituting this relation into the interest rate pass-through above and re-arranging terms gives

$$\begin{aligned}
\frac{\partial r_L}{\partial r} &= \frac{2r_L'}{2r_L' + \frac{1}{n} r_L''}, \\
&= \frac{2}{2 + \frac{1}{n}\frac{r_L''}{r_L'}}.
\end{aligned}$$

Define the *concavity* of the demand curve as

$$\omega\left(\Delta\right) \equiv \frac{r_L''\left(\Delta\right)}{r_L'\left(\Delta\right)}.$$

The interest rate pass-through is then

$$\frac{\partial r_L}{\partial r} = \frac{2}{2 + \frac{\omega(\Delta)}{n}}.$$

With a downward sloping demand curve, a larger concavity implies a lower interest rate pass-through. Also, as the number of banks tends to infinity $n \to \infty$, the market reaches perfect competition and features perfect pass-through.

Returning to a bank's uncertainty over the slope of its demand curve, the slope of a bank's subjective demand curve given its beliefs about being a competitor or local monopolist is

$$\begin{aligned} r_L'(\Delta) &= -2\tau \times h(\Delta) - \tau \times (1 - h(\Delta)) \\ &= -\tau - \tau h(\Delta) \\ &= -\tau(1 + h(\Delta)). \end{aligned}$$

The second derivative is

$$r_L''(\Delta) = -\tau h'(\Delta).$$

Therefore, the concavity of the demand curve for loans is

$$\omega(\Delta) = \frac{h'(\Delta)}{1 + h(\Delta)}.$$

Because h is a probability, I require $h(\Delta) \geq 0$ for all Δ. I also assume that $h' \geq 0$, making the bank increasingly believe it is competing as it expands. These assumptions make $\omega(\Delta) \geq 0$. Furthermore, a bank knows that its headquarters is located at $\Delta = 0$. It also knows that its loan portfolio reaches the neighboring bank's headquarters when $\Delta = \frac{2}{n}$. Therefore, it is reasonable to assume that $\lim_{\Delta \downarrow 0} h(\Delta) = 0$ and $\lim_{\Delta \uparrow \frac{2}{n}} h(\Delta) = 1$. I define the domain of h to be the closed interval $\left[0, \frac{2}{n}\right]$.

If the concavity is uniquely highest at the point midway between headquarters of banks, then the lowest pass-through would occur at the location of the kink in the original model. The function ω should be globally maximized at $\frac{1}{n}$ and hence satisfy

$$\omega'\left(\frac{1}{n}\right) = 0, \tag{41}$$

$$\omega''\left(\frac{1}{n}\right) < 0. \tag{42}$$

A simple way to ensure the global maximum is uniquely reached at $\Delta = \frac{1}{n}$ is to assume that the function $\omega(\Delta)$ is strictly concave along the entire interval. A sufficient but not necessary condition for the global maximum is

$$\omega''(\Delta) < 0, \qquad \forall \Delta \in \left[0, \frac{2}{n}\right]. \tag{43}$$

The restrictions on the belief function h imposed by the conditions (41)-(42) will smooth the kink in the demand curve in a way to minimize the interest rate pass-through at the kink. If condition (43) is also imposed, interest rate pass-through is uniquely minimized at that point.

B.5.2 An example h

Given the assumptions for h, a cumulative distribution function with non-negative bounded support that satisfies (41)-(42), and where the maximum is unique, would deliver an appropriate example.

I use the beta distribution $h(x) = \text{Beta}(x; \alpha, \beta)$, where I make the linear transformation $x = \frac{\Delta}{\left(\frac{2}{n}\right)}$ that adjusts the support to $[0, 1]$. Thus, a bank that has a portfolio size $x = 1$ believes its loan portfolio has reached the headquarters of the two neighboring banks.

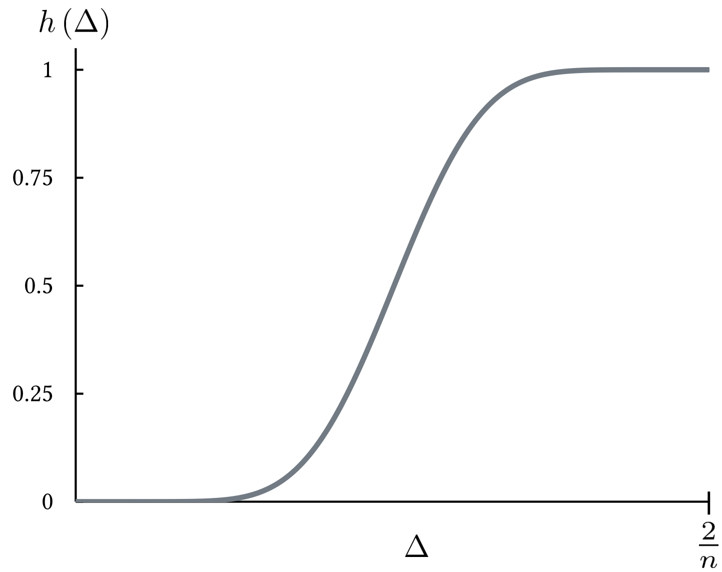
Choosing parameters α and β that deliver a global maximum for ω at $x = \frac{1}{2}$ would give what is needed, which is the highest curvature at the kink in the baseline model. I find those parameters numerically. I search for the parameter vector (α, β) that minimizes the function $|\text{argmax}_{x(\alpha,\beta)} \omega(x(\alpha, \beta)) - \frac{1}{2}|$ over the unit interval and confirm graphically that the solution is the parameter vector that delivers the global maximum of $\omega(x)$. I use 10,000 starting points for the optimization routine.

The optimal solution from the search is $\alpha = 1059$ and $\beta = 1046$. Nevertheless, the differences between the smooth and original demand curves under these parameters are difficult to see in a graph. Therefore, in the figures below, I use the parameters $\alpha = 10.59$ and $\beta = 10.46$, which make the differences clearer and deliver a concavity function ω that approximately is maximized at $x = \frac{1}{2}$.

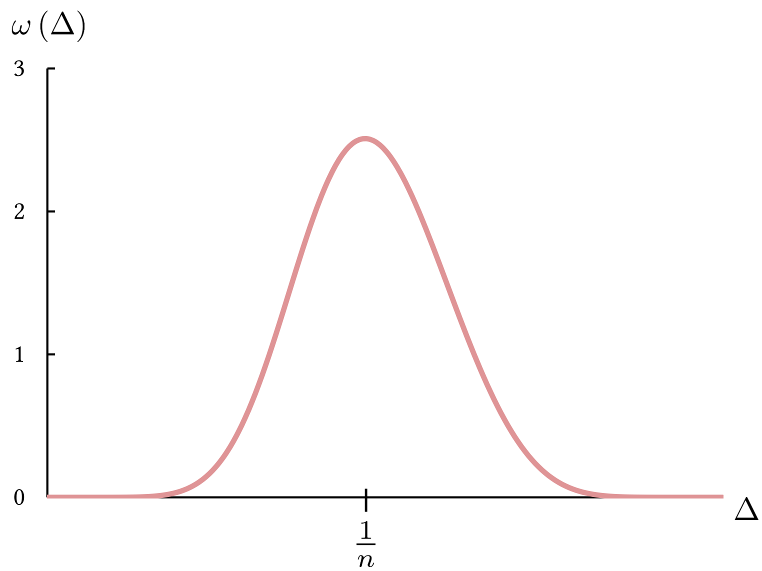
Figures 9(a)-9(b) plot the cumulative distribution function $h(\Delta)$ and the concavity function $\omega(\Delta)$.

Figure 9: Uncertainty Over the Demand Curve for Loans

(a) Cumulative distribution function $h(\Delta)$



(b) Concavity function $\omega(\Delta)$



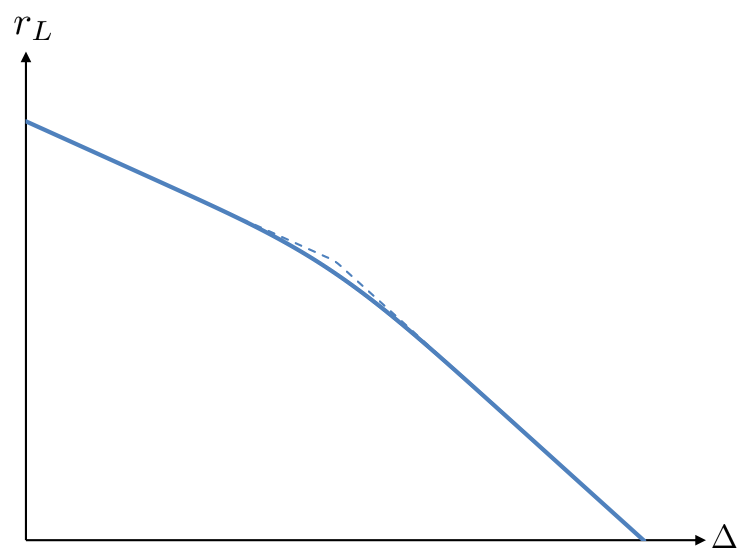
Notes: The function $h(\Delta)$ represents the probability a bank believes it is competing with a neighboring bank. This uncertainty affects the weight a bank places on the two possible slopes in the demand curve for loans. For h, I use a Beta $(x; \alpha, \beta)$ distribution. The function $\omega(\Delta)$ is the concavity of the subjective demand curve implied by the bank's beliefs. The support of the beta distribution has been transformed from the unit interval to $\left[0, \frac{2}{n}\right]$ in the figures. The scale parameters for the beta distribution are $\alpha = 10.59$ and $\beta = 10.46$.

Figure 9(b) reveals that the concavity of the subjective demand curve is maximized at $\frac{1}{n}$, which is the location of the kink. A plot of the probability density function associated with h would show that the uncertainty about the slope of the loan demand curve is concentrated around $\frac{1}{n}$, which is the location a bank could easily believe is still part of its local monopoly market or the beginning of its neighbor's market.

Figure 10 illustrates the original sharp kink featuring no uncertainty about the demand curve and the corresponding smoothed kink in which there is uncertainty. For $\Delta < \frac{1}{n}$, the smooth curve begins to deviate from the original curve once the bank starts assigning positive probability to competing with a neighbor. The bank reduces its lending rate faster in order to attract the marginal borrower because the bank believes it might now be competing for that customer. As Δ approaches $\frac{1}{n}$, the original kink is entirely "rounded out" and the smooth demand curve displays the most concavity. Interest rate pass-through will be lowest in that region, though not zero.

As the bank extends its loan portfolio further past $\frac{1}{n}$, it becomes more confident that it is competing with the neighbor, so the bank puts more weight on the slope of the demand curve being -2τ. Eventually, the original and smooth demand curves converge.

Figure 10: Smooth and Original Demand Curves



Notes: The dashed curve is the original demand curve that features a kink at $\Delta = \frac{1}{n}$. The solid curve is the smooth demand curve that is derived from a bank being uncertain about the slope of the demand curve it faces. The uncertainty is captured by a beta distribution with scale parameters $\alpha = 10.59$ and $\beta = 10.46$.

B.6 Deposit market power

In this extension, I assign market power to banks in the deposit market in addition to the loan market. I show that imperfect competition in deposits does not change the impaired interest rate pass-through to lending rates in a kinked loan market. Later, I add a reserve requirement, which also leaves that result unchanged.

I model market power among deposits using monopolistic competition in a spatial context akin to Besanko and Thakor (1992). Entrepreneur preferences and bank market structure in the loan market is the same as the main text. Hence, the bank credit demand curve will be identical to the one in the main text, which is displayed in Figure 2. Bank equity shareholders in the public capital market are modeled the same as well. The supply of equity is fixed again at ξ.

Unlike the main model, there is now a unit-measure continuum of households along the circle. Households have resources that they can either save via bank deposits or a risk-free asset in the capital market (a bond) that earns the gross monetary policy rate r (i.e., the effective federal funds rate). Households value deposits for both the return on savings and a liquidity benefit, which the bond lacks. Savers do not treat bank deposits as homogeneous financial products, but as differentiated ones. Like entrepreneurs, households prefer to deposit their funds in banks that are closest to their preferred location.

A natural interpretation for this spatial preference among depositors is an inclination to save with a bank geographically nearby in order to reduce physical transportation costs. But the spatial preference can also correspond to enjoying ancillary services like financial planning, cash management, technology at branches, etc. that varies across banks.

B.6.1 Deposit supply curve

If opting to save using bank deposits, households first choose which bank to use and second how much to deposit. Deposits here are interest-bearing accounts such as time and savings deposits. Suppose bank i offers a gross deposit rate $r_{d,i} = r - \delta_{d,i}$, where r is the federal funds rate and $\delta_{d,i}$ is the spread that bank i charges on its deposits. By this definition, if $\delta_{d,i} > 0$, the deposit rate $r_{d,i}$ is below the policy rate $r_{d,i} < r$.

If a depositor chooses to save with this bank, he or she will deposit an amount $s\left(r_{d,i}\right)$ to maximize the sum of an increasing and concave liquidity benefit $U\left(s\right)$ and the gross return on deposits $r_{d,i} \times s$. Hence, the amount saved with bank i ensures

$$s\left(r_{d,i}\right) = \arg\max_{s} \left\{U\left(s\right) + s r_{d,i}\right\}.$$

From the implicit function theorem, because $U' > 0$ and $U'' < 0$, the amount saved satisfies $s' > 0$.

Let $v\left(r_d\right) \equiv U\left(s\left(r_d\right)\right) + r_d s\left(r_d\right)$ denote the value function of a saver who faces a deposit rate r_d. For a depositor, the disutility per unit distance from saving with a less-than-ideal bank is χ. (Recall that for entrepreneurs, the per-unit-distance disutility is τ.) The utility for a household from saving in the bond yielding the federal funds rate r is $rs\left(r\right)$.

Denote by $l_{i,i+1}$ the distance from bank i's headquarters in the clockwise direction such that the household located at that distance is indifferent between depositing with bank i and saving in the bond via the capital market. Formally, $l_{i,i+1}$ satisfies

$$v\left(r_{d,i}\right) - \chi l_{i,i+1} = rs\left(r\right).$$

Solving for $l_{i,i+1}$ and multiplying by $2s\left(r_{d,i}\right)$—in order to reflect savers on either side of the bank that deposit $s\left(r_{d,i}\right)$—gives the bank's *monopoly* deposit supply function $s_{i,m}\left(r_{d,i}, r\right)$:

$$s_{i,m}\left(r_{d,i},r\right)=\frac{s\left(r_{d,i}\right)}{\chi/2}\left(v\left(r_{d,i}\right)-rs\left(r\right)\right). \tag{44}$$

The supply of deposits for a typical monopoly bank (in the deposit market) is increasing in its own deposit rate, decreasing in the federal funds rate r, and decreasing in the cost to travel to the bank χ. The bank implicitly competes against the alternative form of savings available to households.

When banks compete, bank deposits are a dominant investment over the bond for households. The utility from the liquidity benefit, plus a high deposit rate exceeds the utility from a bond investment. In this case, savers choose between banks instead of between deposits and bonds. By the same analysis in the main text for borrowers, the *competitive* deposit supply function $s_{i,c}\left(r_{d,i},r_d,n\right)$ for bank i offering deposit rate $r_{d,i}$ when neighboring banks are offering deposit rate r_d is

$$s_{i,c}\left(r_{d,i},r_d,n\right)=\frac{s\left(r_{d,i}\right)}{\chi}\left(v\left(r_{d,i}\right)-v\left(r_d\right)+\frac{\chi}{n}\right). \tag{45}$$

The supply of deposits for a typical competitive bank increases in its own deposit rate, decreases in the deposit rate of neighboring banks, and decreases in the number of banks n.

B.6.2 Bank problem

Just as in the main text, a typical bank's objective is to maximize profits from lending, taking as given the policies of the other banks in the credit market. The distinction here is that the bank must not only set its lending rate but also its deposit rate.

Banks have three forms of financing sources. The first is the public equity capital market, which supplies equity perfectly elastically at rate r_e. The second is the deposit market, which households supply at a deposit rate set by the bank $r_{d,i}$. The third is the federal funds market, which supplies reserves perfectly elastically at the policy (reserve) rate $r<r_e$. Since the 2007-2008 global financial crisis, the main supplier of reserves are institutions who are ineligible to receive interest on their reserve balances, particularly Government-sponsored enterprises (GSEs) (Bech and Klee (2011)). Therefore, the GSEs should be thought of as the suppliers (sellers) of federal funds to banks in the model, and this form of debt financing is an alternative to deposits.

With this in mind, the profit function of bank i is

$$\pi_i=\sigma r_{L,i}\Delta_i+(1-\sigma)\,\underline{\kappa}\Delta_i-FC\left(\Delta_i\right),$$

which is identical to that in the main text. The only difference is in the financing cost function $FC\left(\Delta_i\right)$, which now includes the endogenous deposit rate $r_{d,i}$, the quantity of deposits raised s_i, and the quantity of reserves raised b_i:

$$FC\left(\Delta_i\right)=r_{d,i}s_i+rb_i+r_e\left(e_{0,i}+e_i\right), \tag{46}$$

The first part of the financing cost function is the total cost of deposit financing: deposits raised s_i multiplied by the deposit rate $r_{d,i}$. The deposits raised will come off the monopoly $s_{i,m}$ or competitive $s_{i,c}$ deposit supply function. The second part is the total cost of reserve financing: the quantity of reserves b_i multiplied by the federal funds rate r. The third part of (46) is the total cost of bank equity capital that includes the starting capital $e_{0,i}$ and the amount raised by the bank e_i. Each bank faces an equity capital requirement that mandates the fraction λ of bank assets $\Delta_i + f$ is financed from equity:

$$e_{0,i} + e_i \geq \lambda \left(\Delta_i + f\right). \tag{47}$$

Banks also face a "reserve borrowing constraint," which requires that financing from reserves is non-negative:

$$b_i \geq 0. \tag{48}$$

This constraint prevents a bank from investing in reserves. Whether this constraint binds determines whether banks finance themselves on the margin with short term debt from the federal funds market or with deposits. In section B.6.4, I add a reserve requirement to a bank's problem, which requires a bank to maintain a certain fraction of deposits rather than lend that amount to entrepreneurs. The results are unchanged.

The bank's problem is to choose the lending and deposit rates $\{r_{L,i}, r_{d,i}\}$, the amount of external equity raised e_i, and the amount of reserves raised b_i that maximize profits, subject to (47) and (48), while considering strategies of every other bank on the circle.

B.6.3 Monetary transmission

To study monetary transmission, I again look at symmetric equilibria. I hold fixed the number of banks n and the cost of equity r_e. With price-setting banks in the deposit market, I examine interest rate pass-through of the policy rate r to both the equilibrium loan rate r_L and deposit rate r_d.

The economy effectively has two circular markets: one in bank credit and the other in bank deposits. Depending on parameter values and the quantity of external bank equity ξ, either circular market could be competitive or kinked. Therefore, four possible equilibria are possible: (1) competitive loan-competitive deposit, (2) competitive loan-kinked deposit, (3) kinked loan-competitive deposit, (4) kinked loan-kinked deposit.

If the loan market is kinked, the loan rate in equilibrium will again feature no component of a bank's marginal cost of financing, deposits or otherwise. Therefore, the loan rate in scenario (3) and (4) will be the kinked loan rate in (8) from the main text and feature no monetary transmission.

Competitive loan - kinked deposit Now suppose that scenario (2) prevails. This scenario is simpler to solve because the deposit rate is not determined from an optimality condition, but rather taken from the monopoly deposit supply curve. Because both reserves and bank equity are supplied perfectly elastically and $r < r_e$, the equity capital requirement in (47) always binds. Hence, $e_{0,i} + e_i = \lambda \left(\Delta_i + f\right)$. Using the binding capital requirement and the balance sheet identity $\Delta_i + f \equiv s_i + b_i + e_{0,i} + e_i$, one can express the amount of reserves raised as

$$b_i = (1 - \lambda)\left(\Delta_i + f\right) - s_i.$$

Hence, the profit function of a bank can be written as

$$\pi_i = \sigma r_{L,i} \Delta_i + (1 - \sigma)\, \underline{\kappa} \Delta_i + \left(r - r_{d,i}\right) s_i - r_\lambda \left(\Delta_i + f\right), \tag{49}$$

where $r_\lambda \equiv (1-\lambda) r + \lambda r_e$, just as in the main text. The first two terms in the profit equation are loan revenues from repayments and recoveries from liquidated loans. The third term is the cost savings from market power in the deposit market, which permits a lower cost of financing overall because $r_{d,i} \leq r$. The fourth term is the cost of loanable funds if the bank had no access to deposits. Let μ be the Lagrange multiplier on the reserve borrowing constraint. The complementary slackness conditions for a bank's problem are

$$
\begin{aligned}
&s_i \leq (1-\lambda)(\Delta_i + f), \\
&\mu \geq 0, \\
&0 = \mu\left((1-\lambda)(\Delta_i + f) - s_i\right).
\end{aligned}
$$

The equilibrium competitive loan rate $r_{L,c}$ is derived using the loan rate optimality condition for each bank and the loan market clearing condition $\Delta_i = \frac{1}{n}$. That competitive loan rate is

$$
r_{L,c} = \frac{1}{\sigma}\left(r_\lambda - \mu(1-\lambda) - (1-\sigma)\underline{\kappa} + \frac{\tau}{n}\right). \tag{50}
$$

The competitive loan rate is identical to that in the main text, save for the extra term $\mu(1-\lambda)$, which is the Lagrange multiplier times the share of bank debt financing $1-\lambda$. In the main text—where the deposit supply curve was perfectly elastic at deposit rate r—the marginal cost to a bank from lending was the weighted average funding cost r_λ. Here, the marginal cost includes a discount $\mu(1-\lambda)$ that captures the marginal cost savings from deposit financing.

If the reserve borrowing constraint is non-binding, the multiplier $\mu = 0$, and banks raise a positive amount of reserves. They do so because the amount of debt they need to finance their loan portfolios exceeds the amount they can raise from deposits at a deposit rate r_d that is less that the federal funds rate r. On the margin, banks use reserve financing rather than deposit financing. In this case, the marginal cost of loanable funds passed onto entrepreneurs is r_λ.

If the reserve borrowing constraint is binding, the multiplier $\mu > 0$, and banks can finance their loan portfolios entirely with the equity they are required to raise and the deposits they can raise at a deposit rate $r_d < r$. On the margin, banks use deposit financing, which lowers the cost of loanable funds. Note that the cost of loanable funds can be written as

$$
r_\lambda - \mu(1-\lambda) = (1-\lambda)(r-\mu) + \lambda r_e,
$$

so that μ is the marginal cost savings from deposit financing over reserve financing. Finally, note that a more stringent capital requirement (larger λ) forces banks to issue more equity capital and moves their marginal cost of funding closer to r_e.

If banks raise reserves in addition to deposits, the multiplier $\mu = 0$, the marginal cost of loanable funds is r_λ, and monetary transmission to lending rates $\frac{\partial r_L}{\partial r}$ is identical to that in the main text. The same applies in the section below with a competitive deposit market. Whenever $\mu = 0$ and banks tap the reserve market, transmission in a competitive loan market is unimpaired.

When the reserve borrowing constraint binds, $\mu > 0$, and banks raise only deposits without tapping the reserves market. In this case, monetary transmission is determined using a comparative statics approach. To pin down this sensitivity, differentiate the first order condition and the capital requirement with respect to the policy rate r to get

$$2\sigma\frac{\partial r_L}{\partial r}+(1-\lambda)\frac{\partial \mu}{\partial r}=1-\lambda$$
$$\left((1-\lambda)\frac{\partial \Delta}{\partial r_L}\right)\frac{\partial r_L}{\partial r}=\frac{\partial s_c}{\partial r_d}\frac{\partial r_d}{\partial r},$$

where $\frac{\partial s_c}{\partial r_d}=\frac{1}{\chi}\left(s^2\left(r_d\right)+v\left(r_d\right)\frac{\partial s}{\partial r_d}\right)>0$ is the change in a bank's deposit base after a marginal increase in the deposit rate when that bank's deposit rate is at the equilibrium value r_d, holding fixed all other banks' deposit rates at r_d.

These equations represent a linear system in $\left(\begin{array}{cc}\frac{\partial r_L}{\partial r}, & \frac{\partial \mu}{\partial r}\end{array}\right)'$. Solving the system gives

$$\frac{\partial r_L}{\partial r}=-\frac{\tau}{\sigma\left(1-\lambda\right)}\frac{\partial s_c}{\partial r_d}\frac{\partial r_d}{\partial r}, \tag{51}$$
$$\frac{\partial \mu}{\partial r}=1+\frac{2\tau}{\left(1-\lambda\right)^2}\frac{\partial s_c}{\partial r_d}\frac{\partial r_d}{\partial r}. \tag{52}$$

Because $\frac{\partial s_c}{\partial r_d}>0$, one can see that monetary transmission to loan rates depends on the sign of monetary transmission to deposit rates $\frac{\partial r_d}{\partial r}$. Consider (52) first. Recall that μ is the difference between the marginal cost of reserves (r) and the marginal cost of deposits. As I explain below, the kinked deposit rate is increasing in the policy rate, such that $\frac{\partial r_d}{\partial r}>0$. Therefore, when the deposit market is kinked *and* banks are financed only with deposits and equity, $\frac{\partial \mu}{\partial r}>1$, which implies that a federal funds rate hike *lowers* the marginal cost of deposits.

Though this comparative static may be counterintuitive, here is the explanation: The marginal cost of deposits is the deposit rate a bank must offer to expand its arc of deposits around the circle by a single increment. So, the deposit rate is *not* the marginal cost of deposits. (This idea is the "dual" of the price not being the marginal revenue of a monopolist.) Given household preference for proximity, the marginal cost of deposits is increasing as the bank draws deposits around the circle. Because a policy rate hike raises the deposit rate for all households, it increases the supply of deposits at every location on the circle. Hence, a bank does not need to expand as far (by raising its deposit rate on the margin) in order to raise deposits. And so, the marginal cost of deposits declines after an increase in r.

Next consider the interest rate pass-through to loan rates $\frac{\partial r_L}{\partial r}$. One can observe that transmission to loan rates is a multiple of the transmission to deposit rates. This is not surprising given that the policy rate influences bank loan rates via the cost of bank funding. With the equity rate r_e constant, transmission can only work through the deposit market, which is the marginal source of financing for the bank when the reserve borrowing constraint binds. Notice that bank loan rates react in opposite directions than deposit rates after policy rate changes when the deposit market is kinked. Hence, a federal funds rate hike lowers the cost of bank credit to firms because it lowers the marginal cost of deposits, as explained earlier.

Finally, the multiple $\frac{\tau}{\sigma(1-\lambda)}$ that converts deposit rate transmission to loan rate transmission (51) is increasing in the degree of bank market power τ, which is counterintuitive. More market power in the loan market leads banks to decrease loan rates by more after a deposit rate increase. The reason is similar to the negative pass-through in the main text in the kinked equilibrium upon bank entry. When banks compete against an outside option rather than with each other, a larger distance cost τ forces them to *lower* loan rates more in order to entice borrowers to borrow from a less preferred bank rather than pursue an outside option. Interestingly, perverse features of a kinked loan market enter a competitive loan market because the *deposit* market is kinked.

Kinked deposit To complete the picture on the competitive loan-kinked deposit equilibrium, I describe next a kinked deposit market. Again, the deposit rate is not set using a first order condition of optimality for the bank, but is taken off the monopoly portion of the deposit supply function. In a symmetric equilibrium, the deposit rate of each bank satisfies $r_{d,i} = r_d$, and each bank raises $s(r_d)/n$ in deposits. Setting the monopoly deposit supply function in (44) to this amount and re-arranging gives

$$U(s(r_d)) + r_d s(r_d) - rs(r) + \frac{\chi}{2n} = 0. \tag{53}$$

The deposit rate is defined implicitly in (53). To study monetary transmission to this rate, recall that the deposit rate is a spread over the policy rate: $r_d \equiv r - \delta_d$. The full measure of monetary transmission to the kinked deposit rate is therefore

$$\frac{\partial r_d}{\partial r} = 1 - \frac{\partial \delta_d}{\partial r}. \tag{54}$$

Monetary transmission in the deposit market will rely on the response of the deposit spread to changes in the policy rate.

Define the function F as the left-hand-side of (53):

$$F(\delta_d, r) \equiv U(s(r_d)) + r_d s(r_d) - rs(r) + \frac{\chi}{2n}.$$

Applying the implicit function theorem to F gives for the comparative static on the spread:

$$\frac{\partial \delta_d}{\partial r} = \frac{s(r_d) - s(r) - rs'(r)}{s(r_d)}.$$

Because $s' > 0$, and $r_d < r$—as banks would never post a deposit rate that exceeded the marginal cost of reserves—the sign of the spread change is $\frac{\partial \delta_d}{\partial r} < 0$. In this case, if the central bank undertakes a rate hike, banks compress the spread between the deposit rate and the policy rate. The monetary pass-through $\frac{\partial r_d}{\partial r} > 1$, implying a greater than one-for-one transmission. Banks pass through the initial rate hike and compress the spread in order to compete more effectively at drawing deposits away from the risk-free asset.

With monetary transmission to deposit rates established, we can fully specify the interest rate pass-through to loan rates. Substituting the transmission of deposit rates in (54) into that of loan rates in (51) gives the transmission to competitive loan rates when the deposit market is kinked and banks raise only deposits and equity instead of reserves:

$$\frac{\partial r_{L,c}}{\partial r} = -\frac{\tau}{\sigma\left(1-\lambda\right)} \frac{\partial s_c\left(r_d\right)}{\partial r_d} \left(\frac{s\left(r\right) + r s'\left(r\right)}{s\left(r_d\right)}\right).$$

As discussed earlier, the transmission to loan rates is opposite in direction to the change in the federal funds rate when the deposit market is kinked and banks rely entirely on deposits and equity financing.

Competitive loan - competitive deposit Finally, suppose both the loan market and deposit market are competitive. In this case, the deposit rate is set using the first order condition of optimality for a bank. The competitive deposit supply function $s_{i,c}$ for bank i is (45). Using the optimality condition for the deposit rate decision, the competitive deposit rate $r_{d,i}$ is pinned down from the condition:

$$r_{d,i} + \frac{s_{i,c}}{\partial s_{i,c}/\partial r_{d,i}} = r - \mu_i. \tag{55}$$

The left-hand-side of (55) is the marginal cost of raising a dollar of deposits. Because each bank faces an upward sloping supply curve for deposits, this marginal cost exceeds the deposit rate $r_{d,i}$. The right-hand-side of (55) is the marginal cost of raising a dollar of debt. If the bank does not tap the reserve market, then $\mu_i > 0$ and bank debt is entirely deposits. So, its marginal cost of debt financing would be less than the federal funds rate. If the bank accesses the federal funds market, $\mu_i = 0$, and the marginal cost of debt will be the federal funds rate. When selecting an optimal deposit rate, banks choose a rate $r_{d,i}$ that equates the marginal cost of deposits and the marginal cost of reserves.

To study the second term in the left-hand-side of (55), note that the increase in deposit supply to the bank after a marginal increase in the deposit rate $r_{d,i}$ is

$$\frac{\partial s_{i,c}}{\partial r_{d,i}} = \frac{1}{\chi}\left(s\left(r_{d,i}\right)\frac{\partial v}{\partial r_{d,i}} + v\left(r_{d,i}\right)\frac{\partial s}{\partial r_{d,i}}\right).$$

The first term is the increase in deposits on the *extensive* margin as the bank expands its deposit base around the circle, holding fixed the supply of deposits for the household at each location. The second term is the increase in deposits on the *intensive* margin as the bank increases the supply of deposits per household at every location, holding fixed the bank's deposit arc length. The change in deposits on these two margins will influence monetary transmission.

To determine the pass-through $\frac{\partial r_{d,i}}{\partial r} = 1 - \frac{\partial \delta_{d,i}}{\partial r}$, start with the optimality condition for loan rates, the equity capital constraint, and the optimality condition for deposits in (55):

$$\frac{\partial \Delta_i}{\partial r_{L,i}}\left(\sigma r_{L,i} + (1-\sigma)\underline{\kappa} - r_\lambda + \mu_i\left(1-\lambda\right)\right) + \sigma\Delta_i = 0, \tag{56}$$

$$\left(1-\lambda\right)\left(\Delta_i + f\right) - s_{i,c} = 0, \tag{57}$$

$$\left(r - r_{d,i} - \mu_i\right)\frac{\partial s_{i,c}}{\partial r_{d,i}} - s_{i,c} = 0, \tag{58}$$

Differentiating (56)-(58) with respect to r delivers a linear system in $x \equiv \left(\frac{\partial r_{L,i}}{\partial r}, \quad \frac{\partial \delta_{d,i}}{\partial r}, \quad \frac{\partial \mu_i}{\partial r} \right)'$. That linear system is

$$Ax = b,$$

where

$$A \equiv \begin{bmatrix} 2\sigma & 0 & 1-\lambda \\ (1-\lambda)\frac{\partial \Delta_i}{\partial r_{L,i}} & \frac{\partial s_{i,c}}{\partial r_{d,i}} & 0 \\ 0 & -\phi\left(r, r_{d,i}\right) & -\chi \frac{\partial s_{i,c}}{\partial r_{d,i}} \end{bmatrix}$$

and

$$b \equiv \begin{bmatrix} 1-\lambda \\ \frac{\partial s_{i,c}}{\partial r_{d,i}} \\ -\left(\chi \frac{\partial s_{i,c}}{\partial r_{d,i}} + \phi\left(r, r_{d,i}\right)\right) \end{bmatrix}.$$

The function

$$\phi\left(r, r_{d,i}\right) \equiv \left(r - r_{d,i} - \mu\right)\left(3s\left(r_{d,i}\right)\frac{\partial s}{\partial r_{d,i}} + v\left(r_{d,i}\right)\frac{\partial f}{\partial r_{d,i}}\right) - 2\chi \frac{\partial s_{i,c}}{\partial r_{d,i}},$$

where $f\left(r_{d,i}\right) \equiv \frac{\partial s}{\partial r_{d,i}} > 0$ is the increase in the supply of deposits at a fixed location from a rise in the deposit rate. Solving the system using Cramer's rule shows that when the reserve borrowing constraint binds and banks are financed entirely from deposits and equity without reserves, $\frac{\partial r_{L,c}}{\partial r} = 0$, which implies no transmission to loan rates. Recall that a non-binding reserve borrowing constraint in a competitive equilibrium ($\mu_c = 0$) would make transmission in a competitive loan market identical to that in the main text. So, the case here of no transmission is a consequence of banks relying entirely on deposit and equity financing and not accessing reserves.

One can observe where the absence of pass-through originates using the closed-form solution for the loan rate given in (50). Banks that price loans in a competitive equilibrium start from the marginal cost of banking financing $(1-\lambda)\left(r - \mu_c\right) + \lambda r_e$. Monetary transmission after a change in the policy rate is

$$\frac{\partial r_{L,c}}{\partial r} = \frac{1}{\sigma}(1-\lambda)\left(1 - \frac{\partial \mu_c}{\partial r}\right).$$

A non-binding reserve borrowing constraint would have $\frac{\partial \mu_c}{\partial r} = 0$, and there would be transmission. When the constraint binds, on the other hand, a policy rate hike not only increases the marginal cost of reserves (r) but it also raises the marginal *value* of deposits over reserves (μ) by an offsetting amount, which leaves the marginal cost of debt financing for banks unchanged and shuts off transmission.

The change in the cost savings of deposits over reserves $\frac{\partial \mu_c}{\partial r}$ is also solved using Cramer's rule and comes to what is expected:

$$\frac{\partial \mu_c}{\partial r} = 1.$$

As mentioned earlier, the marginal cost savings of deposits over reserves $\frac{\partial \mu_c}{\partial r}$ moves one-for-one with policy rate changes. Finally, the transmission to the competitive deposit rates is

$$\frac{\partial r_{d,c}}{\partial r} = \frac{\frac{\sigma\chi}{\tau}\left(1-\lambda\right)^2 \frac{\partial s_c}{\partial r_{d,c}}}{2\sigma\chi\left(\frac{\partial s_c}{\partial r_{d,c}}\right)^2 - \frac{\sigma}{\tau}\left(1-\lambda\right)^2 \phi\left(r, r_{d,c}\right)}. \tag{59}$$

The numerator of (59) is strictly positive, whereas the denominator sign is ambiguous. The sign of transmission to competitive deposit rates is therefore indeterminate. To get a better sense of the transmission, differentiate the deposit optimality condition in (55) with respect to r and use $\frac{\partial \mu_c}{\partial r} = 1$ to get

$$\frac{\partial r_{d,c}}{\partial r} + \frac{\partial\left(\frac{s_c}{\partial s_c/\partial r_{d,c}}\right)}{\partial r} = 0.$$

The term $\frac{s_c}{\partial s_c/\partial r_{d,c}}$ is the inverse semi-elasticity of a bank's deposit base s_c to changes in its deposit rate. The monetary transmission to deposit rates $\frac{\partial r_{d,c}}{\partial r}$ is opposite in sign to the sensitivity of that elasticity to federal funds rate changes.

B.6.4 Reserve requirement

Suppose now a bank faces a reserve requirement. This new constraint requires a bank to maintain at least a fraction φ of deposits on its balance sheet rather than lend that fraction out to entrepreneurs. I study a bank's problem in two cases: (1) when it faces a perfectly elastic supply of deposits at rate r, and (2) when it faces an upward sloping deposit supply curve.

Perfectly elastic deposit curve In its decision problem, a bank again selects a lending rate $r_{L,i}$. A bank also selects an amount s_i of deposits to raise, an amount l_i of those deposits to maintain as reserves, and an amount e_i to raise from the external equity market. Suppose reserves on a bank's balance sheet are held in a non-interest-bearing account. The profits of a typical bank i are

$$\pi_i = \sigma r_{L,i}\Delta_i + \left(1-\sigma\right)\underline{\kappa}\Delta_i + l_i - r s_i - r_e\left(e_{0,i} + e_i\right).$$

The reserve requirement of a bank is

$$l_i \geq \varphi s_i.$$

The bank also faces an equity capital requirement that forces it to finance a fraction λ of lending assets in equity:

$$e_{0,i} + e_i \geq \lambda\left(\Delta_i + f\right).$$

A bank maximizes profits subject to the reserve and equity capital requirements.

Any bank would rather lend than maintain reserves, so the reserve requirement binds. Hence, $l_i = \varphi s_i$. Like in the main text, the capital requirement binds as well. The balance sheet identity of a bank is

$$\Delta_i + f + l_i \equiv s_i + e_{0,i} + e_i,$$

which allows the capital requirement to be re-written as

$$\Delta_i + f - \left(1-\varphi\right)s_i = \lambda\left(\Delta_i + f\right).$$

Substitute the binding constraints into the bank's profit function to get

$$\pi_i = \sigma r_{L,i}\Delta_i + (1-\sigma)\,\underline{\kappa}\Delta_i - r_\varphi\,(\Delta_i + f)\,,$$

where a bank's marginal cost of funding is

$$r_\varphi \equiv \left(\frac{r-\varphi}{1-\varphi}\right)(1-\lambda) + r_e\lambda.$$

The marginal cost r_φ is identical to the one in the main text r_λ, save for the modified cost of deposit funding $\frac{r-\varphi}{1-\varphi}$ in place of r. The new marginal cost of deposit funding accounts for the reserve requirement. For every dollar of deposits raised, only a fraction $1-\varphi$ can be used to finance lending, which raises the marginal cost of deposits. The change to using r_φ instead of r_λ in a bank's profit function would not alter the central results on monetary transmission from the main text.

Upward sloping supply curve Suppose now banks have market power in the deposits market, so they face an upward sloping supply curve of deposits. Banks can raise equity, deposits, or tap the reserves market. Here, banks are required to maintain a fraction φ of the deposits they raise on their balance sheets. Suppose again that any reserves on a bank's balance sheet are kept in a non-interest-bearing account. Otherwise, if banks were allowed to invest their reserves in the reserve market and earn interest rate r, the effect of a reserve requirement would wash out.

One way to allow a bank to invest reserves and preserve effects from a reserve requirement is to have the bank borrow from the reserve market at one rate and lend at a different rate. In the U.S. federal funds market, this is not uncommon, interest rates in the federal funds market are negotiated between institutions. For example, in its 2017 Annual Report (pp. 278), J.P. Morgan Chase, over the course of the year, borrowed federal funds at an average interest rate of 0.86% and lent federal funds at an average interest rate of 1.21%. To keep things simple here, banks can only invest their reserves from the reserve requirement in accounts that earn no interest.

Rather than a constraint that requires banks to borrow on *gross* from the reserve market $(b_i \geq 0)$, the constraint now is that banks borrow on *net* $(b_i \geq l_i)$. Let μ be the Lagrange multiplier on the "net reserve borrowing constraint." Like before, the reserve requirement will bind, making $l_i = \varphi s_i$. The equity capital requirement binds as well. With this in mind, the profit function of a bank is

$$\pi_i = \sigma r_{L,i}\Delta_i + (1-\sigma)\,\underline{\kappa}\Delta_i + (r\,(1-\varphi) - r_{d,i})\,s_i + \varphi s_i - r_\lambda\,(\Delta_i + f)\,.$$

The only change from the profit function in (49) from Section B.6.3 is the adjustment to the reserve rate $r\,(1-\varphi)$ and the contribution of the reserves on balance sheet φs_i to firm profits. The only change in the complementary slackness conditions is that s_i is now replaced with $(1-\varphi)\,s_i$.

With these differences, the solutions for the sensitivity of r_L and μ to a policy rate change, compared to (51) and (52), are

$$\frac{\partial r_L}{\partial r} = -\frac{\tau\left(1-\varphi\right)}{\sigma\left(1-\lambda\right)}\frac{\partial s_c}{\partial r_d}\frac{\partial r_d}{\partial r},$$
$$\frac{\partial \mu}{\partial r} = 1 + \frac{2\tau\left(1-\varphi\right)}{\left(1-\lambda\right)^2}\frac{\partial s_c}{\partial r_d}\frac{\partial r_d}{\partial r}.$$

These sensitivities now account for the reserve requirement, but the transmission results are unaffected. The kinked loan rate does not reflect any part of bank balance sheets, so that too will be unchanged by a reserve requirement.

In the competitive loan-competitive deposit equilibrium, the only meaningful change is to the optimality condition that pins down the deposit rate. Instead of (55), the condition is

$$r_{d,i} + \frac{s_{i,c}}{\partial s_{i,c}/\partial r_{d,i}} = \left(r-\mu\right)\left(1-\varphi\right) + \varphi.$$

The equilibrium deposit rate now reflects the reserve requirement. Nevertheless, this change will not alter the monetary transmission results from Section B.6.3.

B.7 Alternative types of equilibria

In this section, I study both pure-strategy asymmetric equilibria and mixed-strategy equilibria for both the competitive and kinked cases. The possible mixed-strategy equilibria I examine are both symmetric and asymmetric. For simplicity, I focus on equilibria in which all banks price loans either at the kink or the competitive part of their demand curves. I refrain from studying equilibria in which some banks are at the kink, whereas others are at the competitive part, though such types of equilibria are interesting as well.

B.7.1 Pure-strategy asymmetric equilibria

To reduce notation, let the firm recovery value $\underline{\kappa} = 0$. In this case, the profit function (3) of a typical bank i with the capital constraint binding reduces to

$$\pi_i = \left(\sigma r_{L,i} - r_\lambda\right)\Delta_i - r_\lambda f,$$

where $r_\lambda = \left(1-\lambda\right) r + \lambda r_e$. The monopoly demand curve the bank faces is unchanged from the main text. It is reprinted here:

$$\Delta_{i,m} = \frac{\sigma\left(\overline{\kappa} - r_{L,i}\right) - w}{\tau/2}.$$

When banks can post heterogeneous loan rates, the competitive demand curve differs from (5) in the main text, but only slightly. Like before, let $i+1$ be the location of the neighboring bank in the clockwise direction of bank i, and let $i-1$ be the location of the counterclockwise neighbor.

Equidistant locations Suppose first that every bank is located a distance $\frac{1}{n}$ from its two neighbors. Despite being located equidistantly, banks can still have asymmetric market shares Δ_i in equilibrium. The competitive demand curve now accounts for the potentially different loan rates of bank $i+1$ and $i-1$:

$$\Delta_{i,c} = \frac{\sigma}{2\tau}\left(r_{L,i+1} + r_{L,i-1} - 2r_{L,i}\right) + \frac{1}{n}.$$

If either neighboring bank charges a higher loan rate, bank i gains market share for a fixed loan rate $r_{L,i}$.

In a kinked equilibrium, monopoly demand curves of each bank just touch. Therefore, when banks are located equidistantly, it must be that $\Delta_{i,m} = \frac{1}{n}$ for all banks. Making this substitution into the monopoly demand curve and solving for $r_{L,i}$ gives

$$r_{L,i} = \overline{\kappa} - \frac{w}{\sigma} - \frac{\tau}{2\sigma n}.$$

Because $r_{L,i} = r_{L,i+1} = r_{L,i-1}$ for all i, only symmetric, pure-strategy equilibria exist in this kinked case.

In a competitive equilibrium, we need to examine the optimality condition of a bank. From the profit function above, the first order condition is

$$\sigma\Delta_i + \frac{\partial\Delta_i}{\partial r_{L,i}}\left(\sigma r_{L,i} - r_\lambda\right) = 0,$$

whereas the second order condition is

$$\frac{\partial\pi_i^2}{\partial r_{L,i}^2} = 2\sigma\frac{\partial\Delta_i}{\partial r_{L,i}} + \frac{\partial^2\Delta_i}{\partial r_{L,i}^2}\left(\sigma r_{L,i} - r_\lambda\right).$$

Using the competitive demand curve above, one can observe

$$\frac{\partial\Delta_{i,c}}{\partial r_{L,i}} = -\frac{\sigma}{\tau},$$
$$\frac{\partial\Delta_{i,c}^2}{\partial r_{L,i}^2} = 0.$$

Making these substitutions into the first and second order conditions reveals

$$\sigma r_{L,i} - r_\lambda = \tau\Delta_i,$$
$$\frac{\partial\pi_i^2}{\partial r_{L,i}^2} < 0.$$

Hence, the profit function of a bank in the competitive case is strictly concave for fixed loan rates $r_{L,i+1}$ and $r_{L,i-1}$. Substituting the competitive demand curve into the bank's optimality condition gives

$$\sigma r_{L,i} - r_\lambda, = \frac{\sigma}{2}\left(r_{L,i+1} + r_{L,i-1} - 2r_{L,i}\right) + \frac{\tau}{n},$$

which is a system of linear equations. This system yields a unique solution $r_{L,i} = r_{L,i+1} =$

$r_{L,i-1} = r_L$,with

$$r_L = \frac{1}{\sigma}\left(r_\lambda + \frac{\tau}{n}\right).$$

Hence, in the competitive case as well, only a symmetric equilibrium exists and is unique among pure strategies when banks are located a distance $\frac{1}{n}$ from one another.

Heterogeneous distances Suppose next that banks are located around the circle so that they potentially are varied distances from each other. In the kinked case, the loan rate is again taken off the monopoly demand curve:

$$r_{L,i} = \overline{\kappa} - \frac{w}{\sigma} - \frac{\tau}{2\sigma}\Delta_{i,m}.$$

Rather than the kinked equilibrium market share of each bank equaling $1/n$, let each bank's monopoly market share be $\Delta_i = \delta_i(n)$, where $\delta_i(n) > 0$ for all i and $\delta_i(n) \neq \delta_j(n)$ for banks $i \neq j$. The market shares are positioned on the circle such that each bank lends a distance $\delta_i(n)/2$ on each side of it. So that the monopoly markets of each bank just touch, $\sum_i^n \delta_i(n) = 1$. In this setting, the loan rate of each bank in a kinked equilibrium is

$$r_{L,i} = \overline{\kappa} - \frac{w}{\sigma} - \frac{\tau}{2\sigma}\delta_i(n), \tag{60}$$

which characterizes an asymmetric pure-strategy equilibrium.

The full equilibrium is defined by both entry and exit in the loan market (to determine n) and the clearing of the equity market (to determine r_λ). The equity market clearing condition is

$$ne_0 + \xi = \sum_i^n \lambda\left(\delta_i(n) + f\right).$$

Because $\sum_{i=1}^n \delta_i(n) = 1$, the condition becomes

$$\xi - \lambda = n\left(f\lambda - e_0\right) \tag{61}$$

The profit condition for entry and exit is

$$\left(\sigma r_{L,i} - r_\lambda\right)\delta_i(n) - r_\lambda f = v, \qquad \forall i.$$

This profit condition, however, is over-determined: a single variable n needs to satisfy n equations (one for each $\delta_i(n)$). A solution to this problem relies on recalling that the entry decision is the first stage of a two-stage game played by the banks (the second stage is the loan rate-setting stage). With this in mind, we can weaken the profit condition by treating $\delta_i(n)$ as a random variable and requiring *expected* profits to equal v. The revised profit condition becomes

$$E\left(\left(\sigma r_{L,i} - r_\lambda\right)\delta_i(n)\right) - r_\lambda f = v$$

Let each bank before entering have an equal probability of drawing $\delta_i(n)$ as its market share. These possible market shares are functions of the number of banks n that actually enter, which is something an individual bank does not know before making its entry decision. I assume rational expectations on behalf of banks: The number of competitors that a bank anticipates will enter is confirmed in equilibrium. Market shares are simultaneously assigned at random to entering banks, so each bank expects controlling a share $\delta_i(n)$ with probability $\frac{1}{n}$. Finally, although expected profits equal v in equilibrium, some banks will have profits above or below v upon entry and the allocation of market shares. I assume v is large enough (or f small enough) so that profits are still positive for the bank assigned the smallest market share.

A simple function for the heterogeneous market shares is $\delta_i(n) = \frac{\delta_i}{n}$. To illustrate possible values for δ_i, consider the example $n = 3$, with $\delta_1 = 2.1$, $\delta_2 = 0.5$, and $\delta_3 = 0.4$. This example satisfies the restriction $\sum_{i=1}^{n} \frac{\delta_i}{n} = 1$. Using this specification and substituting the kinked loan rate from (60), the expected profit condition becomes

$$(v + fr_\lambda) n^3 - (\sigma\overline{\kappa} - w - r_\lambda) n^2 + \frac{d\tau}{2} = 0,$$

where $d \equiv \sum_{i=1}^{n} \delta_i^2$, and I have used the properties

$$E(\delta_i(n)) = \frac{1}{n},$$
$$E\left(\delta_i^2(n)\right) = \frac{d}{n^3}.$$

We can characterize the properties of the cubic polynomial from the profit condition to better understand the equilibrium number of banks n. Let $G(n)$ denote the cubic polynomial. The sign of the discriminant

$$\Delta_0 = (\sigma\overline{\kappa} - w - r_\lambda)^2$$

determines the number of critical points. Because $\Delta_0 > 0$, $G(n)$ has a local minimum and a local maximum. Also, because $G(n)$ tends to $+\infty$ as $n \to \infty$ and $-\infty$ as $n \to -\infty$,the function is first concave down and then concave up as n increases. The "falling" inflection point of the cubic (where it switches from concave down to concave up) is

$$n_{\text{inflection}} = \frac{\sigma\overline{\kappa} - w - r_\lambda}{3(v + fr_\lambda)},$$

which is positive when $\sigma\overline{\kappa} - w - r_\lambda > 0$.

The two critical points of the cubic are

$$n_{1,c} = 0,$$
$$n_{2,c} = \frac{2}{3}\left(\frac{\sigma\overline{\kappa} - w - r_\lambda}{v + fr_\lambda}\right) > 0.$$

The values of the polynomial at these critical points are

$$G(n_{1,c}) = \frac{d\tau}{2} > 0,$$
$$G(n_{2,c}) = -\frac{2(\sigma\overline{\kappa} - w - r_\lambda)^3}{27(v + fr_\lambda)^2} + \frac{d\tau}{2}.$$

Provided $G\left(n_{2,c}\right)<0$, the cubic will have two positive roots $n_1^*<n_2^*$. For the same reason given in the main text, the larger positive root will be the unique number of banks that enter in equilibrium. This number then pins down the asymmetric equilibrium market shares and kinked loan rates among banks.

In the competitive case, asymmetric equilibria are possible as well. For example, Vogel (2008) shows the existence of these type of equilibria when firms are allowed to choose both their prices *and* their locations on a circle. Firms turn out to choose heterogeneous distances between themselves in a way to increase differentiation and reduce competition. See also Kats (1995); Gong et al. (2016) for discussions of asymmetric competitive equilibria in circular models.

B.7.2 Mixed-strategy equilibria

In this section, I examine mixed-strategy equilibria in which banks are equidistant. I look for a symmetric, mixed-strategy equilibrium in which banks randomize over the choice of their loan rates. (See also Ishida and Matsushima (2004) for a discussion on mixed strategy equilibria in circular models.)

Assume banks follow mixed strategies according to a distribution $F\left(r_L\right)$ with support $[l,h]$. So that banks draw customers at all, it must be that $h\leq\overline{\kappa}-\frac{w}{\sigma}$; otherwise, all entrepreneurs would prefer the outside option to borrowing at some loan rates in the support. Like before, assume the recovery value $\underline{\kappa}=0$. The profit function of a bank again is

$$\pi_i=\left(\sigma r_{L,i}-r_\lambda\right)\Delta_i-r_\lambda f,$$

with the monopoly and competitive demand curves as

$$\Delta_{i,m}=\frac{\sigma\left(\overline{\kappa}-r_{L,i}\right)-w}{\tau/2},$$
$$\Delta_{i,c}=\frac{\sigma}{\tau}\left(r_L-r_{L,i}\right)+\frac{1}{n}.$$

If $r_{L,i}\geq r_L$, then bank i draws customers according to $\Delta_{i,m}$; whereas, when $r_{L,i}<r_L$, the banks steals customers away from its two neighboring banks and draws customers according to $\Delta_{i,c}$. The expected profit $\overline{\pi}_i$ of the bank is then

$$\overline{\pi}_i=\int_l^{r_{L,i}}\left(\sigma r_{L,i}-r_\lambda\right)\Delta_{i,m}dF\left(r_L\right)+\int_{r_{L,i}}^h\left(\sigma r_{L,i}-r_\lambda\right)\Delta_{i,c}dF\left(r_L\right)-r_\lambda f.$$

To randomize, bank i must be indifferent between any $r_{L,i}$ in $[l,h]$. Thus, $F\left(r_L\right)$ must be such that $\frac{\partial\pi_i}{\partial r_{L,i}}=0$ for any $r_{L,i}$. The first order condition of bank i with respect to its loan rate $r_{L,i}$ is

$$q_1\left(r_{L,i}\right)F'\left(r_{L,i}\right)+q_2\left(r_{L,i}\right)F\left(r_{L,i}\right)+q_3\left(r_{L,i}\right)+\frac{\sigma^2}{\tau}\int_{r_{L,i}}^h r_L dF\left(r_L\right)=0,$$

where the terms

$$q_1(r_{L,i}) \equiv -\frac{2\sigma}{\tau}\left(\sigma r_{L,i}^2 - \left(\varphi - \frac{1}{n}\right) r_{L,i} + \frac{r_\lambda}{\sigma}\left(\sigma\overline{\kappa} - w - \frac{1}{n}\right)\right),$$
$$q_2(r_{L,i}) \equiv -\frac{2\sigma}{\tau}\left(\sigma r_{L,i} - (\sigma\overline{\kappa} - w) - \frac{r_\lambda}{2} + \frac{1}{2\tau n}\right),$$
$$q_3(r_{L,i}) \equiv -\frac{2\sigma}{\tau}\left(\sigma r_{L,i} - \frac{r_\lambda}{2}\right) + \frac{\sigma}{n}.$$

The term $\varphi \equiv \sigma\overline{\kappa} - w + r_\lambda$. The profit optimality condition represents a first-order, ordinary integro-differential equation. If a solution exists that is strictly positive and non-decreasing, denote it by $F^*(r_L)$. This solution will have a single constant of integration k, which can be solved for by recognizing

$$F^*(h) = 1.$$

The lower bound l must also satisfy

$$F^*(l) = 0,$$

which will characterize the lower bound as a function of h. To solve for the upper bound h, note that if all other banks abide by the strategy $F^*(r_L)$, bank i must be indifferent between playing any $r_{L,i}$ in $[l, h]$, including the strategy $r_{L,i} = h$. When bank i plays the upper bound, no other bank charges more, so bank i operates only off the monopoly demand curve. Bank i's expected profit from this strategy is

$$\overline{\pi}_i(h) = \int_l^h (\sigma h - r_\lambda)\left(\frac{\sigma(\overline{\kappa} - h) - w}{\tau/2}\right) dF^*(r_L) = (\sigma h - r_\lambda)\left(\frac{\sigma(\overline{\kappa} - h) - w}{\tau/2}\right).$$

The expected profit $\overline{\pi}_i(h)$ is strictly concave in h, so it has a unique maximum h^*. Taking the first order condition and solving for h^* gives

$$h^* = \frac{1}{2}\left(\overline{\kappa} - \frac{w}{\sigma} + \frac{r_\lambda}{\sigma}\right).$$

When $\sigma\overline{\kappa} - w - r_\lambda > 0$, this upper bound is strictly less than $\overline{\kappa} - \frac{w}{\sigma}$. Rather than randomizing over a region that includes loan rate values that would yield zero borrower demand, banks can do strictly better by lowering the upper bound of the allowable loan rates in any mixed strategy. Hence, the symmetric, mixed-strategy equilibrium is characterized by the distribution $F^*(r_L)$ having bounded support $[l(h^*), h^*]$.

As a final note, care must be taken to ensure that a bank does not push neighboring banks out of the market via *predatory pricing.* The predatory loan rate is

$$r_{L,i}^{pr} = r_L - \frac{2c}{n}.$$

At this loan rate, even the two entrepreneurs located at the position of the neighboring banks would prefer borrowing from bank i instead of the neighbors. Predatory pricing can be ruled out as long as the lowest possible loan rate banks can offer exceeds the predatory price against a bank that charges the highest possible rate. That condition is $l(h^*) \geq h^* - \frac{2\tau}{n}$. Let $\underline{l} \equiv h^* - \frac{2\tau}{n}$. If $l(h^*) < \underline{l}$, then the lower bound of F^* is replaced with $\underline{l}$ and the function must be adjusted to become a distribution again. The adjusted function $\hat{F}$ would be

$$\hat{F}\left(r_L\right)=\int_{\underline{l}}^{r_L}\left(\frac{dF^*}{\int_{\underline{l}}^{h^*} dF^*}\right).$$